Welfare Effects of Income Taxation in a Model of Stochastic Growth

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Abstract

This paper analyzes growth and welfare effects of income taxation in a stochastic endogenous growth model with externalities in human–capital accumulation. The government participates in individual income risks by the collection of a flat–rate income tax that affects the mean and the variance of after–tax income. We examine the implications of a tax–transfer policy for the macroeconomic equilibrium of the economy. An increase in the tax rate on mean income has an unambiguously negative effect on the expected growth rate. Paradoxically, this may induce welfare gains. The opposite results can be derived for a rise in the tax rate on transitory income. These counter–intuitive results of the stochastic Arrow–Romer model can be ascribed to the specific interaction of consumption and portfolio choice in the determination of growth and welfare.

Kurzfassung

In diesem Beitrag werden die Wachstums– und Wohlfahrtseffekte einer Einkommensbesteuerung im Rahmen eines stochastischen Wachstumsmodells analysiert. Eine Humankapitalexternalität in der Produktion führt zu anhaltendem Wachstum. Die lineare Einkommensteuer reduziert sowohl den Erwartungswert als auch die Varianz des verfügbaren Einkommens, so daß der Staat einen Teil des Produktionsrisikos übernimmt. Wir untersuchen die Auswirkungen eines Steuer–Transfer Systems auf das makroökonomische Gleichgewicht. Eine Erhöhung des Steuersatzes auf deterministische Einkommen reduziert eindeutig die Wachstumsrate, kann jedoch paradoxerweise zu Wohlfahrtsgewinnen führen. Im Gegensatz dazu kann die Besteuerung stochastischer Einkommensteile trotz gestiegener Wachstumsrate Wohlfahrtsverluste verursachen. Diese kontra–intuitiven Ergebnisse lassen sich auf das Zusammenwirken von Konsum– und Portfoliowahl zurückführen.

1 Introduction

This paper is concerned with the linkage of uncertainty, growth, and income taxation within the context of a dynamic general equilibrium model. The analysis is based on Stiglitz (1969), who found that income taxation under uncertainty tends to smooth the intertemporal income flow. For that reason a lump–sum tax is not necessarily intertemporally optimal. This result contradicts the outcomes for the deterministic setting, e. g. Chamley (1986), who claims that a first–best optimum can only be achieved with non–distortionary taxation. In particular, in the long run the tax rate on capital income has to be zero. This conclusion is supported by recent work on taxation in endogenous growth models, e. g. Lucas (1990), King and Rebelo (1990), Rebelo (1991) or Jones, Manuelli, and Rossi (1993a, b; 1997). In contrast to this, under uncertainty the government may participate in the stochastic income gains and losses, as presented in Eaton and Rosen (1980), Abel (1988) and Mazur (1989). With an appropriate intertemporal tax–transfer policy, it is possible to spread the excess burden across time. This property of taxation was already discussed by Eaton (1981), Zhu (1992) and Smith (1996) within the theory of stochastic growth or by Bizer and Judd (1989) in a general equilibrium model, as well as by Kimball and Mankiw (1989) for the case of heterogenous agents.

In the economics of uncertainty two major types of income risk are to be distinguished: idiosyncratic and aggregate income risks. In the absence of informational asymmetries the individual–specific risks can be shared via private arrangements, for example within insurance or capital markets. Contrary to this, aggregate income risks cannot be diversified completely within national boundaries. A risk averse individual will respond to uncertainty by her intertemporal savings and consumption decision. In the presence of capital markets, the agent may have a motive for precautionary saving to self–insure against future income risk. This aspect was first discussed by Leland (1968) and Sandmo (1970), or more recently by Caballero (1990), Weil (1993) and van der Ploeg (1993). Another means to smooth lifetime consumption is a tax–transfer policy, which itself impacts on the intertemporally optimal decision, as described above. The interdependence between uncertainty, savings and taxation gains a broader interpretation if it is discussed within the theory of endogenous growth.

In modern growth theory, the growth rate of the economy is endogenously determined by individual intertemporal optimization. For sufficiently risk averse agents a decrease in risk leads to a decrease in (precautionary) savings and thus lowers the endogenously determined growth rate. As a consequence, two opposite welfare effects can be identified: The decrease in uncertainty induces a positive primary welfare effect, whereas a decrease in the growth rate has a welfare diminishing secondary effect. It is possible that the second effect outweighs the first, and consequently the agent may suffer welfare losses, although the economy has become less risky. This outcome contradicts the common intuition that a risk averse individual always prefers a safer environment to a riskier one. It was first discussed by Devereux and Smith (1994) and Femminis (1995). The contributions of Greenwood and Jovanovic (1990), Obstfeld (1994) and van Wincoop (1994) discuss risk sharing in models of endogenous growth. They unanimously find positive net welfare effects. This result can be derived within a setting of intertemporal optimization that leads to competitive equilibria that are identical to the Pareto–optimal allocation of a benevolent social planner. In particular, the authors considered the linear, AK–type technology as in Jones and Manuelli (1990).

The unambiguity of welfare effects vanishes if one allows for the class of endogenous growth models with positive external effects in human capital accumulation as in Romer (1986). In this type of model the individual neglects her own contribution to the aggregate capital stock. The stock of knowledge is regarded as a constant in individual optimization and the private marginal product of capital falls short of the social return. Thus, the competitively chosen growth rate of the economy is less than the Pareto–optimal one. If we apply the argument from above, the following results can be stated: A decrease in risk reduces savings and drives the suboptimally low competitive growth rate further away from the optimal one. This gives rise to a situation where the welfare enhancing effects of a less risky environment are outweighed by the welfare diminishing effects of a reduction in the expected growth rate, as was shown in Clemens and Soretz (1997).

In this context, the results regarding the intertemporal tax–transfer policy described above have to be reexamined. It is questionable, whether the insurance effect of fiscal policy leads to welfare gains for this class of models.

Our paper is related to a recent contribution by Smith (1996), who used a setting quite similar to ours but did not consider two significant aspects: First, he neglected labor in production. Second, in his model, regular government expenditure exhibits the same stochastic characteristics as tax receipts. Relaxing these assumptions and allowing for public debt, we are not able to confirm his results with regard to welfare effects or the optimal tax rate, respectively. Our model set–up, constructed purposely to ease comparison, is close to Eaton (1981) and Turnovsky (1993). In particular, we consider a differential tax rate on the stochastic and deterministic components of income. We demonstrate that an increase in the tax rate on mean income unambiguously reduces the endogenously determined growth rate, but unexpectedly may lead to welfare gains. Moreover, an increase in the tax rate on random income parts increases growth, but may lead to welfare losses. These results contradict the usual findings in the literature of taxation under uncertainty in growing economies.

The paper is organized as follows. Section 2 describes the model and the results from individual intertemporal optimization. Section 3 determines the competitive macroeconomic equilibrium, while section 4 discusses growth, portfolio, and welfare effects of a change in the income tax rates. Section 6 briefly summarizes the results.

2 The Model

The economy is populated by identical infinitely–lived individuals who maximize the intertemporal objective

$$\mathcal{U}(0) = \mathrm{E}_0 \int_0^\infty U[C(t)]\, e^{-\beta t}\, dt. \tag{1}$$

E_0 denotes the mathematical expectation, conditional on time 0 information, $C(t)$ is time t consumption, $e^{-\beta t}$ represents the discount factor with the instantaneous rate of time preference $\beta \in (0,1)$. The current period utility function $U[C(t)]$ is strictly concave and takes the constant relative risk aversion (CRRA) form

$$U[C(t)] = \begin{cases} \frac{1}{1-\rho} C(t)^{1-\rho} & \rho > 0, \rho \neq 1 \\ \ln C(t) & \rho = 1 \end{cases} . \tag{2}$$

ρ denotes the measure of relative risk aversion and equals the reciprocal of the intertemporal elasticity of substitution.[1]

The representative firm produces a homogenous good according to the stochastic Cobb–Douglas function

$$dY(t) = \gamma K(t)^\alpha (L(t)A(t))^{1-\alpha}(dt + dy(t)) \qquad \alpha \in (0,1), \gamma > 0. \tag{3}$$

Labor L is supplied inelastically and normalized to unity. $K(t)$ is the stock of physical capital. Using the learning–by–doing setting developed by Arrow (1962), $A(t)$ represents the stock of knowledge and displays the characteristics of a public good. It acts as a Harrod–neutral growth parameter and is enhanced by investment in physical capital. Following Romer (1986), in equilibrium, $A(t)$ equals $K(t)$, the economy–wide stock of capital. Due to the knowledge spillover, aggregate production is linear in capital, thus inducing ongoing growth. For simplicity, depreciation is neglected. The Wiener process $dy(t)$ is a continuous stochastic Markov process, i. e. the disturbances are serially uncorrelated and $dy \sim N(0, \sigma^2 dt)$. In each time increment the production is affected by a Hicks–neutral technological disturbance.

The government levies a flat–rate income tax on wage and capital incomes. The tax rates τ_i, $i = d, s$ are set separately, as in Eaton (1981), in order to disentangle the effects of taxation of deterministic and random income parts

$$dT(t) = \gamma K(t)^\alpha (L(t)A(t))^{1-\alpha} (\tau_d\, dt + \tau_s\, dy(t)) \qquad \tau_i \in (0,1). \tag{4}$$

Furthermore the agents receive a deterministic transfer in a constant fraction θ of wealth $W(t)$

$$d\Theta(t) = \theta W(t)\, dt \qquad \theta \in (0,1). \tag{5}$$

[1] The empirical difficulties concerning the estimation of these two parameters and the resulting consequences for economic modeling will be neglected in this paper. For a comprehensive discussion of non–expected utility models, the reader is referred to Epstein and Zin (1989) and Weil (1990).

The government balances its budget by issuing bonds. The individual thus has two ways of saving income: investing in risky physical capital or investing in financial capital that pays a sure nominal interest rate.[2] However, the market value of these bonds $B(t)$ is stochastic and reflects the random disturbances in the economy. Wealth $W(t)$ is the sum of holdings of the two assets

$$W(t) = B(t) + K(t), \tag{6}$$

with the portfolio shares n_B, n_K, the initial values $K(0) = K_0, B(0) = p_0\, b_0$ and the initial price $p(0) = p_0$. The stochastic processes for the real rates of return dR_K, dR_B on the two assets and the stochastic wage process dR_L are given by

$$\begin{aligned} dR_K &= r_K\, dt + dz_K \\ dR_B &= r_B\, dt + dz_B \\ dR_L &= \omega\, dt + dz_L. \end{aligned} \tag{7}$$

The expected rates of return are denoted by r_K, r_B and the wage rate by ω respectively.

The objective of the representative agent is to select her rate of consumption, as well as her portfolio of assets, in order to maximize the expected value of lifetime utility over an infinite planning horizon subject to the stochastic wealth accumulation equation

$$\max_{C, n_K, n_B, W} \quad \mathrm{E}_0 \int_0^\infty U[C(t)]\, e^{-\beta t} dt \tag{8}$$

$$\text{s. t.} \quad dW = [(n_B r_B + n_K(1-\tau_d) r_K + \theta) W + (1-\tau_d)\,\omega - C]\, dt + dw \tag{9}$$

$$\text{and} \quad n_B + n_K = 1 \tag{10}$$

with $K_0, b_0, y(0)$ given, and $dw = W(n_B dz_B + n_K(1-\tau_s) dz_K) + (1-\tau_s) dz_L$.

The value function $V[W(t),t]$ denotes the maximum feasible level of lifetime utility if the individual starts in time t with wealth $W(t)$. It is assumed to be time–separable $V[W(t),t] = e^{-\beta t}\, G[W(t)]$. To solve the optimization problem above, one has to consider that the stochastic process of wealth is continuous but not differentiable with respect to time. The stochastic differential of the value function is derived by application of Itô's Lemma and allows for the set–up of the following Lagrangean

$$\begin{aligned} \mathcal{L} &= e^{-\beta t} \Big\{ U(C) - \beta G + G_W \left[(n_B r_B + n_K(1-\tau_d) r_K + \theta) W + (1-\tau_d)\,\omega - C \right] \\ &\quad + \frac{1}{2} G_{WW}\, \sigma_W^2 + \lambda [1 - n_B - n_K] \Big\} \end{aligned} \tag{11}$$

where the subscripts denote the partial derivatives for the respective argument and the variance of wealth is given by $\sigma_W^2 = \frac{\mathrm{E}\,(dw)^2}{dt}$.

[2]For simplicity, the bonds are considered to be perpetuities.

The first–order conditions are given by

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial C} &= U'(C) - G_W = 0 && (12)\\
\frac{\partial \mathcal{L}}{\partial n_B} &= G_W r_B W + \frac{1}{2} G_{WW} \frac{\partial \sigma_W^2}{\partial n_B} - \lambda = 0 && (13)\\
\frac{\partial \mathcal{L}}{\partial n_K} &= G_W r_K (1-\tau_d) W + \frac{1}{2} G_{WW} \frac{\partial \sigma_W^2}{\partial n_K} - \lambda = 0 && (14)\\
\frac{\partial \mathcal{L}}{\partial \lambda} &= 1 - n_K - n_B = 0 && (15)\\
\frac{\partial \mathcal{L}}{\partial W} &= G_W \left(n_B r_B + n_K (1-\tau_d) r_K + \theta - \beta\right) + \frac{1}{2} G_{WW} \frac{\partial \sigma_W^2}{\partial W} && (16)\\
&\quad + G_{WW} \left[(n_B r_B + n_K (1-\tau_d) r_K + \theta) W + (1-\tau_d)\omega - C\right] + \frac{1}{2} G_{WWW} \sigma_W^2 .
\end{aligned}
$$

Equation (12) displays the well known result from intertemporal optimization of equalized marginal utility of consumption across time. It determines the accumulation process together with (16). Equations (13) and (14) form the usual consumer asset pricing relationship. (16) replaces the Bellman equation which is commonly used in this kind of model. Since we are dealing with suboptimal competitive outcomes, Bellman's Principle of Optimality cannot be applied here . A feasible consumption program, i. e. convergence of lifetime utility (8), has to satisfy a transversality condition

$$\lim_{t \to \infty} \mathrm{E_t} \left[G(W)\, e^{-\beta t} \right] = 0. \tag{17}$$

This condition is usually met if the propensity to consume out of wealth is positive, as shown in Merton (1969).

The optimal values for consumption and the portfolio shares are functions of the derivatives G_W and G_{WW} of the value function and together form a stochastic differential equation in $G(W)$. To proceed, a function $G(W)$ has to be found that solves the first–order conditions and thus maximizes the stochastic integral (8). We guess that in equilibrium consumption and wealth grow at a common rate. Hence, as Merton (1971), we assume the consumption–wealth ratio to be a constant fraction of wealth

$$C(t) = \mu W(t). \tag{18}$$

Under this assumption the functional form of lifetime utility $G(W)$ corresponds to current period utility $U(C)$. The propensity to consume out of wealth μ, as well as the other endogenous variables, are to be determined in macroeconomic equilibrium.

3 Macroeconomic Equilibrium

The results from individual optimization of the preceeding section can now be employed to determine the overall macroeconomic equilibrium. The parameters from the utility function ρ and β, from the production function α and γ as well as the tax/transfer parameters τ_d, τ_s and θ are exogenous to the system. Furthermore the stochastic process dy is exogenous while the stochastic processes for the real rates of return, wealth and the wage process dz_K, dz_B, dw, dz_L have to be determined. The endogenous parameters are r_B, r_K, ω. In addition the portfolio shares, the consumption–wealth ratio, the endogenous growth rate, φ, the initial market value of bonds, and initial wealth have to be determined.

We assume perfect competition in the factor markets so that the wage rate and the rental rate on physical capital are given by the usual marginal productivity conditions. Additionally, in equilibrium the stock of knowledge equals the economy–wide stock of physical capital and the real pre–tax rates of return are [3]

$$\begin{aligned} dR_K &= \alpha\gamma(dt+dy) \\ dR_L &= (1-\alpha)\gamma K(dt+dy). \end{aligned}$$

To derive macroeconomic equilibrium conditions, it is necessary to establish the government budget constraint. Government debt is the residual from the stochastic tax receipts and the deterministic transfer payment. The market value of government bonds evolves according to[4]

$$\frac{dB}{B} = \left[r_B + \frac{\theta}{n_B} - \gamma\tau_d\frac{n_K}{n_B}\right] dt + dz_B - \gamma\tau_s\frac{n_K}{n_B}\, dy. \tag{19}$$

The market clearing condition $dK = dY - C\,dt$ immediately leads to the accumulation equation of physical capital. The stock of physical capital evolves according to

$$\frac{dK}{K} = \left[\gamma - \frac{C}{n_K W}\right] dt + \gamma dy \tag{20}$$

and determines the equilibrium stochastic growth rate of the economy.

It is reasonable to assume that portfolio shares just as well as the consumption–wealth ratio are constant in equilibrium. The steady state is then characterized by non–stochastic functions C/W and n_B, n_K of the underlying parameters. As will be demonstrated below, this statement is equivalent to all assets growing at a common rate. Constancy of portfolio shares implies not only that $K = n_K W$ and $B = n_B W$ in the static sense, but furthermore $dK = n_K\, dW$ and $dB = n_B\, dW$. Hence,

$$\frac{dW}{W} = \frac{dK}{K} = \frac{dB}{B}. \tag{21}$$

[3] The results for the social planner can simply be derived by setting $\alpha = 1$.

[4] Note that there is no restriction imposed on the size of the portfolio share of government bonds.

To solve for the equilibrium, we first consider the stochastic components of the wealth constraint (9) and the accumulation equations (19) and (20). The stochastic component of the real rate of return on bonds is given by

$$dz_B = \frac{\gamma}{n_B}(n_B + \tau_s n_K)\, dy \tag{22}$$

After this, a solution for the stochastic component of the wealth accumulation equation is obtained

$$dw = \gamma W\, dy. \tag{23}$$

From this can be seen that finally all endogenous stochastic processes are driven by the single exogenous productivity shock. This is the only source of uncertainty in the economy. Utilizing the properties of the stochastic process, the variance of wealth can be derived from the budget constraint and (23) as[5]

$$\frac{\mathrm{E}\,(dw)^2}{dt} = \sigma_W^2 = \gamma^2\sigma^2 W^2. \tag{24}$$

The results obtained so far can be used to determine the real rate of return on bonds. From the consumer asset pricing relationship (13) and (14) and from (18) follows

$$r_B = \alpha\gamma(1-\tau_d) + \rho\gamma^2\sigma^2\left[(1-\alpha)(1-\tau_s) + \frac{\tau_s}{n_B}\right]. \tag{25}$$

Equation (25) displays the well-known result from finance theory: The expected real (after–tax) rates of return on physical capital and bonds differ by the amount of the risk premium, i. e. the second term on the right-hand side of (25). The risk premium is of negative sign if capital is the riskier asset, and vice versa. The relationship for the real interest rate on bonds is combined with the results for the stochastic processes (dw, dz_B, dz_K, dz_L), the guess (18), and the first–order condition on portfolio shares (15) and consumption (12) to derive the consumption–wealth ratio from (16)

$$\begin{aligned}\frac{C}{W} = \mu \quad = \quad & \frac{\beta}{\rho} + \frac{\rho-1}{\rho}\left[\alpha\gamma(1-\tau_d)+\theta\right] + n_K\,\gamma(1-\tau_d)\,(1-\alpha) \\ & + \gamma^2\sigma^2\left(\frac{1-\rho}{2} + (\rho-1)\,\tau_s - (1-\alpha)\,(1-\tau_s)\,(1-\rho n_B)\right). \end{aligned} \tag{26}$$

The propensity to consume out of wealth is a function of the parameters of the model, some of them a priori considered constant, the others shown to be constant in equilibrium. Consequently, the consumption–wealth ratio is also constant in steady state, as we conjectured above.

[5]The covariances of the real rates of return are implicitly contained in the first-order conditions (13), (14) and (16). They are given by $\mathrm{cov}[dw, dz_B] = \frac{\gamma^2}{n_B}\sigma^2(n_B + n_K\tau_s)\, dt$ and $\mathrm{cov}[dw, (1-\tau_s)dz_K] = \alpha\gamma^2\sigma^2(1-\tau_s)\, dt$.

The relation for optimum consumption together with the wealth constraint (9) can now be applied to solve for wealth accumulation. Taking expectations allows for the determination of the expected growth rate. Note that due to the properties of the underlying stochastic process $\mathrm{E}[dy] = 0$. Hence, the term dw from (23) vanishes. The expected growth rate of the economy $\varphi = \left[\frac{1}{W}\frac{\mathrm{E}\,dW}{dt}\right]$ is then given by

$$\varphi = \frac{1}{\rho}\left[\alpha\gamma(1-\tau_d) + \theta - \beta\right] + \gamma^2\sigma^2\left(\frac{\rho+1}{2} - \alpha(1-\tau_s)\right). \tag{27}$$

The expected growth rate is the sum of a deterministic and a stochastic term. From (27) we conclude, that the expected growth rate deviates from the growth rate of the deterministic Arrow–Romer model.[6] Additionally, second-order effects from the variance of the technological disturbance have to be taken into account.

The expected growth rate is determined endogenously and depends on the factors that affect aggregate savings. Higher values of β imply lower savings and consequently a lower growth rate. Additionally, the impact from transfer payments can be learned from (27). A rise in transfer payments increases the mean growth rate, as the deterministic part of income increases. The analysis of a change in the tax parameters τ_d and τ_s and the impact of the measure of relative risk aversion, ρ, capital productivity, α, and the variance of the technological disturbance σ^2 on the expected growth rate will be deferred to the next section.

Finally, the capital share of wealth and the portfolio share of government bonds can be derived

$$n_K = \frac{\beta - (1-\rho)\left(\varphi - \frac{1}{2}\rho\gamma^2\sigma^2\right)}{\gamma\tau_d + \alpha\gamma(1-\tau_d) - \varphi + \rho\gamma^2\sigma^2(1-\alpha)(1-\tau_d)} \tag{28}$$

and residually $n_B = 1 - n_K$. The definition of n_K and n_B allows for the determination of initial wealth and the initial market value of bonds in terms of the initial capital stock K_0 and the endogenously determined portfolio shares

$$W(0) = \frac{K_0}{n_K}, \qquad \text{and} \qquad B(0) = \frac{n_B}{n_K}K_0. \tag{29}$$

The law of motion of wealth is described by a geometric diffusion process, i. e. wealth is log–normally distributed and follows a random walk with positive drift. By Itô's Lemma time t wealth, starting from initial wealth $W(0)$ at time 0, is given by

$$W(t) = W(0)\,e^{\left(\varphi - \frac{1}{2}\gamma^2\sigma^2\right)t + \gamma\sigma[y(t)-y(0)]}. \tag{30}$$

[6] The growth rate of the deterministic setting corresponds the the case $\sigma = 0$.

4 Differential Tax Policy

The macroeconomic equilibrium determined in the previous section is now subject to a comparative dynamic analysis. The main objective is now, to examine whether the stochastic model replicates the main results from the theory of taxation under certainty. Following Chamley (1981, 1986) and Judd (1985) the long run tax rate on capital income should be zero. On the other hand, from Domar and Musgrave (1944) and Stiglitz (1969), we have the notion that taxation provides some sort of insurance against intertemporal income risk. Our aim is now to analyze the macroeconomic effects of a differential tax policy. We demonstrate that in the general equilibrium context considered here, the question of tax incidence is not a trivial one. The central point of interest is whether an increase in the tax rates causes welfare gains or welfare losses. In particular, we derive ambiguous welfare effects of taxation. Of importance in explaining this result will be the interdependence between the attitude towards risk, the tax parameters and the capital productivity in the determination of the expected growth rate and the welfare of the economy.

Recalling the set–up of individual optimization (8), the maximized value of lifetime utility in the economy, starting from initial wealth $W(0) = \frac{K_0}{n_K}$, is given by

$$G[W(K_0), 0] = \frac{K_0^{1-\rho}}{1-\rho} \cdot \frac{\mu^{1-\rho} \cdot n_K^{\rho-1}}{\beta - (1-\rho)\left(\varphi - \frac{1}{2}\rho\gamma^2\sigma^2\right)}. \tag{31}$$

From (31) it is obvious that the key way in which taxes impact on the economy is through consumption and portfolio choice and the growth rate, respectively. Ceteris paribus, a rise in the propensity to consume as well as in the expected growth rate induces welfare gains. In contrast to this, a rise in the portfolio share of capital causes welfare losses due to a simultaneous fall in the endogenously determined market value of the stock of financial wealth (29). The marginal propensity to consume out of wealth, the portfolio shares as well as the growth rate depend on the tax rates. Consequently, for each variable the partial derivative with respect to the tax rate has to be determined.

Proposition 1 (Growth Effects) *A rise in the tax rate on mean income leads to a decline in expected growth. A rise in the tax rate on random income parts increases the expected growth rate.*

$$\frac{\partial\varphi}{\partial\tau_d} = -\frac{1}{\rho}\alpha\gamma < 0 \tag{32}$$

$$\frac{\partial\varphi}{\partial\tau_s} = \alpha\gamma^2\sigma^2 > 0 \tag{33}$$

As labor is inelastically supplied, only the effects on capital accumulation have to be considered. The negative growth effect in (32) can be explained as follows: The real after–tax rate of return on physical capital is reduced, causing physical capital to be less attractive. In this case the distortionary impact of a flat–rate income tax on intertemporal allocation can be observed. The change of the growth rate is equal to the change in the interest rate multiplied by the elasticity of intertemporal substitution.

Contrary to this, an increase of τ_s stimulates growth, because it changes the riskiness of disposable income. In particular, it reduces the variance of return on capital, whereas the expected return remains unchanged. In terms of Rothschild and Stiglitz (1970), taxation of transitory incomes displays the characteristic of an inverted mean preserving spread. This result reflects the introductory statement that taxation provides a sort of insurance. Government participates in income risk and physical capital becomes more attractive.[7]

The change in optimal portfolio choice with respect to the tax rates is given by[8]

$$\frac{\partial n_K}{\partial \tau_d} = \frac{\frac{1-\rho}{\rho}\alpha\gamma n_B - \gamma n_K}{\{D\}} \tag{34}$$

$$\frac{\partial n_K}{\partial \tau_s} = \frac{\alpha\gamma^2\sigma^2(\rho-1)n_B + \rho\gamma^2\sigma^2 n_K}{\{D\}} \tag{35}$$

with $\{D\} \equiv \gamma\tau_d + \alpha\gamma(1-\tau_d) - \varphi + \rho\gamma^2\sigma^2(1-\alpha)(1-\tau_d)$. In the following, all results derived for the portfolio share of physical capital apply to financial wealth n_B with opposite sign.

Proposition 2 (Portfolio Effects I) *The portfolio share of physical capital decreases (increases) with a rise in the tax rate on permanent (transitory) income, if the coefficient of relative risk aversion or n_K are sufficiently high:*

$$\rho \geqslant 1 \vee n_K \geqslant 1 \quad \Longrightarrow \quad \begin{cases} \frac{\partial n_K}{\partial \tau_d} & < 0 \\ \frac{\partial n_K}{\partial \tau_s} & > 0. \end{cases} \tag{36}$$

Whether the portfolio share of physical capital rises with a change in the differential tax rates first of all depends on the attitude towards risk being greater or less than unity. Second, it is of importance whether the agents are net lenders or net borrowers to the state, i. e. whether the portfolio share of capital falls short of or exceeds unity, since we have imposed no restrictions on the portfolio share of government bonds.

With an index of relative risk aversion sufficiently high ($\rho \geqslant 1$), the portfolio share of capital decreases with a rise in the tax rate on mean income τ_d and increases with the tax

[7] With a uniform tax rate, the growth diminishing effect of taxation would outweigh the growth enhancing effect of a rise in the tax rate only if $1/\rho > \gamma\sigma^2$. This condition ensures that the certainty equivalent of portfolio return is of positive sign.

[8] The denominator $\{D\}$ of n_K in (28) has to be positive for feasible solutions of the model as the transversality condition has to be satisfied.

rate on random income τ_s independently of the size of the portfolio shares. This reaction expresses the change in return on capital, as was already discussed for the growth effects above. On the one hand taxation causes physical capital to be a less attractive asset, on the other hand it provides an insurance against risk.

With $n_K > 1$, the government is a net–lender to the public. Although this situation is quite unrealistic, it is nevertheless feasible. The present values of primary surpluses have to suffice to pay off existing debt, which corresponds to $B(0) < 0$. Otherwise, the intertemporal budget constraint will not be met. With financial wealth growing at the constant expected growth rate φ, this implies a negative stock of bonds at all points of time. Hence, the insights from debt policy under certainty, as discussed e. g. in Turnovsky (1993, 1996), extend to the stochastic context. The government uses its interest income from being a net–creditor to the private sector to finance its continuous deficit.

Considering the implications of proposition 2 the portfolio responds to changes in the net–returns of assets in a way one would have expected. Due to taxation the composition of the portfolio changes: in favour of bonds with a rise in τ_d, in favour of physical capital with a change in τ_s.

Proposition 3 (Portfolio Effects II) *The portfolio share of physical capital increases (decreases) with a rise in the tax rate on permanent (transitory) income, if n_K is within the unit interval and the degree of relative risk aversion is sufficiently low:*

$$n_K \in (0,1) \ \wedge \ \rho \leqslant \frac{\alpha n_B}{n_K + \alpha n_B} \quad \Longrightarrow \quad \begin{cases} \frac{\partial n_K}{\partial \tau_d} & > 0 \\ \frac{\partial n_K}{\partial \tau_s} & < 0. \end{cases} \tag{37}$$

Taking regard of the equilibrium value of the real rate of return on bonds (25) together with (28), we have to consider that a change in the tax rate on mean income, does not only affect the first component, but also the risk premium. The latter reflects a change in the riskiness of return on financial wealth. Thus, under the conditions of proposition 3 the portfolio responds to taxation of mean income with an increase in the share of physical capital. The opposite results are obtained for an increase in the tax rate on random income parts, as first demonstrated by Eaton (1981).

Finally, the consumption effects of a change in tax rates have to be examined. Whether a tax policy increases or decreases the consumption–wealth ratio (26), depends of course on the tax rates as taxation affects the intertemporal allocation of consumption and saving. Moreover, the effect of a change in optimal portfolio choice have to be taken into account. Hence,

$$\frac{d\mu}{d\tau_i} = \frac{\partial \mu}{\partial \tau_i} + \frac{\partial \mu}{\partial n_K} \cdot \frac{\partial n_K}{\partial \tau_i} \gtrless 0. \tag{38}$$

The partial derivative $\frac{\partial \mu}{\partial \tau_i}$ measures the direct impact of a change in tax rates. In detail, it is given by

$$\frac{\partial \mu}{\partial \tau_d} = \frac{1-\rho}{\rho}\alpha\gamma - n_K\gamma(1-\alpha) \tag{39}$$

$$\frac{\partial \mu}{\partial \tau_s} = \gamma^2\sigma^2(\alpha(\rho-1)+(1-\alpha)\rho n_K) \tag{40}$$

From this follows immediately:

Proposition 4 (Consumption Effects I) *The propensity to consume out of wealth decreases (increases) ceteris paribus with a rise in the tax rate on mean (random) income, if the Arrow–Pratt–index of relative risk aversion is sufficiently high. The opposite results apply for a relative risk aversion sufficiently low.*

$$\left.\begin{array}{ll} \frac{\partial \mu}{\partial \tau_d} & \lesseqgtr 0 \\ \frac{\partial \mu}{\partial \tau_s} & \gtreqless 0 \end{array}\right\} \iff \rho \gtreqless \frac{\alpha}{n_K + \alpha n_B}. \tag{41}$$

A high degree of risk aversion corresponds to a low intertemporal elasticity of substitution $(1/\rho)$. In this case the agent dislikes deviations from a uniform pattern of consumption over time. An increase in the tax rate on mean income leads to a decrease in expected capital return. In order to compensate this effect and to smooth the intertemporal consumption flow, the individual increases savings. The consumption–wealth ratio reacts conversely to a change in the two tax rates and there is a trade–off in the size of the attitude towards risk and the factor income shares.

Due to the aggregate productivity shock, individual income is stochastic. As long as the agents are sufficiently risk averse, there is a motive for precautionary saving. Following Leland (1968) and Sandmo (1970), this is defined as additional savings caused by the uncertainty of future income flows. A precautionary motive arises with decreasing absolute risk aversion. A constant degree of relative risk aversion ($\rho > 0$) satisfies this condition. In contrast to this, uncertainty in interest rates only causes precautionary saving for a degree of relative risk aversion sufficiently high ($\rho \geqslant 1$). In our model setting, the agents are confronted with risky wage income as well as uncertain interest rates. Thus, the fraction on the right hand side of (41) is less than unity. Whether the agent demands higher savings out of precautionary motives — which immediately induces higher growth — first of all depends on the degree of risk aversion. As can be seen from the definition of the growth rate (27), the sign of the second term is also influenced by the size of capital income share, the tax rate on stochastic incomes and the variance of the random shock. Individual optimization leads to precautionary saving if the intertemporal income effect dominates the intertemporal substitution effect, i. e. if the diffusion component in the expected growth rate is of positive sign. In this case, the insurance effect of a rise in the tax rate on random income parts induces an increase in consumption. The opposite, i. e. lower savings, follows from the domination of the substitution effect.

Additionally, from (38) can be seen that the consumption-wealth ratio responds to changes in the optimal portfolio. The partial derivative $\frac{\partial \mu}{\partial n_K}$ is also of ambiguous sign and mainly depends on the relation of the two tax rates and the size of the technological shock, that is

$$\frac{\partial \mu}{\partial n_K} = \gamma(1-\alpha)(1-\tau_d) - \gamma^2\sigma^2\rho(1-\alpha)(1-\tau_s). \tag{42}$$

Proposition 5 (Consumption Effects II) *Given the ratio of the tax rates, the propensity to consume increases (decreases) ceteris paribus with a rise in the portfolio share of physical capital, if the impact from income risk is sufficiently weak (strong).*

$$\frac{\partial \mu}{\partial n_K} \gtrless 0 \quad \Longleftrightarrow \quad \frac{1-\tau_d}{1-\tau_s} \gtrless \rho\gamma\sigma^2 \tag{43}$$

An increase in the portfolio share of physical capital leads to higher expected growth thus enlarging not only future income flows but also the volatility of future income. Whether this increases current consumption depends on the intertemporal elasticity of substitution as well as on the optimal response to risk.

Considering the sign of the partial derivative $\frac{\partial n_K}{\partial \tau_i}$ from (38), the statements of proposition 2 and 3 apply.

We now examine the impact of a differential tax policy on lifetime utility. We will demonstrate that it is impossible to derive unambigous welfare effects. Neither welfare losses due to an increase in τ_d nor welfare gains due to a rise in τ_s can be determined with certainty. Two of the implications are of major interest: An increase in the mean tax rate reduces growth, thus driving the growth rate away from the Pareto–optimal one, but possibly inducing welfare gains. On the contrary, an increase in the tax rate on stochastic income components unambiguously leads to a higher growth rate, reduces income uncertainty, but may cause welfare losses.

The market clearing condition (20) can now be employed to rewrite the expression for lifetime utility (31). With $\mu = n_K(\gamma - \varphi)$, the welfare effects of a change in the tax rates $\tau_i, i = d, s$ is given by

$$\frac{\partial G}{\partial \tau_i} = \frac{K_0^{1-\rho}\,(\gamma-\varphi)^{-\rho}}{\left[\beta-(1-\rho)\left(\varphi-\frac{1}{2}\rho\gamma^2\sigma^2\right)\right]^2}\cdot\frac{\partial \varphi}{\partial \tau_i}\cdot\left[\gamma-\beta-\rho\,\varphi-\frac{1}{2}\,(1-\rho)\,\rho\gamma^2\sigma^2\right]. \tag{44}$$

The first term is positive, as we require a positive propensity to consume and likewise a positive portfolio share of capital ($\mu, n_K > 0$) to obtain feasible solutions for the model. The second term represents the growth effects discussed with proposition 1. The third term is of ambiguous sign and reflects the optimal response in consumption and portfolio choice.

Proposition 6 (Welfare Effects) *The welfare effect of differential taxation is ambiguous. Taxation of permanent as well as of transitory income may cause welfare gains or losses.*

$$\left.\begin{array}{l} \frac{\partial G}{\partial \tau_d} \lesseqgtr 0 \\ \frac{\partial G}{\partial \tau_s} \gtreqless 0 \end{array}\right\} \quad \Longleftrightarrow \quad \gamma - \beta - \rho\,\varphi - \frac{1}{2}\,(1-\rho)\,\rho\gamma^2\sigma^2 \gtreqless 0 \tag{45}$$

If the third expression in equation (44) is positive, the welfare effects parallel the growth effects: An increase in the tax rate on mean income τ_d reduces the expected growth rate and leads to welfare losses. A rise in the tax rate on random income parts τ_s increases growth and induces welfare gains. These outcomes confirm the known results from literature on tax incidence and will be referred to as 'normal' welfare effects. An increase in the tax rate on deterministic income components has a distortionary effect on accumulation. An increase in the tax rate on stochastic income components lowers the variance of wealth, thus providing a kind of insurance.

Otherwise, if the third term is negative, the impact from tax policies on the growth rate are reversed due to consumption and portfolio effects: A higher tax rate τ_d increases welfare, whereas a higher tax rate τ_s reduces maximum lifetime utility. In the following, these counter–intuitive results will be denoted as 'paradoxical' welfare effects. They can be ascribed first of all to the main features of the stochastic Arrow–Romer–Model. The agents underestimate the marginal return on capital. The competitive allocation is characterized by a suboptimally low growth rate and suboptimally high propensity to consume. Thus, any tax policy is to be discussed within a second–best setting. Additionally, all fiscal policy parameters are relevant to individual optimization. Taxation does not only distort the allocation of consumption over time, but also affects the optimal composition of the portfolio. In particular, the entire consumption effect of a change in tax rates is ambiguous due to the interdependence of consumption and portfolio choice. Moreover, an increase in n_K simultaneously induces a reduction in the initial market value of government bonds. This immediately leads to a devaluation of wealth and causes welfare losses. Vice versa, a decrease in n_K causes welfare gains.

Summarizing these results, welfare gains from an increase in the propensity to consume out of wealth accompanied by a decreasing portfolio share of capital may outweigh the welfare losses from lower growth due to taxation of mean income. On the contrary, a reduction in the consumption–wealth ratio or an increase in the portfolio share of capital leads to negative welfare effects that possibly offset the positive welfare effects of a growth enhancing taxation of transitory income. It is important to stress that paradoxical welfare effects of both tax policies occur within the same parameter setting, as can be seen from proposition 6.

5 A Special Case: Logarithmic Preferences

To illustrate our results and for the sake of transparency, we now discuss the case of logarithmic preferences. Summarizing the propositions 1 to 5, we learn that growth, portfolio, and consumption effects may impact on welfare in opposite directions. Assuming logarithmic preferences simplifies some of the effects. This allows for a more rigorous understanding of the interaction of the key macroeconomic variables in the determination of welfare changes.

With $\rho = 1$, the value function takes the form

$$G[W(K_0),0] = \frac{1}{\beta^2} \cdot \left(\beta \ln(\gamma - \varphi) + \varphi - \frac{1}{2}\gamma^2\sigma^2 + \beta \ln K_0 \right). \tag{46}$$

The welfare effects of a change in the differential tax rates given by (44) simplify to

$$\frac{dG}{d\tau_i} = \frac{(\gamma-\varphi)^{-1}}{\beta^2} \cdot \frac{\partial \varphi}{\partial \tau_i} \cdot \underbrace{(\gamma - \varphi - \beta)}_{E}. \tag{47}$$

The first term is positive for feasible solutions, as discussed in the previous section. Term 'E' still is of ambiguous sign. The growth effects remain unchanged, i. e. the results from proposition 1 apply. The portfolio share of physical capital definitely decreases with a rise in the tax rate on mean income. The opposite reaction can be obtained for an increase in the tax rate on random income. Both outcomes can be derived from proposition 2. The entire consumption effects remain ambiguous. Taking regard of the direct effect from proposition 4, the result is now clear–cut: The consumption–wealth ratio decreases with a rise in τ_d, while it increases with a rise in τ_s. Whereas this effect is definite, the response of consumption to a change in portfolio composition is indeterminate. The agents are confronted with income risk as well as with uncertain future returns. From this follows immediately that even with logarithmic preferences the consumption–wealth ratio responds to changes in the tax rates, as already discussed above. Mutual independence of consumption and portfolio choice is no longer preserved. The latter is usually predicted within the intertemporal C–CAPM setting for the assumption of logarithmic preferences, as in Merton (1969). Consequently, our outcomes differ from the corresponding results of the linear stochastic growth model as in Turnovsky (1995).

Substitution for the expected growth rate in (47) leads to the following expression for term 'E'

$$E = \gamma[1 - \alpha(1-\tau_d)] - \theta - \gamma^2\sigma^2[1 - \alpha(1-\tau_s)]. \tag{48}$$

The question of a specific policy mix is a crucial one. The proportion of fiscal expenditure and revenue parameters decides upon the sign of the term 'E' and hence upon the direction of welfare effects. Furthermore, the size of the external effect of human capital accumulation plays an important role. Without this externality, that is $\alpha = 1$, the sign of 'E' would entirely depend on the portfolio share of government bonds to be greater or less than zero. Paradoxical welfare effects in this case may only arise for $n_B < 0$.[9]

Because it is not possible to restrict the formal analysis any further, we perform some numerical computations. The coefficient α is assumed to be a broad measure of physical capital including also human capital. We set $\alpha = 0.7$. Let furthermore the rate of time preference be $\beta = 0.05$. The standard deviation of the technological disturbance as well as the output/capital ratio were set according to Smith (1996) with $\sigma = 0.76$ and $\gamma = 0.5$.

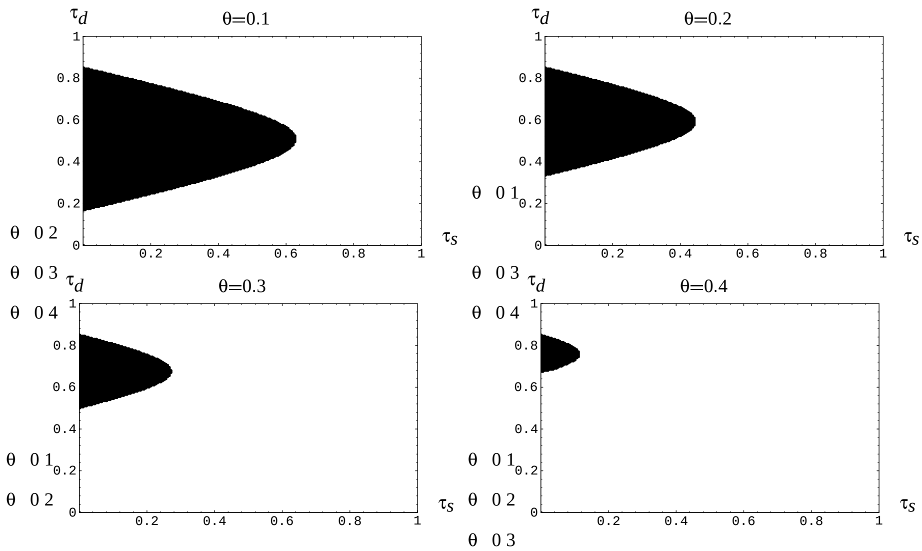

Figure 1: 'Normal' welfare effects of a differential tax policy

The black shaded area represents all tax–transfer policies generating the so–called 'normal' welfare effects of taxation ('E'> 0) which are compatible with feasible consumption programs. That is the propensity to consume out of wealth, the expected growth rate, the portfolio shares, and the expected interest rate on government bonds are positive and the transversality condition is satisfied. In contrast to this, in the white area at least one of these conditions is violated. The space of feasible solutions decreases with a rise in transfer payments. With a large transfer rate, high tax rates on mean income only go along with relatively low tax rates on stochastic income components.

Figure 2 displays parameter combinations of the two tax rates and the transfer rate where $G_{\tau_d} > 0$ and $G_{\tau_s} < 0$ within feasible solutions of the model. It can be seen from the plot, that positive welfare effects of increasing τ_d and vice versa in τ_s arise for a

[9]There is no possibility for Ponzi–games as we require the transversality condition (17) to be satisfied. The argument related to proposition 2 applies.

combination of relatively high transfers together with a very high tax rate on deterministic income components and a negligible rate on stochastic ones. Given the variance of the technological disturbance, counter–intuitive welfare effects may only be derived if there is a tax policy with a high tax rate in mean income, $\tau_d \to 1$, and a relatively low income tax rate on random income parts, $\tau_s \to 0$, as already discussed for proposition 5.

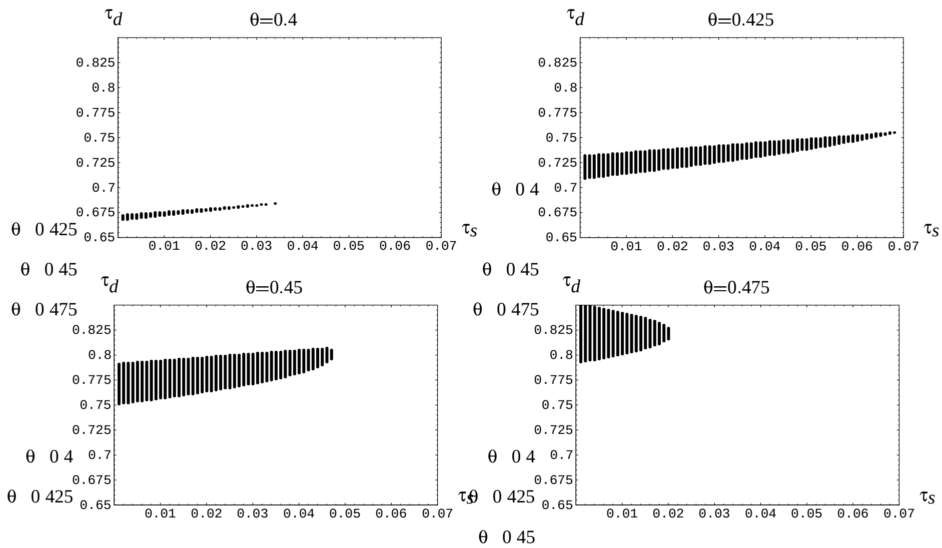

Figure 2: ‘Paradoxical’ welfare effects of a differential tax policy

6 Conclusion

In this paper we developed a continuous–time stochastic endogenous growth model with a positive production externality due to human capital accumulation. Assuming differential tax rates with respect to deterministic and stochastic components of income, we analyzed growth and welfare effects of a change in proportional income taxation. We found that increases in the respective tax rate have contrary effects on the expected growth rate, thus confirming the well–known results from literature on tax incidence. Taxation of mean income reduces expected growth. Taxation of transitory income increases the growth rate, as the government shares part of the income risk. We demonstrated that a differential tax policy may have the welfare effects usually expected in this setting. Under certain conditions, the distortionary effect of taxation of permanent income reduces welfare, whereas the insurance effect of taxation of transitory income leads to welfare gains. But what is more interesting, we derived welfare effects that contradict the common economic intuition. A decrease in the expected growth rate due to taxation may occur together with welfare gains, whereas one would usually relate this situation to welfare losses. The opposite results were obtained for an increase in the tax rate on stochastic income parts: An increase in the growth rate is possibly accompanied by a decrease in welfare. These findings can mainly be ascribed to the interaction of consumption and portfolio choice in the determination of equilibrium growth and welfare.

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