Martingales, Taxes, and Neutrality

By Andreas Löffler*
Fachbereich Wirtschaftswissenschaften
Universität Hannover
Königsworther Platz 1, 30169 Hannover, Germany
Email AL@wacc.de
Fax +49-511-762 4670.

and

Dirk Schneider
Fachbereich Wirtschaftswissenschaft
Freie Universität Berlin
Boltzmannstr. 20, 14195 Berlin, Germany

Abstract

We analyze a general business tax in an uncertain economy. Our tax system allows for a time–dependent tax rate and to this end we incorporate a generalized allowance for corporate equity (ACE). The generalized allowance is given by a fraction of the product of interest rate and book value of the project and this fraction can be time–dependent. We determine conditions under which taking this tax into account does not distort investment decisions, i.e. under which the tax system will be neutral.

To allow for investors with arbitrary risk attitude we make use of the martingale approach. We show that the after–tax capital market is arbitrage-free and complete if it is arbitrage-free and complete in a world without taxation. We furthermore derive a valuation equation under taxes that we use to specify neutral tax systems. Our tax system generalizes two well–known neutral tax systems: the taxation of economic rent and the tax with allowance on corporate equity as introduced by Boadway and Bruce (1979) and Wenger (1983).

The taxation of economic rent even remains neutral if the tax rate is time–dependent and if there is a generalized allowance on corporate equity. This reveals that the assertion of Bond and Devereux (1995) that a constant tax rate is an indispensable condition for neutrality is wrong. This assertion is found to be wrong even in the model of Bond and Devereux (1995).

The taxation with a generalized allowance on corporate equity even remains neutral if the tax rate is time–dependent and if the parameters which determine the fraction of the allowance are chosen in an appropriate manner. This shows that the ACE concept of neutrality is far more general then stated in the literature.

Keywords: arbitrage–free valuation, equivalent martingale measure, taxes, uncertainty.

JEL H21, G12

Nowadays, three tax systems are known to be neutral with respect to investment decisions under certainty in partial equilibrium.[1] The taxation of economic rent introduced by Preinreich (1951), Samuelson (1964), and Johansson (1969) has a tax base equal to economic rent, i.e. net cash–flows plus capital gains. The taxation of cash–flows going back to Brown (1948), is characterized by an immediate write–off of investment expenses and a tax base equal to cash–flows. The tax with an allowance of corporate equity proposed by Boadway and Bruce (1979), Wenger (1983), and Boadway and Bruce (1984), differs from the traditional income–tax (in the sense of Schanz, Haig, and Simons) by a deduction of interest payments on the book value of the investment project. This deduction is called allowance for corporate equity (ACE).[2] All papers mentioned so far use a setup where cash flows from the investment are certain.

If the returns of an investment project are uncertain the important question arises whether the neutrality of the above mentioned tax systems will be preserved. This question was tackled by Fane (1987) and Bond and Devereux (1995). Fane (1987) showed that the cash–flow tax and the taxation of economic rent indeed preserve neutrality. Furthermore, Fane (1987) pointed out that this is true for the cash–flow tax even if there are timing differences between tax payments and accruals. Later Bond and Devereux (1995) extended Fane's results by analyzing a business tax that is neutral under uncertainty including bankruptcy, wind–up decisions, and default outcomes. The results of Fane (1987) and Bond and Devereux (1995) were derived under the assumption of a tax rate that is constant in time. Furthermore, Bond and Devereux (1995, p. 69) argue in their model that a constant tax rate is an indispensable condition for tax neutrality of the taxation of economic rent and give an example where a time varying tax rate is incompatible with neutrality.

Fane (1987) and Bond and Devereux (1995) used the technique of arbitrage-free valuation. In particular they assume that the capital market under taxation is arbitrage–free but not necessarily complete. Because of their assumption that the capital-market is arbitrage-free under taxation it is not obvious whether the capital market is arbitrage-free without taxation. The question whether a capital market with neutral taxation offers opportunities for tax arbitrage was investigated by Jensen (2002). But Jensen does not look at a general income tax but only on taxation of capital gains. Hence, his neutrality concept relies on holding period neutrality and is completely different from ours.

In the present paper we look at a general tax system which incorporates the taxation of economic rent and the tax with ACE as special cases.[3] The general tax system is characterized by two particular features: a time–dependent tax rate and a generalized allowance for corporate equity. This allowance is more general than in the literature since it is a time–dependent fraction of the product of book value of the project and interest rate. By strictly using the martingale approach we establish a connection between the existence (and uniqueness) of the equivalent martingale measure in a world with and without taxes. In contrast to Fane (1987) and Bond and Devereux (1995) we are able to show that the capital market with taxation is arbitrage–free if this is already the case for the market without taxation.

Furthermore, we are able to derive a valuation equation which can be used to specify conditions under which our general tax system is neutral.[4] In particular, the above mentioned assertion of Bond and Devereux (1995) that the taxation of economic rent needs a time–constant tax rate to be neutral is found to be wrong in our model.

The paper is organized as follows: In section I the model is specified and the tax system is introduced. In section II we derive our valuation equations and proceed with the neutrality analysis. In section III we extend our findings to a continuous–time model. Section IV concludes the paper. All proofs are in the appendix.

I. The model

A. The capital market

Consider a model in discrete time $t = 0, 1, \ldots, T$ with uncertainty. The probability space is denoted by $(\Omega, \mathcal{F}, P)$. The filtration $\mathcal{F}$ need not be finitely generated, it consists of the σ–algebras $\mathcal{F}_0 \subseteq \mathcal{F}_1 \subseteq \ldots \subseteq \mathcal{F}_T$ that describe the information set of every investor.[5] There are $N + 1$ tradeable financial assets that pay dividends (adapted random variables)

$$\widetilde{X}_{1,t}, \ldots, \widetilde{X}_{N,t}$$

The prices – also called values – of the risky assets at time t are adapted random variables

$$\widetilde{V}_{1,t}, \ldots, \widetilde{V}_{N,t}.$$

There is one risk–free asset, labelled $n = 0$. The prices of the risk–free asset are given by

$$V_{0,t} = \begin{cases} 1 & \text{if } t < T \\ 0 & \text{if } t = T \end{cases} \tag{1}$$

and the cash–flows of the risk–free asset are given by

(2) $$X_{0,t} = \begin{cases} r_f & \text{if } t < T \\ 1 + r_f & \text{if } t = T \end{cases}$$

where r_f is the risk–free interest rate.[6]

At time $t = 0$ the investor selects a portfolio consisting of the available financial assets. This portfolio will be changed at every subsequent trading date $t = 1, \ldots, T$. The portfolio held during period t, denoted by $\widetilde{H}_{t-1}$, has a value of

$$\widetilde{H}_{t-1} \cdot \widetilde{V}_t = \sum_{n=0}^{N} \widetilde{H}_{n,t-1} \widetilde{V}_{n,t}.$$

At time t the investor can withdraw the amount $\delta_t(\widetilde{H})$ given by

(3) $$\delta_t(\widetilde{H}) = \widetilde{H}_{t-1} \cdot (\widetilde{X}_t + \widetilde{V}_t) - \widetilde{H}_t \cdot \widetilde{V}_t.$$

Denote that $H_{-1} = \widetilde{H}_T = 0$ (see figure 1).

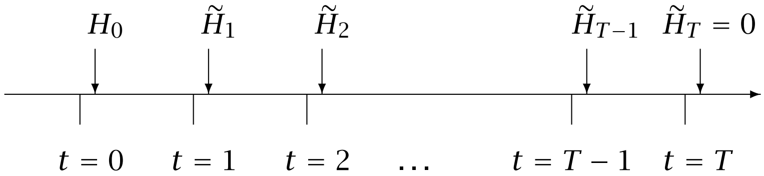

Figure 1: The time structure of the model

Let the capital market be arbitrage–free in the following sense.

Assumption 1 (Arbitrage–free capital market). *There exists no trading strategy $\widetilde{H}$ that satisfies*

(4) $$\delta_t(\widetilde{H}) \geq 0$$

for all t and

(5) $$P(\delta_t(\widetilde{H}) > 0) > 0$$

for at least one t.

According to Harrison and Kreps (1979) assumption 1 implies the existence of an equivalent martingale measure Q (for a proof see e.g. Kabanov and Kramkov (1995)).

Proposition 1 (Fundamental pricing lemma). *If assumption 1 holds there exists a probability measure Q such that*

$$\tilde{H}_t \cdot \tilde{V}_t = \frac{E_Q[\tilde{H}_t \cdot (\tilde{X}_{t+1} + \tilde{V}_{t+1})|\mathcal{F}_t]}{1 + r_f}. \tag{6}$$

We now introduce the tax system.

B. The tax system

We have to distinguish between the market value of a risky asset and the value that will be the underlying for the tax base. The underlying tax base will not be determined by the market alone but by the tax law. We denominate it "the book value" of a financial asset. The book value of asset n at time t will be denoted by $\tilde{B}_{nt}$ and is a random variable. We assume that the book value is an adapted random variable and that will be zero at time $t = T$. It is not necessary for our model to incorporate other details from any actual tax law. The portfolio $\tilde{H}_{t-1}$ has the book value

$$\tilde{H}_{t-1} \cdot \tilde{B}_t = \sum_{n=1}^{N} \tilde{H}_{n,t-1} \tilde{B}_{n,t}. \tag{7}$$

Using the book value we define the depreciation of a portfolio as follows:

Definition 1 (Depreciation). *The depreciation of portfolio $\tilde{H}_{t-1}$ in period t is given by the difference of the book values of all containing financial assets*

$$\tilde{D}_t(\tilde{H}_{t-1}) = -\tilde{H}_{t-1} \cdot (\tilde{B}_t - \tilde{B}_{t-1}). \tag{8}$$

This immediately leads us to the definition of the gain of a portfolio.

Definition 2 (Gain). *The gain of portfolio $\tilde{H}_{t-1}$ is given by the difference of cash-flow and depreciation in t*

$$\tilde{G}_t(\tilde{H}_{t-1}) = \tilde{H}_{t-1} \cdot \tilde{X}_t - \tilde{D}_t(\tilde{H}_{t-1}). \tag{9}$$

The concepts of depreciation and gain are determined by the tax code and may differ from economic depreciation or profit.

Now we are able to define the tax base. We use an idea already developed by Boadway and Bruce (1979) and Wenger (1983) that the tax base will be given by the gain of a portfolio with an allowance on interest on book value (ACE or "allowance on corporate equity"). In our model the allowance can be time-dependent, making our model slightly more general.

Definition 3 (ACE–tax base). *The tax base $\widetilde{U}_t$ of the portfolio $\widetilde{H}_{t-1}$ in $t > 0$ is given by the difference between the gain and a time-dependent, but deterministic fraction $1 - \alpha_t$ of interest in t on book value in $t - 1$*

$$\widetilde{U}_t(\widetilde{H}_{t-1}) = \widetilde{G}_t(\widetilde{H}_{t-1}) - (1 - \alpha_t) \cdot r_f \cdot \widetilde{H}_{t-1} \cdot \widetilde{B}_{t-1}. \tag{10}$$

If the tax base is negative, there is an immediate and full loss offset. In $t = 0$ no tax is paid.

The parameter α_t is exogenous and can take positive as well as negative values. In the case of a negative α_t there is a tax relief on interest on book value.

We assume a proportional tax which is time-dependent but deterministic. Therefore, the tax payments in t are given by

$$\widetilde{T}_t(\widetilde{H}_{t-1}) = \tau_t \cdot \widetilde{H}_{t-1} \cdot \left(\widetilde{X}_t + \widetilde{B}_t - (1 + r_f \cdot (1 - \alpha_t)) \cdot \widetilde{B}_{t-1}\right). \tag{11}$$

Summing up, our tax system has two new features not incorporated in the tax system studied by Boadway and Bruce (1984): a time-dependent allowance on corporate equity and a time-dependent tax rate. We now turn to the characterization of the financial assets.

C. Characterization of financial assets

We need an assumption concerning the book value of a financial asset. This assumption is motivated by considering a riskless bank account with a closing balance equal to the book value. In every period the interest payment is added to and the cash–flow (withdrawal) is subtracted from the opening balance. The evolution of the bank account from t to $t+1$ is as follows:

	book value at the beginning of period $t+1$	$B_{0,t}$
+	interest at $t+1$	$r_f B_{0,t}$
−	withdrawal at $t+1$	$X_{0,t+1}$
=	book value at the end of period $t+1$	$B_{0,t+1}$

We get

$$(1 + r_f)B_{0,t} = X_{0,t+1} + B_{0,t+1}, \tag{12}$$

which resembles to the fundamental pricing lemma (6). Since at $t = T$ book value and market will be equal to zero we conclude that this equation implies by induction that book value and market value are the same at every time t. Although other rules for the determination of book value could be incorporated, we make the assumption that the tax law requires investors to mark their assets to market in each period and the tax law is applied to that measure of value:

Assumption 2 (Book value of a financial asset). *The book value $\widetilde{B}_{n,t}$ of a financial asset is equal to its value $\widetilde{V}_{n,t}$*

$$\widetilde{V}_{n,t} = \widetilde{B}_{n,t}. \tag{13}$$

The existing American tax law states under the Statements of Financial Accounting Standards (SFAS) 115 that "unrealized holding gains and losses for trading securities shall be included in earnings". Hence, the American tax system contains elements similar to our assumption.

II. Pricing of real assets

A. Valuation of a real asset

We start this section by showing that our tax system has no arbitrage opportunities. Therefore it is possible to value risky cash–flows in a world with taxation without recurring to personal utility functions.

Proposition 2 (Fundamental Theorem of Asset Pricing under taxes). *Under assumption 1 the following holds:*

(i) The capital market is arbitrage-free and there is an equivalent martingale measure Q such that

$$\widetilde{H}_t \cdot \widetilde{V}_t = \frac{E_Q[\widetilde{H}_t \cdot (\widetilde{X}_{t+1} + \widetilde{V}_{t+1}) - \widetilde{T}_{t+1}(\widetilde{H}_t)|\mathcal{F}_t]}{1 + (1 - \alpha_{t+1}\tau_{t+1})\, r_f} \tag{14}$$

(ii) If the capital market without taxes is complete then the capital market is also complete with taxes.

The non-existence of arbitrage opportunities as well as the completeness of the capital market carry over if taxes are taken into account. Furthermore, the equivalent martingale measure is the same under both circumstances. This result will be used to value the real asset. If the cash–flow of a real asset can be duplicated using a portfolio of financial assets both values must coincide.

Suppose that beyond the $N + 1$ financial assets there is one real asset, indexed by $n = N+1$. We assume that this real asset pays cash–flows $\widetilde{X}_{N+1,t}$ and has a book value $\widetilde{B}_{N+1,t}$. The investor has to make investment expenses of $\widetilde{I}_{N+1,t}$ for the real asset. In contrast to our assumption concerning the financial assets we do not presuppose that this real asset has a book value equal to expenses $\widetilde{I}_{N+1,t}$. We assume that the real asset is taxed in the same way as the financial assets, i.e. according to equations (8) to (11) for $n = N + 1$.

As we assume the capital market to be complete it is possible to duplicate the cash–flows of the real asset by a trading strategy using the financial assets. Our arbitrage assumption enables us to compute a fair value of the real asset. This fair value $\widetilde{V}_{N+1,t}$ (taking taxes into account) might be different from the value evaluated without taxes and designated as $\widetilde{V}^*_{N+1,t}$. We can show the following equation for both values.

Proposition 3 (Fundamental valuation equation). *The value $\widetilde{V}_{N+1,t}$ of the real asset taking taxes into account and the value $\widetilde{V}^*_{N+1,t}$ ignoring taxes satisfy*

$$\widetilde{V}_{N+1,t} = \widetilde{V}^*_{N+1,t} - c_t \cdot (\widetilde{V}^*_{N+1,t} - \widetilde{B}_{N+1,t}) + \sum_{s=t+1}^{T-1} C_{s,t} \cdot E_Q[\widetilde{V}^*_{N+1,s} - \widetilde{B}_{N+1,s} | \mathcal{F}_t], \tag{15}$$

where

$$c_t = \frac{\tau_{t+1} \cdot (1 + r_f \cdot (1 - \alpha_{t+1}))}{1 + r_f \cdot (1 - \alpha_{t+1}\tau_{t+1})}$$

and

$$C_{s,t} = \frac{\alpha_{s+1}\tau_{s+1}r_f \cdot (1 - \tau_s) + (1 + r_f) \cdot (\tau_s - \tau_{s+1})}{\prod_{k=t+1}^{s+1} (1 + r_f \cdot (1 - \alpha_k\tau_k))}$$

for $s = t + 1, \ldots, T - 1$.

B. Neutrality and neutral tax systems

Richter (1986) distinguished between static and dynamic neutrality. Static neutrality refers only to the time the investment is undertaken, i.e. $t = 0$. A tax system that satisfies the criterion of static neutrality has no distorting effects in $t = 0$ but there might be some future date $t = 1, \ldots, T$ where the early investment decision will not be maintained if taxes are taking into account. Dynamic neutrality refers to the time when the investment is undertaken as well as to all future dates. Therefore a tax system that satisfies the criterion of dynamic neutrality has no distorting effects in $t = 0$ nor in $t = 1, \ldots, T$. The criterion of dynamic neutrality is stronger than the criterion of static neutrality. Obviously, static neutrality is satisfactory if the investment project is irreversible and not tradeable at future dates. But these conditions do not necessarily hold in general. Consequently we focus on dynamic neutrality.

In defining neutrality we follow Fane (1987) and Bond and Devereux (1995) and focus on the net present value of an investment.[7] We say a tax system is neutral iff the ordering of the net present value of two investment projects in a world with taxation is the same as the ordering of the net present value in a world without taxation. This criterion only makes sense if the real assets are traded at a net present value not equal to zero, i.e. the investment expenses $\widetilde{I}_{N+1,t}$ are different from the fair value $\widetilde{V}_{N+1,t}$. In order to analyze whether a tax is neutral or not the net present value in a world with taxation

$$\widetilde{\mathrm{NPV}}_t = \widetilde{V}_{N+1,t} - \widetilde{I}_{N+1,t} \tag{16}$$

has to be compared with the net present value in a world without taxation

$$\widetilde{\mathrm{NPV}}_t^* = \widetilde{V}_{N+1,t}^* - \widetilde{I}_{N+1,t}. \tag{17}$$

Now, our aim is to show necessary and sufficient conditions for the following form of dynamic neutrality:

Definition 4 (Dynamic neutrality). *A tax system is dynamically neutral iff*

$$\widetilde{\mathrm{NPV}}_t = (1 - a_t) \cdot \widetilde{\mathrm{NPV}}_t^* \tag{18}$$

for some $a_t < 1$.

Since we have not assumed that the parameters a_t are positive, taxation may increase the net present value.

Comparing equations (15) and (18) the following inequality must be satisfied for a neutral tax system for all $t = 0, \dots, T-1$:

$$c_t \cdot \widetilde{V}_{N+1,t}^* - \widetilde{B}_{N+1,t} - \widetilde{I}_{N+1,t} - \sum_{s=t+1}^{T-1} C_{s,t} \cdot E_Q[\widetilde{V}_{N+1,t}^* - \widetilde{B}_{N+1,t} | \mathcal{F}_t] = a_t \, \widetilde{\mathrm{NPV}}_t^* < 1, \tag{19}$$

which can be done by either setting the coefficients c_t and $C_{s,t}$ or by setting the book value in an appropriate manner. This leads to a variety of neutral tax systems. Two particular systems, which can be regarded as generalizations of well–known neutral systems, will now be discussed in detail.

Proposition 4 (Neutral tax systems). *The following two tax systems are dynamically neutral:*

- *(Generalization of taxation on economic rent) For all $t \geq 0$ the book values of the real asset equal the fair market values without taxes*

  $$\widetilde{V}^*_{N+1,t} = \widetilde{B}_{N+1,t}$$

  regardless whether there is allowance on corporate equity or not (α_t arbitrary). Then a_t is equal to zero: $a_t = 0$.

- *(Tax with ACE) For all $t \geq 0$ the book values equal the investment expenses*

  $$\widetilde{B}_{N+1,t} = \widetilde{I}_{N+1,t},$$

  and the parameters α are set according to

  $$\alpha_{t+1} = \frac{1 + r_f}{r_f} \cdot \frac{\tau_{t+1} - \tau_t}{\tau_{t+1} \cdot (1 - \tau_t)}$$

  and

  $$\alpha_1 = \frac{1 + r_f}{r_f} \cdot \frac{\tau_1 - a_0}{\tau_1 (1 - a_0)}$$

  for all $t > 0$. In this case $a_t = \tau_t$ for all $t > 0$.

Since in the first tax system depreciation is equal to economic depreciation, it is a generalization of the taxation of economic rent.[8] Therefore, the taxation of economic rent preserves neutrality under conditions of a time-dependent tax-rate and/or a partial allowance on corporate equity with a time-dependent parameter.

The second tax system is characterized by a tax base with allowance on corporate equity in combination with depreciations that only have to sum up to the investment expenses. Therefore, it is a generalization of the tax with an ACE of Boadway and Bruce (1979) and Wenger (1983).[9] Hence, this tax remains neutral under conditions of a time-dependent tax rate if the interest allowance is chosen appropriately (i.e. time–dependent and with a factor $1 - \alpha_t$ not necessarily equal to one). Both tax systems generalize well known neutral tax systems to the case of a time–dependent tax rate and uncertain cash–flows.

C. Neutrality and a time-dependent tax rate: Bond & Devereux's error

Our findings concerning the neutrality of the taxation of economic rent in the case of a time-dependent tax rate is contrary to the assertion of Bond and Devereux (1995), that in this case neutrality is impossible.[10] In the following we prove that their assertion is wrong. In order to do so we will look at their arguments using their notation.[11] Bond and Devereux (1995) state that the tax base of the taxation of economic rent can be written as

$$B_1 = R_1 - \delta_1 - r_1 - \left(K_1 - (1 - z_1)\right) \text{ with } \delta_1 = 1 - K_1 \text{ and } z_1 = \delta_1 \tag{20}$$

and

$$B_2 = R_2 - \delta_2 - r_2 K_1 - (1 + r_2)\left(K_1 - (1 - z_1)\right) \text{ with } \delta_2 = K_1 - K_2 \text{ and } z_2 = \delta_2. \tag{21}$$

Notice that for the tax base to be written as above it is necessary to assume that an allowance on interest payments exist.

Now Bond and Devereux state that the net present value of an investment with and without taxes takes the form

$$\widetilde{\mathrm{NPV}}^{*} = S_1 + S_2 \text{ and } \widetilde{\mathrm{NPV}} = (1 - \tau_1)\, S_1 + (1 - \tau_2)\, S_2 \tag{22}$$

with

$$S_1 = \frac{B_1}{1 + r_1} \text{ and } S_2 = \frac{B_2}{(1 + r_1)\,(1 + r_2)} \tag{23}$$

as the discounted tax bases. Based on these expressions for the net present value Bond and Devereux construct an example of a violation of neutrality, given by $S_1 < 0$ and $S_2 = -S_1 > 0$. If now the tax rate is time–dependent then a counterexample is established.

The flaw in Bond and Devereux is based on the fact that with a tax on economic rent the tax base is much simpler: Since $z_1 = \delta_1$ and $\delta_1 = 1 - K_1$ we have $K_1 - (1 - z_1) = 0$, and the tax base described by equations (20) and (21) reduces to

$$B_1 = R_1 - \delta_1 - r_1 \text{ and } B_2 = R_2 - \delta_2 - r_2 K_1. \tag{24}$$

Bond and Devereux assumed $K_0 = 1$ for the present value in 0.[12] Using now the recursive equations for the present value

$$(25) \qquad 1 = \frac{K_1 + R_1}{1 + r_1} \text{ and } K_1 = \frac{K_2 + R_2}{1 + r_2}$$

this leads to tax bases of

$$(26) \qquad B_1 = R_1 - \delta_1 - r_1 = \big((1 + r_1) - K_1\big) - (1 - K_1) - r_1 = 0$$

and

$$(27) \qquad B_2 = R_2 - \delta_2 - r_2 K_1 = \big((1 + r_2) K_1 - K_2\big) - (K_1 - K_2) - r_2 K_1 = 0.$$

Therefore the further assumption $S_1 < 0$ and $S_2 > 0$ in Bond and Devereux is impossible. On the contrary, the taxation of economic rent can be neutral even if the tax rate is time–dependent as our model has shown.

III. Continuous time model

In this section we will consider a continuous time setup. Our model has to be modified as follows. The value of the riskless asset evolves over time according to the differential equation (the instantaneous risk free rate is constant)

$$(28) \qquad dV_{0,t} = rV_{0,t}dt,$$

and the value of the risky assets evolves according to the stochastic differential equation[13]

$$(29) \qquad dV_t = \mu_t V_t dt - dX_t + \sigma_t V_t dW_t,$$

where σ_t represents the volatility and μ_t the drift. The depreciation is given by the differential $-dB_t$, and the tax payments in t are equal to

$$(30) \qquad dT_t = \tau_t \left(dX_t + dB_t - (1 - \alpha_t) r B_t dt\right).$$

We further assume that the tax rate τ_t is differentiable with respect to t. Then we get the following valuation equation:

Proposition 5 (Fundamental valuation equation in continuous time). *In a continuous time setup the fundamental valuation equation (15) reads*

$$V_t - (1-\tau_t)V_t^* - \tau_t B_t = \\ = E^Q\left[\int_t^T e^{-\int_t^s r(1-\alpha_u\tau_u)du}\left(r\alpha_s\tau_s(1-\tau_s) - \dot{\tau}_s\right)(V_s^* - B_s)ds \mid \mathcal{F}_t\right]. \tag{31}$$

Equation (31) enables us to derive neutral tax systems which are essentially analogous to the discrete time case discussed in section II.B. The real asset trades at investment expenses I_t. Again the tax on economic rent is characterized by the equality of book value and value before taxes, i.e. $B_t = V_t^*$. Then equation (31) reduces to

$$V_t - V_t^* = 0, \tag{32}$$

and so the tax is neutral. To get the second neutral tax system the parameters α_t must satisfy

$$r\alpha_t\tau_t(1-\tau_t) = \dot{\tau}_t. \tag{33}$$

In this case equation (31) is of the form

$$V_t - I_t - (1-\tau_t)(V_t^* - I_t) = 0, \tag{34}$$

which proves neutrality.

IV. Conclusion

The present paper is a first step in characterizing neutral tax systems under uncertainty with a time–dependent tax rate. It was shown that the tax on economic rent and a tax with deductible interest payments are neutral with respect to investment decisions and that the results also hold in a continuous time model. Since we used the martingale theory it was not necessary to assume that the investors are risk–neutral. The distributive effects of the neutral tax systems in an equilibrium model were ignored in this paper. This aspect is left for future research.

V. Appendix

A. Proof of Proposition 2

(i): We show the first part of proposition 2 (no arbitrage opportunities). Suppose, the trading strategy $\widetilde{H}$ is an arbitrage opportunity in a world with taxation. This is

$$H_0 \cdot V_0 \le 0 \tag{A1}$$

for $t = 0$,

$$-H_0 \cdot V_0 \ge 0 \;\text{ and }\; \widetilde{H}_t \cdot (\widetilde{X}_{t+1} + \widetilde{V}_{t+1}) - \widetilde{T}_{t+1}(\widetilde{H}_t) - \widetilde{H}_{t+1} \cdot \widetilde{V}_{t+1} \ge 0 \tag{A2}$$

for all $t = 0, \dots, T-1$, and at least one inequality must be strict with positive probability. For $t = T$ we have

$$\widetilde{H}_T = 0. \tag{A3}$$

After taking the expectation with respect to Q and using Lemma 1 and Proposition 1 we get

$$\begin{aligned} E_Q[\widetilde{H}_t \cdot (\widetilde{X}_{t+1} + \widetilde{V}_{t+1}) - \widetilde{T}_{t+1}(\widetilde{H}_t)] - E_Q[\widetilde{H}_{t+1} \cdot \widetilde{V}_{t+1}] \ge 0 \\ E_Q[\widetilde{H}_t \cdot (\widetilde{X}_{t+1} + \widetilde{V}_{t+1} - \tau_{t+1}\alpha_{t+1} r_f \widetilde{V}_t)] - E_Q[\widetilde{H}_{t+1} \cdot \widetilde{V}_{t+1}] \ge 0 \\ (1 + r_f(1 - \alpha_{t+1}\tau_{t+1}))E_Q[\widetilde{H}_t \cdot \widetilde{V}_t] \ge E_Q[\widetilde{H}_{t+1} \cdot \widetilde{V}_{t+1}]. \end{aligned}$$

Together with (A1) and the fact that at least one inequality must be strict with positive probability these inequalities imply by induction

$$0 \ge E_Q[\widetilde{H}_t \cdot \widetilde{V}_t] \;\text{ for all }\; 0 \le t \le T \tag{A4}$$

and

$$0 > E_Q[\widetilde{H}_t \cdot \widetilde{V}_t] \;\text{ for all }\; t \ge t',$$

i.e. t' is the first time at which the inequality is strict. Thus

$$0 > E_Q[\widetilde{H}_T \cdot \widetilde{V}_T], \tag{A5}$$

but this contradicts (A3).

Now we are able to prove the fundamental pricing lemma under taxes (14). The expected tax payments $E_Q[\widetilde{T}_{t+1}(\widetilde{H}_t)|\mathcal{F}_t]$ are given by

$$\begin{aligned} E_Q[\widetilde{T}_{t+1}(\widetilde{H}_t)|\mathcal{F}_t] &= \tau_{t+1}E_Q[\widetilde{H}_t\cdot(\widetilde{X}_{t+1}+\widetilde{B}_{t+1}-(1+r_f(1-\alpha_{t+1}))\,\widetilde{B}_t)|\mathcal{F}_t] && \text{by (11)}\\ &= \tau_{t+1}E_Q[\widetilde{H}_t\cdot(\widetilde{X}_{t+1}+\widetilde{V}_{t+1}-(1+r_f(1-\alpha_{t+1}))\,\widetilde{V}_t)|\mathcal{F}_t] && \text{by (13)}\\ &= \tau_{t+1}E_Q[\widetilde{H}_t\cdot((1+r_f)\widetilde{V}_t-(1+r_f(1-\alpha_{t+1}))\,\widetilde{V}_t)|\mathcal{F}_t] && \text{by (6)}\\ &= \tau_{t+1}E_Q[\alpha_{t+1}r_f\,\widetilde{H}_t\cdot\widetilde{V}_t|\mathcal{F}_t], \end{aligned}$$

and since $\widetilde{H}_t$ and $\widetilde{V}_t$ are adapted, we have

$$E_Q[\widetilde{T}_{t+1}(\widetilde{H}_t)|\mathcal{F}_t] = \tau_{t+1}\alpha_{t+1}r_f\,\widetilde{H}_t\cdot\widetilde{V}_t. \tag{A6}$$

Then it follows using equations (A6) and (6)

$$\begin{aligned} E_Q[\widetilde{H}_t\cdot(\widetilde{X}_{t+1}+\widetilde{V}_{t+1})-\widetilde{T}_{t+1}(\widetilde{H}_t)|\mathcal{F}_t] &= (1+r_f)\widetilde{H}_t\cdot\widetilde{V}_t-\alpha_{t+1}\tau_{t+1}r_f\widetilde{H}_t\cdot\widetilde{V}_t\\ &= (1+r_f\cdot(1-\alpha_{t+1}\tau_{t+1}))\widetilde{H}_t\cdot\widetilde{V}_t, \end{aligned}$$

which was to be shown.

(ii): The last part of proposition 2 covers the uniqueness of the martingale measure. We show that if the equivalent martingale measure in (6) is unique then there is only one martingale measure satisfying (14). Assume the contrary and consider

$$E_{Q_1}[\widetilde{H}_t\cdot(\widetilde{X}_{t+1}+\widetilde{V}_{t+1})-\widetilde{T}_{t+1}(\widetilde{H}_t)|\mathcal{F}_t] = E_{Q_2}[\widetilde{H}_t\cdot(\widetilde{X}_{t+1}+\widetilde{V}_{t+1})-\widetilde{T}_{t+1}(\widetilde{H}_t)|\mathcal{F}_t] \tag{A7}$$

Using (A6) we get

$$\begin{aligned} &E_{Q_1}[(1-\tau_{t+1})\widetilde{H}_t\cdot(\widetilde{X}_{t+1}+\widetilde{V}_{t+1})+(1+r_f(1-\alpha_t))\tau_{t+1}V_t|\mathcal{F}_t] =\\ &\qquad E_{Q_2}[(1-\tau_{t+1})\widetilde{H}_t\cdot(\widetilde{X}_{t+1}+\widetilde{V}_{t+1})+(1+r_f(1-\alpha_t))\tau_{t+1}V_t|\mathcal{F}_t]. \end{aligned} \tag{A8}$$

Since V_t is $\mathcal{F}_t$–measurable the expectation is equal to the variable itself and we have for all $\widetilde{H}$

$$E_{Q_1}[\widetilde{H}_t\cdot(\widetilde{X}_{t+1}+\widetilde{V}_{t+1})|\mathcal{F}_t] = E_{Q_2}[\widetilde{H}_t\cdot(\widetilde{X}_{t+1}+\widetilde{V}_{t+1})|\mathcal{F}_t] = (1+r_f)\widetilde{H}_t\cdot V_t \tag{A9}$$

and therefore Q_1 and Q_2 cannot be different. This completes the proof of proposition 2. ∎

B. Proof of Proposition 3

We first show that equation (14) also holds for the real asset.

Lemma 1 (Real asset). *If the market is complete and free of arbitrage, then the following equation holds for the real asset:*

$$\text{(A10)} \quad \widetilde{V}_{N+1,t} = \frac{E_Q[\widetilde{X}_{N+1,t+1} + \widetilde{V}_{N+1,t+1}|\mathcal{F}_t]}{1 + (1 - \alpha_{t+1}\tau_{t+1})\, r_f} - \\ - \frac{E_Q[\tau_{t+1} \cdot (\widetilde{X}_{N+1,t+1} + \widetilde{B}_{N+1,t+1} - (1 + r_f(1 - \alpha_{t+1}))\widetilde{B}_{N+1,t})|\mathcal{F}_t]}{1 + (1 - \alpha_{t+1}\tau_{t+1})\, r_f}.$$

Proof. Due to our assumptions the after-tax cash-flows of the real asset can be duplicated by a trading strategy consisting of financial assets. Then the after-tax cash-flows of the real asset are equal to the dividends of the trading strategy. Therefore the value of the real asset must be equal to the value of the trading strategy. Otherwise there would exist an arbitrage opportunity. ∎

We proceed with proving Proposition 3. In the absence of taxes equation (A10) has the form

$$\text{(A11)} \quad \widetilde{V}^*_{N+1,t} = \frac{E_Q[\widetilde{X}_{N+1,t+1} + \widetilde{V}^*_{N+1,t+1}|\mathcal{F}_t]}{1 + r_f}.$$

Substituting this in equation (A10) and thereby eliminating $\widetilde{X}_{N+1,t+1}$ leads to

$$\text{(A12)} \quad (1 + r_f \cdot (1 - \alpha_{t+1}\tau_{t+1}))\, \widetilde{V}_{N+1,t} - E_Q[\widetilde{V}_{N+1,t+1}|\mathcal{F}_t] = \\ = (1 - \tau_{t+1}) \left((1 + r_f)\widetilde{V}^*_{N+1,t} - E_Q[\widetilde{V}^*_{N+1,t+1}|\mathcal{F}_t]\right) + \\ + \tau_{t+1} \left((1 + (1 - \alpha_{t+1})r_f)\widetilde{B}_{N+1,t} - E_Q[\widetilde{B}_{N+1,t+1}|\mathcal{F}_t]\right).$$

Equivalently

$$\text{(A13)}\quad \left\{\widetilde{V}_{N+1,t} - (1-\tau_{t+1})\widetilde{V}^{*}_{N+1,t} - \tau_{t+1}\widetilde{B}_{N+1,t}\right\} = \\ = \frac{E_Q[\{\widetilde{V}_{N+1,t+1} - (1-\tau_{t+2})\,\widetilde{V}^{*}_{N+1,t+1} - \tau_{t+2}\,\widetilde{B}_{N+1,t+1}\}\,|\,\mathcal{F}_t]}{1+r_f\cdot(1-\alpha_{t+1}\tau_{t+1})} + \\ + \frac{r_f\alpha_{t+1}\tau_{t+1}(1-\tau_{t+1})(\widetilde{V}^{*}_{N+1,t} - \widetilde{B}_{N+1,t}) + (\tau_{t+1}-\tau_{t+2})E_Q[\widetilde{V}^{*}_{N+1,t+1} - \widetilde{B}_{N+1,t+1}|\mathcal{F}_t]}{1+r_f\cdot(1-\alpha_{t+1}\tau_{t+1})}.$$

In this equation the term in curly brackets on the right is just the term on the left shifted to one period later. So we get via induction

$$\text{(A14)}\quad \widetilde{V}_{N+1,t} - (1-\tau_{t+1})\widetilde{V}^{*}_{N+1,t} - \tau_{t+1}\widetilde{B}_{N+1,t} = \\ = \sum_{s=t}^{T-1}\frac{r_f\alpha_{s+1}\tau_{s+1}(1-\tau_{s+1})}{\prod_{k=t+1}^{s+1}(1+r_f\cdot(1-\alpha_k\tau_k))}\cdot E_Q[\widetilde{V}^{*}_{N+1,s} - \widetilde{B}_{N+1,s}|\mathcal{F}_t] + \\ + \sum_{s=t+1}^{T-1}\frac{(\tau_s-\tau_{s+1})}{\prod_{k=t+1}^{s}(1+r_f\cdot(1-\alpha_k\tau_k))}\cdot E_Q[\widetilde{V}^{*}_{N+1,s} - \widetilde{B}_{N+1,s}|\mathcal{F}_t].$$

Equivalently

$$\text{(A15)}\quad \widetilde{V}_{N+1,t} - \widetilde{V}^{*}_{N+1,t} - \left(\tau_{t+1} - \frac{r_f\alpha_{t+1}\tau_{t+1}(1-\tau_{t+1})}{1+r_f(1-\alpha_{t+1}\tau_{t+1})}\right)(\widetilde{B}_{N+1,t} - \widetilde{V}^{*}_{N+1,t}) = \\ = \sum_{s=t+1}^{T-1}\frac{r_f\alpha_{s+1}\tau_{s+1}(1-\tau_{s+1}) + (\tau_s-\tau_{s+1})(1+r_f(1-\alpha_{s+1}\tau_{s+1}))}{\prod_{k=t+1}^{s+1}(1+r_f\cdot(1-\alpha_k\tau_k))}\cdot E_Q[\widetilde{B}_{N+1,s} - \widetilde{V}^{*}_{N+1,s}|\mathcal{F}_t].$$

Now

$$\text{(A16)}\quad r_f\alpha_{s+1}\tau_{s+1}(1-\tau_{s+1}) + (\tau_s-\tau_{s+1})(1+r_f(1-\alpha_{s+1}\tau_{s+1})) = \\ = r_f\alpha_{s+1}\tau_{s+1}(1-\tau_s) + (1+r_f)(\tau_s-\tau_{s+1})$$

and

$$\text{(A17)}\qquad -\tau_{t+1} + \frac{r_f\alpha_{t+1}\tau_{t+1}(1-\tau_{t+1})}{1+r_f(1-\alpha_{t+1}\tau_{t+1})} = -\frac{\tau_{t+1}(1+r_f(1-\alpha_{t+1})}{1+r_f(1-\alpha_{t+1}\tau_{t+1})}.$$

and we have finished our proof. ■

C. Proof of Proposition 4

It is apparent that our neutrality condition (19) is satisfied if the book value equals the value without taxes and that $a_t = 0$ in this case.

If the parameters α_{s+1} satisfy

$$\text{(A18)} \qquad \forall s = t+1, \dots \qquad \alpha_{s+1} = \frac{1+r_f}{r_f} \cdot \frac{\tau_{s+1} - \tau_s}{\tau_{s+1} \cdot (1-\tau_s)} \quad \Longrightarrow \quad C_{s,t} = 0,$$

then the relationship between the value with taxes and the value without taxes for the real asset is

$$\text{(A19)} \qquad \widetilde{V}_{N+1,t} = \widetilde{V}^*_{N+1,t} - \tau_t \cdot (\widetilde{V}^*_{N+1,t} - \widetilde{B}_{N+1,t}).$$

Subtracting the investment expenses $\widetilde{I}_{N+1,t}$ and using neutrality condition (18) yields

$$\text{(A20)} \qquad (1-a_t) \cdot \widetilde{\text{NPV}}^*_t = \widetilde{\text{NPV}}_t = \widetilde{\text{NPV}}^*_t - \tau_t \cdot (\widetilde{V}^*_{N+1,t} - \widetilde{B}_{N+1,t}).$$

Since book value and investment expenses are equal

$$\text{(A21)} \qquad \widetilde{B}_{N+1,t} = \widetilde{I}_{N+1,t}$$

we get $a_t = \tau_t$. For $t = 0$ we find

$$\text{(A22)} \qquad \widetilde{V}_{N+1,0} = \widetilde{V}^*_{N+1,0} - a_0 \cdot (\widetilde{V}^*_{N+1,0} - \widetilde{B}_{N+1,0}).$$

Using

$$\text{(A23)} \qquad \alpha_1 = \frac{1+r_f}{r_f} \cdot \frac{\tau_1 - a_0}{\tau_1(1-a_0)}$$

and equation (A21) gives the neutrality condition for $t = 0$. This completes the proof.
■

D. Proof of Proposition 5

The price process under the martingale measure is given by[14]

(A24) $$dV_t = r_t V_t dt - dX_t + \sigma_t V_t dW_t^Q.$$

Using this equation and $B_t = V_t$ we get for the tax payments

$$\begin{aligned} dT_t &= \tau_t dX_t + \tau_t dB_t - \tau_t(1-\alpha_t) r B_t dt \\ &= \tau_t \alpha_t r V_t dt + \tau_t \sigma V_t dW_t^Q. \end{aligned}$$

For $dV_t + dX_t - dT_t$ we get

$$\begin{aligned} dV_t &= rV_t dt - dX_t + \sigma V_t dW_t^Q \\ &= r(1-\alpha_t\tau_t)V_t dt - (dX_t - dT_t) + (1-\tau_t)\sigma V_t dW_t^Q. \end{aligned}$$

This equation represents the continuous time analog of the fundamental pricing lemma under taxes (14). Q is also the martingale measure under taxes. The drift reduces to $r(1-\alpha_t\tau_t)$ and the volatility to $(1-\tau_t)\sigma$.

We are now able to derive a valuation equation for the real asset. We have the two equations

(A25) $$\begin{aligned} dV_t^* &= rV_t^* dt - dX_t + \sigma V_t^* dW_t^Q, \\ dV_t &= r(1-\alpha_t\tau_t)V_t dt - (dX_t - dT_t) + (1-\tau_t)\sigma V_t dW_t^Q. \end{aligned}$$

Substituting dT_t into the second equation gives according to (30)

(A26) $$\begin{aligned} dV_t - \tau_t dB_t = r(1-\alpha_t\tau_t)\,(V_t - \tau_t B_t)\,dt + \\ + r\alpha_t\tau_t(1-\tau_t)B_t dt - (1-\tau_t)dX_t + (1-\tau_t)\sigma V_t dW_t^Q. \end{aligned}$$

Multiplying equation (A25) with $-(1-\tau)$ and adding to (A26) we get

(A27) $$\begin{aligned} dV_t - (1-\tau_t)dV_t^* - \tau_t dB_t = r(1-\alpha_t\tau_t)\big(V_t - (1-\tau_t)V_t^* - \tau_t B_t\big)\,dt + \\ + r\alpha_t\tau_t(1-\tau_t)(B_t - V_t^*)dt + (1-\tau_t)\sigma(V_t - V_t^*)dW_t^Q. \end{aligned}$$

Defining $\zeta_t = V_t - (1-\tau_t)V_t^* - \tau_t B_t$ the last equation can be formulated as

$$\text{(A28)} \quad d\zeta_t = r(1-\alpha_t\tau_t)\zeta_t dt + (1-\tau_t)\sigma\zeta_t dW_t^Q + \\ + (r\alpha_t\tau_t(1-\tau_t) - \dot{\tau}_t)(B_t - V_t^*)dt + \tau_t(1-\tau_t)\sigma(B_t - V_t^*)dW_t^Q.$$

where $\dot{\tau}_t = \frac{d\tau_t}{dt}$. Then for $Z_t = e^{\int_t^T r(1-\alpha_u\tau_u)du}\zeta_t$ we get using Itô's Lemma (see Karatzas and Shreve (1988, p. 159))

$$\text{(A29)} \quad dZ_t - e^{\int_t^T r(1-\alpha_u\tau_u)du}(r\alpha_t\tau_t(1-\tau_t) - \dot{\tau}_t)(B_t - V_t^*)dt = \\ = (1-\tau_t)\sigma Z_t dW_t^Q + e^{\int_t^T r(1-\alpha_u\tau_u)du}\tau_t(1-\tau_t)\sigma(B_t - V_t^*)dW_t^Q.$$

So the left hand side is a martingale with respect to W^Q. This implies

$$\text{(A30)} \quad E^Q\left[Z_T - Z_t - \int_t^T e^{\int_s^T r(1-\alpha_u\tau_u)du}(r\alpha_s\tau_s(1-\tau_s) - \dot{\tau}_s)(B_s - V_s^*)ds \mid \mathcal{F}_t\right] = 0.$$

Since $Z_T = 0$, we get substituting back

$$\text{(A31)} \quad e^{\int_t^T r(1-\alpha_u\tau_u)du}(V_t - (1-\tau_t)V_t^* - \tau_t B_t) = \\ = E^Q\left[\int_t^T e^{\int_s^T r(1-\alpha_u\tau_u)du}(r\alpha_s\tau_s(1-\tau_s) - \dot{\tau}_s)(V_s^* - B_s)ds \mid \mathcal{F}_t\right].$$

This proves the claim. ■

References

Boadway, Robin and Neil Bruce, ”Depreciation and Interest Deductions and the Effect of the Corporation Income Tax on Investment”, *Journal of Public Economics*, 1979, 19, pp. 93–105.

Boadway, Robin and Neil Bruce, ”A general proposition on the design of a neutral business tax”, *Journal of Public Economics*, 1984, 24, pp. 231–239.

Bond, Steven R. and Michael P. Devereux, ”On the design of a neutral business tax under uncertainty”, *Journal of Public Economics*, 1995, 58, pp. 57–71.

Brown, Cary E. ”Business–income and investment incentives”, Essays in honor of Alvin Hansen, pp. 300–316, in L. Metzler (ed.), *Income, Employment and Public Policy*, 1948.

Duffie, Darrell *Security Markets*, Academic Press, Inc., San Diego, 1988.

Fane, George ”Neutral taxation under uncertainty”, *Journal of Public Economics*, 1987, 33, pp. 95–105.

Harrison, J. Michael and David M. Kreps, ”Martingales and arbitrage in multiperiod securities markets”, *Journal of Economic Theory* 1979, 20, pp. 381–408.

IFS Capital Taxes Group ”Equity for Companies: A Corporation Tax for the 1990s”, Commentary No. 26, London: Institute for Fiscal Studies.

Jensen, Bjarne Astrup ”On valuation before and after tax in no arbitrage models: Tax neutrality in the discrete time model”, Working Paper, Copenhagen Business School, 2002.

Johansson, Sven–Erik ”Income taxes and investment decisions”, *Swedish Journal of Economics*, 1969, 71, pp. 104–110.

Kabanov, Yuri M. and Dmitry O. Kramkov, ”No–arbitrage and equivalent martingale measures: An elementary proof of the Harrison/Pliska–Theorem”, *Theory Probabilty and its Applications*, 1995, 39, pp. 523–527.

Karatzas, Ioannis and Steven E. Shreve, *Brownian Motion and Stochastic Calculus*, 2nd ed., Springer, Berlin, 1988.

Panteghini, Paolo ”On corporate tax asymmetries and neutrality”, *German Economic Review*, 2001, 2, pp. 269–286.

Preinreich, Gabriel ”Models of taxation in the theory of the firm”, *Econ. Int.* 1951, 4, pp. 372–397.

Richter, Wolfram ”Comprehensive versus neutral income taxation”, pp. 281–296, in D. Bös, M. Rose, and C. Seidl (eds.): *Welfare and efficiency in public economics*, Springer–Verlag, Heidelberg, 1986.

Rose, Manfred and Rolf Wiswesser ”Tax Reform in Transition Economies: Experiences from the Croatian Tax Reform Process of the 1990s” in: Chr. P.B Sorensen (eds.), *Public Finance in a Changing World*, 1998, pp. 257–278.

Ross, Stephen A. ”Arbitrage and Martingales with Taxation”, *Journal of Political Economy*, 1987, 95, pp. 371–393.

Samuelson, Paul A. ”Tax deductability of economic depreciation to insure invariant valuations”, *Journal of Political Economy*, 1964, 72, pp. 604–606.

Wenger, Ekkehard ”Gleichmäßigkeit der Besteuerung von Arbeits– und Vermögenseinkünften” (in German), *FinanzArchiv*, 1983, 41, pp. 207–252.

Notes

[0]Corresponding author. We are indebted to Dorothea Schäfer and the Deutsche Forschungsgemeinschaft for (not only financial) support and to Jochen Hundsdörfer, Lutz Kruschwitz, and Rainer Niemann for helpful comments.

[1]All other known neutral tax systems are only descendants of these three tax systems and therefore do not incorporate a fundamental new idea.

[2]For more details on the ACE concept see for example IFS Capital Taxes Group (1991). The tax with allowance for corporate equity as well as the cash-flow tax were quite often proposed as serious alternatives to the presently implemented traditional income-tax. Moreover, the tax with allowance for corporate equity was actually implemented from 1994 to 2000 the republic of Croatia had a tax system with an ACE. The implementation of this idea was done with the substantial help of three German scientists, for more details see Rose and Wiswesser (1998).

[3]The tax system can easily be modified to incorporate the cash-flow-tax as a special case as well.

[4]The valuation equation proved by Ross (1987) is based on a much more general tax system with a nonlinear tax. Hence, a characterization of neutral tax systems cannot be provided.

[5]These are standard assumptions in an uncertain economy, see Duffie (1988).

[6]All our findings are also valid in an economy with a time–dependent interest rate. Therefore, interest is assumed to be constant in time without loss of generality.

[7]This is to say that our model is not an equilibrium model and that there are no lump sum payments to the investors.

[8]Setting $\alpha_t = 1$ and $\tau_t = \tau$ for all $t = 1, \ldots, T$ we get the standard form of the taxation of economic rent.

[9]Setting $\alpha_t = 0$ and $\tau_t = \tau$ for all $t = 1, \ldots, T$, we get the standard form of the tax

with an ACE which was introduced by Boadway and Bruce (1979).

[10]This assertion is also contained in the recent paper of Panteghini (2001).

[11]The meaning of the symbols is the following: R_1, R_2: Cash-flows in periods 1, 2; r_1, r_2: interest rate in periods 1, 2; K_1, K_2: present value in 1, 2; δ_1, δ_2: true economic depreciation in periods 1, 2; z_1, z_2: depreciation in 1, 2.

[12]As they also assumed $I_0 = 1$, their assertion is restricted to the case of investments with a net present value before taxes of $\widetilde{\text{NPV}}^* = 0$.

[13]The conditions under which this PDE is solvable are described in Duffie (1988, p. 95ff.).

[14]See Duffie (1988, chapter 6).