Entrepreneurship and Growth

— An Overlapping Generations Approach —

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Abstract

This paper discusses a two–sector neoclassical overlapping generations economy with intermediate and final goods in the spirit of Romer (1990). The risk averse agents engage in one of two alternative occupations: either firm–ownership in the intermediate goods sector, characterized by monopolistic competition, or employment as a worker in this sector. The occupational choice under risk endogenizes the number of firms and products in the intermediate goods industry. Since entrepreneurial profits are stochastic, an inefficiently low number of agents chooses firm–ownership. We find that expected profits of monopolists do not vanish in equilibrium and that the level of economic performance is inefficiently low due to the presence of risk. This result carries over to a suboptimally low growth rate in an enodgenous growth context.

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1 Introduction

This paper investigates occupational choice under risk in the context of an overlapping generations growth model. The analysis is embedded in a two–sector economy with intermediate and final goods in the spirit of Romer (1990). We are especially interested in the question of how entrepreneurial risk–taking interacts with long–run growth of the economy.

In this sense, the framework presented here combines the two predominant views on the role of entrepreneurship in modern economies: The Knightian view, which considers risk–bearing to be an essential task of entrepreneurs (Knight, 1921), and the Schumpeterian view, which stresses the creative and innovative capacity of firms in the process of economic development (Schumpeter, 1930). These notions on the importance of entrepreneurship have spend a live in the shadows for quite a long time, while formal growth theory in the tradition of Solow (1956) relied on the long–run substitutability of factors and constant returns to scale in perfectly competitive markets, altogether implying that the number of firms as well as their activities beyond mere production do not matter.

Only recently, with the pioneering work of Romer (1987, 1990), the Schumpeterian view that entrepreneurship might be a key factor in economic growth has experienced its revival in the course of the development of modern growth theory. Referring to Schumpeter's emphasis on the innovative role of entrepreneurs, many contributions focused on R&D, product imitation, technological spillovers, human capital formation, patents, intellectual property rights and institutions to preserve the latter, and the respective consequences of theses factors for economic growth; see Schmitz (1989), Romer (1990), Chou and Shy (1991), Aghion and Howitt (1992, 1993), Grossman and Helpman (1991).

These approaches share the common feature that the technological progress no longer is assumed to be exogenous and quasi costlessly available. Instead, it is argued that technological improvements absorb costly resources and therefore are subject to economic decision–making. In order to motivate research, markets have to provide incentives for potential innovators. Generally these are assumed to consist of property rights guaranteeing excludability and rewarding the entrepreneur with monopoly profits. But even if innovations are considered to follow random processes (cf. Aghion and Howitt, 1992; Grossman and Helpman, 1991), firms undertaking research are usually assumed to be indifferent towards risk. These models neglect the Knightian view on entrepreneurship, consequently blinding out the possibility of (safe) outside options and occupational choice. Contrary, if we consider modern life–cycle formulations of occupational choice under risk, we find that these are more concerned with the distributional consequences (Banerjee and Newman, 1991), or with portfolio choice (Rosen and Willen, 2002*a*,*b*) than with economic growth.

The majority of contributions focusing on occupational choice and growth also are more interested in the distributional side of the problem and the possible emergence of poverty traps; see for instance Banerjee and Newman (1993, 1994), Galor and Zeira (1993), Aghion and Bolton (1997), Piketty (1997), Aghion *et al.* (1999), and Ghatak and Jiang (2002). These approaches share the common assumption that inequalities in the income and wealth distribution originate from capital market imperfections, such as borrowing constraints and credit rationing or costs of market entry. Only recently, Ghatak *et al.* (2001) developed a model, where agents can become entrepreneurs when old by working hard and saving when young in order to cover the costs of market entry. Keuschnigg and Nielsen (2002) focus on venture capital, but do so only in a static context.

Lazear (2002) and Irigoyen (2002) follow a different line of argument by stressing the skill and ability oriented notion of entrepreneurship and the importance of entrepreneurial human capital, where individuals first learn the key aspects of business in order to set–up their own firms later on.

The analysis of Iyigun and Owen (1998) is motivated by the empirical observation that, as an economy develops, its occupational structure changes towards a lower ratio of employers to employees (Kuznets, 1971; Blau, 1987). They explain this development with the greater risk inherent in self–employment compared to the relatively safe return of schooling. While Banerjee and Newman (1993) demonstrate that economic development may be associated with increased entrepreneurship, Iyigun and Owen (1998) derive the opposite result which can be ascribed to the riskiness of entrepreneurial ventures.

Regarding empirical evidence, there is strong support for the hypothesis that capital market imperfections are an impediment to entrepreneurship even after controlling for entrepreneurial ability; see Evans and Leighton (1989), Quadrini (1999), Blanchflower and Oswald (1998). Desai *et al.* (2003) investigate the impact of institutions and capital constraints on entrepreneurial firm dynamics in Europe. They find that a greater protection of property rights increases entry rates, thus supporting theoretical predictions of modern growth theory already reviewed above. Empirical evidence underlining the importance of entrepreneurial risk–taking is provided by Cramer *et al.* (2002) and Ilmakunnas *et al.* (1999), whereas Audretsch and

Thurik (2000), Audretsch *et al.* (2002) and Carree *et al.* (2002) find that entrepreneurship is a vital determinant of economic growth.

This last observation represents the starting point for our analysis. In what follows, we develop an approach which combines occupational choice under risk in the tradition of Kihlstrom and Laffont (1979), or Kanbur (1979*a*,*b*, 1980) with elements of modern growth theory as sketched above. The business owners of our model receive monopoly profits not because of their innovative potential, but because of their willingness to bear the risks of production. We demonstrate that the presence of risk and risk–sensitive behavior of individuals results in an inefficiently low level of economic performance in the basic neoclassical growth model, and in a suboptimally low long–run growth rate of the economy in an extended context, where we also allow for an endogenous growth mechanism. Welfare in both economies could be higher, if more agents chose the entrepreneurial profession.

The paper is organized as follows: Section 2 develops the model and derives the optimality conditions of the consumer problem and the firm problems in the two sectors of production. Section 3 determines the general equilibrium and the equilibrium occupational distribution. Section 4 is devoted to the comparative static analysis. Section 5 then discusses transitional dynamics for the special case of logarithmic preferences and extends the analysis to an endogenous growth context. Section 6 concludes.

2 The Model

Households We consider a discrete time overlapping generations economy in the tradition of Diamond (1965). The identical households live for two periods. We normalize the population size of each cohort to unity. There is no population growth. Each member of the young generation is endowed with one unit of labor, which she supplies inelastically. At the beginning of their life, citizens choose between two alternative types of occupation. They can decide either to set up a firm and become a monopolistic entrepreneur in the intermediate goods industry, or they become employed in this sector. λ denotes the population share of entrepreneurs. The corresponding population share of workers is given by $1-\lambda$. While employment is payed the riskless wage income w, self–employment yields risky profits π_j per monopoly j. The risk stems from a idiosyncratic technology shock. By the time the households choose between the occupations, they do not know the realization of the shock. By the time they compose their intertemporal consumption profile, the income realization is known and the agents act under perfect foresight. We assume the costs of switching between occupations to be prohibitively high, such that the employment decision once made is irreversible. All individuals retire after the first period. When old, savings and interest payments are used to finance retirement consumption. There are no bequests.

The individuals spend their income on a single final good, which can be consumed or invested respectively. Lifetime utility of a member of a cohort i is additively–separable and given by

$$U\left(c_{i,t}, c_{i,t+1}\right) = \frac{1}{1-\rho}\left[c_{i,t}^{1-\rho} + \beta\, c_{i,t+1}^{1-\rho}\right], \quad \rho > 0\,. \tag{1a}$$

The current period utility functions are characterized by constant relative risk aversion, measured by the parameter ρ. For simplicity, the attitude towards risk is assumed to be identical for all agents, although Kihlstrom and Laffont (1979), Kanbur (1981), and Cramer *et al.* (2002) stress, that the entrepreneurial occupation is more likely to be chosen by agents who are less risk averse.[1] The agents discount future consumption. The discount factor $0 < \beta < 1$ is related to the intertemporal rate of time preference δ via $\beta = 1/(1+\delta)$.

Let $y_{i,t}$ denote the period t income of a member of generation i and an occupation generating an income of type $y_i \in \{w, \pi_j\}$. Then, the intertemporal budget constraint can be written as follows

$$c_{i,t} = y_{i,t} - s_{i,t}\,, \tag{1b}$$

$$c_{i,t+1} = s_{i,t}(1 + r_{t+1})\,. \tag{1c}$$

r_{t+1} is the interest rate paid on saving held from period t to period $t+1$. Define with $R_{t+1} \equiv 1 + r_{t+1}$ the return factor on saving. Because we assumed the income realizations to be known by the time of intertemporal choice, optimization is performed under certainty and yields the familiar Euler condition

$$U'(c_{i,t}) = \beta R_{t+1} U'(c_{i,t+1})\,. \tag{1d}$$

Given the functional form of utility (1a), substituting $c_{i,t} = y_{i,t} - s_{i,t}$, and $c_{i,t+1} = R_{t+1}\, s_{i,t}$ implies the following savings function

$$s_{i,t} = \frac{y_{i,t}}{1 + \beta^{-1/\rho}\, R_{t+1}^{(\rho-1)/\rho}}\,, \tag{1e}$$

and optimal consumption $c_{i,t}$, $c_{i,t+1}$

$$c_{i,t} = \frac{\beta^{-1/\rho}\, R_{t+1}^{(\rho-1)/\rho}\, y_{i,t}}{1 + \beta^{-1/\rho}\, R_{t+1}^{(\rho-1)/\rho}}\,, \qquad c_{i,t+1} = \frac{R_{t+1}\, y_{i,t}}{1 + \beta^{-1/\rho}\, R_{t+1}^{(\rho-1)/\rho}} \tag{1f}$$

[1] Incorporating heterogeneity with respect to the degree of risk aversion is a worthwhile extension of the model, but beyond the scope of this paper.

Incorporating these relationships into (1a) yields the following expression for maximized lifetime utility of a household of generation i and profession with income of type $y_i \in \{w, \pi_j\}$

$$U\left(c_{i,t}, c_{i,t+1}\right) = \frac{\beta R_{t+1}^{1-\rho}}{1-\rho}\left[1+\beta^{-1/\rho} R_{t+1}^{(\rho-1)/\rho}\right]^{\rho} y_{i,t}^{1-\rho}\,. \tag{1g}$$

Occupational choice is related to the labor market equilibrium and will be discussed below.

Final goods sector The representative firm of the final goods sector produces a homogeneous good Q_t using capital K_t and varieties of a differentiated intermediate good $\{x_{j,t}\}_{j=0}^{\lambda}$ as inputs. Production in this sector takes place under perfect competition and the price of Q_t is normalized to unity. We assume a production function of the generalized CES–form; see Spence (1976), Dixit and Stiglitz (1977) and Ethier (1982):

$$Q_t = K_t^{1-\alpha} \int_0^{\lambda} x_{j,t}^{\alpha}\ \mathrm{d}\,j\,, \tag{2a}$$

where $0 < \alpha < 1$. The production function displays positive but diminishing marginal productivity for each input K and x_j, and constant returns to scale in all inputs together. The capital stock depreciates completely in each period. Additive–separability of (2a) in intermediate goods ensures that the marginal product of input j is independent of the quantity employed of j'. The intermediate goods are close but not perfect substitutes in production, with the elasticity of substitution between goods j and j' given by $\varepsilon = 1/(1-\alpha)$.

The time t profit of the representative firm in the final goods sector is

$$\Pi_t = Q_t - r_t K_t - \int_0^{\lambda} p_{j,t} x_{j,t}\ \mathrm{d}\,j\,, \tag{2b}$$

where p_j denotes the price of intermediate good j. Optimization yields the standard conditions from marginal productivity theory

$$\frac{\partial Q_t}{\partial K_t} = r_t \qquad \Longrightarrow \qquad r_t = (1-\alpha)\frac{Q_t}{K_t}\,, \tag{2c}$$

$$\frac{\partial Q_t}{\partial x_{j,t}} = p_{j,t} \qquad \Longrightarrow \qquad x_{j,t} = K_t\left(\frac{\alpha}{p_{j,t}}\right)^{1/(1-\alpha)}\,. \tag{2d}$$

Condition (2d) represents the demand function, which the producer of the intermediate good x_j faces. It is isoelastic, with the direct price elasticity of demand given by

$$\eta_{x_j,p_j} = \frac{\partial x_j}{\partial p_j} \times \frac{p_j}{x_j} = -\frac{1}{1-\alpha} = -\,\varepsilon\,.$$

Intermediate goods sector The intermediate goods sector is populated by a large number λ of small firms, each producing a single variety j of a differentiated good. The producers engage in monopolistic Bertrand competition. Labor L_t is the single input of production. We assume that all the monopolists of the intermediate sector produce according to the identical constant returns to scale technology of the form

$$x_{j,t} = \theta_{j,t} \, L_{j,t} \, . \tag{3a}$$

Firms differ only with respect to the realization of the idiosyncratic (firm specific) productivity shock θ_j with density $\theta_j \in \Theta \subset \mathbb{R}^{++} : f(\theta)$, which is assumed to be non–diversifiable, uncorrelated across firms and lognormally distributed, with mean $\mathrm{E}[\ln\theta] = \bar{\theta}$ and variance $\mathrm{Var}[\ln\theta] = \sigma^2$. Similar to Kanbur (1979*b*), we posit that the entrepreneurs hire labor after the draw of nature has occurred. Recall that earlier we assumed the costs of changing occupations to be prohibitively high, such that agents are prevented from switching between groups in case of unfavorable realizations of the shock.

Given (2d) and (3a), the time t profit of a typical producer in this sector then reads as

$$\pi_{j,t} = K_t \left(\frac{\alpha}{p_{j,t}} \right)^{1/(1-\alpha)} \left[p_{j,t} - \frac{w_t}{\theta_{j,t}} \right] \, . \tag{3b}$$

The firm problem essentially is a static one. Under perfect competition on the labor market, the producer treats the wage rate w_t as exogenously given. Price setting behavior implies the following solution for the monopoly price

$$p_{j,t} = \frac{w_t}{\alpha\theta_{j,t}} \, . \tag{3c}$$

The profit maximizing price of a typical entrepreneur in the intermediate goods market is the markup $1/\alpha > 1$, $\forall\alpha \in (0,1)$ over the marginal costs of production.

3 Market Equilibrium

Market for intermediate good of type j The demand for intermediate good j is given by equation (2d) and can be rewritten as follows $p_{j,t} = \alpha(K_t/x_{j,t})^{1-\alpha}$. Equating this expression with condition (3c) yields the market clearing amount of good j

$$x_{j,t} = \left(\frac{\alpha^2\theta_{j,t}}{w_t} \right)^{\frac{1}{1-\alpha}} K_t \, . \tag{4}$$

By substitution into (3a), we derive the labor demand of entrepreneur j as follows

$$L_{j,t} = \left(\frac{\alpha^2 \theta_{j,t}^{\alpha}}{w_t} \right)^{\frac{1}{1-\alpha}} K_t \, . \tag{5}$$

Labor market The labor market is characterized by perfect competition. The equilibrium wage rate can then be derived by equating the aggregate labor supply with expected labor demand. If we take account of (5), the i. i. d. property of the firm–specific technology shock and the characteristics of the underlying distribution, the aggregate labor demand is given by

$$\begin{aligned} L_t &= K_t \left(\frac{\alpha^2}{w_t} \right)^{\frac{1}{1-\alpha}} \int_0^{\lambda_t} \int_{\theta \in \Theta} \theta_{j,t}^{\frac{\alpha}{1-\alpha}} \, f(\theta) \; \mathrm{d}\theta \; \mathrm{d}j = \lambda \mathrm{E}[L_{j,t}] \\ &= \lambda_t K_t \left(\frac{\alpha^2}{w_t} \right)^{\frac{1}{1-\alpha}} \exp\left[\frac{\alpha}{1-\alpha} \left(\bar{\theta} + \frac{1}{2} \frac{\alpha \sigma^2}{1-\alpha} \right) \right] \, . \end{aligned} \tag{6}$$

The aggregate labor supply equals the population share of workers, $L_t = 1 - \lambda_t$, due to the normalization of population size. Equating this expression with (6) and integrating, allows us to solve for the market clearing wage rate w_t

$$w_t = \alpha^2 K_t^{1-\alpha} \left(\frac{\lambda_t}{1-\lambda_t} \right)^{1-\alpha} \exp\left[\alpha \bar{\theta} + \frac{1}{2} \frac{\alpha^2 \sigma^2}{1-\alpha} \right] \, . \tag{7}$$

The equilibrium wage rate is a function of the yet undetermined population shares of workers and entrepreneurs. Since we are dealing with a general equilibrium model, each change in the number of firms simultaneously affects aggregate labor supply and thereby the market clearing wage rate.

Given the equilibrium wage rate, it is now possible to derive a closed–form solution for the expected output level, which is identical for all firms in the intermediate goods industry. Substituting (7) into (4) and taking expectations gives

$$x_t = \left(\frac{1-\lambda_t}{\lambda_t} \right) \exp\left[\bar{\theta} + \frac{(1+\alpha)\sigma^2}{2(1-\alpha)} \right] \, . \tag{8}$$

We conclude with the determination of the equilibrium profit income of monopolist j in the intermediate goods market. Substituting (7) and (3c) into (3b) yields

$$\pi_{j,t} = \theta_{j,t}^{\frac{\alpha}{1-\alpha}} \, \alpha(1-\alpha) K_t^{1-\alpha} \left(\frac{1-\lambda_t}{\lambda_t} \right)^{\alpha} \exp\left[-\frac{\alpha}{1-\alpha} \left(\alpha \bar{\theta} + \frac{\alpha^2 \sigma^2}{2(1-\alpha)} \right) \right] \, . \tag{9}$$

The profit income of a typical producer j in the intermediate goods industry also depends on the equilibrium distribution of agents over occupations.

Additionally, entrepreneurial incomes are positively related to the existing capital stock and the realization of firm–specific technology shock.

Equilibrium occupational choice An equilibrium distribution of households over the two types of occupation is characterized by a situation, where the marginal agent ex ante does not benefit from switching between occupations, that is, if expected lifetime utility from being an entrepreneur equals lifetime utility of a worker.

Since the equilibrium wage rate is safe, lifetime utility $U(c_{i,t}, c_{i,t+1}|w)$ of a worker of generation i, can simply be derived by substituting (7) into (1g)[2]

$$U(c_{i,t}, c_{i,t+1}|w_t) = A\left(\alpha^2 K_t^{1-\alpha}\left(\frac{\lambda_t}{1-\lambda_t}\right)^{1-\alpha}\exp\left[\alpha\bar{\theta}+\frac{\alpha^2\sigma^2}{2(1-\alpha)}\right]\right)^{1-\rho} \tag{10}$$

The expected lifetime utility $\mathrm{E}[U(c_{i,t}, c_{i,t+1}|\pi_{j,t})]$ of being a monopolist of cohort i in the intermediate goods sector can be determined as follows

$$\begin{aligned}
\mathrm{E}[U(c_{i,t}, c_{i,t+1}|\pi_{j,t})] &= A\,\mathrm{E}\left[\pi_{j,t}^{1-\rho}\right] \\
&= A\left(\alpha(1-\alpha)K_t^{1-\alpha}\left(\frac{1-\lambda_t}{\lambda_t}\right)^{\alpha}\exp\left[-\frac{\alpha}{1-\alpha}\left(\alpha\bar{\theta}+\frac{\alpha^2\sigma^2}{2(1-\alpha)^2}\right)\right]\right)^{1-\rho} \\
&\quad\times\int_{\theta\in\Theta}\theta_{j,t}^{\frac{\alpha(1-\rho)}{1-\alpha}}\,f(\theta)\;\mathrm{d}\theta \\
&= A\left(\alpha(1-\alpha)K_t^{1-\alpha}\left(\frac{1-\lambda_t}{\lambda_t}\right)^{\alpha}\exp\left[\alpha\bar{\theta}+\frac{\alpha^2\sigma^2(1-\rho-\alpha)}{2(1-\alpha)^2}\right]\right)^{1-\rho}
\end{aligned} \tag{11}$$

Equating (10) with (11) finally yields the equilibrium population share of monopolists in the intermediate goods industry

$$\lambda_t = \frac{1-\alpha}{1-\alpha+\alpha\exp\left[\frac{\rho\alpha^2\sigma^2}{2(1-\alpha)^2}\right]}, \tag{12}$$

and $1-\lambda_t$ residually. The population shares are constant in equilibrium and depend on the primitives of the model, which are the degree of risk aversion ρ, the variance of the technology shock σ^2 and the elasticity of substitution between two arbitrary intermediate goods j and j', implicitly measured by α. We find $0 < \lambda_t < 1$, $\forall \alpha, \sigma, \rho$. Note that λ_t is independent of the mean $\bar{\theta}$ of the productivity shock. This results can be ascribed to the assumption of CRRA preferences, where the degree of risk aversion does not depend on the level of income.

[2]For notational simplicity, we define $A \equiv \frac{\beta R_{t+1}^{1-\rho}}{1-\rho}\left[1+\beta^{-1/\rho}R_{t+1}^{(\rho-1)/\rho}\right]^{\rho}$, such that lifetime utility is given by $U(c_{i,t}, c_{i,t+1}) = A y_{i,t}^{1-\rho}$.

Proposition 1 *The occupational choice of risk averse households endogenizes the number of firms in the intermediate goods industry in terms of a population share, thereby simultaneously determining the range of intermediate goods employed in the production of the final good.*

Proposition 2 *Due to the endogenized firm number, profits in monopolistic competition do not vanish. If agents are risk averse, the expected profit in the intermediate goods industry, as given by*

$$\pi_t = \alpha(1-\alpha)K_t^{1-\alpha}\left(\frac{1-\lambda}{\lambda}\right)^{\alpha}\exp\left[\alpha\bar{\theta}+\frac{\alpha^2\sigma^2}{2(1-\alpha)}\right], \tag{13}$$

exceeds the riskless wage rate. The expected risk premium, which the entrepreneurs demand as a compensation for bearing the production risk, is positive

$$\phi_t \equiv \pi_t - w_t = K_t^{1-\alpha}\left(\frac{1-\lambda_t}{\lambda_t}\right)^{\alpha}\frac{\alpha(1-\alpha-\lambda_t)}{1-\lambda_t}\exp\left[\alpha\bar{\theta}+\frac{\alpha^2\sigma^2}{2(1-\alpha)}\right]. \tag{14}$$

Proof: (14) can be rewritten as

$$\phi_t = \alpha^{1+\alpha}\left[(1-\alpha)K_t\right]^{1-\alpha}\exp\left[\alpha\bar{\theta}-\frac{\alpha^2\sigma^2(1-\rho)}{2(1-\alpha)}\right]\left(\exp\left[\frac{\rho\alpha^2\sigma^2}{2(1-\alpha)^2}\right]-1\right).$$

From this follows immediately that $\pi_t \gtreqless w_t$, if

$$\exp\left[\frac{\rho\alpha^2\sigma^2}{2(1-\alpha)^2}\right] \gtreqless 1 \quad \Longleftrightarrow \quad \rho \gtreqless 0 \quad \text{for} \quad \alpha \in (0,1), \sigma > 0\,. \qquad \square$$

In what follows, it will be convenient to define the function

$$\Omega(\lambda) = \lambda_t^{1-\alpha}(1-\lambda_t)^{\alpha}\exp\left[\alpha\bar{\theta}+\frac{\alpha^2\sigma^2}{2(1-\alpha)}\right], \tag{15}$$

which, on the one hand, measures the effects of shifts in the distribution of agents over occupations. On the other hand, via the exponential term, it captures the impact of the lognormally distributed technology shock on the level of economic activity.

We conclude this paragraph with the derivation of the equilibrium level of income payed to the members of the young generation, who represent the economically active share of the the time t population. The total income of the young generation is identical to the mean income generated in the intermediate goods industry

$$Y_t = (1-\lambda_t)w_t + \int_0^{\lambda_t}\pi_{j,t}\,\mathrm{d}j = (1-\lambda_t)w_t + \lambda\pi_t = \alpha\Omega(\lambda)K_t^{1-\alpha} \tag{16}$$

and depends linearly on $\Omega(\lambda)$.

Final goods sector The market for intermediate goods is cleared, if aggregate demand for goods $x_{j,t}$ equals aggregate supply. By utilizing the demand function (4) for intermediate goods j, the equilibrium output of the final good can be derived as follows

$$Q_t = \Omega(\lambda) K_t^{1-\alpha}\,, \tag{17}$$

with λ given by (12). The representative firm of the final goods sector then spends the amount of

$$\alpha Q_t = \left(\frac{\alpha}{w_t}\right)^{\frac{\alpha}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}} K_t \int_0^{\lambda} \int_{\theta\in\Theta} \theta_{j,t}^{\frac{\alpha}{1-\alpha}}\, f(\theta)\,\mathrm{d}\theta\,\mathrm{d}j = Y_t \tag{18}$$

on the purchase of intermediate goods, and

$$(1-\alpha) Q_t = r_t K_t \tag{19}$$

on physical capital, thereby fixing the overall income of those, who belong to the old generation of period t. (18) and (19) also display the well–known result of zero profits under the regime of perfect competition.

Income distribution Equipped with the equilibrium conditions of the income variables (7), (13), (16), (18), and (19), we are now able to express the functional distribution of income in terms of income shares, which are related either to aggregate or to sectoral income.

Aggregate output is distributed between the young generation, who claim that part of national income, which is generated in the intermediate goods sector, and those being old in period t, who own the capital stock. The respective income shares are:

$$\frac{Y_t}{Q_t} = \alpha \qquad \text{and} \qquad \frac{r_t K_t}{Q_t} = 1-\alpha\,. \tag{20}$$

Sectoral income in the intermediate goods industry is claimed by workers and entrepreneurs, according to the following income shares:

$$\frac{\lambda_t \pi_t}{Y_t} = 1-\alpha \qquad \text{and} \qquad \frac{(1-\lambda_t) w_t}{Y_t} = \alpha\,,$$

which also can be expressed in relation to aggregate output

$$\frac{\lambda_t \pi_t}{Q_t} = \alpha(1-\alpha) \qquad \text{and} \qquad \frac{(1-\lambda_t) w_t}{Q_t} = \alpha^2\,. \tag{21}$$

The income shares are constant and entirely determined by the productivity parameter α, thereby reflecting the standard implications of neoclassical growth theory for the functional distribution of income.

Capital market The market clearing interest rate on capital is determined by marginal productivity theory and can be derived as

$$r_t = (1-\alpha)\,\Omega(\lambda)\,K_t^{-\alpha}\,. \tag{22}$$

The capital stock was assumed to depreciate completely at the end of each period. Intertemporal market clearing then requires the capital stock of time $t+1$, which equals investment I_t, to equal aggregate savings S_t of period t, that is

$$K_{t+1} \equiv I_t = S_t\,. \tag{23}$$

By taking account of (1e), aggregate savings of period t can be determined as the weighted average of savings out of the two types of income of the young

$$\begin{aligned} S_t &= \frac{1}{1+\beta^{-1/\rho}R_{t+1}^{(\rho-1)/\rho}}\left[(1-\lambda_t)w_t + \int_0^{\lambda_t}\int_{\theta\in\Theta}\pi_{j,t}\ f(\theta)\ \mathrm{d}\theta\ \mathrm{d}\,j\right] \\ &= \frac{1}{1+\beta^{-1/\rho}R_{t+1}^{(\rho-1)/\rho}}\left[(1-\lambda_t)w_t + \lambda_t\pi_t\right], \end{aligned}$$

which, by utilizing (16), reduces to

$$S_t = s\left[R(K_{t+1})\right]\,Y_t\,, \tag{24}$$

where $s\left[R(K_{t+1})\right]$ denotes the propensity to save, which depends on the time $t+1$ capital stock and preference parameters.

Equation (23) can then be rewritten to obtain the following nonlinear first–order difference equation for the evolution of the capital stock over time

$$K_{t+1} = \alpha\Omega(\lambda)\,s\left[R(K_{t+1})\right]\,K_t^{1-\alpha}\,. \tag{25}$$

Equation (25) implicitly defines K_{t+1} as a function of K_t and reflects the well–known dynamics of the capital stock of the neoclassical growth model. A stationary point is characterized by $K_t = K_{t+1} = K$. In the special case of logarithmic utility ($\rho = 1$), the propensity to save is independent of the interest rate. In this case, (25) can be solved explicitly for the associated steady state value K of the capital stock

$$\tilde{K} = \left(\frac{\alpha\beta\,\Omega(\lambda)}{1+\beta}\right)^{1/\alpha}. \tag{26}$$

The steady state level of the capital stock is determined by the capital income share (see (20)), the discount factor, and additionally — via $\Omega(.)$ — by the mean and variance of the technology shock as well as the degree of risk aversion.

We conclude with the following result on the distribution of wealth:

Proposition 3 *The average entrepreneur owns a larger share of the aggregate capital stock than the representative worker.*

By Proposition 2, expected profits exceed wage incomes, when households are risk averse. With identical propensities to save, the mean entrepreneur contributes a larger amount to the aggregate capital stock.

4 Comparative Statics

The macroeconomic equilibrium of the previous section is now subject to a comparative static analysis. The state of the economy at a given point of time is entirely governed by the law of motion of the capital stock (25) and the primitives of the model. The latter essentially determine the equilibrium distribution of agents over occupations, which, by (12), is time–invariant and independent of the rate of time preference.

In a first step, we are interested in the question of how changes in the model primitives affect the agents' equilibrium occupational choice. The population shares of workers and entrepreneurs are a major component of the equilibrium conditions for the wage rate, expected profits, the risk premium, sectoral output, aggregate and sectoral income as well as aggregate savings. This directly leads us to the question of how adjustments in occupational choice, due to changes in the model parameters, transmit to the conditions for a macroeconomic equilibrium. By (16), (17), (22), (24), and (25), it becomes obvious that most effects can be assessed via the associated response of the function $\Omega(\lambda)$. For this reason, we will first state the results related to $\Omega(\lambda)$ before turning to the corresponding consequences for the macroeconomic variables.

Proposition 4 *The equilibrium population share of entrepreneurs decreases with a rise in the elasticity of substitution between two intermediate goods j and j', a rise in the degree of risk aversion, and a rise in risk, the latter* *measured by the variance of the technology shock*

$$\text{(i)} \qquad \frac{\partial\lambda}{\partial\alpha} = -\frac{\lambda(1-\lambda)}{\alpha(1-\alpha)} \times \left[1 + \frac{\rho\alpha^2\sigma^2}{(1-\alpha)^2}\right] < 0\,, \tag{27a}$$

$$\text{(ii)} \qquad \frac{\partial\lambda}{\partial\rho} = -\frac{\lambda(1-\lambda)\,\alpha^2\sigma^2}{2(1-\alpha)^2} < 0\,, \tag{27b}$$

$$\text{(iii)} \qquad \frac{\partial\lambda}{\partial\sigma^2} = -\frac{\lambda(1-\lambda)\,\rho\alpha^2}{2(1-\alpha)^2} < 0\,. \tag{27c}$$

A rise in the parameter α corresponds to an increase in the elasticity of substitution between intermediate goods. Other things equal, this increase in competition is accompanied by a decrease in expected profits, which makes the entrepreneurial profession less attractive. We observe an equivalent effect on the population share of entrepreneurs for rises in ρ or σ^2. In the first case, households develop a greater disliking for risk, while, in the latter, the riskiness of profit incomes increases, both (ex ante) causing risk averse agents to switch away towards safe wage incomes.[3]

Proposition 5 $\Omega(\lambda)$ *responds to a change in* λ *and a variation in the parameters* α, ρ *and* σ *as follows:*

(i) $\Omega(\lambda)$ *is maximized, if the population share of firms equals the profit income share in the intermediate goods industry, which only occurs in case of 'certainty equivalence'.* $\Omega(\lambda)$ *unambiguously increases with a rise in the population share of entrepreneurs, if the agents of the economy are risk averse. Define* $\lambda^* \equiv 1-\alpha$. *Then,*

$$\frac{\partial\Omega(\lambda)}{\partial\lambda} = \Omega(\lambda) \times \frac{\lambda^* - \lambda}{\lambda(1-\lambda)} > 0 \quad \textit{for} \quad \lambda < \lambda^*\,, \tag{28a}$$
$$\lambda = \lambda^* \quad \Longleftrightarrow \quad \rho = 0\,, \quad \forall \alpha \in (0,1)$$
$$\left.\frac{\partial^2\Omega(\lambda)}{\partial\lambda^2}\right|_{\lambda=\lambda^*} < 0\,.$$

(ii) $\Omega(\lambda)$ *decreases with a rise in the degree of risk aversion*

$$\frac{\partial\Omega(\lambda)}{\partial\rho} = \frac{\partial\Omega(\lambda)}{\partial\lambda} \times \frac{\partial\lambda}{\partial\rho} < 0\,. \tag{28b}$$

[3]It is important to remember that we assumed agents to be risk averse. The results of Proposition 4 do not necessarily extend to a risk loving or risk neutral society.

(iii) $\Omega(\lambda)$ *increases with a rise in the variance of the technology shock, as long as the degree of risk aversion does not become too large*

$$\frac{\partial\Omega(\lambda)}{\partial\sigma^2} = \frac{\Omega(\lambda)\alpha^2\left[\lambda^* - \rho(\lambda^* - \lambda)\right]}{2(1-\alpha)^2} \gtreqqless 0 \quad \textit{for} \quad \frac{1}{\rho} \gtreqqless \frac{\lambda^* - \lambda}{\lambda^*}\,. \tag{28c}$$

$\Omega(\lambda)$ *unambiguously increases with a rise in the variance of the technology shock for all* $\rho \leqslant 1$.

(iv) The response of $\Omega(\lambda)$ *to changes in* α *is of ambiguous sign*

$$\frac{\partial\Omega(\lambda)}{\partial\alpha} = \frac{\partial\Omega}{\partial\lambda} \times \frac{\partial\lambda}{\partial\alpha} + \Omega(\lambda)\left[\bar{\theta} + \frac{\sigma^2\left(1-(1-\alpha)^2\right)}{2(1-\alpha)^2} + \ln\left(\frac{1-\lambda}{\lambda}\right)\right] \tag{28d}$$

The ambiguous response of $\Omega(\lambda)$ to changes in the variance of the technological shock stems from the counteracting effects of the latter on the population share λ and the exponential productivity component in $\Omega(\lambda)$. While, by (27c), the first one declines with a rise in σ, the second one increases. The diminishing effect of adjustments in occupational choice dominates the other, if the households are sufficiently risk averse.

By (27a) and (28a), the first term in (28d) is negative, whereas the impact from the exponential term in $\Omega(\lambda)$ is entirely positive. The sign of the logarithm depends on the relative size of population shares. It is more likely to be positive. In a realistic environment, the population share of workers rather exceeds the share of entrepreneurs than otherwise.

Since $\Omega(\lambda)$ is a nonlinear polynomial in α, it is not possible to explicitly solve for the maximizing value of α. This can only be obtained by numerical approximation.

Proposition 6 (Change in λ) *A rise in the population share of entrepreneurs causes an increase in aggregate and sectoral income, the equilibrium values of the wage and the real interest rate, and savings, if the agents of the economy are risk averse, whereas expected profits, the expected risk premium, and mean output per firm* j *in the intermediate sector decline.* Q_t, Y_t, r_t*, and* S_t *are maximized under the condition of certainty equivalence, that is, if the population share of firms equals the profit income share in the intermediate goods sector.*

The risk premium vanishes in this case.

$$\text{(i)} \quad \frac{\partial Q_t}{\partial \lambda} = K_t^{1-\alpha}\,\frac{\partial \Omega(\lambda)}{\partial \lambda} > 0\,, \qquad \text{(ii)} \quad \frac{\partial Y_t}{\partial \lambda} = \alpha K_t^{1-\alpha}\,\frac{\partial \Omega(\lambda)}{\partial \lambda} > 0\,,$$

$$\text{(iii)} \quad \frac{\partial w_t}{\partial \lambda} = \frac{(1-\alpha)\,w_t}{\lambda(1-\lambda)} > 0\,, \qquad \text{(iv)} \quad \frac{\partial \pi_t}{\partial \lambda} = -\frac{\alpha \pi_t}{\lambda(1-\lambda)} < 0\,,$$

$$\text{(v)} \quad \frac{\partial \phi_t}{\partial \lambda} = -\frac{\alpha \pi_t + (1-\alpha)\,w_t}{\lambda(1-\lambda)} < 0\,, \qquad \text{(vi)} \quad \frac{\partial r_t}{\partial \lambda} = (1-\alpha)K_t^{-\alpha}\frac{\partial \Omega(\lambda)}{\partial \lambda} > 0\,,$$

$$\text{(vii)} \quad \frac{\partial x_t}{\partial \lambda} = -\frac{x_t}{\lambda(1-\lambda)} < 0\,. \tag{29a}$$

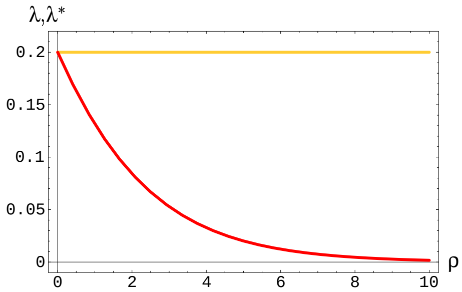
The time $t+1$ capital stock equals time t savings by assumption. The response of (25) *to changes in λ is determined by application of the implicit function theorem*

$$\text{(viii)} \quad \frac{\partial K_{t+1}}{\partial \lambda} = \frac{\lambda^* - \lambda}{\lambda(1-\lambda)} \times \frac{K_{t+1}\left[1 + (\beta R_{t+1})^{-1/\rho}\left(1 + \frac{r_{t+1}}{\rho}\right)\right]}{1 + (\beta R_{t+1})^{-1/\rho}\left(R_{t+1} + \frac{\alpha(1-\rho)}{\rho}\right)} > 0\,. \tag{29b}$$

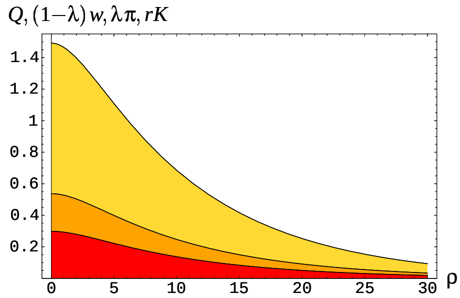
One of the main insights of Proposition 6 is that the overall level of economic activity could be raised if more households chose to be an entrepreneur. To this extent, our model displays one the standard results of the literature on entrepreneurial risk–taking (cf. Kanbur, 1979*b*; Kihlstrom and Laffont, 1979). The population share of firms in the intermediate sector is inefficiently low in an economy where agents are risk averse, when compared to a risk neutral environment, thereby resulting in a suboptimal level of output. The inefficiently low level of economic performance in this sector then carries over to the production of final goods and is also reflected in aggregate savings and the real interest rate.

The wage rate is increasing with a rise in λ. An increase in the number of monopolistic firms goes along with a decrease in aggregate labor supply, which then drives the market clearing wage rate upwards. Simultaneously, a larger population share of firms aggravates competition in the sector for intermediate goods, thereby reducing the market share falling to the single firm, which is accompanied by a decline in firm–specific and expected profits. With wages rising and profits falling, the risk premium diminishes.

Except for the propensity to save, the consequences of a change in the individual degree of risk aversion for the macroeconomic equilibrium can entirely be traced back to the corresponding reaction of the population share λ and the associated adjustments in the economic relationships. By utilizing Propositions 4, 5, and 6, we find:



(a) Population share of firms



(b) Income variables

Figure 1: *Inefficiency and risk aversion*

Proposition 7 (Change in ρ) *An increase in the degree of risk aversion causes a decline in aggregate and sectoral income, the equilibrium values of the wage and the real interest rate, and savings, if the agents of the economy are risk averse, whereas expected profits, the expected risk premium, and mean output per firm j in the intermediate sector rise.*

$$\text{(i)}\quad \frac{\partial Q_t}{\partial \rho} = \frac{(\lambda-\lambda^*)\,\alpha^2\sigma^2 Q_t}{2(1-\alpha)^2} < 0\,, \qquad \text{(ii)}\quad \frac{\partial Y_t}{\partial \rho} = \frac{(\lambda-\lambda^*)\,\alpha^2\sigma^2 Y_t}{2(1-\alpha)^2} < 0\,,$$

$$\text{(iii)}\quad \frac{\partial w_t}{\partial \rho} = -\frac{\alpha^2\sigma^2\, w_t}{2(1-\alpha)^2} < 0\,, \qquad \text{(iv)}\quad \frac{\partial \pi_t}{\partial \rho} = \frac{\alpha^3\sigma^2\, \pi_t}{2(1-\alpha)^2} > 0\,,$$

$$\text{(v)}\quad \frac{\partial \phi_t}{\partial \rho} = \frac{\alpha^2\sigma^2\,[\alpha\pi_t + (1-\alpha)\,w_t]}{2(1-\alpha)^2} > 0\,, \quad \text{(vi)}\quad \frac{\partial r_t}{\partial \rho} = \frac{(\lambda-\lambda^*)\,\alpha^2\sigma^2 r_t}{2(1-\alpha)^2} < 0\,,$$

$$\text{(vii)}\quad \frac{\partial x_t}{\partial \rho} = \frac{\alpha^2\sigma^2 x_t}{2(1-\alpha)^2} > 0\,. \tag{30a}$$

The future capital stock declines with a rise in ρ

$$\begin{aligned}\text{(viii)}\quad \frac{\partial K_{t+1}}{\partial \rho} &= \frac{\partial K_{t+1}}{\partial \lambda} \times \frac{\partial \lambda}{\partial \rho} - K_{t+1}\,(1-s[R(K_{t+1})])\;\frac{\ln(\beta R_{t+1})}{\rho^2}\\ &= -K_{t+1}\left[\frac{(\lambda^*-\lambda)\,\alpha^2\sigma^2\left[1+(\beta R_{t+1})^{-1/\rho}\left(1+\frac{r_{t+1}}{\rho}\right)\right]}{2(1-\alpha)^2\left[1+(\beta R_{t+1})^{-1/\rho}\left(R_{t+1}+\frac{\alpha(1-\rho)}{\rho}\right)\right]} + \right.\\ &\qquad \left. +\,[1-s(.)]\;\frac{\ln(\beta R_{t+1})}{\rho^2}\right] < 0\,.\end{aligned} \tag{30b}$$

Regarding the adjustments of the macroeconomic equilibrium to an increase in the degree of risk aversion, the arguments of Proposition 6 are simply reversed. An greater disliking for risk causes more agents to seek safe wage incomes (wage rate falls), reduces competition in the intermediate goods industry (profits and market shares rise), and drives the overall economic activity in the present as well as in the future further away from its efficient level.

A variation in the attitude towards risk impinges on capital accumulation twofold: Besides the diminishing effects from a change in occupational choice we already discussed above, the valuation of consumption at different points of time is affected via the intertemporal elasticity of substitution, $1/\rho$. This, by itself, reduces the propensity to save.

Figure 1 illustrates the results from Propositions 6 and 7. The dark curve in Figure 1(a) displays the response of the population share of entrepreneurs to a variation in the coefficient of risk aversion, which is contrasted with the efficient value λ^*. Figure 1(b) shows the associated decline in aggregate output. It also differentiates between the wage sum and aggregate profit incomes (light areas), as well as aggregate capital incomes (dark area), the first two representing the time t income of the young generation, the latter being the income of the old. Note that, although expected profits are increasing with a rise in ρ, overall profits decrease due to the shrinking population share of entrepreneurs.

The model was calibrated to reflect empirical estimates for wage incomes to amount to approximately 65% of aggregate income, which, by (21), implies a value of $\alpha = 0.8$. Furthermore, if we assume an empirically plausible value for the attitude towards risk in the interval $\rho \in (0.5, 3)$, the population share of the monopolistic firms should lie around 10%($\pm$3 percentage points). We set $\overline{\theta} = 1.0$, $\sigma^2 = 0.06$ and normalized the time t capital stock to unity.

Proposition 8 (Change in σ^2) *An increase in the variance of the productivity shock causes a decline in aggregate and sectoral income, the equilibrium values of the wage and the real interest rate, as well as in aggregate savings, if the agents of the economy are sufficiently risk averse.*

$$\left.\begin{aligned}
&\textit{(i)} & \frac{\partial Q_t}{\partial \sigma^2} &= K_t^{1-\alpha}\frac{\partial \Omega(\lambda)}{\partial \sigma^2} \\
&\textit{(ii)} & \frac{\partial Y_t}{\partial \sigma^2} &= \alpha K_t^{1-\alpha}\frac{\partial \Omega(\lambda)}{\partial \sigma^2} \\
&\textit{(iii)} & \frac{\partial r_t}{\partial \sigma^2} &= (1-\alpha) K_t^{-\alpha}\frac{\partial \Omega(\lambda)}{\partial \sigma^2} \\
&\textit{(iv)} & \frac{\partial K_{t+1}}{\partial \sigma^2} &= \frac{\partial K_{t+1}}{\partial \Omega(\lambda)} \times \frac{\partial \Omega(\lambda)}{\partial \sigma^2}
\end{aligned}\right\} \gtreqless 0 \quad \textit{for} \quad \frac{1}{\rho} \gtreqless \frac{\lambda^* - \lambda}{\lambda^*} \tag{31a}$$

$$(v)\quad \frac{\partial w_t}{\partial \sigma^2} = \frac{w\alpha^2(1-\rho)}{2(1-\alpha)} \gtreqless 0 \quad for \quad \rho \lesseqgtr 1\,,$$

Expected profits, the expected risk premium, and mean output per firm j in the intermediate sector unambiguously increase with a rise in σ

$$(vi)\quad \frac{\partial \pi_t}{\partial \sigma^2} = \frac{\pi_t\alpha^3\,(\rho + \frac{1-\alpha}{\alpha})}{2(1-\alpha)^2} > 0\,, \qquad (vii)\quad \frac{\partial x_t}{\partial \sigma^2} = \frac{x_t\,[1+\alpha^2(\rho-1)]}{2(1-\alpha)^2} > 0\,,$$

$$(viii)\quad \frac{\partial \phi_t}{\partial \sigma^2} = \frac{\alpha^2\,[(\rho-1)(\alpha\pi_t + (1-\alpha)w_t) + \pi_t]}{2(1-\alpha)^2} > 0\,. \tag{31b}$$

From (31a) it becomes obvious that the counteracting effects of the variance of the technology shock on $\Omega(\lambda)$ already observed in (28c) are passed through to the corresponding changes in the output variables, capital accumulation, and the interest rate. The sign of the derivatives depends on the relative deviation of equilibrium occupational choice from its efficient value in relation to the individual attitude towards risk. For the wage rate, however, the direction of change depends solely on the coefficient of risk aversion.

By (27c), a larger ρ aggravates the reduction in λ as σ increases. The decline in output in the course of firms leaving the market only dominates the augmenting effect stemming from the exponential factor in $\Omega(\lambda)$, if agents are sufficiently risk averse, thereby inducing an overall reduction of economic activity. Also by (27c), the population share of entrepreneurs decreases with a rise in σ. This explains rising profits and the increase in expected output per firm. The expected income differential, as expressed by the risk premium, unambiguously grows with an increase in the variance of the technology shock, independent of the associated change in the wage rate. This means that, even if the wage rate is rising too (for small ρ), expected profits grow even faster.

We conclude this section with deriving the effects of a variation in the productivity parameter α, which also indirectly measures the pairwise elasticity of substitution between intermediate goods.[4]

Proposition 9 (Change in α) *The response of aggregate and sectoral income, the real interest rate, and aggregate savings to an increase in the productivity*

[4]The results are derived for a given time t stock of capital, which means that adjustments of K_t to changes in α are neglected.

parameter α is of ambiguous sign.

$$\left.\begin{aligned}
&\textit{(i)} & \frac{\partial Q_t}{\partial \alpha} &= K_t^{1-\alpha}\,\frac{\partial \Omega(\lambda)}{\partial \alpha}\\
&\textit{(ii)} & \frac{\partial Y_t}{\partial \alpha} &= \alpha K_t^{1-\alpha}\left[\frac{\partial \Omega(\lambda)}{\partial \alpha} + \frac{\Omega(\lambda)}{\alpha}\right]\\
&\textit{(iii)} & \frac{\partial r_t}{\partial \alpha} &= (1-\alpha)\,K_t^{-\alpha}\left[\frac{\partial \Omega(\lambda)}{\partial \alpha} - \frac{\Omega(\lambda)}{1-\alpha}\right]\\
&\textit{(iv)} & \frac{\partial K_{t+1}}{\partial \alpha} &= \frac{\partial K_{t+1}}{\partial \Omega(\lambda)} \times \frac{\partial \Omega(\lambda)}{\partial \alpha}
\end{aligned}\right\} \gtreqless 0 \tag{32}$$

The wage rate grows in α, if the degree of risk aversion is sufficiently low. Otherwise, the sign is ambiguous.

$$\textit{(v)}\quad \frac{\partial w_t}{\partial \alpha} = w\left[\bar{\theta} + \frac{1}{\alpha} + \frac{\alpha\sigma^2\left(2(1-\rho)-\alpha\right)}{2(1-\alpha)^2}\right] > 0 \quad \textit{for} \quad 2(1-\rho) \geqslant \alpha\,, \tag{33}$$

Expected profits, the expected risk premium, and mean output per firm j in the intermediate sector grow with an increase in α

$$\begin{aligned}
&\textit{(vi)} & \frac{\partial \pi_t}{\partial \alpha} &= \pi_t\left[\bar{\theta} + \frac{1}{\alpha} + \frac{\rho\alpha^2\sigma^2}{(1-\alpha)^3} + \frac{\sigma^2\left(1-(1-\alpha)^2\right)}{2(1-\alpha)^2}\right] > 0\,,\\
&\textit{(vii)} & \frac{\partial x_t}{\partial \alpha} &= x_t\left[1 + \frac{\sigma^2\left(1+\rho\alpha^2\right)}{(1-\alpha)^2}\right] > 0\,,\\
&\textit{(viii)} & \frac{\partial \phi_t}{\partial \alpha} &= \phi_t\left[\bar{\theta} + \frac{1}{\alpha} + \frac{\sigma^2\left(1-(1-\alpha)^2\right)}{2(1-\alpha)^2}\right] + \frac{\rho\alpha^2\sigma^2\left[\pi_t + \frac{1-\alpha}{\alpha}\,w_t\right]}{(1-\alpha)^3} > 0\,.
\end{aligned} \tag{34}$$

By recalling the results stated in Proposition 5, the ambiguous outcomes in (32) once more can be explained by the negative impact from a decline in λ, due to an increase in the elasticity of substitution, versus the positive impact from the exponential term. Both are counteracting in the determination of $\Omega(\lambda)$, such that the derivatives *(i)* to *(iv)* of (32) are positive if $\Omega(\lambda)$ increases in α, negative otherwise, and equal to zero, if the effects exactly offset.

As before in Proposition 8, the response of the wage rate to changes in the primitives is related to the size of the coefficient of risk aversion. We also observe counteracting effects. On the one hand, the wage rate is reduced via the decline in λ, which is amplified by a larger coefficient of risk aversion. On the other hand, here, too, is a positive impact from the exponential factor.

Only for expected profits, the risk premium, and the firms' mean output in the intermediate sector, the isolated effects work into the same direction. As before, a reduction in λ lets the expected profits and market shares grow, which adds to the positive impact stemming from the exponential term.

5 Convergence and Endogenous Growth

Convergence The overlapping generations model presented here displays the typical characteristics commonly observed for this class of neoclassical growth models regarding the dynamics of the system. Changes in the model primitives only affect the steady state level of economic activity and the convergence speed. Due to diminishing returns, the model is not able to generate long–run growth of per capita incomes, and changes in the parameters do not have a persistent impact on the growth rate of the economy.

We now focus on the special case of logarithmic preferences, that is $\rho = 1$. Recalling the argument from above, this is the only case, where the propensity to save is independent of the next period's interest rate and it is possible to derive an explicit expression for the steady state value of the capital stock. Under this condition, the equation of motion for the capital stock and the associated steady state value $\tilde{K}$ are given by

$$K_{t+1} = \frac{\alpha\beta\Omega(\lambda)}{1+\beta} K_t^{1-\alpha} \tag{35}$$

$$\tilde{K} = \left(\frac{\alpha\beta\Omega(\lambda)}{1+\beta}\right)^{1/\alpha} . \qquad \text{(by (26))}$$

Linearization around the balanced growth path requires a first–order approximation around $K = \tilde{K}$, that is

$$K_{t+1} \simeq \tilde{K} + \left(\left.\frac{\mathrm{d}K_{t+1}}{\mathrm{d}K_t}\right|_{K_t=\tilde{K}} \right) \left(K_t - \tilde{K}\right) ,$$

which, by utilizing (26), can be rearranged to yield

$$\frac{K_t - \tilde{K}}{\tilde{K}} \simeq (1-\alpha)^t \, \frac{K_0 - \tilde{K}}{\tilde{K}} , \tag{36}$$

where K_0 is the initial value of K. From (36) it becomes obvious that the convergence speed is solely determined by the capital income share $1-\alpha$, a result representing a standard outcome for this type of model. Of course, this simultaneously implies that the rate of change in the case of logarithmic preferences is independent of occupational choice and the distribution of the productivity shock.

Endogenous growth We finally want to give a notion of how occupational choice in a risky environment might permanently influence the long–run growth rate of the economy by extending the framework applied here with a simple endogenous growth mechanism. The easiest way to incorporate ongoing growth of per capita incomes is to introduce human capital externalities à la Romer (1986) in the final goods sector, such that, in the aggregate, production is linear in the capital stock and displays increasing returns to scale. The production technology of the final goods sector (2a) is modified as follows

$$Q_t = K_t^{1-\alpha}\bar{K}_t^{\alpha}\int_0^{\lambda} x_{j,t}^{\alpha}\,\mathrm{d}j\,, \tag{37}$$

where $\bar{K}$ denotes the aggregate stock of capital, which the individual firm takes as exogenously given and therefore neglects in optimization. Under these conditions, the real interest rate is constant in equilibrium, such that (22) changes to

$$r = (1-\alpha)\Omega(\lambda)\,. \tag{38}$$

Contrary to the commonly employed endogenous growth models with R&D and monopolistic competition by Romer (1990) or Grossman and Helpman (1991), the equilibrium value of the real interest rate (38) is not implicitly determined by fixed costs of research and the present value of profits earned in the intermediate goods sector. Instead, it follows directly from the usual marginal productivity conditions of the firm problem in the production of the final good. Note also that the equilibrium value of the population share of entrepreneurs and hence $\Omega(\lambda)$, as defined by (15), is not affected by the model extension. Unchangingly, it solely depends on the model primitives and not on the level of the capital stock.

The real interest rate (38) is suboptimally low for two reasons: on the one hand, due to insufficient entrepreneurial risk–taking of risk averse agents, on the other hand, due to the fact that the individual savings decision is based on the private and not on the (higher) social return on investment. By (15) and Proposition 5, the efficient capital return can be derived as

$$r = \Omega(\lambda^*) = (1-\alpha)\left(\frac{\alpha}{1-\alpha}\right)^{\alpha}\exp\left[\alpha\bar{\theta} + \frac{\alpha^2\sigma^2}{2(1-\alpha)}\right].$$

Given the altered technology (37), the equilibrium expressions for the macroeconomic relationships, such as aggregate output (17), sectoral income (16), the wage rate (7), expected profits (13), the expected risk premium (14), and capital accumulation (25) become linear functions in the economy–wide capital stock. Since labor is inelastically supplied, there is no labor–leisure choice, and the economy immediately enters the long–run growth path. The growth rate, ψ, of the economy is implicitly given by the growth factor

$$1+\psi = \frac{\alpha\,\Omega(\lambda)}{1+\beta^{-1/\rho}\big[1+(1-\alpha)\,\Omega(\lambda)\big]^{(\rho-1)/\rho}}\,. \tag{39}$$

The growth factor is restricted to $1+\psi > 1$ for feasibility reasons.

In what follows, denote for notational simplicity the propensity to save with $s(R)$ and define $B \equiv 1+(\beta R)^{-1/\rho}(1+r/\rho) > 0$. The Proposition stated below shows that the qualitative results derived above for the equilibrium values of the macroeconomic relationships now extend to the long–run growth rate:

Proposition 10 (Growth effects) *The long–run growth rate of the economy responds to a change in* λ *and a variation in the parameters* α, ρ *and* σ *as follows:*

(i) The growth rate increases for a rise in the population share of firms, if the agents of the economy are risk averse. Long–run growth in a risky environment is inefficiently low, as the growth rate is maximized under the conditions of certainty equivalence

$$\frac{\partial(1+\psi)}{\partial\lambda} = (1+\psi)\,s(R)\,B\;\frac{\lambda^*-\lambda}{\lambda(1-\lambda)} > 0, \quad \textit{for} \quad \rho > 0\,. \tag{40a}$$

$\partial(1+\psi)/\partial\lambda = 0$ *for* $\lambda = \lambda^* = 1-\alpha$*, which only occurs for* $\rho = 0$.

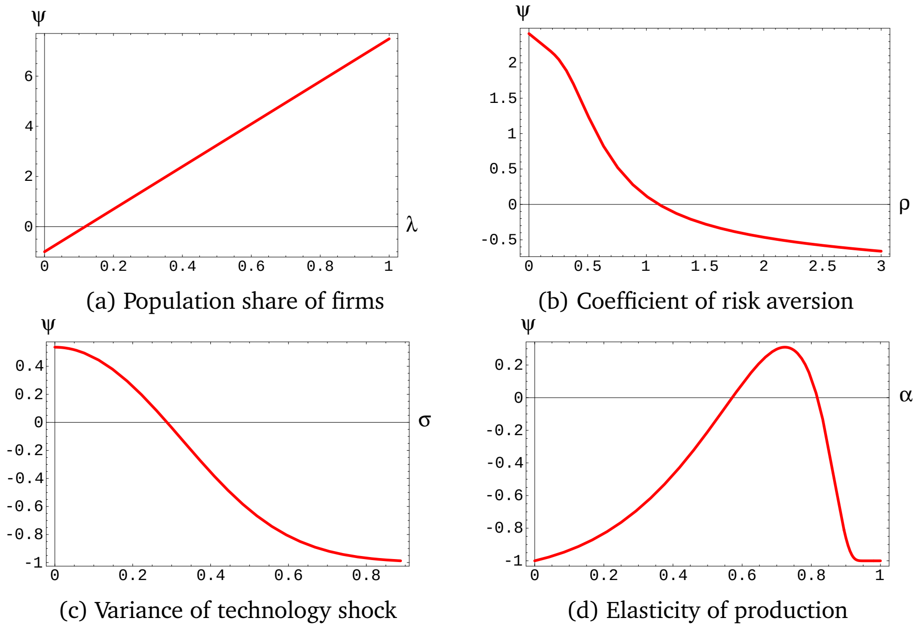
(ii) The growth rate declines with a rise in the coefficient of risk aversion

$$\frac{\partial(1+\psi)}{\partial\rho} = -(1+\psi)\left[\frac{s(R)\,\alpha^2\sigma^2 B\,(\lambda^*-\lambda)}{2(1-\alpha)^2} + \frac{[1-s(R)]\ln(\beta R)}{\rho^2}\right] < 0\,. \tag{40b}$$

(iii) The growth rate declines with a rise in the variance of the productivity shock, if the agents of the economy are sufficiently risk averse

$$\frac{\partial(1+\psi)}{\partial\sigma^2} = (1+\psi)s(R)\,B\;\frac{\alpha^2\,[(1-\alpha)(1-\rho)+\lambda\rho]}{2(1-\alpha)^2}$$

$$\frac{\partial(1+\psi)}{\partial\sigma^2} \gtreqless 0 \quad \textit{for} \quad \frac{1}{\rho} \gtreqless \frac{\lambda^*-\lambda}{\lambda^*}\,. \tag{40c}$$



(a) Population share of firms

(b) Coefficient of risk aversion

(c) Variance of technology shock

(d) Elasticity of production

Figure 2: *Growth effects of occupational choice*

(iv) The response of the growth rate to a change in α *is of ambiguous sign*

$$\frac{\partial(1+\psi)}{\partial\alpha} = \frac{(1+\psi)}{\alpha\,(1-\alpha)}\left[s(R)\,B\left((1-\alpha)\,\varepsilon_{\Omega(\lambda),\alpha}-\alpha\right)+1\right], \tag{40d}$$

where $\varepsilon_{\Omega(\lambda),\alpha}$ *denotes the elasticity of* $\Omega(\lambda)$ *with respect to* α*, which, by* (28d) *in Proposition 5 is of ambiguous sign.*

We observe that the growth rate of the economy is driven further away from its efficient 'certainty equivalent' level, if either the households become more risk averse, or if they are characterized by a comparably low risk tolerance and the riskiness of their environment increases. This result matches empirical result of Ramey and Ramey (1995), who find a negative relationship between the mean growth rate of output and its standard deviation. Again, changes of α yield ambiguous results, depending on whether or not the impact of a declining population share λ dominates the positive effect stemming from the exponential productivity factor.

Figure 2 illustrates the results from Proposition 10. The model is calibrated to yield a growth rate below 10% for a wage income share of $\alpha^2 \approx 0.65$, a degree of risk aversion around $\rho \approx 1$, a variance of the technology shock around $\sigma^2 \approx 6\%$, which altogether implies a population share of entrepreneurs of about $\lambda \approx 10\%$. As displayed by Figure 2(c), the degree of risk aversion is large enough to let the growth rate of the economy fall with a rise in risk, as measured by the variance of the random disturbance. Via $\Omega(\lambda)$, the growth rate, too, is a nonlinear polynomial in α. Therefore, the growth maximizing value of the partial elasticity of production cannot be determined analytically. For the parameterization chosen here it can be determined numerically with $\alpha = 0.72$, which yields a corresponding value of $\lambda = 0.24$, a wage income share of $\alpha^2 = 0.52$, and a implausibly high growth rate of 39%.

6 Conclusion and Directions for further Research

This paper investigated occupational choice under risk in the context of an overlapping generations growth model. The major part of the analysis focused on the neoclassical growth model in the tradition of Diamond (1965). To give an impression of how occupational choice affects the long–run growth rate of the economy, we also considered endogenous growth due to externalities in human capital accumulation, based on the original work of Romer (1986).

The economy consists of two sectors, producing intermediate and final goods in the spirit of Romer (1990). The homogeneous final good was assumed to be generated by differentiated intermediate goods and physical capital in a perfectly competitive market, whereas the intermediate good is produced under the regime of monopolistic competition by employing labor only. The homogeneous risk averse agents decide between being a worker, earning riskless wages, or being an entrepreneur in the intermediate goods sector, bearing the production risk and therefore receiving random profits.

An equilibrium occupational distribution is characterized by a situation, where entrepreneurs receive the same expected utility as workers. From this follows immediately that, given a linear production technology in the intermediate goods sector, monopolistic profits are essential for the feasibility of the underlying allocation. An indefinite increase in the degree of competition, as measured by the pairwise elasticity of substitution between goods, lets the population share of firms shrink towards its lower bound of zero (see Proposition 4). Perfect competition would then be characterized by a degenerate distribution with workers only and a non–producing equilibrium.

An important feature of our model is that, by endogenizing occupational choice, this determines the population share of entrepreneurs and simultaneously controls the range of intermediate products available in the final goods sector. We find that the entrepreneurial activity in the intermediate sector takes place at a suboptimally low level, an insufficient number of the risk averse agents choosing firm ownership, which is a well–documented result for decision–making under uncertainty. This outcome then carries over to final output, aggregate consumption and saving, such that the overall economic performance in equilibrium is inefficiently low. An efficient allocation is characterized by a situation, where the aggregate share of income received by entrepreneurs equals their weight in the population. In the absence of appropriate tax–transfer–schemes implemented by a public sector and providing incentives for firm ownership, the efficient allocation is characterized by certainty equivalence and attained in the risk neutral society only.

In general, occupational choice also affects the transitional dynamics towards the steady state. This impact vanishes in the special case of logarithmic preferences discussed here. The neoclassical OLG model is characterized by the standard result, that changes in the model primitives only affect the equilibrium level of economic activity, but do not have a long–lasting impact on the growth rate of the economy, which is zero in equilibrium, since we also abstracted from population growth. For this reason, occupational choice, too, does not have a permanent effect on the growth rate.

Things are different in the endogenous growth framework, where we found the real interest rate to be suboptimally low, first, because of inefficient risk–taking, and second, due to technological spillovers. This ends up in a suboptimal low growth rate, which shows that the results regarding the inefficiency of the underlying allocation from the neoclassical model carry over to the endogenous growth setting. The economy would experience welfare gains and a permanently higher growth rate, if more individuals chose firm ownership in the intermediate sector.

Regarding the distribution of wealth we observe that the average entrepreneur owns a larger fraction of the aggregate capital stock than a representative worker. This finding can be the starting point of further research introducing heterogeneity into the framework, for instance, by allowing for bequests or by extending the model with market entry costs and associated capital market imperfections (credit rationing, borrowing constraints) similar to the analysis of Banerjee and Newman (1991), Aghion and Bolton (1997), or more recently Ghatak *et al.* (2001).

If it comes to the public sector, the impact of linear, progressive or differential taxation on occupational choice, entrepreneurial risk–taking, and growth remains to be investigated, following the research of Kanbur (1981, 1982), Boadway *et al.* (1991), or more recently Poutvaara (2002). Taxing labor incomes and using the revenues to subsidize capital formation, for instance, could pay some kind of 'double dividend'. On the one hand, the interest rate rises due to the investment subsidy, while on the other hand, capital returns also increase, as the wage tax induces more agents to choose the entrepreneurial profession, thereby stimulating savings and growth, and subsequently closing the efficiency gap.

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