Tests of Bias in Log-Periodogram Regression

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Zusammenfassung: Diese Arbeit schlgt einfache Hausman Tests vor, die auf Verzerrung der Log-Periodogrammregression bei Zeitreihen, die mglicherweise langes Gedchtnis haben, testen. Die Teststatistiken sind unter der Nullhypothese, dass keine Verzerrung vorliegt, asymptotisch standardnormalverteilt. Sie sind zudem konsistent. Der Nutzen der Tests in Verbindung mit Tests auf die Signifikanz des Gedchtnisparameters wird in einer Monte Carlo Studie dargelegt.

Abstract: This paper proposes simple Hausman-type tests to check for bias in the log-periodogram regression of a time series believed to be long memory. The statistics are asymptotically standard normal on the null hypothesis that no bias is present, and the tests are consistent. The use of the tests in conjunction with tests of significance of the long memory parameter is illustrated by Monte Carlo experiments.

KEYWORDS: Long memory, log-periodogram estimation, Hausman test

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1 Introduction

Long memory models, and specifically fractionally integrated models, are an increasingly popular method of representing the persistence characteristics of time series. If the modeller believes that her data may exhibit long memory, a popular strategy is to estimate the integration parameter d using a semi-parametric estimator such as the Geweke and Porter-Hudak (1983) log-periodogram regression (GPH). The issue of whether long memory exists in a time series is a fundamental one. It is not unknown for practitioners to compute the GPH estimator of d and make a decision on this question based on the significance of the estimate.

The properties of the GPH estimator and variants thereof have been studied in Hassler (1993), Hurvich and Beltrao (1994), Robinson (1995), Hurvich, Deo and Brodsky (1998) and Velasco (2000), inter alia. Akliagloglou, Newbold and Wohar (1993) first pointed out that when the serial dependence has a substantial 'short-run' component, the GPH estimator can be severely biased. The existence of autoregressive roots not too far from unity pose a special hazard in this respect, in view of the fact that the fractional difference filter $(1-L)^d$, $d > 0$, and the autoregressive filter $(1-\lambda L)$, $\lambda > 0$, both approach the simple difference $1-L$ as their respective parameters approach 1. This poses a well-known problem for the identification of ARFIMA(p, d, q) models.

Various methods have been proposed for reducing the bias, such as the truncation of the high-frequency periodogram points advocated by GPH. However, these methods impose a substantial efficiency penalty. The optimal trade-off between bias and variance depends on the short-run component, but since the use of a nonparametric method implies ignorance of this component, the best choice of bandwidth for the GPH estimator is a problem without a ready solution.

These considerations make it very desirable to have a means of checking whether significant bias is present in the log-periodogram regression, and in this paper we propose and evaluate some simple tests. These are of the Hausman (1978)-type in which alternative estimates are compared, one of which is expected to exhibit less bias when the null is false, albeit lower efficiency when the null is true. Since the log-periodogram regression is strictly unbiased only where there is no short-run dependence (in a sense to be defined precisely), our tests can also be considered as tests for the null hypothesis that no short-run component is present.

The most critical case is where the true dependence is "short-run" albeit relatively persistent. There is a risk of detecting spurious long memory when none is in fact present. Our tests can be suitably paired with conventional tests of the null hypothesis $d = 0$, in which short memory is the null hypothesis, and long memory the alternative. Performing tests in both directions gives, in effect, a non-nested testing procedure with four possible outcomes; rejecting one null or the other, or both, or neither. In the first two cases we may infer that the short memory representation encompasses (is encompassed by) the long memory representation. In the other cases the test results would be less conclusive, and we would need to be cautious about categorizing the series in question as either long or short memory.

The paper is organized as follows. Section 2 describes the log-periodogram regression estimator and proposed variants in more detail. As an alternative to the "narrow band" GPH approach, we consider the "broad band" augmented regression proposed by Soulier and Moulines (1999). Section 3 describes the test statistics and shows them to be consistent and asymptotically standard normal under the null hypothesis. Section 4 reports Monte Carlo evidence, Section 5 gives an empirical example, and Section 6 concludes.

2 Log-Periodogram Regression

We consider the class of covariance stationary processes $\{Y_t\}$ whose spectrum takes the form

$$f(\lambda) = |1 - \exp(-i\lambda)|^{-2d} f^*(\lambda), \quad -\pi \leq \lambda \leq \pi \tag{2.1}$$

where $|d| < 0.5$ is the fractional integration parameter, and $f^*(\lambda)$ represents the short-term correlation structure of the model. We assume that Y_t is Gaussian, $f^*(\lambda)$ is an even, positive and continous function on $[-\pi, \pi]$ which is bounded away from zero and satisfies $f^{*'}(0) = 0$, with the second and third derivative is bounded in a neighborhood of the origin. Given a sample $Y_1, \dots, Y_N$, the idea of log-periodogram regression is to estimate d in (2.1) from the regression

$$\log I(\lambda_k) = c + dX_k + \varepsilon_k,\ \ k = 1, \dots m \tag{2.2}$$

for $m \leq [N/2]$ where $X_k = -2\log(\sin\lambda_k)$, $\lambda_k = 2\pi k/N$ denotes the kth Fourier frequency, and

$$I(\lambda) := \frac{1}{2\pi N}\left|\sum_{t=1}^{N} Y_t \exp(-it\lambda)\right|^2.$$

We note that some authors such as Robinson (1995) propose replacing X_k by $-2\log\lambda_k$, which behaves similarly in the neighbourhood of the origin.

To minimise bias due to the omission of the unknown function f^*, it was suggested by GPH to set $m = O(T^{1/2})$, in the hope that in this narrow band of low frequencies, the variations in f^* are small. However, discarding so much data imposes an efficiency cost to be traded against the reduction in bias. Hurvich, Deo and Brodsky (1998) (henceforth, HDB) derive the bias expression

$$\begin{aligned} E(\hat{d} - d) &= \frac{1}{S_{XX}}\sum_{k=1}^{m} a_k \log f_k^* + \frac{1}{S_{XX}}\sum_{k=1}^{m} a_k E(\varepsilon_k) \\ &= -\frac{2\pi^2}{9}\frac{f^{*''}(0)}{f^*(0)}\frac{m^2}{N^2} + o\left(\frac{m^2}{N^2}\right) + O\left(\frac{\log^3 m}{m}\right) \end{aligned} \tag{2.3}$$

where $a_j = X_j - \bar{X}$ where $\bar{X} = m^{-1}\sum_{j=1}^{m} X_j$, and $S_{XX} = \sum_{k=1}^{m} a_j^2$. For example, in the case of the ARFIMA$(1, d, 0)$ model $(1 - \phi L)(1 - d)^d Y_t = u_t$ where u_t is i.i.d., we find $f^*(\lambda) = (1 - 2\cos\lambda + \phi^2)^{-1}$ and

$$\frac{f^{*''}(0)}{f^*(0)} = \frac{-2\phi}{(1-\phi)^2}. \tag{2.4}$$

For ϕ close to 1 the bias can clearly be substantial in finite samples. HDB go on to derive a formula for the mean squared error, and show that the MSE is minimised by setting $m = CT^{4/5}$ where, however, the constant C depends on the inverse of the ratio in (2.4). Also, it turns out that asymptotic centered-normality of the estimator holds only for $m = o(T^{4/5})$. Hurvich and Deo (1999) propose a plug-in estimator of C, but their Monte Carlo evidence shows that the asymptotic optimality criteria are in practice successful only when there is a limited amount of short-run dependence. When the bias component is large enough to dominate the MSE, the GPH narrow-bandwidth proves more effective.

More recently, several authors have proposed alternative log periodogram estimators, using dummy regressors to account more fully for the neglected short run dependence. Phillips and Shimotsu (2002) advocate a frequency-grouping approach with a fixed-effects treatment of the intercept, Andrews and Guggenberger (2003) suggest including polynomial terms in λ_k in the narrow-band regression, while Hurvich and Deo (1999) propose a direct bias correction. On the other hand, Moulines and Soulier (1999) (henceforth MS) suggest a broad-band approach, using the whole range of frequencies. They propose expanding $\log f^*$ in Fourier series to pth order, yielding terms of the form h_j, $j = 1, ..., p$ where $h_j(\lambda_k) = \cos(j\lambda_k)/\sqrt{\pi}$. Assuming the expansion

$$\log f^*(\lambda) = \sum_{j=1}^{\infty} \theta_j h_j(\lambda)$$

they show that if the coefficients decline exponentially, such that $|\theta_j| = O(\rho^j)$ for $0 < \rho < 1$, then if $p = p_n = O(\log n)$, the mean squared error of the estimator of d is of $O(\log n/n)$. Taking the AR(1) model as an example again, note that

$$\begin{aligned} \ln\{(1+\phi^2 - 2\phi\cos\lambda)^{-1}\} + \ln(1+\phi^2) &= -\ln(1-\rho\cos\lambda) \\ &= \rho\cos\lambda - \frac{\rho^2}{2}\cos^2\lambda + \frac{\rho^3}{6}\cos^3\lambda \ldots \end{aligned}$$

where $\rho = 2\phi/(1+\phi^2)$. The expansion of $\cos^j\lambda$ in terms of $h_0, \ldots, h_j$ has weights $2^{-j}\binom{j}{1}$, so that the θ_j decline exponentially in this case.

3 The Test Statistics

To test the null hypothesis $d = 0$, we employ the usual t statistics for the estimates of d from the GPH and MS regressions,[3] denoted respectively by $\hat{d}_{GPH}$ and $\hat{d}_{MS}$. HDB show that

$$\sqrt{m}(\hat{d}_{GPH} - d) \xrightarrow{d} N\left(0, \frac{\pi^2}{24}\right)$$

for $m = o(T^{4/5})$, whereas MS show that

$$\sqrt{T/p_n}(\hat{d}_{MS} - d) \xrightarrow{d} N\left(0, \frac{\pi^2}{6}\right)$$

Note that $\pi^2/6$ is the limiting variance of the regression disturbances. In practice, the approach to these limits is slow, and in finite samples t ratios are best constructed using conventional standard error formulae based on the regressor second moments.

The hypothesis of zero bias is conveniently stated as:

$$H_0 : f^*(\lambda) = \text{constant}, \; 0 \le \lambda \le \pi$$

The test statistics we derive for this case are of the general form

$$\text{TS} = \frac{|\hat{d}_1 - \hat{d}_2|}{\text{SE}(\hat{d}_1 - \hat{d}_2)}. \tag{3.1}$$

where $\hat{d}_1$ and $\hat{d}_2$ are alternative estimators of d, and SE($\cdot$) denotes a suitable estimator of the standard error of the argument, to be defined. Note that this notation is generic, and will be applied to the GPH and MS cases in turn.

In the GPH case the alternatives we consider are computed with different amounts of truncation, using respectively m_1 and $m_2 < m_1$ periodogram ordinates. Applying expression (2.3) yields

$$E(\hat{d}_1 - \hat{d}_2) = \frac{2\pi^2}{9}\frac{f^{*''}(0)}{f^*(0)}\frac{m_1^2}{N^2}\left(1 - \frac{m_2^2}{m_1^2}\right) + o\left(\frac{m_1^2}{N^2}\right) + O\left(\frac{\log^3 m_2}{m_2}\right). \tag{3.2}$$

Also,letting $S_i = \sum_{j=1}^{m_i} a_{ij}^2$ for $i = 1$ and 2 respectively, we can adapt HDB's formula (5) to give

$$\text{Var}(\hat{d}_1 - \hat{d}_2) = E\left(\frac{1}{S_1}\sum_{j=1}^{m_1} a_{1j}\varepsilon_j - \frac{1}{S_2}\sum_{j=1}^{m_2} a_{2j}\varepsilon_j\right)^2$$

[3] MS propose smoothing the periodogram by using non-overlapping averages of groups of m periodogram points. In this paper we set $m = 1$ for simplicity. Robinson (1995) has proposed a similar variant of the GPH estimator, and both variants might be compared in a similar manner to the present analysis.

$$
\begin{aligned}
&= E\left(\sum_{j=1}^{m_2}\left(\frac{a_{1j}}{S_1}-\frac{a_{2j}}{S_2}\right)\varepsilon_j+\frac{1}{S_1}\sum_{j=m_2+1}^{m1}a_{1j}\varepsilon_j\right)^2 \\
&= \frac{\pi^2}{6}\left(\sum_{j=1}^{m_2}\left(\frac{a_{1j}}{S_1}-\frac{a_{2j}}{S_2}\right)^2+\frac{1}{S_1^2}\sum_{j=m_2+1}^{m1}a_{1j}^2\right)+o\left(\frac{1}{m_2}\right) \\
&= \frac{\pi^2}{6}\left[\left(\frac{1}{m_1}-\frac{1}{m_2}\right)^2 m_2+\frac{m_1-m_2}{m_1^2}\right]+o\left(\frac{1}{m_2}\right) \qquad (3.3)
\end{aligned}
$$

Now, set $m_1 = CT^\beta$ in this expression for some $\beta \leq 1$ and also $m_2 = Km_1$, where $K \in (0,1)$ is another constant to be chosen. Substituting these choices into (3.2) and (3.3) yields respectively

$$
E(\hat{d}_1-\hat{d}_2)=\frac{2\pi^2}{9}\frac{f^{*''}(0)}{f^*(0)}T^{2\beta-2}\left[1-K^2\right]+o(T^{2\beta-2})+O\left(\frac{\log^3 T^\beta}{T^\beta}\right),
$$

$$
\mathrm{Var}(\hat{d}_1-\hat{d}_2)=\frac{\pi^2}{6T^\beta}\left[\frac{1}{K}-1\right]+o(T^{-\beta}).
$$

Compare the leading terms in these expressions. Note first that a consistent test, such that TS diverges under the alternative it is necessary that $\beta/2+2\beta-2>0$, or in other words that $\beta>4/5$, just the opposite constraint proposed in the narrow-band regression. However, under the null hypothesis the full broad band regression, setting $\beta=1$, is consistent and asymptotically normal. This can be deduced as a special case of HDB's Theorem 2 , but is most directly seen as a consequence of Moulines and Soulier's (1999) Theorem 1. $m_1 = [T/2]$ is therefore the natural choice. On the other hand, choosing K to make $E(\hat{d}_1-\hat{d}_2)/\sqrt{\mathrm{Var}(\hat{d}_1-\hat{d}_2)}$ as large as possible requires maximizing the expression

$$
\frac{1-K^2}{\left(\frac{1}{K}-1\right)^{1/2}}.
$$

The solution on $[0,1]$ can be verified numerically to be about $K = 0.64$. The test statistic is computed as a t ratio using the penultimate member of (3.3), less the small-order term, to estimate the standard error.

To obtain the form of statistic TS appropriate to the MS estimator $\hat{d}_1$ is the estimator computed in the regression of the log periodogram points $\log I(\lambda_k)$ onto $(X_k,1)$. Following Moulines and Soulier (2000) is the efficient procedure under the null hypothesis of pure long memory. $\hat{d}_2$ is the estimator computed from the regression of $I(\lambda_k)$ onto $(X_k, 1, h_1(\lambda_k), \ldots, h_{p_n}(\lambda_k))$. The asymptotic variance of $\hat{d}_1-\hat{d}_2$ under H_0 is straightforwardly shown to be the difference of their asymptotic variances, by the usual Hausman (1978) variance formula. Asymptotic normality follows from Moulines and Soulier (1999), Theorem 1.

4 Monte Carlo Evidence

We conducted a set of simulations of 500 observations of the Gaussian ARFIMA(1,d,0) model, estimating d by four different methods. These are the GPH estimator with bandwidths $m = 127 = [T^{0.78}]$ and $m = 22 = [T^{0.5}]$ (Cases 1 and 2), and the MS estimator with $p = 4 = [T^{0.25}]$ and $p = 8 = [T^{0.35}]$ (Cases 3 and 4). The results are reported graphically in Figures 1-4. The cells of the chart represent the results of 2000 replications of the experiment, with cases of d in rows, and cases of ϕ in columns. Each plot consists of a bar chart showing the root mean squared error(top, shaded light gray) and bias (bottom, shaded dark gray) on a scale with maximum of 0.6, and a pie chart showing the percentages in the replications of each outcome for the pair of tests, each conducted at the nominal 5% level. The lightly shaded regions show non-rejection; the broad up-sloping bars show rejections of the significance test only; the narrow down-sloping bars rejection on the bias test only; and the heavily shaded regions the rejections on both tests. Thus, total rejections for each test are represented by combining each barred region with the shaded region.

The first point to note, from the first column of each figure, is that the bias test is generally correctly sized in each case, the number of rejections falling close to 5%. This outcome is independent of the value of d. However, the top rows of the figures show clearly the proportion of incorrect rejections in the significance test, of the null hypothesis $d = 0$. The proportions vary with the estimator used, and are clearly unacceptably high in Case 1, and also in Case 3 when $\phi = 0.75$. These are of course the estimators with the generally smaller mean squared error when there is little short-run dependence. However, it is reassuring to see that the number of rejections in the bias test increase correspondingly with these false rejections, even though the smaller power of the bias test is evident in those cases where both null hypotheses are false.

Note that the bias test is actually the same in Cases 1 and 2, whereas in Cases 3 and 4 it is different because the test is based on $\hat{d}_2$ estimated with corresponding number of Fourier terms. It is interesting to note that because of the lower efficiency of $\hat{d}_2$ in Case 4, the bias test actually displays lower power in this case than in Case 3. However, Case 3 (Moulines-Soulier with 4 Fourier terms) appears to offer a good compromise of mean squares error and bias, somewhat better than "classic" GPH (Case 2).

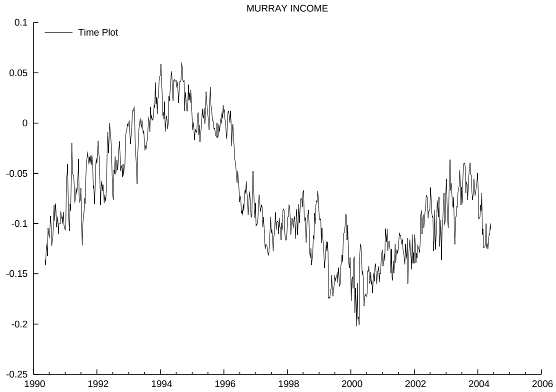
5 Empirical examples

In this section we apply the significance test as well as the bias test to the discount process of UK closed - end mutual funds.

It is a much discussed problem that there is a discount on closed-end mutual funds. Presently the discount in the UK is more than 11%. The discount is the difference between the net asset value and the actual price of the fund. Copeland (2005) discusses the time-series properties of UK discount data, being mainly concerned with the question whether the discount is mean-reverting to an equilibrium value. He also discusses the long-memory properties of this data, showing that for most funds the memory parameter d is between 0.5 and 1 with an average of $d = 0.75$. Copeland argues that the long memory in this data might be spurious due to bounds for the discount process given by factors such as arbitrage costs, management fees etc.

There are, therefore, three hypotheses that we might consider concerning the generation process of these series. The first is that the series are mean-stationary, and even weakly dependent. This is the hypothesis of relatively rapid mean reversion, call it H_1. The second hypothesis, is that there is no mean reversion, and hence that the series contain unit roots; call this H_2. The third hypothesis, H_3, is that the series are highly persistent, and nonstationary, but nonetheless mean reverting. H_1 is tested by a 1-sided test of the hypothesis $d = 0.5$ against $d > 0.5$. H_2 is tested by a 1-sided test of $d = 1$ against $d < 1$. If both of these hypotheses are rejected, we accept H_3. However, we wish to guard against incorrectly rejecting either H_1 or H_2 due to the presence of neglected short-memory components. A stable autoregressive root that is nonetheless close enough to 1 to make the process look highly persistent in a finite sample, could result in biased estimates of d and a false rejection of either H_1 or H_2. However, such a data feature should result in rejection in our bias test.

To perform our tests, we consider weekly data of discounts of five UK closed-end mutual


MURRAY INCOME
Time Plot
0.1
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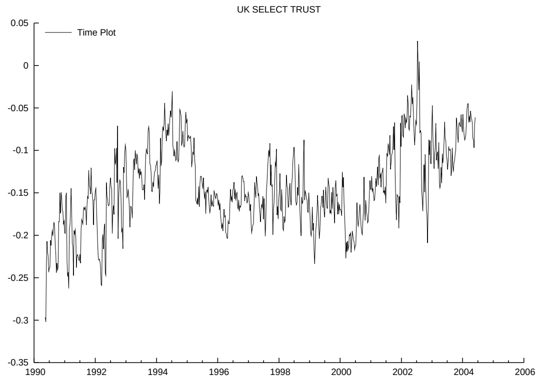
UK SELECT TRUST
Time Plot
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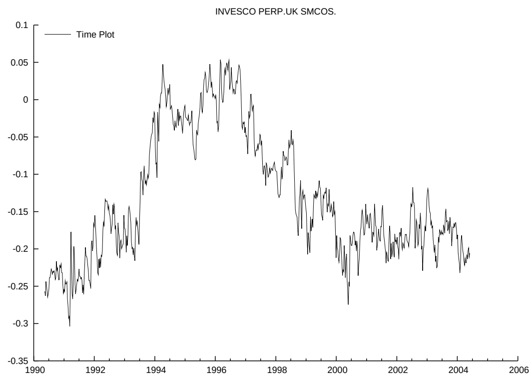
INVESCO PERP.UK SMCOS.
Time Plot
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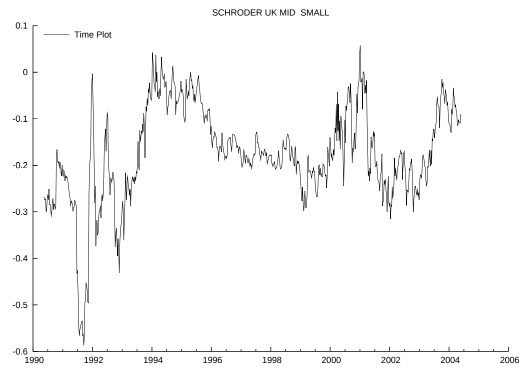
SCHRODER UK MID SMALL
Time Plot
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-0.2
-0.3
-0.4
-0.5
-0.6
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	GPH			*M-S*		
Series	$\hat{d}_{GPH}$	Signif. Test	Bias Test	$\hat{d}_{MS}$	Signif. Test	Bias Test
Murray	0.175 (0.074)	2.369 {0.009}	0.464 {0.642}	0.186 (0.088)	2.105 {0.0178}	0.702 {0.482}
UK Select	0.176 (0.074)	2.383 {0.0085}	$0.-783$ {0.433}	0.112 (0.088)	1.272 {0.102}	1.554 {0.12}
Invesco	0.264 (0.074)	3.562 {0}	1.977 {0.047}	0.238 (0.088)	2.685 {0.0036}	2.668 {0.007}
Schroder	0.354 (0.074)	4.776 {0}	0.552 {0.58}	0.343 (0.088)	3.882 {0}	1.249 {0.211}
Archimedes	0.463 (0.074)	6.253 {0}	-0.343 {0.731}	0.389 (0.088)	4.397 {0}	2.661 {0.007}

Table 1: Significance and bias tests for half-differenced series (p-values in curly brackets)

funds from May 1990 to May 2004[4]. These funds are Murray Income, UK Select Trust, Invesco Perp UK SMCOS, Schroder UK Mid & Small and Archimedes Capital SUSP. Figures 1-5 show the time plots of these series. First, each series was subjected to the transformation $(1-L)^{0.5}$, to obtain series that are $I(0)$ under H_1.[5] We refer to these as 'half-differenced'. The one-sided tests for $d = 0.5$, with alternative $d > 0.5$, and the associated bias tests, were performed based on the GPH-estimator with a bandwidth of $m_2 = 0.68$ and the Moulines-Soulier broadband estimator based on $p = 4$ Fourier terms. These results are shown in Table 1, where for each series the rows of the table show the point estimates of d, $\hat{d}_{GPH}$ and $\hat{d}_{MS}$ respectively, with asymptotic standard errors in parentheses, together with the test statistics with asymptotic p-values associated with the one-tailed tests in curly brackets. Table 2 shows the corresponding results for the regularly-differenced series, such that the one-sided tests are of a unit root (i.e., no mean reversion tendency) against alternative $d < 1$, so that and the critical region lies in this case in the lower of the test distribution.

While the bias tests are nominally sensitive to any departure from the 'pure fractional' model, in practice we concieve that their primary object in Table 1 is to check for the existence of a stable

[4]Data is from Datastream

[5]Following this transformation, the first 50 observations of each series was discarded to eliminate start-up effects. For the sake of comparability, the same observations were discarded in the differenced data.

	GPH			*M-S*		
Series	$\hat{d}_{GPH}$	Signif. Test	Bias Test	$\hat{d}_{MS}$	Signif. Test	Bias Test
Murray	−0.232 (0.074)	−3.137 {0.001}	0.393 {0.694}	−0.252 (0.085)	−2.964 {0.001}	0.115 {0.908}
UK Select	−0.349 (0.074)	−4.716 {0}	−0.941 {0.346}	−0.419 (0.085)	−4.926 {0}	1.820 {0.068}
Invesco	−0.337 (0.074)	−4.550 {0}	2.234 {0.025}	−0.337 (0.085)	−3.965 {0}	3.062 {0.002}
Schroder	−0.135 (0.074)	−1.825 {0.068}	1.593 {0.111}	−0.132 (0.085)	−1.552 {0.12}	1.427 {0.153}
Archimedes	−0.223 (0.074)	−3.01 {0.001}	0.387 {0.698}	−0.298 (0.085)	−3.500 {0}	4.684 {0}

Table 2: Significance and bias tests for differenced series (p-values in curly brackets)

autoregressive root large enough to mimic nonstationary long memory. In Table 2, on the other hand, we are interested to check whether 'mean reversion' is being mimicked by a short-range negative autocorrelation in the differences of a unit root process. A simple case in point would be an integrated moving average (IMA) with MA coefficient negative. Each of these cases would contradict the 'long memory' characterization of the processes that would otherwise be indicated by a rejection.

In general, our results suggest that the 'nonstationary but mean reverting' characterization of these processes is supported by the data. Of the 20 significance tests reported, most reject decisively, and the largest p-value obtained is 0.12. Considering the bias test, the evidence against the fractional integration model is mainly weak at best. The Moulines-Soulier bias statistics are generally larger (absolutely) than their GPH counterparts, but this does not imply greater power, of course, since these tests may equally exhibit more size distortion in finite samples. The fact that the two estimators point to different results cannot be taken to support one or the other. However, the estimates agree in suggesting some bias in the Archimedes and Invesco cases. Remember that a rejection in the bias test should not be taken to necessarily contradict the significance test result. It simply casts some doubt on it by suggesting that there are additional unmodelled features in the data, that ought to be taken into account.

6 Conclusions

In this paper we have proposed some diagnostic procedures to try to detect bias in general, and the presence of spurious long memory in particular. The research strategy suggested by our results may be summarised as follows. Use an efficient estimator where possible, but compute the bias statistic, and experiment with a more conservative estimation procedure in cases where it rejects. If both the bias test and the significance test reject, apply particular caution in deciding whether to treat the series as long memory. It may be helpful to compare the relative p-values of the two tests in deciding which result to give most credence to.

We mention in conclusion that more informal diagnostic procedures can also be helpful, such as visual inspection of the residuals in the log-periodogram regression. Figures 5 and 6 show two "Actual/Fitted" plots from broad-band regressions ($T = 500$) the regression slope coefficients being in each case close to 0.35. However, while in Figure 5 the series was generated by the process $(1 - L)^{0.35} Y_t = NID(0,1)$ in Figure 6 the model used process used was the AR(1), $(1 - 0.5L)Y_t = NID(0,1)$. The inferior fit of the regression is evident in the second case, in particular the cluster of negative residuals close to the origin, where the periodogram fails to increase as fast as it should. This pattern is quite characteristic of spurious long memory.

7 References

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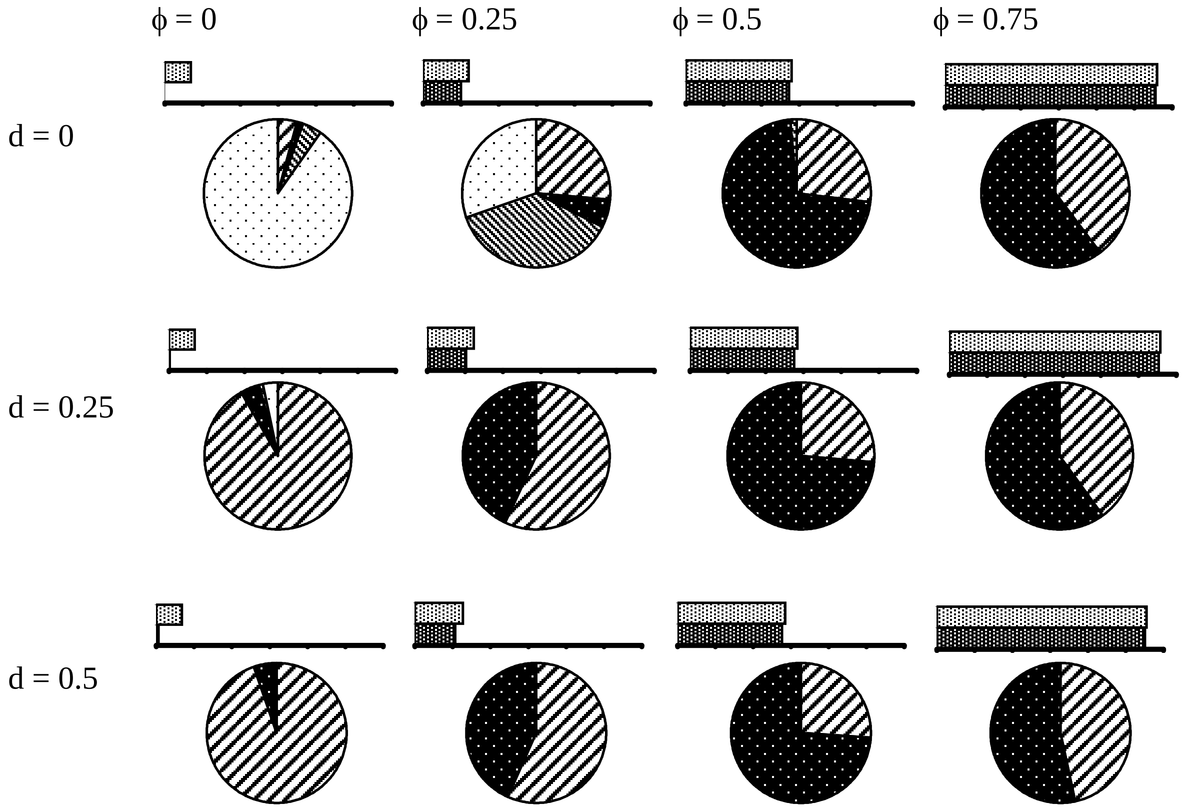

Figure 1. Geweke-Porter-Hudak Estimator, β=0.78

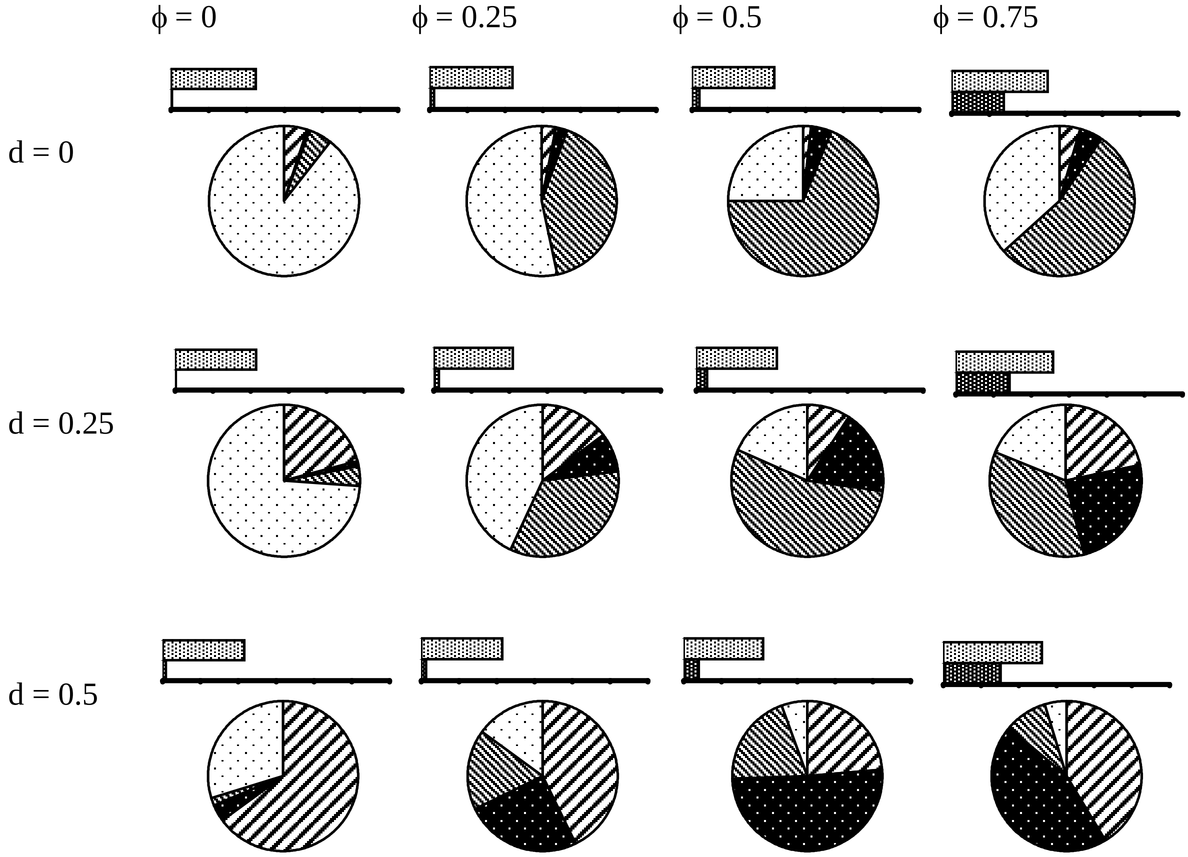

Figure 2. Geweke-Porter-Hudak Estimator, β=0.5

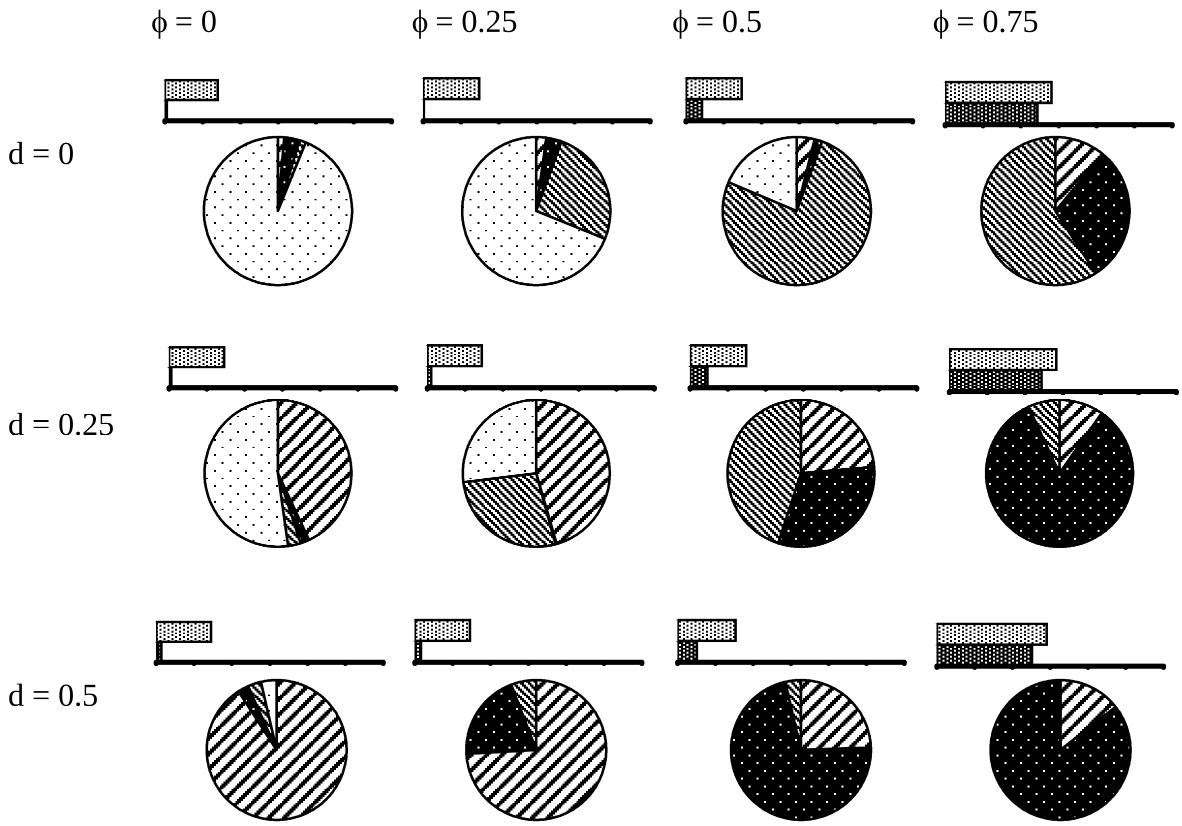

Figure 3. Moulines-Soulier Estimator, $p = 4$

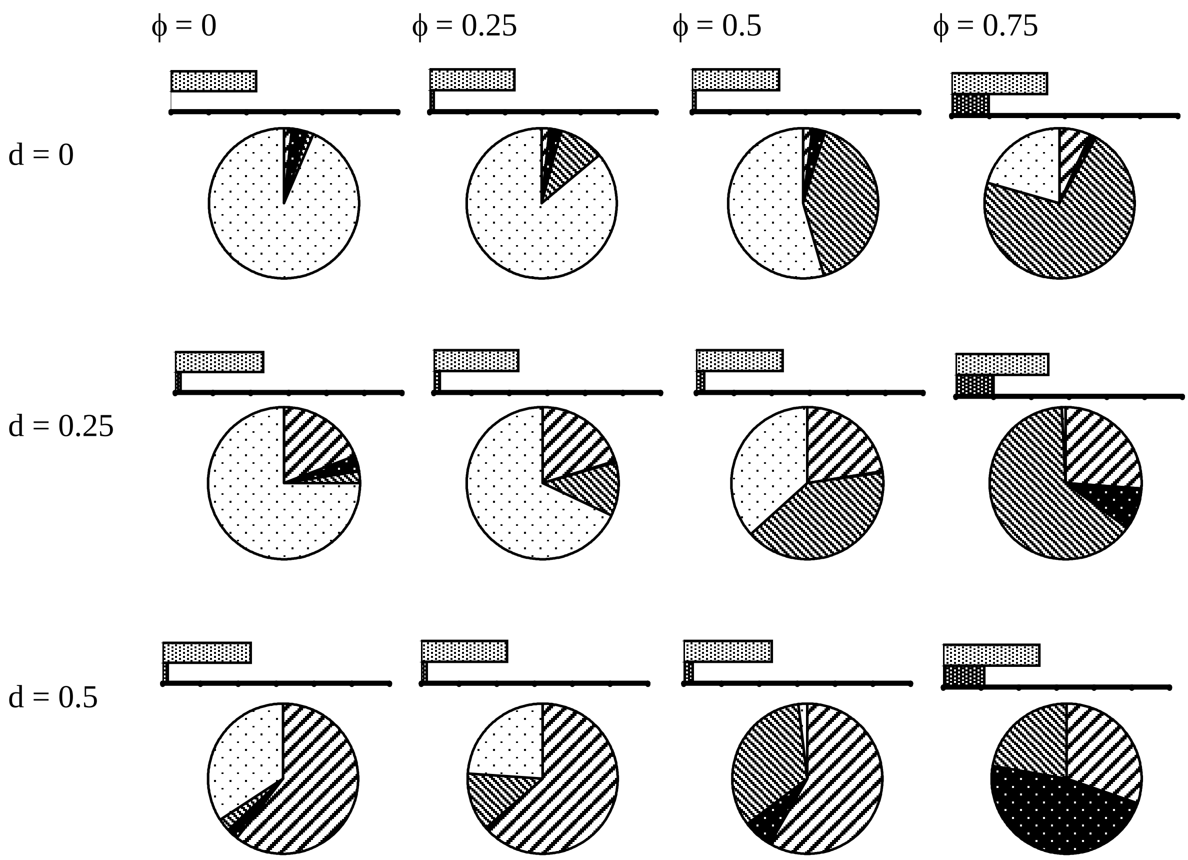

Figure 4. Moulines-Soulier Estimator, $p = 8$

Chart Key

Estimator performance (value axis limit = 0.6)

Root mean squared error

Bias

Test outcomes at 5% level (% cases)

Neither test rejects

Significance test rejects

Hausman (bias) test rejects

Both tests reject

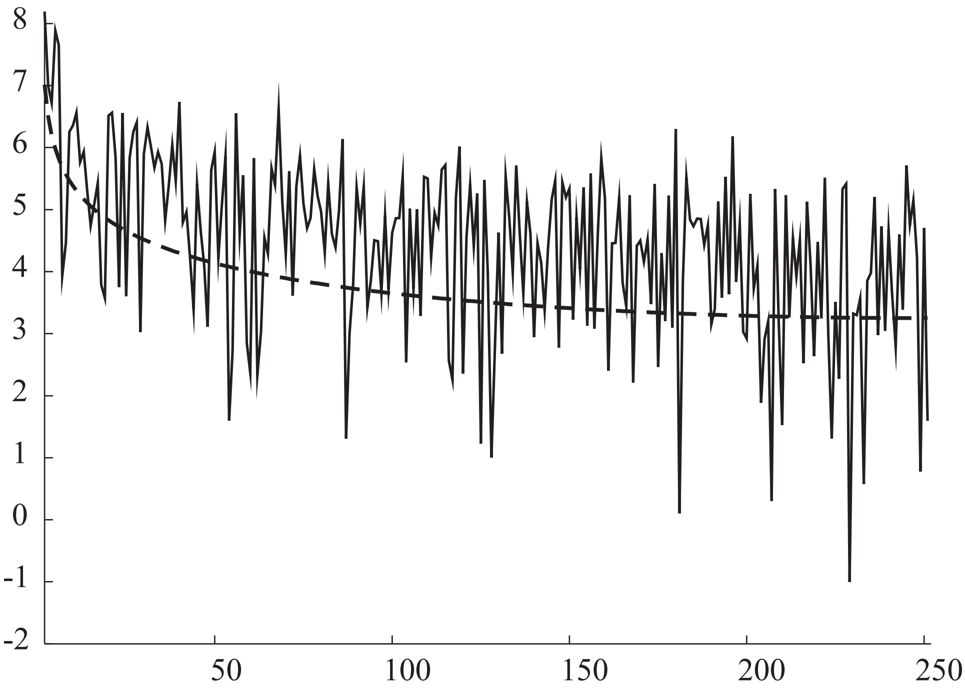

Figure 5. Actual and fitted log-periodogram, FI model, $d = 0.35$.

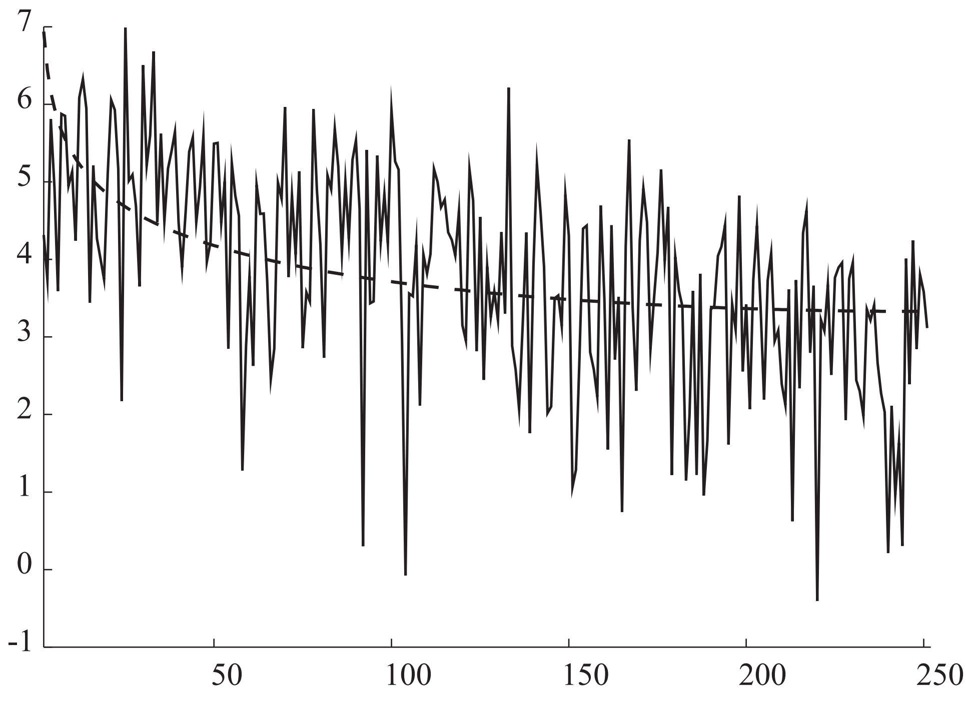

Figure 6. Actual and fitted log-periodogram, AR(1) model, $\phi = 0.75$