Does Democratization Benefit the Environment in the Long-Run in the Presence of Inequality?

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Abstract

Political economy may provide an important link between inequality and pollution. This paper studies the dynamic relationship between inequality and redistributive policy leading to differing transitional paths of pollution to the steady state, using a pollution-augmented framework developed by Bénabou and employing numerical simulations. The results indicate that democratization can be beneficial for the environment in the long run if the share of redistributive transfers devoted to abatement is relatively high. Otherwise, less wealth-biased and more democratic regimes display highest income and pollution levels, differing in transitional paths contingent on initial inequality levels. Sustainable development, defined as non-declining level of utility over time, is achieved for a high degree of democracy when initial inequality is low. The representative agent with average wealth does not provide sustainability, which emphasizes the importance of heterogeneity in power and income for sustainability debates.
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1 Introduction

Political and institutional factors are widely considered as the crucial deep determinants of economic development, playing a decisive role in the quest for the wealth and prosperity of nations.[1] An ever expanding amount of research on political economy, public choice, the role of constitutions, political systems, voting rules etc., is currently being done to better understand the interplay of political and economic forces. Political economy, in particular, is concerned with heterogeneous agents resolving their conflicts on economic issues with the aid of specified collective choice mechanisms (Drazen 2000, pp. 15-16). These conflicts often pertain to income or wealth redistribution and have been closely examined by the literature analyzing the relationship between inequality and economic growth, especially during the last two decades.[2] Perhaps surprisingly, there has not yet been much research effort devoted to environmental problems linking unequal power and resource distribution to environmental damage via the politico-economic channel, notable theoretical exceptions being Eriksson and Persson (2003) and Drosdowski (2005). Under the condition that pollution affects everyone equally, the former shows that democratization, which shifts the identity of the decisive voter to a poorer individual, provides her with higher marginal utility from consumption relative to marginal utility from better environmental quality, leading her to lowering of a pollution standard generating more aggregate pollution. In a perfect democracy, an exogenous inequality increase has similar implications. The latter paper obtains some opposite results: a poorer decisive individual opts for more progressive redistribution that can reduce aggregate consumption and provide active abatement, decreasing pollution. Marginally increased inequality in a perfect democracy also increases the progressivity of redistribution, leading to improved environmental quality in the same way. While earlier contributions addressed the issues in static frameworks, this paper is perhaps the first attempt to discuss some dynamic implications. Its main objective is obtaining an answer to the central question whether democratization can be beneficial for environmental quality in the long run. Here, we understand democratization as a reduction of the wealth bias between richer decisive voters and the median voter in the politico-economic setting. Furthermore, we try to determine conditions under which sustainable development could be achieved, concentrating on the role of democratization.

The paper draws upon Bénabou's (2000) OLG model providing a useful basis for the analysis. We employ its redistribution scheme, intergenerational wealth-transmission mechanism and dynamic feedback mechanism.[3]

[1] Seminal contributions are e.g. Acemoglu et al. (2001) and Hall and Jones (1999).

[2] A good survey is Bertola (1999).

[3] This article abstracts from numerous features found in Bénabou (2000) such as labor in the utility and production functions, deadweight loss from redistribution, measures of

Environmental aspects enter the model in the simple form of an aggregate polluting flow externality, causing equal disutility to every agent. Pollution is generated by the aggregate consumption of a single produced good and affected by a special form of technology financed by a constant share of redistributed funds. It can decrease pollution when taxation is progressive and exacerbate environmental quality when income is distributed in a regressive way. The former is to be interpreted as abatement, while the latter is an abstraction for polluting subsidies. There is no specified incentive-driven instrument of environmental policy involved. Rather, it could be conceived as a fixed command-and-control governmental measure having distributional consequences for the economy.[4] A consumption-based approach to pollution is taken, to reflect the notion that "consumption is the principal driving force behind environmental impact" (Rothman 1998, p. 178).
Each OLG-family uses only its own wealth endowment in production, because a credit market is assumed away. Individual incomes are redistributed by the government in a progressive or regressive fashion, depending on the political preferences of the decisive voter. A constant share of redistributed income is consumed, and the savings are devoted to enhance the child's inherited wealth endowment, which underlies an i.i.d. shock. Both consumption and bequest are utility-providing.
Due to myopic agents' behavior the bequests are weakly altruistic, as opposed to representative agents of infinite-lifetime dynastic models. As already mentioned, the political process, which is completed before individual utility maximization, reflects the preferences of the decisive agent taking pollution into account in his indirect utility function. The politically determined tax rate is an increasing function of inequality for every feasible wealth bias, which is a different result from Bénabou's U-shaped relationship.[5] Taxation is ceteris paribus declining in wealth-bias, meaning that individuals richer than the median opt for less progressivity of redistribution. Furthermore, pollution is a declining function of progressivity, i.e. more wealth-biased regimes are responsible for more pollution, or equivalently, democratization is good for the environment, all other things being equal.[6] These particular

risk aversion or technological shocks. Moreover, we do not analyze endogenous growth, restricting growth considerations to the familiar neoclassical setting without technological progress.

[4] A similar way of modeling environmental pollution has been developed in Drosdowski (2005), where pollution stems from the production sector and is abated with the aid of funds raised by wealth redistribution prior to productive activities. In the present paper however, income is redistributed.

[5] Figini (1999) provides empirical evidence for this alleged shape. However, Bassett et al. (1999) obtain a significantly positive relationship between inequality and the share of transfers in GDP in a cross-section of 13 OECD countries. Milanovic (2000) also unveils a positive relationship between inequality and redistribution.

[6] Drosdowski (2005) discusses the possibility of an inverted U-shaped relationship for a certain range of parameters.

comparative-statics results which have already been confirmed in earlier literature build the foundation of the long-run analysis.
The dynamics of the model are derived from the movement equation for individual wealth as well as the optimal tax function conditional on a given wealth bias. Concentrating on policies chosen by the median voter or richer decisive groups, poorer pivotal voters are excluded by parameter restrictions, in a way that the median voter chooses the highest feasible level of progressive redistribution, in order to correct the credit-market imperfection. The model's long-run solutions are obtained numerically, using empirically plausible parameters.
Initial inequality is shown to play a crucial role during the transition to the unique and stable steady state. In case of high initial inequality, surpassing all eventual stationary values, that differ across political regimes, inequality decreases over time, which is typical of neoclassical technology with diminishing returns to reproducible capital and very similar to the well-known σ-convergence. Decreasing wealth and income dispersion across individuals reduces the progressivity of redistribution. In turn, low initial inequality implies dynamics leading to increasing inequality and progressivity of taxation over time. These alternative scenarios have consequences for the model's other endogenous variables. In the high-inequality scenario the initial aggregate income level is high but very unequally distributed. In moderately or not biased political regimes, a strong income convergence effect leads to negative income growth during some early stages of development, surpassing the gains from redistribution relaxing the credit constraints of the poor. However, the strength of both effects is reversed over time, due to sustained progressive redistribution and slower convergence. Diminishing returns to investment then cause the growth to cease, and the steady state income ends up being below its initial level. Very wealth-biased regimes exacerbate the credit restrictions with regressive taxation and the economy shrinks over time. In the alternative scenario initial income is lower and it is more equally distributed. While inequality increases, the taxation becomes more progressive. Hence, the aggregate income rises and the economy converges to the zero-growth equilibrium. Since income growth is maximized by moderately wealth-biased regimes, they generate the highest long-term income.
Aggregate pollution is generated by consumption which is a constant share of output. Therefore, the intertemporal paths of pollution resemble the paths of income. For high initial inequality, baring initial effects, pollution increases over time in relatively democratic regimes, while decreasing in more wealth-biased regimes. In the case of low initial inequality, however, it is growing over time in all but the most biased regime. Hence, dynamically, inequality is positively correlated with pollution. Stationary pollution is low in wealth-oriented regimes promoting little income growth and relatively high in more democratic societies. While in these settings democracy does not perform well with respect to environmental degradation, the situation

is reversed when the share of redistributed incomes directed to abatement is significantly increased (by the factor ten). Then, in both scenarios, progressively redistributing and strongly democratic regimes reduce long-run pollution, while wealth-biased regimes increase it, since progressive taxation reduces income (and consumption) growth via higher abatement spending, which further reduces pollution. Regressive taxation yields opposite results. In the perfect democracy pollution falls by around one-third, while in the regime with the highest wealth bias it rises by the same proportion. In each time-period pollution is higher the more wealth bias exists. The transitional paths differ with respect to initial inequality. In the benchmark case the pollution generated in perfect democracy decreases, as opposed to the alternative scenario where pollution increases (from a lower initial level).

The examination of the economy's transitional behavior indicates that democratic societies are arguably more effective in the provision of better quality of life for future generations. The sustainability criterion used in the paper is non-declining utility over time. This condition is not satisfied in any of the cases pertaining to high initial inequality. Conversely, low initial inequality entails cases in which sustainability can be attained. Merely scratching the surface of the issue by analyzing the intertemporal welfare levels of the decisive individuals alone would tempt to conclude that for low initial inequality all regimes, including the all-familiar representative individual, can achieve this elusive goal. However, an analysis of the repercussions of their preferred policies for the utilities of several agents located in different parts of the wealth distribution reduces the set of possible sustainable regimes. Taking wealth heterogeneity into account strengthens the point that only perfect democracy or moderately wealth-biased regimes can be credited for sustainable policies. The representative individual with average wealth, does not guarantee sustainability in the present framework.[7] This finding may put a caveat on discussing sustainability without reflecting social heterogeneities.

The paper is organized as follows. Section 2 describes the structure of the economy and the economic decisions made in its sectors. Section 3 introduces the environmental problem, while section 4 is devoted to political economy. In section 5 the characteristics of the steady state are discussed. Section 6 contains the numerical calculations yielding the paper's main results. Section 7 concludes.

[7] This paper is not concerned with normative choices of a utilitarian social planner. Its ethical core is intergenerational fairness.

2 Economy

2.1 Basic structure

The economy consists of a continuum of overlapping-generations agents/families denoted by i ($i \in [0,1]$) and differing in their wealth endowments w_{it}. Each one produces output y_{it} using the same technology, as well as her individual wealth and one unit of labor. A credit market which could alleviate wealth inequality is absent. Production generates income, which is redistributed by the government. The after-tax income υ_{it} is then allocated between consumption c_{it} and investment k_{it} contributing to the wealth of the next generation, provided by the parent's bequest. Aggregate consumption generates pollution that reduces individual lifetime well-being. A fraction of the redistributed incomes is used for a public abatement technology or polluting subsidies which affect the polluting externality, whose net level is denoted as X_t. Prior to consumption-saving decisions the households determine the degree of redistribution τ. Finally, there is no population growth.

2.2 Households

Member i of generation t derives utility U_{it} from consumption and discounted expected wealth $E[w_{it+1}]$ of his offspring, as well as disutility from the polluting externality:[8]

$$U_{it} = \ln c_{it} + \rho \ln E[w_{it+1}] - \ln X_t. \tag{1}$$

ρ is a positive discount factor, not exceeding unity, and

$$w_{it+1} = \kappa \epsilon_{it+1} w_{it}^{\delta} k_{it}^{\nu}, \tag{2}$$

with κ, δ and ν as positive constants. ϵ is an i.i.d. intergenerational shock with $\ln \epsilon_{it} \sim \mathcal{N}(-\sigma^2/2, \sigma^2)$.[9] The wealth of an individual in generation $t+1$ is thus a part of her parent's wealth supplemented by the parent's additional investment. The individual budget restriction reads as follows:

$$\upsilon_{it} = c_{it} + k_{it}. \tag{3}$$

[8]Note that labor is not included in the utility function in order to simplify the analysis. In addition, we set the relative risk aversion parameter from Bénabou (2000, p. 102) to zero, which does not matter in the analysis, since we are not interested in the insurance aspects of redistribution.

[9]Its mean is one, because $\ln \epsilon = -\sigma^2/2 + \sigma^2/2 = 0$. Hence, $\epsilon = e^0 = 1$. The uninsurable idiosyncratic shock simply represents luck.

2.3 Production

Individual output is generated according to production function

$$y_{it} = Aw_{it}^{\beta}. \tag{4}$$

A is a constant productivity parameter, while β is a positive production elasticity not exceeding unity. Hence, it is the standard neoclassical production function with diminishing returns to its accumulable input, satisfying Inada conditions. Every firm uses one unit of labor in production, which is therefore suppressed in the notation.

2.4 Intertemporal utility maximization

Households maximize their lifetime utility in the second stage of the decision-making process by their choice of optimal consumption, given the redistribution rate τ which is determined in the first stage. The solution is obtained by backward induction. The individual utility maximization is subject to budget restriction (3). After the substitution of equation (2) the problem reads:

$$\max_{c_{it}} \quad \ln c_{it} + \rho \ln E[\kappa \epsilon_{it+1} w_{it}^{\delta} k_{it}^{\nu}] - \ln X_t$$

$$\text{s.t. } k_{it} = \upsilon_{it} - c_{it}.$$

We obtain the first order condition

$$c_{it}^{-1} - \rho\nu \left[\upsilon_{it} - c_{it}\right]^{-1} = 0,$$

which, solved for optimal consumption yields:

$$c_{it} = \frac{1}{1+\rho\nu}\upsilon_{it}. \tag{5}$$

Optimal consumption is then a constant fraction of redistributed (disposable) income. Substituting (5) in the budget restriction gives the expression for optimal saving:

$$k_{it} = \frac{\rho\nu}{1+\rho\nu}\upsilon_{it} \equiv s\upsilon_{it}. \tag{6}$$

s is a constant fraction of disposable income saved for investment, which is independent from redistribution. Consequently, optimal consumption is the fraction $(1-s)$ of the same income.

2.5 Government

The government redistributes incomes according to the following scheme:

$$y_t = \int_0^1 y_{it} di = E[(y_{it})^{1-\tau_t}]\tilde{y}_t^{\tau_t} = \underbrace{E[(y_{it})^{1-\tau_t}]\tilde{y}_t^{(1-\alpha)\tau_t}}_{\upsilon_t} B_t. \tag{7}$$

y_t is the aggregate income that equals the average income due to normalization. $\tilde{y}_t$ denotes the break-even level of income, i.e. the level of income not affected by redistribution, and α is the fraction of transfers used for environmental purposes. The parameter can be interpreted as an expression of public preferences for environmental policy or, as Pearce and Turner (1990, p. 237) note, community-regarding values reflected in the political process, not necessarily for economic reasons. It follows from the equation that the expected pre-tax income must exactly match the expected post-tax income, in order to satisfy the balanced-budget condition. The redistribution rate τ is restricted to values not exceeding unity.[10] Taxation is progressive for positive and regressive for negative values. The funds devoted to environmental policy are then:

$$B_t = \tilde{y}_{it}^{\alpha\tau_t}. \tag{8}$$

This fraction of government transfers is used for abatement in case of progressive taxation and polluting subsidies in case of regressive redistribution.[11]

2.6 Distribution and growth

The wealth distribution is lognormal, which means that the logarithms of individual wealth levels are normally distributed:

$$\ln w_{it} \sim \mathcal{N}(m_t, \Delta_t^2). \tag{9}$$

m indicates the median wealth level and Δ^2 is the variance of the distribution, a variable corresponding to the extent of wealth inequality. From the properties of lognormal distribution it follows that $\ln w_t = m_t + \Delta_t^2/2$ and $\ln \tilde{w}_t = m_t + (2 - \tau_t)\Delta_t^2/2$. Wealth is normalized to unity, hence w is the average/aggregate level.[12] $\tilde{w}$ is the break-even level of wealth. The income distribution is also lognormal and can be expressed using the parameters of wealth distribution. Taking the aggregate production function $y_t = Aw_t^\beta$ into account, income is distributed as follows:

$$\ln y_{it} \sim \mathcal{N}(\ln A + \beta m_t, \beta^2\Delta_t^2), \tag{10}$$

with $\ln y_t = \ln A + \beta m_t + \beta^2\Delta_t^2/2$ and $\ln \tilde{y}_t = \ln A + \beta m_t + (2 - \tau_t)\beta^2\Delta_t^2/2$. The growth rate of income is defined as[13]

$$g(\tau) \equiv \ln\left(\frac{y_{t+1}}{y_t}\right) = \beta\left[(m_{t+1} - m_t) + \beta(\Delta_{t+1}^2 - \Delta_t^2)/2\right]. \tag{11}$$

Taking logs in equation (2) and then the expected values, substituting k from equation (6) and using the fact that $\upsilon_i = y_i^{1-\tau}\tilde{y}^{(1-\alpha)\tau}$ it is possible to

[10] Later, we also restrict τ to be bigger than -1 in the steady state.

[11] Drosdowski (2005) provides the rationale for this formulation of environmental policy.

[12] $E[w_{it}] = w_t = \int_0^1 w_{it} di$.

[13] $\ln\left(\frac{y_{t+1}}{y_t}\right)$ is approximately equal to the usual growth rate $\frac{y_{t+1}-y_t}{y_t}$.

obtain expressions for m_{t+1} and Δ^2_{t+1} that constitute a difference-equation system:

$$m_{t+1} = \ln \kappa s^\nu - \sigma^2/2 + \nu(1-\alpha\tau_t)\ln A + [\delta + \nu\beta(1-\alpha\tau_t)]m_t \\ +(1-\alpha)\nu\beta^2\tau_t(2-\tau_t)\Delta_t^2/2, \quad (12)$$

$$\Delta^2_{t+1} = [\delta + \nu\beta(1-\tau_t)]^2\Delta_t^2 + \sigma^2. \quad (13)$$

The growth rate is then

$$g(\tau) = \beta\ln \kappa s^\nu - \beta(1-\beta)\sigma^2/2 + \nu\beta(1-\alpha\tau_t)\ln A + [\delta + \nu\beta(1-\alpha\tau_t) - 1]m_t \\ +\beta^2[(1-\alpha)\nu\beta\tau_t(2-\tau_t) + [\delta + \nu\beta(1-\tau_t)]^2 - 1]\Delta_t^2/2. \quad (14)$$

It can be equivalently expressed as

$$g(\tau) = const - (1-\delta-\nu\beta)y_t \\ -\nu\beta\ln B_t - \left[\delta + \nu\beta(1-\tau_t)^2 - (\delta + \nu\beta(1-\tau_t))^2\right]\beta^2\Delta^2,$$

the constant being $\ln \kappa s^\nu - \sigma^2/2 + (1-\delta)\ln A$. The growth rate is similar to Bénabou's (2000, p. 104), except for the presence of abatement (polluting subsidies) in the equation, that exert a negative (positive) influence on growth in case of progressive (regressive) taxation. The expression before y_t is the convergence effect, whereas the last term represents a growth loss due to the missing credit market, which is U-shaped in redistribution. Differentiating (14) with respect to τ and solving the first order condition yields the growth-maximizing tax rate τ^g for $1 > \alpha + \nu\beta$:

$$\tau^g = 1 - \frac{\alpha(\beta m_t + \ln A) + \delta\beta^2\Delta_t^2}{(1-\alpha-\nu\beta)\beta^2\Delta_t^2}. \quad (15)$$

The second derivative of $g(\tau)$ with respect to τ is negative, which is the sufficient condition for a maximum. Growth is maximized when the marginal gain from relaxing the credit restrictions is equal to the marginal gain from abatement.

3 Environment

The polluting externality is generated by the aggregate consumption of the produced output. Net pollution X perceived by the agents, which enters their utility functions, is the pollution diminished by abatement or increased by polluting subsidies. It is a flow, for it does not accumulate over time. An example for this kind of short-lived environmental problems could be smog stemming from car engines that can be reduced by appropriate devices or augmented by subsidizing transportation. The logarithmic measure of net pollution is then:

$$\ln X_t = \gamma \ln c_t - \mu \ln B_t, \quad (16)$$

with $c = \int_0^1 c_i di = (1-s)v = (1-s)y/B$. γ is a parameter indicating the strength of the polluting activity, whereas μ is the strength of the abatement technology or the impact of polluting subsidies. Both parameters are positive. Substituting the logarithmic expressions for B, y and $\tilde{y}$, obtained previously, in equation (16) and rearranging terms leads to the following function:

$$\begin{aligned} \ln X_t(\tau) &= \gamma \ln(1-s) + [\gamma - \alpha\tau_t(\gamma+\mu)](\beta m_t + \ln A) \\ &+ [\gamma - \alpha\tau_t(2-\tau_t)(\gamma+\mu)]\beta^2\Delta_t^2/2. \end{aligned} \tag{17}$$

To examine how pollution depends on redistribution it is necessary to differentiate the function twice with respect to τ:

$$\begin{aligned} \ln X_t{}'(\tau) &= -\alpha(\gamma+\mu)[\beta m_t + \ln A + (1-\tau_t)\beta^2\Delta_t^2], \\ \ln X_t{}''(\tau) &= \alpha(\gamma+\mu)\beta^2\Delta_t^2. \end{aligned} \tag{18}$$

The first derivative is negative for every feasible tax rate, while the second derivative is positive. Hence log-pollution is always declining with progressivity of redistribution and there is no pollution-maximizing tax rate within the feasible range of redistribution.[14] The reason is the ambiguous influence of environmental policy on the pollution level. It is easily seen in the logarithmic expression derived from (8):

$$\ln B_t = \alpha\tau_t \left[\ln A + \beta m_t + (2-\tau_t)\beta^2\Delta^2/2\right].$$

The expression is positive for progressive taxation, zero for an absence of redistribution and negative for regressive taxation. Thus, more progressive taxation reduces polluting subsidies and eventually generates funds needed to reduce pollution, that also reduce economic growth (see the previous subsection). This trade-off between environmental quality and growth is maintained for regressive taxation: pollution is higher and so is the income growth rate. The growth rate of net pollution $p(\tau)$ is defined as follows:

$$\begin{aligned} p(\tau) &\equiv \ln\left(\frac{X_{t+1}}{X_t}\right) \\ &= \alpha(\gamma+\mu)[\tau_t - \tau_{t+1}]\ln A \\ &+ [\gamma - \alpha\tau_{t+1}(\gamma+\mu)]\beta m_{t+1} - [\gamma - \alpha\tau_t(\gamma+\mu)]\beta m_t \\ &+ [\gamma - \alpha\tau_{t+1}(2-\tau_{t+1})(\gamma+\mu)]\beta^2\Delta_{t+1}^2/2 - [\gamma - \alpha\tau_t(2-\tau_t)(\gamma+\mu)]\beta^2\Delta_t^2/2. \end{aligned} \tag{19}$$

which is a very complicated expression. Even for a time-invariable tax rate its derivative with respect to τ contains cubic terms, indicating that there

[14] A pollution minimum would be achieved at $\tau^X = 1 + \frac{\ln A + \beta m_t}{\beta^2\Delta^2}$, which exceeds the feasible bound of full expropriation.

are three solutions, which are not necessarily real. Therefore, we are not able to analyze the path of pollution growth depending on redistribution at this stage. Further discussion requires numerical calculations which will be exercised later.

4 Political economy

Knowing how income growth and pollution level depend on redistribution, the next logical step of the analysis is its actual politico-economic determination. Substituting the optimal expressions for c_{it} and k_{it} in the individual utility function gives the indirect utility function of the individual i:[15]

$$U_{it}(\tau) = \ln c_{it} + \rho \ln E[\kappa \epsilon_{it+1} w_{it}^{\delta} k_{it}^{\nu}] - \ln X_t, \tag{20}$$

which, after some simple algebra, can be expressed as:

$$\begin{aligned} U_{it}(\tau) = \overline{u}_t + [\rho\delta + (1+\rho\nu)\beta(1-\tau_t)](\ln w_{it} - m_t) \\ -(1+\rho\nu)(1-\tau_t)^2\beta^2\Delta_t^2/2 \\ -\alpha\tau_t[1+\rho\nu-\gamma-\mu](\ln A + \beta m_t + (2-\tau_t)\beta^2\Delta_t^2/2), \end{aligned} \tag{21}$$

with

$$\overline{u}_t = (1-\gamma)\ln(1-s) + \rho\ln\kappa s^{\nu} + \rho\delta m_t + (1+\rho\nu-\gamma)(\ln A + \beta m_t + \beta^2\Delta_t^2/2),$$

independent from redistribution. The next two terms can be interpreted as in Bénabou (2000, pp. 104-5). The first one represents the redistributive effects of taxation and contains the intergenerational wealth persistence $\delta + \nu\beta(1-\tau)$, which declines with progressivity, while the second one is the aggregate welfare loss from inequality, diminishing with progressive taxation, reallocating funds to the poor with higher returns to investment. The last term, equivalent to $[1+\rho\nu-\gamma-\mu]\ln B$, captures the counteracting effects of environmental policy on individual welfare: it can decrease utility from consumption, while increasing welfare from reduced pollution, and vice versa, depending on the progressivity of taxation and the sign of the expression in square brackets.
Maximization of the indirect utility function yields the first-order condition:

$$\begin{aligned} -(1+\rho\nu)\beta(\ln w_{it} - m_t) + (1+\rho\nu)(1-\tau_t)\beta^2\Delta_t^2 \\ -\alpha[1+\rho\nu-\gamma-\mu](\ln A + \beta m_t + (1-\tau_t)\beta^2\Delta_t^2) = 0 \end{aligned} \tag{22}$$

[15]In the voting process the individuals take into account pollution, for they can influence its level if they are the decisive ones. They have no incentive to care about pollution in their consumption-saving decisions, however.

that can be solved for the individually optimal redistribution rate, τ^i, balancing the positive and negative marginal welfare effects of redistribution and abatement:

$$\tau^i = 1 - \frac{\alpha[1+\rho\nu-\gamma-\mu](\ln A + \beta m_t) + (1+\rho\nu)\beta(\ln w_{it} - m_t)}{[1+\rho\nu-\alpha(1+\rho\nu-\gamma-\mu)]\beta^2\Delta_t^2}. \qquad (23)$$

The second derivative is

$$U_{it}''(\tau) = -[1+\rho\nu-\alpha(1+\rho\nu-\gamma-\mu)]\beta^2\Delta_t^2,$$

which is negative for $1 > \alpha[1+\rho\nu-\gamma-\mu]-\rho\nu$. Hence, τ^i is a maximum of a strictly concave function. The optimal tax rate depends on the distance between the decisive individual's wealth $\ln w_i$ and the median wealth m.[16] Since the denominator in (23) is positive, greater power inequality($\ln w_i - m$) makes the fraction bigger and the optimal tax rate decreases. Using the cumulated probability function of the standard normal distribution it is possible to express the wealth bias in a more convenient way:

$$prob = \Phi\left(\frac{\ln w_i - m}{\Delta}\right).$$

$prob$ is equal to the probability attached to a given $\ln w_i$ in the standard normal distribution. By inversion the function is transformed into λ, being the wealth bias normalized by Δ:

$$\lambda \equiv \Phi^{-1}(prob) = \frac{\ln w_i - m}{\Delta}.$$

Then, $\ln w_i = m + \lambda\Delta$.
Substituting λ in (23) the optimal redistribution rate τ^λ is obtained:

$$\tau^\lambda = 1 - \frac{\alpha[1+\rho\nu-\gamma-\mu](\ln A + \beta m_t) + (1+\rho\nu)\beta\lambda\Delta_t}{[1+\rho\nu-\alpha(1+\rho\nu-\gamma-\mu)]\beta^2\Delta_t^2}. \qquad (24)$$

λ is a parameter reflecting the influence of wealth in the politico-economic process, which can be described as a democracy distortion. The reasons may be voluntary (discouragement of voters refusing political participation, educational deficits etc.) or involuntary in nature (dictatorship, vote-buying, wealth restrictions to voting; see Drosdowski (2005) for a discussion. With a positive λ the politically decisive individual or group is richer than the median voter with the wealth level corresponding to m and $\lambda = 0$.[17] In

[16] m is the mean of the normal distribution of log-wealth and corresponds to the median of the lognormal distribution given by e^m. Conversely, $\ln w = m + \Delta^2/2$ is the natural logarithm of the lognormal distribution's mean.

[17] Bénabou (1996) discusses a theoretical possibility of populist bias or left-extremism for $\lambda < 0$. Although we discuss the negative lower bound of λ at this stage, we will abstain from these cases later in the analysis.

order to ensure that the tax rates are between -1 and 1, the bounds for feasible λ-values are given by:

$$\underline{\lambda} = -\frac{\alpha[1+\rho\nu-\gamma-\mu](\ln A+\beta m)}{(1+\rho\nu)\beta\Delta},$$

$$\overline{\lambda} = \frac{2[1+\rho\nu-\alpha(1+\rho\nu-\gamma-\mu)]\beta^2\Delta_t^2-\alpha[1+\rho\nu-\gamma-\mu](\ln A+\beta m)}{(1+\rho\nu)\beta\Delta}.$$

Thus, $\underline{\lambda} < \lambda < \bar{\lambda}$, under the condition that $1+\rho\nu-\gamma-\mu$ is non-negative. The optimal tax rate varies with inequality ceteris paribus in the following way:

$$\frac{\partial\tau^\lambda}{\partial\Delta} = \frac{2\alpha[1+\rho\nu-\gamma-\mu](\ln A+\beta m)+(1+\rho\nu)\beta\lambda\Delta}{[1+\rho\nu-\alpha(1+\rho\nu-\gamma-\mu)]\beta^2\Delta^3}. \quad (25)$$

The first partial derivative is unambiguously positive for every feasible degree of wealth bias, i.e. optimal redistribution for every social group within the given bounds for λ becomes progressive with rising inequality. A formal proof is included in appendix A1. The second partial derivative is negative, indicating that τ is concave in Δ. The reason why higher disparity of wealth always leads to higher progressivity of redistribution is the fact that welfare-maximizing agents derive utility from the expected (aggregate) bequests. Neglecting the implications of environmental policy for welfare, we see that higher inequality reduces the wealth of the next generation, making a resource reallocation to the poor more attractive. Moreover, a more equal distribution of individual consumptions increases average welfare due to the concavity of utility function.[18]

The median voter's preferred policy τ^m is given by:

$$\tau^m = 1-\frac{\alpha[1+\rho\nu-\gamma-\mu](\ln A+\beta m_t)}{[1+\rho\nu-\alpha(1+\rho\nu-\gamma-\mu)]\beta^2\Delta_t^2}. \quad (26)$$

Since τ^m is a unique internal solution, the conditions of the median-voter theorem are satisfied. The growth-maximizing tax rate τ^g is always below the rate preferred by the median voter, which leads to the conclusion that a perfect democracy is characterized by low pollution, since it declines with democratization as discussed in section 3, and suboptimal growth, because the marginal growth losses from abatement exceed the gains from redistribution.[19] In order to determine the redistribution rate that maximizes the welfare of the economy's representative individual with the average wealth, $\ln w = m+\Delta^2/2$, we simply replace λ in equation (24) by $\Delta/2$ and obtain:[20]

$$\tau^R = 1-\frac{\alpha[1+\rho\nu-\gamma-\mu](\ln A+\beta m_t)+(1+\rho\nu)\beta\Delta_t^2/2}{[1+\rho\nu-\alpha(1+\rho\nu-\gamma-\mu)]\beta^2\Delta_t^2}. \quad (27)$$

[18]Bénabou (2000, p. 105) discusses these effects among others. However, in his paper the inequality-redistribution relationship is U-shaped due to various distortions excluded from present work.

[19]The simple condition for $\tau^g < \tau^m$ is $1 > \nu\beta$, which is proved in appendix A2.

[20]Setting $\ln w_i = m+\lambda\Delta$ equal to $\ln w = m+\Delta^2/2$ yields this result.

As noted earlier, empirical research indicates that an analysis of regimes with decisive political decision-makers poorer than the median voter is not supported by data. To reflect its absence in the real world and make the subsequent analysis more convenient, we exclude the cases of $\lambda < 0$ and assume from now on that $1+\rho\nu-\gamma-\mu = 0$. In this way, environmental factors and the state variable m_t do not influence optimal taxation, which also simplifies the model's dynamics. However, the environment is still influenced by endogenously determined redistribution, and environmental policy has an impact on other endogenously determined variables. This simplification does not change any of the qualitative results from numerical simulations (section 6), which would be obtained otherwise. Consequently, the expression for a general politically determined tax rate, (24), reduces to:

$$\tau^{\lambda} = 1 - \frac{\lambda}{\beta \Delta_t}. \tag{28}$$

The median voter now chooses the full expropriation of the produced income ($\tau^m = 1$ in equation (26)), which assures that no populist regimes wishing higher progressivity will be analyzed within the framework, and we can concentrate exclusively on democracy imperfections stemming from positive wealth bias in relation to a perfect democracy. Feasible λ then becomes bounded by zero from below, with the upper bound being:

$$\bar{\lambda} = 2\beta\Delta_t. \tag{29}$$

Since the median voter is assumed to choose the highest feasible degree of redistribution, he will not increase the progressivity of taxation facing marginal inequality increases.[21] The optimal tax rate of the average individual now simplifies to:

$$\tau^{R} = 1 - \frac{1}{2\beta}. \tag{30}$$

It is time-invariant, which means that the representative individual does not wish to change his preferred policy when a mean-preserving spread in the wealth distribution takes place. Similar to the general tax rate from (28), it depends positively on the production elasticity of wealth, β, responsible for the concavity of the production function, and thus for the degree of the poor's liquidity constraints. The higher the parameter the higher is c.p. the attractiveness of redistribution in order to generate growth and welfare.

[21] The right-hand side of equation (25) becomes zero. See also the proof in appendix A1.

5 Steady state

Following Bénabou (2000) we concentrate on the recursive dynamical system exhibiting the joint behavior of policy and inequality:[22]

$$\begin{cases} \tau_t = T(\Delta_t) \\ \Delta_{t+1} = \mathcal{D}(\Delta_t, \tau_t). \end{cases}$$

$T(\Delta_t)$ is a concave function given by (28) and $\mathcal{D}(\Delta_t, \tau_t)$ by (13). In the long run it holds true that $\Delta_{t+1} = \Delta_t = \Delta_\infty$ and from (13) we obtain:

$$\Delta_\infty^2 = \frac{\sigma^2}{1 - [\delta + \nu\beta(1-\tau)]^2} \equiv \mathcal{D}^2(\tau). \tag{31}$$

In the steady state with an invariable tax rate τ the inequality locus $\Delta = \mathcal{D}(\tau)$ intersects with the concave tax function $\tau = T(\Delta)$. The transition to the steady state occurs along the $T(\Delta)$-curve representing the political mechanism for a given wealth-bias. Δ is determined by squaring the equation (31):

$$\mathcal{D}(\tau) = \frac{\sigma}{\sqrt{1 - [\delta + \nu\beta(1-\tau)]^2}}. \tag{32}$$

$\mathcal{D}(\tau)$ is a hyperbolic function declining in τ. Figure 1 illustrates the steady-state equilibrium in the range of feasible redistribution rates marked by dashed lines. The horizontal axis represents Δ, while the vertical axis represents τ. However, plugging the expressions (28) and (32) together indicates

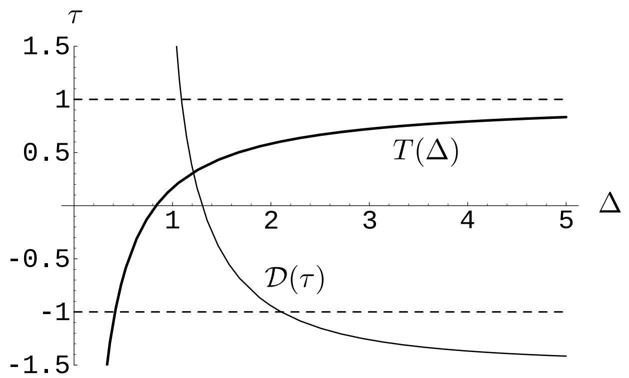

Figure 1: Political long-run equilibrium

that there may be multiple steady-state equilibria:

$$\tau = 1 - \frac{\lambda}{\beta\mathcal{D}(\tau)} = 1 - \frac{\sigma\lambda}{\beta\sqrt{1 - [\delta + \nu\beta(1-\tau)]^2}}, \tag{33}$$

[22] The simplifications undertaken in the previous section allow us to suppress the state variable m_t and make the analysis much easier, without loss of generality.

thus multiple equilibrium values for τ and Δ.[23] To address the issue of uniqueness we first determine the range of inequality values for which $\mathcal{D}$ crosses the range of feasible taxes:

$$\frac{\sigma}{\sqrt{1-\delta^2}} < \Delta < \frac{\sigma}{\sqrt{1-[\delta+2\nu\beta]^2}}, \tag{34}$$

with $1 > [\delta + 2\nu\beta]^2 > \delta^2$. Defining the left fraction from (34) as $\underline{\Delta}$ and the right one as $\overline{\Delta}$ it is possible to determine the threshold value for λ from (29), below which the feasible steady state equilibrium is unique. It is stable, because the curve $\Delta = \mathcal{D}(\tau)$ cuts the curve $\tau = T(\Delta)$ from above, which is proved in Bénabou (2000, pp. 122-123).

6 Dynamic behavior of the economy

In the steady-state equilibrium the growth rates of pollution and income given by equations (11) and (19) are zero, because the differences in median wealth, inequality and redistribution disappear over time. It is easy to determine the stationary levels of logarithmic pollution and income, plugging the stationary values of the relevant variables into suitable equations. In this section, we also discuss the transitional paths of the endogenous variables, including those of both growth rates, their differences and the individual utility levels. Although both growth rates eventually converge to zero, there is a possibility that the pollution growth rate is always higher or lower than the income growth rate. The former would mean the ecological burdens affecting future generations always increase more than their economic well-being and would be a strong hint for unsustainable development. Fortunately, there is a much better way of tracing (un)sustainability within the model, by comparing the respective utility paths over time. Accepting a scenario with (even temporarily) declining utility as unsustainable and irresponsible towards those born in the distant future, allows to ask which policies and, ultimately, which degrees of democracy imperfection would be sufficient to avoid this unwanted development. Therefore, we employ numerical simulations to compare the time-paths in question. We calibrate the model using the following parameters and initial values. β, representing the share of capital (physical and human) is set to 0.6, close to the standard value of two-thirds from Mankiw et al. (1992). The discount factor we choose is $\rho = 0.5$, corresponding to around 0.98 per year in a generation of 30 years and around 0.97 for 25 years. Bénabou (2002, p. 498) reports estimates of intergenerational persistence $(\delta + \nu\beta(1-\tau))$ in the range from 0.3 to 0.6. By setting $\delta = \nu = 0.4$, we allow it to be between 0.4 and 0.88 in the range of feasible taxes. Since the analysis is restricted to $1 + \rho\nu - \gamma - \mu = 0$, $\gamma + \mu$

[23]Some intersections of both loci may occur beyond the range of feasible taxes and positive inequality-values. We are interested in a feasible and unique solution, though.

must equal 1.2, and we set $\gamma = 0.4, \mu = 0.8$. Following Bénabou (2002), $\sigma = 1$, and the feasible range of steady-state inequality, which is calculated with the aid of equation (34), is $[1.09, 2.11]$. Thus, high initial inequality will be $\Delta_0 = 2$ ("benchmark value"), while the low initial value will be one. The share of funds devoted to abatement, α is set to 0.01, reflecting the relatively low abatement expenditure in reality[24], especially when a limited number of flow pollutants is considered. For the sake of comparison we will also employ an alternative value of $\alpha = 0.1$ later. Finally, we arbitrarily set $m_0 = 2$, $A = 2.5$ and $\kappa = 5$. The latter two are scale parameters, chosen to obtain positive growth rates.

6.1 Inequality and redistribution

The initial rate of redistribution is determined endogenously given the initial inequality and the chosen degree of wealth bias, which remains constant over time. The dynamic feedback mechanism (see section 5) allows the determination of inequality in the next period, and so forth. Additionally, we are able to determine the thresholds assuring the uniqueness of the equilibrium. From (29) we know that the wealth bias must not exceed 2.53 in the long-run equilibrium.[25] Table 1 illustrates the evolution of redistribution towards the steady state for differing degrees of wealth bias.[26]

		t : 0	1	2	3	10	∞
λ :	0	1	1	1	1	1	1
	0.5	0.58	0.41	0.34	0.32	0.31	0.31
	1	0.17	-0.07	-0.16	-0.19	-0.21	-0.21
	1.5	-0.25	-0.45	-0.53	-0.56	-0.57	-0.57
	2	-0.67	-0.77	-0.8	-0.82	-0.82	-0.82

Table 1: Time paths of redistribution depending on wealth bias

The median voter always redistributes the entire income, due to assumptions made in section 4, and richer decisive voters enforce less progressive redistribution in each period. The time patterns of redistribution indicate that inequality must be declining over time, because redistribution becomes less progressive, which is confirmed in Table 2, with the exception of perfect democracy with constantly high redistribution. For the wealth bias corresponding to one standard deviation ($\lambda = 1$), the initial taxation is

[24] See Jaffe et al. (1995).

[25] Therefore, the highest degree of wealth bias in the numerical analysis will be $\lambda = 2$.

[26] The steady-state values indicated by the time point "∞" have been actually obtained in the 30th period. Since nothing has changed since the 10th period, we maintain the infinity symbol in the tables.

slightly progressive, only to turn regressive in the next period and henceforth. Regimes with an even higher degree of wealth-bias choose regressive taxation from the beginning, and it becomes even more regressive over time, as inequality falls. While it is entirely plausible that decreasing inequality reduces the progressivity via the political mechanism, the reason for decreasing inequality, especially for regressive redistribution, is more intricate. Equation (13) shows the evolution of inequality over time, reflecting the accumulation mechanism of the economy. Given the initial inequality, its subsequent level positively depends on the variance of the idiosyncratic shock σ^2 and the term in square brackets $\delta + \nu\beta(1 - \tau)$, representing the intergenerational persistence of family wealth, or an inverse measure of social mobility (see Bénabou 2002, p. 486). The parameters indicate respectively the strength of the basic wealth bequest, or family background (δ), and diminishing returns applying to complementary parental investment ($\nu\beta$), which is diminished by progressive taxation, relaxing the liquidity constraints of the poor. Following this logic, increasingly regressive taxation should at first glance increase the wealth persistence and thus the variability of wealth in the next generation. However, production has an equalizing effect due to the concavity of the production function, hence output is less dispersed than wealth, with variance $\beta^2\Delta^2$. For high initial inequality this effect is particularly pervasive. Even though incomes are redistributed regressively, complementary bequests k_i, which are a constant share of redistributed income, are more equally distributed in the next period, and the wealth inequality decreases, despite the inequality-enhancing presence of idiosyncratic shock and family bequests. Thus, the dynamics of the model imply that an initial inequality above the steady state level, which is given in our basic scenario, must fall over time, in the presence of a concave neoclassical technology.[27] Since the discussed term is always smaller than one, the dispersion of wealth and incomes adjusts towards the steady-state value, in a way resembling the standard $\sigma-$convergence, as demonstrated in Barro and Sala-i-Martin (2005, pp. 50-1). For a persistence term equal to one, inequality would increase without a bound. Progressive taxation increases social mobility and the speed of inequality-adjustment, while regressive taxation works in the opposite direction. In democratic environments, relatively high initial inequality requires stark political responses that radically diminish wealth persistence and relocate capital to the poor, which stimulates growth and reduces inequality along the transitional path. Note that in perfect democracy it first decreases by 0.72, corresponding to 36%. The subsequent decreases over time are more moderate. The adjustment to the steady-state values is relatively quick and smooth - the stationary redistribution is reached already in the tenth period and the values obtained after only 3 periods are very close to the steady-state

[27] A formal proof is found in appendix A3.

ones.[28]

		t: 0	1	2	3	10	∞
λ:	0	2	1.28	1.12	1.1	1.09	1.09
	0.5	2	1.41	1.26	1.22	1.21	1.21
	1	2	1.56	1.43	1.4	1.38	1.38
	1.5	2	1.72	1.63	1.6	1.59	1.59
	2	2	1.89	1.85	1.84	1.83	1.83

Table 2: Time paths of inequality depending on wealth bias

Tables 3 and 4 show that the results change when initial inequality is below its steady-state level ($\Delta_0 = 1$).

		t: 0	1	2	3	10	∞
λ:	0	1	1	1	1	1	1
	0.5	0.17	0.29	0.31	0.31	0.31	0.31
	1	-0.67	-0.3	-0.23	-0.21	-0.21	-0.21
	1.5	-1.5	-0.77	-0.63	-0.59	-0.57	-0.57
	2	-2.3	-1.13	-0.91	-0.85	-0.82	-0.82

Table 3: Time paths of redistribution depending on wealth bias for $\Delta_0 = 1$

		t: 0	1	2	3	10	∞
λ:	0	1	1.08	1.09	1.09	1.09	1.09
	0.5	1	1.17	1.2	1.21	1.21	1.21
	1	1	1.28	1.35	1.37	1.37	1.38
	1.5	1	1.41	1.54	1.57	1.59	1.59
	2	1	1.56	1.74	1.8	1.83	1.83

Table 4: Time paths of inequality depending on wealth bias for $\Delta_0 = 1$

While the stationary values in both tables are identical with the benchmark case, there are some significant differences with respect to the transitional paths of both variables. An inequality level below its long-run value reverses the direction of the time-paths with respect to the benchmark case. Lower initial inequality calls for much lower initial degree of progressivity

[28]Usually, a period's length in OLG models is assumed to be somewhere between 25 (e.g. in Bénabou (2002, p. 499)) and 30 years (see De la Croix and Michel (2002, p. 338)).

in the presence of political wealth influence. The three bottom rows indicate highly regressive redistribution in the initial period, that depresses growth due to wealth constraints and increases intergenerational persistence of wealth. Hence, inequality increases and taxation becomes less regressive in the course of time. Inequality slightly increases even in a perfect democracy with the most equalizing policy available. It occurs, because the equalizing reallocation of income towards the poor, proportional to the low inequality-level, is outweighed by the persistence of family wealth and the magnitude of the shock. Again, the reason is the neoclassical production technology. In case of low initial inequality, incomes are much more equally distributed than wealth, but the difference between variances is not substantial. A highly equalizing progressive redistribution is not sufficient to push inequality below the initial level, which coincides with the magnitude of the shock. The representative agent invariantly chooses progressive taxation, which, given by (30), is 0.17. Table 5 shows optimal inequality levels, caused by his preferred policies.

	t : 0	1	2	3	10	∞
Δ :	1	1.17	1.22	1.24	1.25	1.25
	2	1.56	1.37	1.29	1.25	1.25

Table 5: Time paths of inequality depending on initial inequality levels, preferred by representative individual.

Similar to the case of a moderate wealth bias, inequality slowly increases over time for a high initial inequality relative to its steady-state value, while being decreasing for a low initial level, for reasons given above.

6.2 Pollution

As it is known from section 3, pollution is a function of aggregate consumption, which is a constant share of aggregate output, and environmental policy. Logarithmic aggregate income is initially independent from wealth bias and thus taxation[29], but it evolves according to distributional changes brought by redistributive policies, as seen in Table 6. With high initial wealth dispersion, aggregate income is high, and inequality decreases in subsequent periods, as explained above. In regimes choosing progressive taxation, the strong convergence effect, due to the concave technology, together with growth losses from

[29]Recall from subsection 2.6 that logarithmic aggregate income is given by $\ln y_t = \ln A + \beta m_t + \beta^2 \Delta_t^2/2$. The evolution of median wealth (mean log-wealth) essentially follows the development of tax rates for a given regime, because net transfers influencing it decrease with regressive and increase with progressive redistribution. We will not discuss the variable in detail, in order to concentrate on more important factors in the model.

inefficiently high redistribution, outweigh the gains from the improvement of the liquidity conditions of the poor. Therefore, the average/aggregate income becomes lower. However, after few time-periods sustained transfers to the poor become less inefficient due to falling inequality, while the convergence effect becomes smaller in magnitude, and the income growth in a more equal economy picks up, only to cease in the steady state. Wealth-biased regimes implementing regressive taxation permanently reduce aggregate income, by exacerbating liquidity of the poor with higher marginal returns to investment, making wealth positions more persistent and increasing inefficiency, which is minimized in the range of progressive taxes. Calculations of the growth-maximizing tax rate for all discussed cases, using equation (15), reveal that this rate lies between 0.39 and 0.45, which strengthens the result that growth is maximal for a moderate level of democracy imperfection.

		$t:$ 0	1	2	3	10	∞
$\lambda:$	0	2.84	2.68	2.68	2.7	2.73	2.74
	0.5	2.84	2.72	2.71	2.72	2.75	2.75
	1	2.84	2.71	2.67	2.66	2.64	2.64
	1.5	2.84	2.66	2.56	2.51	2.42	2.42
	2	2.84	2.56	2.39	2.28	2.1	2.09

Table 6: Time paths of aggregate income depending on wealth bias

For $\Delta_0 = 1$, aggregate income has a lower initial level of 2.3, which reduces the income convergence effect. For the four top cases it monotonically increases over time, while in the bottom case it decreases towards the steady state (not displayed). The growth rates for $\Delta_0 = 1$ are positive and declining over time in the four top cases.[30] Because inequality is also rising, there is a negative dynamic relationship between both variables. This clearly does not hold true for the bottom case of strong democracy imperfection: the growth rates are negative throughout. For progressive taxation, income growth is created through the relaxation of liquidity constraints and increasing social mobility, despite rising inequality and losses caused by abatement. Regressive taxation reverses the strength of the effects and growth is positive because the positive effect of polluting subsidies offsets the losses created by inefficiencies. Only excessively regressive taxation leads to big inefficiencies and negative growth. As the time patterns of aggregate pollution closely mirror those of aggregate income, declining aggregate production leads at first to falling pollution levels in economies, where progressive taxation is

[30]Some of the results are in line with the empirically estimated half-life of a transition to the steady state of a Solow model of roughly 35 years (see Barro and Sala-i-Martin (2004, p. 59)).

implemented (Table 7). Highly wealth-biased regimes implementing regressive taxation are responsible for much less pollution than for instance the median voter, always opting for the highest possible progressivity. In the perfect democracy the steep fall in inequality leads at first to a sharp decline in aggregate consumption and thus pollution. After the initial adjustment, however, the strength of the effects is reversed, and both aggregate income and pollution catch up to their steady-state values. In the last three cases pollution becomes less rampant from one period to another for the same reasons applying to the evolution of income growth. Table 8 provides the growth rates of aggregate pollution calculated with the aid of equation (19), which can be re-written as a difference between the income growth rate multiplied by the pollution parameter ($\gamma g(\tau)$) and the growth rate of environmentally used funds multiplied by the parameter μ.[31] We can conclude that in virtually all cases, the constant share of income growth rate ($\gamma = 0.4$) accounts for almost the entire growth rate of pollution, because the share of funds devoted to abatement is very small.

		t : 0	1	2	3	10	∞
λ :	0	1	0.93	0.94	0.94	0.96	0.96
	0.5	1	0.97	0.97	0.97	0.98	0.98
	1	1.02	0.98	0.97	0.96	0.96	0.96
	1.5	1.04	0.98	0.94	0.92	0.89	0.88
	2	1.06	0.95	0.88	0.84	0.77	0.76

Table 7: Time paths of log pollution depending on wealth bias (benchmark case)

		t : 1	2	3	4	10	∞
λ :	0	-6.1	0.07	0.68	0.53	0.04	0
	0.5	-4.05	0.04	0.48	0.4	0.03	0
	1	-4.17	-1.16	-0.45	-0.23	-0.01	0
	1.5	-6.48	-3.49	-2.1	-1.31	-0.09	0
	2	-10.97	-6.9	-4.39	-2.81	-0.2	0

Table 8: Time paths of pollution growth depending on wealth bias (in %)

For a low initial inequality aggregate pollution increases in the first four cases and decreases in the last one, following again closely the development

[31] Note that Table 8 displays growth rates beginning with period one. The same applies to every other table below containing growth rates.

of production (Table 9). Pollution-growth rate (Table 10) is declining in the first three cases, inversely U-shaped over time in the fourth one and becoming less negative in the last one, which molds a different pattern from the one observed in Table 8. Again, the production technology is the main source of pollution. Only in highly wealth-biased regimes the pollution growth rate declines in higher proportions in the early periods, because the growth of the share of transfers harmful to the environment (the counterpart of the abatement funds if taxation is regressive) drastically declines with decreasingly regressive taxation.

		$t:$ 0	1	2	3	10	∞
$\lambda:$	0	0.79	0.85	0.89	0.91	0.96	0.96
	0.5	0.81	0.88	0.92	0.94	0.98	0.98
	1	0.83	0.88	0.91	0.93	0.96	0.96
	1.5	0.86	0.87	0.87	0.88	0.88	0.88
	2	0.89	0.83	0.81	0.79	0.76	0.76

Table 9: Time paths of log pollution depending on wealth bias for $\Delta_0 = 1$

		$t:$ 1	2	3	4	10	∞
$\lambda:$	0	6.25	3.89	2.44	1.55	0.1	0
	0.5	6.73	4	2.49	1.57	0.11	0
	1	4.71	2.94	1.78	1.11	0.08	0
	1.5	0.41	0.75	0.4	0.22	0.01	0
	2	-6.18	-2.49	-1.61	-1.07	-0.08	0

Table 10: Time paths of pollution growth depending on wealth bias (in %) for $\Delta_0 = 1$

Now, it is interesting to examine how aggregate pollution would change in the face of a significant increase in funds devoted to environmental policy. For this purpose we multiply the share α by the factor ten. The results included in Table 11 clearly demonstrate that the pollution levels in each period increase with the degree of wealth bias. In a situation of perfect democracy, the tenfold increase in the abatement share reduces long-run pollution by around one third, compared with the benchmark case. Conversely, for $\lambda = 2$, long-run pollution is around one-third higher. As long as redistribution remains progressive, the abatement technology effectively improves environmental quality in each period, while reducing consumption growth, and therefore also the long-run environmental quality. With regressive taxation, subsidies lead to more pollution and a relatively lower consumption growth decline. In a perfect democracy pollution declines over time through the abatement and the growth channel. In the second case of moderate wealth bias with progressive taxation, higher production generated by better investment opportunities of the poor offsets growth losses and the abatement activities over time, leading to slightly increasing pollution. In the middle case, the same time-path of pollution is encountered. However, the reason for this pattern is now the harmful effect of regressive taxation through polluting subsidies which outweighs declining consumption. For $\lambda = 1.5$, the pollution path is slightly inverted U-shaped over time (in fact almost linear), because at first the abatement effect outweighs the growth effect, and they are reversed after just one period. In the last case pollution declines over time, since the economy is shrinking and polluting subsidies provide a much weaker influence on pollution.

		t : 0	1	2	3	10	∞
λ :	0	0.69	0.63	0.62	0.62	0.62	0.62
	0.5	0.81	0.82	0.84	0.85	0.86	0.87
	1	0.96	1	1.02	1.03	1.04	1.04
	1.5	1.14	1.15	1.15	1.15	1.13	1.13
	2	1.35	1.29	1.24	1.2	1.13	1.13

Table 11: Time paths of log pollution depending on wealth bias for $\alpha = 0.1$

There are some transitional differences, when lower initial inequality is chosen (not displayed). The direction of changes in time is different, because taxation becomes more progressive (less regressive) over time, with the exception of a perfect democracy. The relative strength of both described effects is reversed, and in the first cases pollution rises, whereas in the bottom two it declines.

The next table (12) illustrates the differences between the growth rates of income and pollution over time for the benchmark case. It is apparent that only a perfect democracy and a moderately wealth-biased regime are able to generate economic growth surpassing the growth rate of pollution during the adjustment to the steady state. They always implement progressive taxation that increases aggregate consumption, taking care of environmental protection via abatement technology as well. While it is impossible to postulate that a de-linking of economic development and environmental degradation takes place, for both growth rates are positive from the third period onwards, it seems to be legitimate to say that democratization is beneficial for future generations that enjoy the fruits of growth and suffer relatively less from pollution. A deviation from the benchmark case with respect to α

does not change the qualitative pattern observed in Table 12. However, as seen in Table 11 wealth-biased regimes then generate more pollution in each time-period, whereas the more democratic ones provide effective abatement.

		t: 1	2	3	4	10	∞
λ:	0	-9.62	0.11	1.07	0.84	0.06	0
	0.5	-7.95	-0.61	0.55	0.56	0.05	0
	1	-8.61	-2.62	-0.95	-0.42	-0.02	0
	1.5	-11.57	-5.84	-3.3	-1.98	-0.13	0
	2	-16.86	-10.28	-6.43	-4.08	-0.28	0

Table 12: Time paths of growth-rate differences depending on wealth bias (in %)

A much brighter picture emerges when initial inequality is low (Table 13).

		t: 1	2	3	4	10	∞
λ:	0	10.29	6.14	3.85	2.44	0.16	0
	0.5	11.14	6.27	3.81	2.39	0.16	0
	1	9.71	4.93	2.8	1.69	0.11	0
	1.5	6.07	2.22	0.91	0.42	0.02	0
	2	0.21	-1.83	-1.81	-1.38	-0.11	0

Table 13: Time paths of growth-rate differences depending on wealth bias (in %) for $\Delta_0 = 1$

Then, only in the last case pollution grows stronger over time than income. As taxation becomes more progressive, consumption possibilities remain relatively higher compared to pollution burdens.
If the representative individual, constantly choosing progressive taxation, was the decisive force in the politico-economic game, the development of pollution in the economy would resemble the time-paths resulting from policies chosen by a moderately wealth-biased regime (not displayed). For high initial inequality (declining over time) combined with aggregate income surpassing the steady-state value, the convergence effect and growth losses from abatement would outweigh the gains provided by the relaxations of the credit constraints. Therefore, aggregate production would fall at first, as well as pollution. Later, sustained progressive redistribution would reverse the direction of changes: abatement and investment reallocation effect would overcome the strength of convergence (reduced by lower aggregate income), leading to

positive growth of income and pollution. In case of low initial inequality and aggregate income, progressive taxation would lead to monotonically rising production and pollution, with a positive growth-rate difference throughout.

The study of various pollution paths in this subsection has not systematically detected an empirical regularity called the environmental Kuznets curve (EKC), being an inverted U-shaped relationship between income and pollution. An important reason could be the fact that the EKC is mostly found in cross-sectional analyses, while our simulations pertain to developments over time. It is very likely that this cross-sectional non-linearity is created by using single observations for a given point in time that are also parts of monotonically rising or falling curves over time. Another reason may be the absence of efficient instruments of environmental policy in the model.

6.3 Sustainability

Finally, we make a modest attempt to discuss sustainability within the current framework, given its contestable assumptions about technology, the absence of technological progress and the lack of accumulative pollutants. In the literature there appear to be numerous competing definitions pertaining to sustainability.[32] Our reference point is a definition of sustainable development as non-declining utility over time. Table 14, exhibiting intertemporal paths of utility for the baseline values of the calibration, leads to unanimous conclusion that sustainability cannot be achieved under the conditions shaping the structure of the model economy for any decisive individual or group.[33] None of the cases is associated with non-declining utility over time, particularly those of high wealth bias, in which the welfare levels of future generations decline in monotonic fashion. As seen before, pollution growth always surpasses the growth of consumption in such regimes.

		$t:$ 0	1	2	3	10	∞
$\lambda:$	0	3.02	2.99	3.01	3.03	3.08	3.08
	0.5	3.37	3.3	3.3	3.31	3.35	3.35
	1	4.02	3.88	3.84	3.82	3.81	3.8
	1.5	4.97	4.75	4.63	4.57	4.47	4.47
	2	6.22	5.89	5.69	5.56	5.36	5.34

Table 14: Time paths of utility depending on wealth bias

[32] A fair number of concepts is discussed e.g. in Perman et al. (2003), pp. 85-103.

[33] This statement may be contested, because the temporary decline in welfare in both upper cases is due to the strong initial effect. However, we are interested in clear results including this effect.

The development of the welfare levels preferred by the individual with average wealth is given by Table 15. It is definitely unsustainable as well.

$t:$	0	1	2	3	10	∞
	4.02	3.58	3.45	3.42	3.44	3.44

Table 15: Time path of utility preferred by representative individual (benchmark case)

Once again, the results change dramatically when a lower initial variation of individual wealth levels is assumed. Tables 16 and 17 depict the utility levels preferred by different regimes.

		$t:$ 0	1	2	3	10	∞
$\lambda:$	0	2.59	2.77	2.88	2.96	3.07	3.08
	0.5	2.84	3.04	3.15	3.22	3.34	3.35
	1	3.39	3.57	3.66	3.72	3.8	3.8
	1.5	4.24	4.38	4.43	4.45	4.46	4.47
	2	5.39	5.46	5.45	5.42	5.35	5.34

Table 16: Time paths of utility depending on wealth bias for $\Delta_0 = 1$

$t:$	0	1	2	3	10	∞
	2.16	2.68	2.97	3.15	3.43	3.44

Table 17: Time path of utility preferred by representative individual for $\Delta_0 = 1$

The results indicate that all analyzed decisive individuals, excluding the one associated with $\lambda = 2$, would choose policies leading to sustainable development for *themselves* in the presence of relatively equal initial wealth distribution. As the progressivity of taxation remains high or increases over time, consumption possibilities rise relative to welfare losses via pollution. Although the representative individual clearly chooses sustainable welfare levels too, the question is still unanswered whether every group within society benefits from the policies and the preferences for consumption (and bequests) are appropriately balanced with the preferences for better environmental quality. In other words: Is there a possibility of a tyranny of decision-making?[34] Hence, we calculate the time paths of utility for social

[34] Pearce (1997, p. 11) writes about "tyranny of decision-making in the name of sus-

groups identified by their position in the wealth distribution in the range from $\lambda = -2$ ("the poorest") to $\lambda = 2$ ("the richest"), under alternative political regimes. The discussion is possible, since the identities of pivotal voters are invariant to growth and changes in inequality.[35] Thus, the rank in the distribution remains unchanged over time and political influence depends on relative wealth. Tables 18-20 contain the main results. Since high initial inequality is not compatible with sustainability, even for the deciding groups alone, we restrain the analysis to $\Delta_0 = 1$.

		t :					
		0	1	2	3	10	∞
λ :	-2	2.19	2.34	2.45	2.52	2.64	2.64
	-1.5	2.29	2.45	2.56	2.63	2.75	2.75
	-1	2.39	2.56	2.67	2.74	2.86	2.86
	-0.5	2.49	2.66	2.78	2.85	2.97	2.97
	0	2.59	2.77	2.88	2.96	3.07	3.08
	0.5	2.69	2.88	2.99	3.06	3.18	3.19
	1	2.79	2.99	3.10	3.17	3.29	3.3
	1.5	2.89	3.1	3.21	3.28	3.4	3.41
	2	2.99	3.2	3.32	3.39	3.51	3.52

Table 18: Time paths of individual utility for regime with $\lambda = 0$ (perfect democracy).

		t :					
		0	1	2	3	10	∞
λ :	-2	0.84	0.96	1.05	1.12	1.24	1.24
	-1.5	1.24	1.37	1.47	1.54	1.66	1.66
	-1	1.64	1.79	1.89	1.96	2.08	2.08
	-0.5	2.04	2.21	2.31	2.38	2.5	2.51
	0	2.44	2.62	2.73	2.8	2.92	2.93
	0.5	2.84	3.04	3.15	3.22	3.34	3.35
	1	3.24	3.46	3.57	3.65	3.76	3.77
	1.5	3.64	3.87	3.99	4.07	4.19	4.19
	2	4.04	4.29	4.41	4.49	4.61	4.61

Table 19: Time paths of individual utility for regime with $\lambda = 0.5$

tainability". However, in this model, the agents are not credibly committed to pursue sustainability. As Neumayer (2003, p. 11) states, "in terms of political economy this assumption is utterly naive".

[35]See Bénabou (2000, p. 108).

		t: 0	1	2	3	10	∞
λ:	-2	0.84	0.7	0.7	0.73	0.81	0.82
	-1.5	1.24	1.17	1.19	1.23	1.31	1.32
	-1	1.64	1.64	1.68	1.72	1.81	1.82
	-0.5	2.04	2.10	2.17	2.22	2.31	2.32
	0	2.44	2.57	2.66	2.71	2.81	2.82
	0.5	2.84	3.03	3.14	3.21	3.31	3.32
	1	3.24	3.5	3.63	3.7	3.81	3.82
	1.5	3.64	3.97	4.12	4.2	4.31	4.32
	2	4.04	4.43	4.61	4.7	4.81	4.82

Table 20: Time paths of individual utility given policy of the representative individual

The time paths of utility from the first two tables are non-declining for every social group under consideration. Hence, perfect democracy and moderately wealth-oriented regimes fulfill the sustainability criterion.[36] The situation changes significantly for $\lambda = 1.5$ or higher degrees of democracy imperfection (not displayed). Groups poorer than the median voter have declining utility levels over time. One striking result from Table 20 is the fact that the representative individual does not provide sustainable utility development for the poorest members of society. This may weakly support the view that utilitarianism as an ethical principle may not be best suited to guarantee sustainability.[37] The virtue of democratization, among other things, is thus its beneficial treatment of the poor. Table 18 indicates that in a perfect democracy everyone gains utility over time, while the differences in utility among social groups remain roughly constant. It shows that in spite of full expropriation, richer individuals can hardly be considered as the losers of the political game. Of course, moderate departures from the ideal of perfect democracy open up a welfare gap between social classes, but the benefits are still rather equally spread and sustainability is ensured.

7 Conclusions

Unequal distribution of income and political power is a potentially important reason for the differences in observed pollution levels. This paper shows that in more democratic societies, where progressive taxation is implemented, sufficiently high abatement spending directly reduces pollution-generating consumption and improves environmental quality, whereas pollution is expected

[36] A table for $\lambda = 1$, which would also indicate sustainability, is not displayed.

[37] See Pearce and Turner (1990, pp. 234-5).

to be more rampant in excessively wealth-biased regimes. Initial inequality plays an important role during the transition to the steady state shaping the patterns of the dynamic interplay of inequality and politically determined redistribution. When it is high, inequality will decline over time, due to the concavity of production function and the wealth-transmission mechanism, leading to declining progressivity of taxation. Convergence effects combined with growth losses from either abatement or credit constraints are responsible for declining aggregate income, consumption and pollution. On the other hand, low initial inequality associated with low initial pollution and income levels, will lead to increasing levels over time. In democratic societies, the idea of intergenerational fairness appears to be more likely to become reality, in a way that incomes grow faster than pollution, because progressive taxation chosen in democracies relaxes credit constraints of the poor and activates the abatement technology along the transition towards the steady-state. Moreover, individual utility levels are non-declining over time, which is compatible with the notion of sustainable development. The representative agent with average wealth does not provide sustainable policies if the utility levels of other individuals are taken into account, i.e. heterogeneity should matter in the discussion of economic and ecological development. As Neumayer (2003, p. 11) rightly states, "inter-generational fairness questions are at the centre of concern of most proponents of sustainable development, but that is not a good reason to exclude intra-generational conflicts per se."

Appendix A

A1. In order to prove that every feasible politically determined tax rate is ceteris paribus increasing in inequality, we use the expressions for $\underline{\lambda}$ and $\overline{\lambda}$, as well as the earlier assumption that the term in the square brackets in the denominator of (25) is positive. Defining $1 + \rho\nu - \gamma - \mu$ as Z, we are able to examine inequality's influence on optimal taxation across regimes:
1. For $\lambda = \overline{\lambda}$ and $Z \geq 0$ the numerator turns to $\alpha Z(\ln A + \beta m) + 2[1 + \rho\nu - \alpha Z]\beta^2\Delta^2$, which is positive; hence $\partial\tau^\lambda/\partial\Delta > 0$.
2. For $\overline{\lambda} > \lambda \geq 0$ the numerator is positive as well and τ^λ increases.
3. For $\lambda = \underline{\lambda}$ and $Z > 0$ the numerator becomes $\alpha Z(\ln A + \beta m) > 0$. However, at $\underline{\lambda}$ the taxation reaches $\tau = 1$ and no further redistribution is allowed.
4. Therefore, for $\underline{\lambda} < \lambda < 0$ and $Z > 0$ the numerator must be positive too, since τ^λ is monotonous in Δ.
5. Finally, $\underline{\lambda} = 0$ for $Z = 0$, which means that the median voter turns into the most redistributing political force choosing $\tau = 1$. Thus, no further redistribution can take place, and the case $\lambda < 0$ for $Z = 0$ is excluded from the analysis.

A2. The growth-maximizing tax rate is always smaller than the one preferred by the median voter if

$$\frac{\alpha(\beta m + \ln A) + \delta\beta^2\Delta^2}{\beta^2(1-\alpha-\nu\beta)\Delta^2} > \frac{\alpha[1+\rho\nu-\gamma-\mu](\ln A + \beta m)}{[1+\rho\nu-\alpha(1+\rho\nu-\gamma-\mu)]\beta^2\Delta^2}.$$

Simple algebra yields

$$\alpha(\ln A + \beta m)[\nu\beta(1+\rho\nu) + (\gamma+\mu)(1-\nu\beta)] + \delta[1+\rho\nu-\alpha Z]\beta^2\Delta^2 > 0,$$

which is fulfilled for $1 > \nu\beta$.

A3. The dynamics of inequality, showing the direction of change towards the steady state can be inferred from the linear difference equation (13). Substracting Δ_t^2 from both sides yields

$$\Delta_{t+1}^2 - \Delta_t^2 = -(1-[\delta+\nu\beta(1-\tau)])^2\Delta_t^2 + \sigma^2.$$

Inequality increases, i.e. the left hand side of the above equation is positive, if

$$\Delta_t^2 < \frac{\sigma^2}{1-[\delta+\nu\beta(1-\tau)]^2},$$

the right hand side being the stationary inequality level $\mathcal{D}^2$ from equation (31). In turn, inequality must decrease over time if its value is above the stationary level.

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