Riding High –
Success in Sports and the Rise of Doping Cultures

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Abstract. This article develops a socio-economic model that analyzes the doping decision of professional athletes. In their evaluation of whether to use performance enhancing drugs athletes consider not only costs and benefits (through rank improvement) but also approval by their fellow athletes. Peer-group approval is modeled as a lagged endogenous variable depending on the share of doping athletes in the history of a sport. This way, the model can explain an equilibrium of high incidence of doping as a “doping culture”. Besides the comparative statics of the equilibrium (how can a doping culture be eliminated?) the article also investigates how the doping decision is affected by standards set by the respective leader in a sport, e.g. Olympic qualification marks, and by the taste for victory resulting from the disproportionate public veneration of winners.

Keywords: sport, doping, approval, social dynamics, rank loss aversion, taste for victory, superheroes.


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I never cheated.
(Jan Ullrich)

So I did it, but I didn’t feel totally guilty about it because everybody else seemed to be doing it.
(Frankie Andreu)

1. Introduction

This article was heavily inspired by the confessions of drug using professional cyclists in the run-up of the year 2007 Tour de France and the following “scandals” during the event. I frequently refer to cycling and the Tour de France as examples but I hope that the proposed theory is general enough to cover also many other sports in which doping is prevalent.

In particular I got interested in a by then clearly emerging theme exemplified by the two preceding quotes from professional cyclists. Many of the convicted drug users who were definitely violating the official rules of their sport nevertheless maintained that they were not doing anything wrong and justified this view in particular by mentioning that their fellow athletes were using drugs as well. Apparently, the community of professional athletes sustained a different norm about doping than society at large, a fact, which was increasingly addressed by commentators as a doping culture.¹

In the following I provide a brief introduction of how doping behavior and the notion of a doping culture is conceptualized in the scientific literature on the sociology and psychology of sport. In the main text I then try to translate these concepts into economic language and set up a model that is capable to explain the evolution and stability of a doping culture in economic terms. I then use the model to investigate the effectiveness of anti-doping polices and the impact of rules (qualifications marks) and general norms (veneration of winners) on the individual doping decision and on the resulting doping equilibrium assumed by an athletes’ community.

The term “doping culture” initially coined by the news press appears to be indeed quite appropriately chosen against the background of thorough definitions of “culture” in sociology and anthropology. A sports community produces its own rules, own language, a code of practice, which is considered to be “normal”, it shares a common code of honor as well as common symbols (the gold medal, the yellow jersey, the sweeper bus). Technically speaking, sports communities may be

even better described as semi-autonomous fields, a term developed in legal anthropology (Moore, 1973), i.e. as communities that develop their own rules embedded in the general rules of conduct of society at large. Nevertheless I will use the term culture here because it is by far the more familiar one.

Doping has a long history in many sports but it has been (and presumably still is) most prevalent in cycling.\(^2\) At the time of the first Olympics when Baron de Coubertin promoted the motto that competing is more important than winning, cyclists used wine, cocaine, strychnine, ephedrine, and cocoa leaves in order to enhance their performance (de Rose, 2007). Since then the performance enhancing power of drugs has been constantly on the rise therewith increasing the incentive to dope. For example, injections of EPO, available since the early nineties, provide enhancements in endurance performance by 5% or more (Sawka et al., 1996, Birkeland, et al., 2000). This value just exceeds the average gap between the winner’s time and that of the last-place finisher in the last 10 Tours de France, which was 4.75% (Lindsay, 2007).

The popularity and power of the applied doping method varies with the requirements of the sport. Endurance athletes most frequently use methods to increase the oxygen-carrying capacity of blood (EPO, blood doping), Power athletes like sprinters, boxers, and weightlifters, prefer anabolic steroids, and athletes for whom steady action is most important (archers and shooters) prefer sedatives.\(^3\)

The question may arise why doping is apparently most widespread in cycling, in particular compared to other endurance sports for which presumably the same power of drugs is available. The literature offers two explanations. First, cycling, in particular the stage races, is frequently considered to be the hardest sport, a fact that makes it easier to develop individually a pro-doping attitude (Waddington, 2000; Mignon (2003); Kimmage, 2006). Second, cycling is a team-sport, a fact that makes it easier to develop and sustain a pro-doping attitude within the in-group of professional athletes. In the following I develop this argument in more detail.

In many countries it is illegal to prescribe and sell performance enhancing drugs but doping itself is not against the law. Doping athletes must thus not feel guilty from a legal perspective. Doping is

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\(^2\)Cycling is the sport with the highest percentage of adverse findings (3.78% of all samples). Perhaps surprisingly, second place in this statistic is baseball (3.60%), then comes boxing and triathlon. Both, de Rose (2007), from which this information was taken, and Catlin et al provide excellent overviews of the history of doping from medical perspective. According to the latest available statistic (WADA, 2009a) the international cycling organization UCI reported 45 doping rule violations over the last year, followed by the international swimming organization which reported 26 cases. A detailed description of doping cases in cycling since 1886 is available at [http://www.absoluteastronomy.com/topics/List_of_doping_cases_in_cycling](http://www.absoluteastronomy.com/topics/List_of_doping_cases_in_cycling).

\(^3\)See Catlin et al. (2008) for a detailed description of composition, effects, and side effects of EPO and other performance enhancing drugs.
forbidden by rules of many athletes’ organizations and the World-anti-doping-agency (WADA). The WADA defines doping as the “presence of prohibited substances in an athlete’s body” and in order to make this abstract definition manageable it has drawn up a so-called negative list of substances.\textsuperscript{4} If the use of drugs on the list is detected, the deviant athlete is punished with a temporary ban from competition (of two years in many sports). This means that athletes using substances that are not or not yet on the negative list must not feel guilty for violating the rules of their sport. However, irrespective of whether the drug they use is forbidden or not-yet-forbidden, athletes may feel ashamed for cheating on their competitors.

This article argues that the awareness of doping fellow athletes affects the individual doping decision not only through its anticipated effect on rank in competition but also through learning and encouragement from peers (Sutherland and Cressy, 1974). If sufficiently many athletes are doping, the use of drugs may become a norm. Of course, such a doping norm is, if it exits, only operative within the athletes’ community. A doping norm, or doping culture, among professional athletes of a sport can be perfectly compatible with a different and probably directly opposed norm of the dominant culture, i.e. the general public, the spectators, and the journalists. The dominant culture may, in fact, despise “dirty athletes”.

The degree to which professional athletes form closed communities influences how easily they manage to reject society’s hostile attitude against doping and to produce and sustain their own norms. In the case of cycling, for example, Wieting (2000) has observed that “there are two (rather than one) normative frameworks: one of the racers themselves and the other of the surrounding society.” This way, drug use can become widely approved within the group of athletes until it is seen as an essential prerequisite for success. This notion is taken one step further by Coakley and Hughes (1998). They argue that drug use by professional athletes should not be conceptualized as negative deviance but as positive deviance. It expresses an overconformity to key values in sport, most notably the value attached to winning. With respect to professional cycling Mignon (2003) argues that it is not only the desire to win (which is anyway a reasonable goal for only a subgroup of athletes) but also the desire to stay in the game. An athletes’ community has to defend its own norms because “outsiders do not know how hard a rider has to work just to stay in the race”.

\textsuperscript{4}In this article “drug use” is meant as a convenient simplification of “forbidden performance enhancing behavior” and may thus be thought of encompassing activities like blood doping that involve strictly speaking no intake of drugs. The WADA also bans some recreational, non-performance enhancing drugs (like cannabis) if they are “harmful to the health of the athlete” and “against the spirit of the game”. The non-performance enhancing aspects of drug intake are ignored in this article.
This leads to the question of what “fair play” in sports is. In the philosophy of sports there exists a small literature that defines “fair play” as the ethos of a sport. While the rules of a sport distinguish the permissible from the impermissible, the ethos of a sport distinguishes the acceptable from the unacceptable. The rules are written in a book, the ethos is based upon shared experiences of the athletes’ community. This way, the social practice of a sport defines what is conceptualized as “fair play” by the participants.

There exists some recent empirical research supporting this assessment. Petroczi (2007) analyzes interviews on the doping attitudes and behavior of male college athletes. Generally she finds that doping athletes acknowledge their rule breaking behavior but do not consider themselves as cheaters or more cheating than others. Lentillon-Kaestner and Carstairs (2009) analyze interviews of young elite cyclists and conclude that these athletes believe doping to be acceptable at the professional level (but not on the amateur level), that cyclists who recently became professional experienced pressure from teammates to start doping, and that “more experienced cyclists transmitted the culture of doping to the young cyclists: they gave information about which substance to use and taught the young cyclists the methods”. Implicitly the WADA and many sport organizations have recently acknowledged the power of peer group approval and group cohesion by launching educational programmes designed to reduce this influence, e.g. the “Play True Generation” program (WADA, 2009b) and the “True Champion or Cheat” program of the UCI (2009).

So far, a small economic literature has tried to rationalize the use of drugs in sports, mostly focussing on two players competing in a game-theoretic situation with only two outcomes: winner or loser. The present article adds at least two new aspects to this literature: It considers the behavior of many athletes competing about the ranking in their sport and it investigates the role of socially dependent preferences. The focus on ranking instead of winning allows to analyze a richer set of motives for participating professionally in a sport. Many athletes are obviously getting something (utility, prestige, money) out of their rank without managing to be the number one in their sport. While there may be individual contests between just two athletes, the ultimate goal of each athlete is not to win one particular match or tournament but to appear high in the (world) ranking of his or her sport. In other words, in every sport season an athlete competes virtually with all other professional athletes of his peer group although he might not fight for real against everybody.

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6 Berentsen (2002), Berentsen and Lengwiler (2004), Haugen (2004), and Kräkel (2007). Some of the ideas developed in the present article were already mentioned in Bird and Wagner (1997) but they were not formally investigated. Dilger et al. (2007) provide a survey over the theoretical literature in economics and some (sparse) empirical evidence.
The model presented below tries to translate arguments and mechanisms from the sociological and psychological literature on sport into economic terms. Utility of competing athletes is assumed to depend positively on their rank in a sport and positively (negatively) on social (dis-) approval of a pro-doping decision. Social approval consists of peer-group approval and disapproval experienced from society at large. Athletes differ with respect to ability (talent) and susceptibility to approval of their behavior. Peer group approval in turn is derived from the doping history of a sport, i.e. the share of athletes that were using drugs in the past. Depending on the power drugs, peer group cohesion, and the monetary and stigma costs of using drugs, the model is capable to generate different equilibria. In particular it can motivate an equilibrium of high incidence of doping which is assumed and sustained “only” because peer-group approval matters for utility and group cohesion is sufficiently strong.

The article is organized as follows. The next section sets up the basic model and solves the individual decision problem. Section 3 discusses social equilibria, i.e. aggregate dynamics and steady-states for the share of doping athletes. The impact of economic and social changes on aggregate doping behavior are evaluated and discussed. Section 4 modifies the model in order to investigate the effect of rank loss aversion (i.e. disproportionately negative effects on rank from staying clean) and Section 5 investigates how results change when utility is exponentially increasing in rank (reflecting the disproportionately large positive utility derived from finishing among the first ranks). Section 7 shows that the introduction of ability-dependent costs of doping may change doping decisions on the individual level but leaves the aggregate behavior of the model and the conclusions unchanged.

2. The Basic Model

Suppose ability of competitors in a particular professional sport is uniformly distributed in the unit interval. Athlete \( i \) has ability \( A(i) \in [0,1] \). An \( A(i) \) of 1 indicates highest ability and an \( A(i) \) of 0 lowest ability. Suppose that the rank of athletes in competition, for example at the PGA if the sport is golf or at the Tour de France if the sport is cycling, is uniformly and continuously distributed. In an ideal world ability would map one-to-one into rank, i.e. \( R(i) = A(i) \). Yet, there is the possibility of doping to improve rank.\(^8\) Doping is a binary choice, \( d \in \{0,1\} \). An athlete either

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\(^7\) There exists a small economic literature that uses similar ideas to investigate the welfare state (Lindbeck, et al., 1999), out-of-wedlock childbearing (Nechyba, 2001), and occupational choice (Mani and Mullin, 2004).

\(^8\) We ignore the possibility of doping in order to manipulate a competitor’s rank negatively (poisoning), a possibility which is mainly popular in horse racing. Doping a competitor’s horse is excluded from analysis because this behavior is certainly regarded as cheating by the riders’ community and the present article focuses on behavior that is not necessarily regarded as “against the rules” by the athletes, i.e. behavior with potential for peer-group approval.
uses performance enhancing drugs, here denoted by \( d = 1 \), or he stays clean, denoted by \( d = 0 \).

Given the possibility of doping, an athlete’s rank depends not only on ability alone anymore but also on his doping decision and on the doping decision of his competitors. Let \( \theta_t \) denote the share of competitors who are using drugs in season \( t \). It is reasonable to assume that – given ability – an athlete’s rank is positively influenced by the decision to use drugs but that this positive effect vanishes as the share of competitors who are also using drugs increases. One analytically convenient way to implement this notion is by letting rank of athlete \( i \) in season \( t \), \( R_t(i) \), be determined by the following function.

\[
R_t(i) = A(i) \cdot \left\{ 1 + \alpha \left[ d \cdot (1 - \theta_t) - (1 - d) \cdot \theta_t \right] \right\}. \tag{1}
\]

Generalizations and extension of the rank function will be discussed later in the article. According to the rank function there are two situations in which rank equals ability. The first possibility is clean sports: athlete \( i \) stays clean and so do all other athletes, \( d = \theta = 0 \). The other possibility is completely dirty sports: athlete \( i \) uses drugs and so do all others athletes, \( d = \theta = 1 \). In between these borders an athlete can improve his rank through drug intake. This is captured by the first term in brackets. Yet, frequently professional athletes describe their decision on doping with a different twist. They emphasize that they would lose ranking-wise if they opt against doping because (many of) their competitors are using drugs. This side of the argument is captured by the second term in brackets. The loss of rank of a clean athlete is the larger the higher the share of doping athletes in a sport.

The parameter \( \alpha \geq 0 \) measures the power of drugs, i.e. the possibility to manipulate rank. In the limit \( \alpha = 0 \) and using drugs has no effect on rank. The value of \( \alpha \) characterizes the sport under investigation, which may range from golf, which is generally believed to have a low incidence of doping, to cycling, which is possibly associated with the highest value of \( \alpha \).

Implicitly the size of \( \alpha \) also characterizes the skill requirements of a sport. In sports where success relies largely on fine-motor skills as, for example, in golf one can argue that players are competing only with players of similar ability. The power of drugs is (yet) too small for the doping decision of inferior athletes to affect the ranking of the best players in the world. This is certainly different in some endurance sports where the power of drugs is already strong enough such that the set of competitors considered may indeed encompass all professional athletes in a competition (for example, as shown in the Introduction for the power of EPO and arrival times at the Tour de France).

There are also costs of using drugs. For the basic model, we subsume individual costs in a single
parameter, \( c \). These costs include the actual monetary costs of the drugs (which athletes frequently pay out of their personal budget) and the expected costs of losing one’s job if being detected as a doper (losing the licence to compete in tournaments for a while, losing attractive advertising contracts). Costs may also include the fear of consequences later in life, like illness and premature death, caused by the unhealthy use of chemical substances.\(^9\)

In bearing these costs it helps if an athlete observes that many other athletes are also using drugs, i.e. if there is approval of doping by his peers. Let \( S_t \) denote the level of peer-group approval in season \( t \). The magnitude of \( S_t \) is given to the individual athlete but endogenously determined by the share of athletes who were doping in the past such that \( S_t \in [0,1] \). The degree to which athletes are influenced by approval of their actions is individual-specific, also assumed to be uniformly distributed within the unit interval, and denoted by \( \sigma(i) \in [0,1] \). At the highest level of susceptibility \( \sigma(i) = 1 \) and at the lowest level of susceptibility, i.e. socially independent preferences, \( \sigma(i) = 0 \).

Athletes also experience disapproval of doping, in particular from spectators, the press, and the society at large. Let the marginal strength of disapproval be denoted by \( \phi \) so that \( \sigma(i) \cdot \phi \) reflects the “stigma costs” experienced by athlete \( i \) if he is the one and only in his sport who uses drugs. The relative importance of approval experienced from peers (compared to general disapproval) is measured by the parameter \( \beta > 0 \). This parameter thus tries to measure the cohesion and closeness of an athletes’ community. The strength of community cohesion is potentially sport-dependent, varying, for example, between team sports (as for example cycling) where teammates are more likely to rely on each other and to exchange doping experiences and individual sports (track and field) where athletes are potentially less close to and dependent on other athletes. Group cohesion is also potentially situation specific (for example, cycling before and after the Festina scandal) and policy dependent (for example, affected by leniency policies for doping convicts).\(^{11}\)

\(^9\) The basic model assumes that costs do not vary across athletes. Later, in Section 6, the model is extended by assuming that costs are individual-specific and ability-dependent. It will be shown that this extension leaves the previously obtained aggregate results unaffected although it may change the individual characteristics of doping and clean athletes.

\(^{10}\) All arguments made in this article are generally independent from the assumptions about distribution functions and many would be re-enforced by bell-shaped distribution functions. A uniform distribution, however, allows for analytical solution and diagrammatic exposition of equilibria.

\(^{11}\) The Festina scandal, i.e. the doping cases at and around the 1998 Tour France, is generally regarded as the most significant doping affair in sports because it revealed for the first time that an entire athlete community (including trainers, doctors, and officials) were practicing and/or concealing doping (see Waddington (2000), Houlihan (2002) and Mignon (2003)). It has led to the foundation of the WADA and the introduction of much stronger doping controls and punishment. Moreover, Lentillon-Kaestner and Carstairs (2009) provide evidence that it has also changed group cohesion. Before the Festina scandal doping was often organized at the team level. After Festina, doping became a more private and clandestine activity.
In sum, social approval of doping – which may turn to disapproval if negative – experienced by athlete \(i\) in season \(t\) is given by \(\sigma(i) \cdot (\beta S_t - \phi)\). Since the maximum peer-group approval is unity, obtained when all athletes are using drugs, we assume that \(\beta > \phi\) in order to allow peer-group approval to have a positive influence on doping. Sometimes we will also consider a reference case in which social influences play no role at all, i.e. \(\sigma(i) = 0\) for all \(i\). Comparing with the case of \(\sigma(i) > 0\) we can assess the role of peer-group approval in sports and explain the phenomenon of a doping culture.

Choosing \(d \in \{0, 1\}\) athlete \(i\) maximizes his net utility which consists of rank minus costs of doping plus utility obtained from social approval (or disutility caused by disapproval). Athlete \(i\) in season \(t\) maximizes

\[
U(i) = A(i) \cdot \{1 + \alpha [d \cdot (1 - \theta_t) - (1 - d) \cdot \theta_t]\} - d \cdot c + d \cdot \sigma(i) \cdot (\beta S_t - \phi), \quad \beta > \phi. \tag{2}
\]

If athlete \(i\) decides to stay clean and selects \(d = 0\) he receives utility \(A(i) (1 - \alpha \cdot \theta_t)\). Utility of a clean athlete is thus increasing in individual ability and decreasing in the share of doping fellow athletes whereby the magnitude of the loss depends on \(\alpha\), the power of performance enhancing drugs. If athlete \(i\) decides to use drugs and selects \(d = 1\) he receives utility

\[
A(i) \cdot [1 + \alpha \cdot (1 - \theta_t)] - c + \sigma(i) \cdot (\beta S_t - \phi).
\]

Comparing utilities, the athlete decides to stay clean if \(\alpha A(i) \leq c - \sigma(i)(\beta S_t - \phi)\). From this we get the threshold between clean and doping athletes in the two-dimensional ability-susceptibility space given by

\[
A = \frac{c - \sigma(\beta S_t - \phi)}{\alpha}. \tag{3}
\]

To begin with consider the special case where social approval plays no role, i.e. \(\sigma(i) = 0\) for all \(i\). In this case, all athletes with ability below \(c/\alpha\) stay clean. In other words, it are athletes of high ability who are inclined to use drugs in order to further improve their rank. Compared to costs, high-ability athletes get more out of performance enhancing drugs in terms of rank improvement and utility. In later sections we investigate how rank loss aversion and ability-dependent costs affect the generality of this conclusion.

Turning towards the case with socially interdependent preferences, we make the following assumption in order to focus the analysis on interesting cases and to avoid inconvenient case differentiation.
**Assumption 1.** There exists at least one drug using athlete if preferences were independent from social (dis-) approval of doping, i.e. \( c < \alpha \).

We thus focus on cases where, absent social influences, at least one athlete would have an incentive to use performance enhancing drugs and elaborate how the existence of social interdependence aggravates (or perhaps attenuates) the incidence of doping in a sport.

Figure 1: The Threshold between Clean and Drug-Using Athletes

Athletes are distinguished by ability \( A \in [0, 1] \) and by susceptibility to peer-group approval \( \sigma \in [0, 1] \). The share of drug using athletes in season \( t \) is \( \theta_t \). The strength of peer approval is \( S_t \). If \( S_t > S_{\text{high}} \equiv (\phi + c)/\beta \), there exists a critical \( \bar{\sigma} \) above which all athletes dope. If \( S_t < S_{\text{low}} \equiv (\phi + c - \alpha)/\beta \), there exists a critical \( \bar{\sigma} \), above which all athletes stay clean. For intermediate values of \( S_t \) there exist some doping athletes at any level of \( \sigma \).

Figure 1 displays the three qualitatively distinct cases that may occur when social approval affects the doping decision. The threshold (3) is represented by a bold line. A \((\sigma, A)\)-tuple above the threshold identifies a doping athlete. Allowing for social preferences does thus not change the observation that athletes of high ability are at on average more inclined to use doping. At the individual level, however, socially dependent preferences allow for a refined view on doping, since there are some athletes of highest ability who refrain from doping if social disapproval is strong enough as well as some athletes of lowest ability who succumb to doping if they get a lot of peer-group approval for a pro-doping decision.

The panel on the left hand side of Figure 1 shows a case where peer group approval of doping is so strong that there exists a critical level of susceptibility to approval above which all athletes dope irrespective of their ability, Formally, the curve represented by the threshold equation (3) hits the abscissa within the unit interval. This requires that \( S_t > S_{\text{high}} \equiv (c + \phi)/\beta \) such that \( \bar{\sigma} = c/(\beta S_t - \phi) < 1 \). For given strength of approval \( S_t \) this case occurs when monetary costs \( c \) and the stigma costs \( \phi \) are sufficiently low and cohesion of the athlete’s community \( \beta \) is sufficiently high.
The panel on the right hand side of Figure 1 shows the diametrically opposing case where peer approval is so low that there exists a critical level of susceptibility above which all athletes stay clean irrespective of their ability. Formally, the curve represented by threshold equation (3) has positive slope and assumes the value of unity within the unit interval. This requires that \( S_t < S_{\text{low}} \equiv \left( c + \phi - \alpha \right) / \beta \) such that \( \sigma = \left( \alpha - c \right) / \left( \phi - \beta S_t \right) < 1 \). Ceteris paribus, this case occurs when monetary and stigma costs are sufficiently high and peer group cohesion and the power of drugs \( \alpha \) are sufficiently low.

The central panel in Figure 1 shows the intermediate case where at all levels of susceptibility to approval there are some doping athletes (of on average high ability) and some clean athletes (of on average low ability).

The area above the threshold in Figure 1 provides the share of doping athletes, denoted by \( \theta \). For the special case of socially-independent preferences, the threshold is given by the horizontal dashed line in Figure 1 and the size of the area can be immediately read off the figure.

**Lemma 1.** If athletes’ preferences were socially-independent \( (\sigma(i) = 0 \text{ for all } i) \), the share of doping athletes is given by \( \theta_u = 1 - c/\alpha \).

In the case of socially-dependent preferences the share of doping athletes is situation-specific and depends on the strength of current peer-group approval that a pro-doping decision receives. Integrating the area above the threshold, we obtain the following piece-wise defined result.

\[
\theta_t = \theta(S_t) = \begin{cases} 
  1 - \frac{c^2}{2\alpha(\beta S_t - \phi)} & \text{for } S_t \geq S_{\text{high}} \equiv \left( \phi + c \right) / \beta \\
  1 - \frac{1}{\alpha} \left[ c - \frac{1}{2}(\beta S_t - \phi) \right] & \text{for } S_{\text{high}} \geq S_t \geq S_{\text{low}} \\
  \frac{(\alpha-c)^2}{2\alpha(\phi-\beta S_t)} & \text{for } S_t \leq S_{\text{low}} \equiv \left( \phi + c - \alpha \right) / \beta.
\end{cases}
\]  

(4)

The doping–approval association is in detail derived in the Appendix, which also contains a proof of the following Lemma.

**Lemma 2.** The share of doping athletes is everywhere increasing in the strength of peer-group approval, \( \theta'(S_t) > 0 \). The doping–approval association is convex for \( S_t < S_{\text{low}} \), concave for \( S_t > S_{\text{high}} \), and linear for intermediate \( S_t \).

Inspection of (4) provides the following result.

**Proposition 1.** For given strength of peer group approval \( S_t \), the incidence of doping depends
positively on the power of drugs and of peer group cohesion ($\partial \theta / \partial \alpha > 0, \partial \theta / \partial \beta > 0$). It depends negatively on the monetary and stigma costs of doping ($\partial \theta / \partial c < 0, \partial \theta / \partial \phi < 0$).

So far, the observation of $\theta(S_t)$ provides just a snapshot of the current incidence of doping in a sport. In order to investigate in which doping situation a sport will finally end up under the present rules and anti-doping policies, we have to investigate existence, uniqueness, and stability of long-run equilibria.

3. Doping Cultures

The model displays social dynamics because peer-group approval is not an exogenously given constant but itself endogenously explained. The strength of approval $S_t$ depends positively on the fraction of athletes who were actually doping in the (recent) history of the sport. We assume that this behavior is either directly observable at the team level or that athletes exchange their knowledge about doping with fellow athletes. Let $\delta$ denote the time preference rate or rate of oblivion by which the doping history of the sport is depreciated in the backward looking mind of athletes so that current approval is given by:

$$S_t = (1 - \delta) \sum_{i=0}^{\infty} \delta^i \theta_{t-1-i}.$$ Alteratively, this can be written as the period-by-period evolution of approval,

$$S_t = (1 - \delta) \theta_{t-1} + \delta S_{t-1}. \tag{5}$$

A social equilibrium is obtained where approval equals the actual incidence of doping, $S_t = \theta_t$, such that the share of doping athletes stays constant over time. Inserting this equilibrium condition into (4) and solving for $\theta$ provides the following result.

**Proposition 2.** Depending on the size of parameters an athletes’ community is characterized by the following long-run incidence of doping. Let $a \equiv (\alpha - c)\sqrt{2\beta/\alpha}$ and $b \equiv \beta - c\sqrt{2\beta/\alpha}$.

I. If $\phi > a$ and $\phi > b$, then there exists a globally stable equilibrium at $\theta = \theta_{low} < \theta_u$, $\theta_{low} \equiv \phi/(2\beta) - \sqrt{\phi^2/(4\beta^2) - (\alpha - c)^2/(2\alpha \beta)}$.

II. If $a > \phi > b$, then there exists a globally stable equilibrium at $\theta = \theta_{mid} > \theta_u$, $\theta_{mid} \equiv (\alpha - c - \phi/2)/(\alpha - \beta/2)$.

III. If $b > \phi > a$, then there exist two locally stable equilibria $\theta_{low}$ and $\theta_{high}$ separated by an unstable equilibrium $\theta_{mid}$.

IV. If $\phi < a$ and $\phi < b$, then there exists a globally stable equilibrium at $\theta = \theta_{high} > 1/2$, $\theta_{high} \equiv (\beta + \phi)/(2\beta) + \sqrt{(\beta - \phi)^2/(4\beta^2) - c^2/(2\alpha \beta)}$. 


The figure shows the four different cases established in Proposition 2. The share of drug using athletes is given by \( \theta(S_t) \) according to (4). \( S_t \in [0, 1] \) denotes peer-group approval of doping in season \( t \) evolving according to (5). Arrows indicate the direction of motion of \( \theta_t \) over time. A star identifies the incidence of doping if there were no social interaction (\( \theta_u \)). Parameters for construction: all panels: \( c = 0.4 \) and \( \phi = 0.3 \). Panel I: (\( \alpha = 0.5, \beta = 1 \)), Panel II: (\( \alpha = 0.6, \beta = 1 \)), Panel III: (\( \alpha = 0.5, \beta = 1.5 \)), Panel IV: (\( \alpha = 0.6, \beta = 1.5 \)).

The rather lengthy proof of the proposition is delegated to the Appendix. The result can best be intuitively explained with help of Figure 2. The four panels of the Figure display the four qualitatively different curvatures and positions with respect to the identity line that the \( \theta(S_t) \) curve of equation (4) can possibly assume. According to Lemma 2 a common feature of all four panels is that the \( \theta(S_t) \)-curve starts out with convex shape, becomes linear when \( S \) exceeds \( S_{low} \), and becomes concave when \( S \) exceeds \( S_{high} \). Note from inspection of (4) and Assumption 1 that \( \theta(0) > 0 \) and
To alleviate comparisons all four panels have been constructed by holding the individual costs and stigma costs of doping constant ($c = 0.4$ and $\phi = 0.3$ for all panels). Note also that social dynamics lead the athletes’ community towards higher incidence of doping whenever the $\theta(S_t)$-curve lies above the identity line and towards lower incidence of doping when it lies below the identity line. The resulting dynamics for the share of doping athletes are indicated by arrows on the $\theta_t$ axis.

Panel I visualizes Case I of Proposition 2. In this case the power of performance enhancing drugs $\alpha$ as well as group cohesion $\beta$ are relatively low compared to individual and social costs ($\alpha = 0.5$ and $\beta = 1$). As a consequence, $\phi > a$ and $\phi > b$ and there exists just one equilibrium with very low incidence of doping $\theta_{low}$. It turns out that socially-dependent preferences are actually helpful in reducing the incidence of doping. The social equilibrium lies below the one that would result if preferences were socially independent, $\theta_{low} < \theta_u$. (The incidence of doping under socially independent preferences $\theta_u$ is indicated by a star on the $\theta$ axis in all four panels.)

If, for some reason, the community started out at high $\theta$, peer-group approval and group cohesion are not sufficiently strong in order to support such a high incidence of doping and some athletes are motivated to stay clean next period. A bandwagon dynamic (Granovetter, 1978) towards low $\theta$ sets in. At some point approval from peers $\beta S_t$ falls below social stigma $\phi$, and as a result a pro-doping decision receives net social disapproval. As a consequence, the incidence of doping approaches a low value below $\theta_u$. Social disapproval, however, is not enough to eradicate doping entirely. Intuitively, there is always one athlete at the lower boundary of $\sigma$ (i.e. an athlete $i$ with $\sigma(i) = 0$) who is not influenced by social disapproval and thus keeps using drugs as long as individual benefits are larger than individual costs (as long as $\alpha > c$). In conclusion, at $\theta_{low}$ performance enhancing drugs are mainly taken by a few independent-minded athletes who give not much on the social approval of their behavior.

Next consider Panel II which differs from the setup of Panel I only by assuming a higher impact of doping on individual rank ($\alpha$ rises from 0.5 to 0.6). Given the higher power of drugs, peer-group approval is now sufficiently large to exceed stigma costs and the community of drug using athletes is able to generate support for an equilibrium $\theta_{mid}$ at which the incidence of doping exceeds the one obtained if preferences were socially-independent. Social dynamics aggravate the doping problem, $\theta_{mid} > \theta_u$. Recalling the results displayed in Figure 1 we can also infer that it are mostly athletes of high ability who are using drugs while those of low ability are more inclined to stay clean. Since
net approval is positive at $\theta_{mid}$, the situation with respect to susceptibility to approval has reversed compared to Panel I. At $\theta_{mid}$, athletes who are highly influenced by peers are on average more inclined to use drugs.

Panel III differs from Panel I only by assuming a higher strength of group cohesion ($\beta$ rises from 1 to 1.5). While the equilibrium $\theta_{low}$ maintains to exist, another locally stable equilibrium with a very high incidence of doping, $\theta_{high}$, emerges. For two reasons it seems to be appropriate to speak of $\theta_{high}$ as a doping culture. First, a majority of athletes uses performance enhancing drugs, $\theta_{high} > 1/2$. Second, most of the doping athletes are using drugs “only” because their competitors are using drugs as well. In order to see this note that the power of drugs $\alpha$ is the same in Panel I and Panel III. It is thus the high peer-group approval $\beta S_t$ that is generated at $\theta_{high}$ that makes the situation sustainable. To use drugs has become the norm, it belongs to the ethos of the sport.

Panel III reflects the dilemma situation frequently addressed by professional athletes of different sports. At $\theta_{high}$ the improvement in rank through doping is relatively small because the power of drugs is not large and because a majority of competitors enhances their performance with drugs as well. As a result the majority of those using drugs at $\theta_{high}$ would actually prefer to stay clean if only their competitors would refrain from doping as well. Formally, the same set of individual costs and benefits from doping supports also an equilibrium $\theta_{low}$. In order to reach this situation situation, however, a massive collective action effort is needed: a share $(\theta_{high} - \theta_{mid})$ of athletes has to coordinate to stay clean next season. Once the share of drug using athletes has fallen below $\theta_{mid}$, there is enough social disapproval of a pro-doping decision to support the movement towards $\theta_{low}$.

Finally, Panel IV differs from Panel I by both higher performance enhancing effect of drugs and tighter group cohesion ($\alpha$ rises to 0.6 and $\beta$ rises to 1.5), i.e. it differs from Panel III only by the higher power of drugs). Panel IV probably reflects best the situation in professional cycling, at least before the Festina scandal 1998. The power of drugs is strong enough to dispose of the equilibrium of low incidence of doping. A majority of athletes is doping and – with contrast to the situation in Panel III – collective action alone cannot manage to establish an equilibrium $\theta_{low}$. In order to get rid of the doping culture athletes need external help from sport organizations and/or the legislative, i.e. they need new rules of the game.

Turning towards policy, taking the respective derivatives of $\theta(S_t)$ proves the following result.

**Proposition 3.** *Everywhere, i.e. for all $S_t$, the share of doping athletes is decreasing in individual costs ($\partial \theta_t / \partial c < 0$) and stigma costs ($\partial \theta_t / \partial \phi < 0$). It is increasing in the power of drugs ($\partial \theta_t / \partial \alpha > 0$)*
and in the strength of group cohesion \((\partial \theta_t / \partial \beta > 0)\).

Since the result applies everywhere, it holds also at the steady-state(s). These marginal comparative static effects are immediately intuitive. More interesting, however, are non-marginal effects of parameter changes:

**Proposition 4.** Any doping culture \(\theta_{high}\) can be eliminated by a sufficiently large increase of individual costs \(c\) or stigma costs \(\phi\) or by a sufficiently strong reduction of the power drugs \(\alpha\) or the strength of group cohesion \(\beta\).

In order to be effective not only at the marginal level an anti-doping policy has to be sufficiently drastic. This, and other interesting aspects of policy, are visualized in Figure 3. Each of the three panels in the Figure originates from the same initial situation, represented by the solid line, which implies a long-run equilibrium at \(\theta_{high}\). Holding the power of drugs \(\alpha\) constant, the panels consider the effects of increasing individual costs \(c\), stigma costs \(\phi\), and reducing group cohesion \(\beta\).\(^{12}\)

The panel on the left hand side investigates the affects of rising individual costs of doping originating, for example, from higher fines or longer bans from competition when being exposed as a doper. If \(c\) rises from 0.4 to 0.5 (dashed lines), the athlete’s community converges towards a mildly lower

\(^{12}\)Actually, a sports organization can also manipulate the power drugs, at least to a certain degree, by setting upper limits of acceptable substances detected in blood or urine tests.
equilibrium $\theta_{\text{high}}$ but the high incidence of doping in the sport does not disappear. With contrast to the initial situation, the rules under the higher costs of doping would, in principle, also support an equilibrium of low incidence of doping. But coming from a doping culture, an equilibrium $\theta_{\text{high}}$ remains to be (locally) supported by peer-group approval. In order to leave $\theta_{\text{high}}$ a more drastic increase of costs is needed. The dotted line shows the $\theta(S_t)$-curve for $c \to 0.6$. Since the individual benefit is now equalized by individual costs, the only sustainable long-run situation is at $\theta_{\text{low}} \to 0$. Doping has effectively been eliminated.

While increasing individual costs would be the only anti-doping policy available if preferences were socially-independent, social interaction of preferences allows to investigate two alternative (or, in reality, possibly complementing measures). The central panel shows that increasing the social stigma costs of doping can have similar effects as found for individual costs. Yet there are also remarkable differences. The panel shows an increase of $\phi$ from 0.3 to 0.5 (dashed lines) and to 0.75 (dotted lines). Again, the increase of stigma costs must be sufficiently large in order to eliminate the doping culture $\theta_{\text{high}}$. The most salient difference compared with the (c)–Panel is that increasing stigma is less effective in manipulating $\theta_{\text{low}}$, i.e. in effecting doping when the incidence of doping is already low. Intuitively, at $\theta_{\text{low}}$ it are the independent-minded athletes who keep on using drugs. They are less easily convinced by social stigma and need rising individual costs to stop their doping behavior.

A similar and even more pronounced outcome of an asymmetric effect of policy at $\theta_{\text{high}}$ and $\theta_{\text{low}}$ is visible in the right hand side panel, which shows the change of behavior caused by an decrease of peer-group cohesion; $\beta$ is reduced from 1.5 to 1 (dashed lines) and to 0.6 (dotted lines). Conceivable policies bringing about such a change are leniency policies motivating doping convicts to testify against their teammates or a change of rules which reduces the dependence on team-fellows in competition (e.g. the abandoning of team time trials in cycling). While a reduction of $\beta$ is very effective in eliminating a doping culture it is also completely ineffective in changing doping behavior at $\theta_{\text{low}}$. Intuitively, when the incidence of doping and thus peer-group approval is relatively low anyway ($S_t$ is low) it does not matter much how strongly athletes evaluate peer group approval ($\beta S_t$ is low anyway).

The broad conclusion from these exercises is that the model does support the endeavor of the WADA and other sport organizations targeted on influencing the social environment of a sport.\footnote{Acknowledging the power of social influence on the formation of athletes’ attitudes and believes the WADA has recently launched the “Play True Generation” program (WADA, 2008). Similar programs trying to increase an athletes
Measures increasing the awareness of stigmatization of doping by society at large or reducing group-cohesion can be successful in getting rid of a doping culture if they are sufficiently strong. These measures are, however, at the same time insufficient to eliminate doping entirely. For this an increase of individual costs is inevitable.

4. **Rank Loss Aversion**

This and the next sections consider some generalizations of the basic model. Here, we discuss disproportionate effects of doping and not-doping. Although a modeling of symmetric effects of doping and not-doping on rank may be appropriate for many sports, there are some general rules in some sports and some situations in other sports that produce asymmetric effects. This is particularly true when the best performers in the field set standards that imply cut-off thresholds for all other competitors.

A strong variant of such a threshold-rule exists in cycling at the Tour France (and other stage races). At each racing day the stage winner sets a cut off value on arrival time for all competitors. Riders that arrive \( x \) minutes later than the winner are excluded from the competition at further stages, i.e. in our notation they are automatically assigned with rank zero irrespective of their original ability. In particular, sprint specialists who could compete for high rankings (the Green Jersey) at later stages on flat land are threatened by elimination during the mountain stages.

Among the confessing professional cyclists in 2007 the argument that they do not want to end as Tour drop outs was sometimes uttered as the motivation – if not as the justification – for their use of performance enhancing drugs. Actually, they would prefer to stay clean but because of the challenge set by an incredibly strong (and possibly drug using) stage winner they have to give in and join the collective of dirty athletes.\textsuperscript{14} In many other sports the phenomenon occurs not as a general rule as in cycling but is situation-dependent. For example, nationally, the best athlete in a sports field sets a threshold value for participating at the Olympic Games. In all these cases a drug using winner, an athlete of anyway high ability, executes a disproportionately high threat on the career of clean athletes even if are they not aspiring to win but only to participate.

In order to capture the asymmetric effects of a doping culture on rank we introduce the parameter awareness of the size of \( \phi \) and to reduce \( \beta \) are launched on the national level (e.g. the “100% Me” program in the UK) and at the level of sport organizations (e.g. the “True Champion or Cheat” program of the UCI).

\textsuperscript{14} From 1948 to 1999 the average speed at the Tour de France raised from 32.4 km/h to 40.3 km/h. Interestingly, during the same period the variation of performance decreased tremendously. The share of riders managing to stay in the race and complete it in Paris raised from 20 percent to 70 percent.
\[ R_t(i) = A(i) \cdot \{1 + \alpha [d \cdot (1 - \theta_t) - \lambda \cdot (1 - d) \cdot \theta_t]\} \]

The doping motivation brought forward by athletes is reflected by a value of \( \lambda > 1 \). The larger \( \lambda \), the larger the rank loss if an athlete stays clean relative to the rank gain of a doping athlete. Inserting the modified rank function into the utility function provides

\[ U(i) = A(i) \cdot \{1 + \alpha [d \cdot (1 - \theta_t) - \lambda \cdot (1 - d) \cdot \theta_t]\} - d \cdot c + d \cdot \sigma(i) \cdot (\beta S_t - \phi), \tag{6} \]

which replaces (2). Proceeding as in Section 2 we compare for any athlete \( i \) utility when doping and not doping. This provides the modified ability–susceptibility threshold between clean and doping athletes.

\[ A = \frac{c - \sigma(\beta S_t - \phi)}{\alpha[1 + \theta_t(\lambda - 1)]}. \tag{7} \]

Again, athletes characterized by an ability–susceptibility tuple \((A, \sigma)\) above the threshold prefer doping and athletes with an ability–susceptibility tuple \((A, \sigma)\) below the threshold stay clean. Inspecting the threshold (7) we see that the right hand side decreases as \( \lambda \) rises. This implies that a season \( t \) snapshot confirms the “weak athletes’ argument”: the higher the threat of rank loss \( \lambda \) and the higher the share of doping athletes \( \theta_t \), the lower the threshold that has to be crossed in order to enter the club of doping athletes.

In order to obtain the equilibrium incidence of doping we proceed as explained in detail in the last section. We integrate the right hand side of (7) and set it equal to the share of clean athletes, \( 1 - \theta_t \). With contrast to the basic model, however, we can no longer explicitly solve for the equilibrium share of doping athletes \( \theta_t \). Instead we arrive at an implicit function determining \( \theta_t \) for given \( S_t \).

\[ 0 = G(\theta_t, S_t) = \begin{cases} 1 - \theta_t - \frac{c^2}{2\alpha[1+\theta_t(\lambda-1)](\beta S_t - \phi)} & \text{for } S_t \geq S_{\text{high}} \equiv (\phi + c)/\beta \\ 1 - \theta_t - \frac{1}{\alpha[1+\theta_t(\lambda-1)]} \left[c - \frac{1}{2}(\beta S_t - \phi)\right] & \text{for } S_{\text{high}} \geq S_t \geq S_{\text{low}} \\ \frac{(\alpha[1+\theta_t(\lambda-1)]-c)}{2\alpha[1+\theta_t(\lambda-1)](\beta S_t - \phi)} - \theta_t & \text{for } S_t \leq S_{\text{low}} \equiv \frac{\phi + c - \alpha[1+\theta_t(\lambda-1)]}{\beta} \end{cases} \tag{8} \]

The Appendix derives (8) in detail and also proves the following result.

**Proposition 5.** At any potential long-run equilibrium \( \theta^* \) the incidence of doping is increasing in the relative size of rank loss when not doping \( (\partial \theta^*/\partial \lambda > 0) \). As for the basic model the incidence of doping is increasing in the power of drugs \( \alpha \) and group cohesion \( \beta \) and decreasing in individual and social costs \( (c \text{ and } \phi) \).
Rank loss aversion thus, generally, aggravates the doping problem. In fact, rank loss aversion may be causal for doping to exist. The easiest way to see this, is to consider the case of socially-independent preferences ($\sigma = 0$ for all athletes). The threshold (7) then reduces to $A = c / [1 + \theta_t(\lambda - 1)]$. Integrating the area above the threshold provides $1 - \theta_t = c / [1 + \theta_t(\lambda - 1)]$, i.e.

$$\theta_t^2(\lambda - 1) - \theta_t(\lambda - 2) - 1 + c/\alpha = 0.$$  

Given the constraint that $\theta \in [0, 1]$, this provides a unique solution $\theta_u$. The most interesting case is to investigate $c \rightarrow \alpha$. In the basic model, without rank loss aversion, such a rise of costs eliminates doping entirely. With rank loss aversion, however, the solution is $\theta_{u,\lambda} = \max \{0, (\lambda - 2)/(\lambda - 1)\}$, which is strictly positive for $\lambda > 2$. For example, for $\lambda = 3$, $\theta_{u,\lambda} = 1/2$. In conclusion, rank loss aversion is sufficient to explain the incidence of doping.

The fact, that preferences are socially-dependent aggravates the problem of rank loss aversion further. It amplifies the incentive to dope when the incidence of doping is anyway high already and it eliminates potential equilibria of low incidence of doping. These conclusions are illustrated in Figure 4. The (\lambda)-panel on the left hand side resumes the case III of Figure 2 and displays the effect of increasing loss aversion. The solid line reiterates case III of Figure 2, i.e., there is no loss aversion, $\lambda = 1$, and there exist two locally stable equilibria $\theta_{high}$ and $\theta_{low}$. The dashed line shows the consequence of $\lambda = 2$. The emergence of loss aversion, e.g. through the introduction of minimum arrival times at stage trials on the Tour de France, eliminates the equilibrium $\theta_{low}$. The fear of rank loss (and the associated utility loss from, for example, disqualification for further stages of the competition) is sufficient to move an athlete’s society from an equilibrium of low incidence of doping towards a doping culture $\theta_{high}$ where a majority is using drugs.

Intuitively, starting at $\theta_{low}$ there are initially only a few athletes who take up doping after introduction of the new, loss aversion amplifying rules. Diagrammatically at low peer-group approval $S_t$ the dotted lines is just above the 45-degree line, such that $\theta_{low}$ ceases to exist. Next a bandwagon effect sets in. Since more athletes are using drugs, peer group approval rises, and further athletes are motivated to take drugs. The bandwagon affect is much stronger than without rank loss aversion because some athletes who would not be motivated by peer-group approval to take up doping in the basic scenario now start using drugs because they fear rank loss caused by their increasingly doped

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15The introduction of asymmetries in the utility function is thus not sufficient to generate multiple equilibria. Socially-dependent preferences remain to be essential for multiple equilibria to occur.
competitors. Their behavior in turn amplifies peer-group approval further, etc. As consequence the athletes’ community ends up at an equilibrium \( \theta_{\text{high}} \) where the incidence of doping is higher than it would be without rank loss aversion. Further rising rank loss aversion (\( \lambda = 3 \) for dotted lines) exacerbates the problem but does not qualitatively change the picture. For \( \lambda \to \infty \) the \( \theta(S_t) \) curve converges towards a step function assuming the value of unity for all \( S_t \).

Figure 4: The Impact of Rank Loss Aversion on Doping and Policy Effectiveness

The panel on the right hand side of Figure 4 takes up the policy experiment from Figure 3. The solid line reflects the same parameters as the solid lines in Figure 3 except that \( \lambda = 2 \). We thus investigate the impact of rank loss aversion on the effectiveness of anti-doping policy. This is exemplarily shown for an increase of the cost of doping. Comparing the \((c)\) panels of Figure 3 and 4 leads to the conclusion that rank loss aversion drastically reduces the scope of anti-doping policy. An increase of \( c \) to \( c = 0.5 \) (dashed lines) does not change the situation under loss aversion. Without rank loss aversion it has led to the emergence of \( \theta_{\text{low}} \) and the hope that a collective action effort moves the athletes’ community towards a low-doping equilibrium. This possibility arises under rank loss aversion only if costs are further increased toward \( c \to 0.6 \), a policy that has eliminated doping entirely in the basic model without loss aversion. With loss aversion, the doping culture \( \theta_{\text{high}} \) continues to exist at only mildly reduced incidence of doping in equilibrium. Only a collective action effort of athletes which manages to reduce \( \theta \) below the now emerging \( \theta_{\text{mid}} \) could eliminate the doping culture.
Assuming an athletes’ community originally situated at $\theta_{\text{low}}$, it is clear that the take up of doping after an increase of $\lambda$ and the bandwagon dynamics towards a doping culture $\theta_{\text{high}}$ is solely caused by the endeavor to avoid disutility from rank loss if not doping. This implies that the majority of athletes (who were originally not doping) would certainly be better off at the original $\theta_{\text{low}}$ equilibrium. This result implies the conclusion that the athletes’ community itself – not only spectators and politicians – should be in favor for a drastic reform of the rules and norms of their sport, a reform strong enough to allow the system to return to $\theta_{\text{low}}$.

5. The Taste for Victory

It has been frequently argued that the unequal distribution of honors (and money) is at the root of all evil in professional sports. For example, in the Olympic games there is just one gold medal and only three medals altogether for any discipline. Already the fourth in competition returns home with empty hands, more or less just as every other participant. If the Olympic motto “participation is everything” was ever a social norm it is long gone and replaced by “winning is everything” if not by “winning is the only thing”.

We discuss disproportionate effects from the aspiration to stay on top by modifying the utility function so that agent $i$ gets utility $R(i)^\gamma$ out of his rank, $\gamma \geq 1$. The higher $\gamma$ the higher the disproportionate effect of rank. For $\gamma \to \infty$ utility converges towards a step function, where the winner receives a utility value of one and all others athletes receive no utility at all, i.e. preferences converges towards the case where indeed “winning this the only thing.”

In order to investigate how the taste for victory affects the incentive for athletes to use performance enhancing drugs and how this changes the doping culture we rewrite (2) taking the reformulated utility from rank into account.

$$U(i) = \left( A(i) \cdot \left\{ 1 + \alpha [d \cdot (1 - \theta_t) - (1 - d) \cdot \theta_t] \right\} \right)^\gamma - d \cdot c + d \cdot \beta \cdot \sigma(i) \cdot (S_t - \phi).$$

Athletes compare the solutions of (9) for $d = 0$ and $d = 1$ in their decision whether to use drugs or stay clean. From that we see that athlete $i$ refrains from doping if $A(i)^\gamma z_t \leq c - \sigma(i) [\beta S_t - \phi]$ with $z_t \equiv \{ [1 + \alpha (1 - \theta_t)]^\gamma - [1 - \alpha \theta_t]^\gamma \}^{1/\gamma}$. The ability-susceptibility threshold dividing clean and

16 Asked whether they would take a banned substance that guarantees them winning a competition without being caught only 3 of 198 American athletes of Olympic standard said that they would not do it. Asked whether they would take a drug that guarantees them winning every competition for five years without being caught but entails also certain death caused by the drug’s side effects after the five years are over, more than half of the athletes still answered that they would do it. (Andrews, 1998).
doping athletes is thus given by
\[
A = \frac{\left[c - \sigma(\beta S_t - \phi)\right]^{1/\gamma}}{z_t}.
\] (10)
Proceeding as in Section 2, i.e. integrating the right hand side of (10) and equating it with \((1 - \theta)\) provides the equilibrium share of doping athletes in season \(t\). As in the previous section we cannot solve explicitly for the incidence of doping but get it determined by an implicit function.

\[
0 = F(\theta_t, S_t) = \begin{cases} 
1 - \theta_t - \frac{\gamma c^{1+1/\gamma}}{z_t(1+\gamma)(\beta S_t - \phi)} & \text{for } S_t \geq S_{\text{high}} \equiv (\phi + c)/\beta \\
1 - \theta_t - \frac{\gamma \{c^{1+1/\gamma} - (c + \phi - \beta S_t)\}^{1+1/\gamma}}{(1+\gamma)(\beta S_t - \phi)z_t} & \text{for } S_{\text{high}} \geq S_t \geq S_{\text{low}} \\
1 - \frac{1}{\beta - \beta S_t} \left\{ \frac{\gamma c^{1+1/\gamma}}{(1+\gamma)z_t} + \frac{z_t^{1+1/\gamma} - c}{1+\gamma} \right\} - \theta_t & \text{for } S_t \leq S_{\text{low}} \equiv \frac{\phi + c - z_t^{1+1/\gamma}}{\beta}.
\end{cases}
\] (11)

Setting \(S_t = \theta_t\) in (11) we get the long-run social equilibria. Interestingly, a higher taste for victory does not necessarily lead to a higher share of drug-using athletes at the doping equilibrium. The direction of the effect depends on which equilibrium the athletes’ community attained initially and on the total strength of the taste for victory. Generally, a sufficiently high increase of the taste for victory eliminates both an equilibrium with very low incidence of doping \(\theta_{\text{low}}\) and an equilibrium where a majority of athletes is doping \(\theta_{\text{high}}\) and it initiates the move towards a globally stable equilibrium \(\theta_{\text{mid}}\).

This result can best be demonstrated with help of the panel on the left hand side of Figure 5. The solid line reiterates case III from Figure 2. This means that initially, when \(\gamma = 1\), social (dis-)approval supports locally stable equilibria \(\theta_{\text{low}}\) and \(\theta_{\text{high}}\) and the initial situation (the history of the sport) determines which equilibrium is actually attained. Suppose, initially the athletes’ community is situated at or close to \(\theta_{\text{low}}\). The dashed line shows the effect of a rising taste for victory from \(\gamma = 1\) to \(\gamma = 2\). The higher veneration of winners eliminates the \(\theta_{\text{low}}\) equilibrium. Intuitively, a higher share of high ability athletes takes up doping. The motivation of these athletes originates predominantly from the higher desirability of attaining a high rank, i.e. it occurs irrespective of peer-approval. This can be seen in the diagram by the upward shift of the lower part of the \(\theta(S_t)\) curve, i.e. the higher incidence of doping without or with little approval from fellow athletes.

The fact that more (high-ability) athletes are using drugs in turn increases peer-group approval and motivates some additional athletes of middle and lower ability to take up doping. The initiated bandwagon effect move the athletes’ community towards a doping culture \(\theta_{\text{high}}\). The incidence of doping at \(\theta_{\text{high}}\) is somewhat lower then it would be without the higher taste for victory because with higher \(\gamma\) some athletes of low ability are discouraged from taking drugs. They refrain from doping
because they would anyway – with or without assistance of performance enhancing drugs – not be able to reach the highest ranks.

Figure 5: Doping Equilibria: The Impact of the Taste for Victory

Left hand side: $\alpha = 0.5$, $\beta = 1.5$, $c = 0.4$, $\phi = 0.3$ and $\gamma = 1$ (solid lines, as panel III in Figure 2), $\gamma = 2$ (dashed lines) and $\gamma = 10$ (dotted lines), a star on the $\theta_t$ axis marks the solution under socially-independent preferences. Right hand side: $\alpha = 0.6$, $\beta = 1.5$, $\phi = 0.3$, $\gamma = 10$ and $c = 0.4$ (solid lines), $c = 0.5$ (dashed lines), and $c = 0.6$ (dotted lines).

The latter observation begs the question whether a further increase of the taste for victory can actually discourage so many athletes of low ability from taking drugs that the doping culture $\theta_{\text{high}}$ becomes unsustainable. The dotted line in the ($\gamma$)-panel shows that this is indeed the case. To generate the dotted line $\gamma$ has been increased to the value of 10, reflecting a very strong preference for attaining the highest ranks. Indeed many athletes of lesser ability are now discouraged from using drugs and the reduced peer-group approval caused by their abstention induces a bandwagon effect away from $\theta_{\text{high}}$. However, instead of approaching $\theta_{\text{low}}$ the athletes’ community arrives at an intermediate equilibrium $\theta_{\text{mid}}$ where athletes of high ability are using drugs in order to compete for the highest ranks. Their decision is relatively independent from social approval, a fact that can be inferred from the relatively flat slope of the lower part of the $\theta(S_t)$ curve.

If the taste for victory gets very high, one can no longer speak of a doping culture in the strict sense. First, at the equilibrium $\theta_{\text{mid}}$ the majority of athletes is actually not doping. Secondly, the decision to take drugs is only marginally influenced by social approval from fellow athletes or disapproval from society at large. Although preferences are socially-dependent, the incidence of doping is not very different from the one that would occur under socially-independent preferences.
(indicated by a star in Figure 5). For further rising $\gamma$ the lower part of the $\theta(S_t)$ curve approaches a horizontal line and the solutions with and without socially-dependent preferences, $\theta_{\text{mid}}$ and $\theta_u$, coincide. In this case it is probably more appropriate to speak of a doping subculture formed by the athletes of highest ability who are competing for the highest ranks, all of them using drugs no matter how large the social disapproval generated by their behavior is. The situation is sustainable because they are indeed finishing at the highest ranks in each season’s tournaments thereby receiving the public veneration for winners that they need for being motivated to train hard and to use drugs. They are the superheroes of their sport.

The fact that at a superhero-equilibrium the athletes of highest ability are those who try to further push their rank whereas the weak and intermediate athletes stay clean has an empirically verifiable implication. If we look at results in absolute terms (for example, arrival time at a mountain stage at the Tour de France) we should be able to observe a clear structural break between the best athletes in the field and the rest. Interestingly, in the year 2007 this was indeed observed at the Tour de France. The French press spoke of “le cyclisme a deux vitesses”, two-speed cycling: one subset of riders climbed the mountains at incredible speed and sometimes seemingly without visible effort, and the rest of the peloton was left behind.\footnote{A similar phenomenon was observed twice before. In 1999, after the Festina scandal, when the French cycling association unilaterally implemented stricter anti-doping rules, and in 2002, after reliable tests for EPO doping became available after which (allegedly) a few high ability riders (i.e. the team leaders) turned towards the much more elaborate and expensive blood doping causing a huge gap between their arrival times and those of the rest of the peloton.}

Another indication that professional cycling may have approached a superhero equilibrium is that virtually all of the 23 riders disqualified in the course of the infamous Operacion Puerto in May 2006, were regarded as the top contenders of the 2006 Tour de France (Baron et al., 2007).

From the viewpoint of the fight against doping, convergence towards a doping subculture, or superhero equilibrium $\theta_{\text{mid}}$ where only the most determined and talented athletes are using drugs is actually bad news because a superhero equilibrium turns out to be rather resistant against policy. With respect to the parameters comprising social influence, $\beta$ and $\phi$, this should be obvious by now. Given a high taste victory, the highly-able athletes do not give much on social approval of their behavior. Sadly, the panel on the right hand side of Figure 5 demonstrates that this is also true with respect to individual costs. The panel takes up the scenario analyzed by the $(c)$-panels if Figures 3 and 4, keeping the same parameters except that now $\gamma = 10$, indicating a very high taste for victory. The effect of an increase of individual costs is reflected by the dashed line ($c$ rises from 0.4 to 0.5) and the dotted line ($c = 0.6$). One sees that even the formerly quite effective large increase
of costs has now only little effect on the incidence of doping and no power to eliminated $\theta_{mid}$. While the majority of professional athletes continuous to react on largely increasing costs (more controls, heavier punishment) with abandoning doping, a minority of superheroes resists. In the Figure this is reflected by the low impact of policy on the lower, almost horizontal part of the $\theta(S_t)$-curve.

In order to assess the ineffectiveness of rising costs quantitatively, we drop for a moment Assumption 1 and focus on socially-independent preferences. The approximation will be quite accurate because, as indicated, at the superhero equilibrium the decision to dope is in fact largely independent from social approval. Computing (7) for $\sigma = 0$ and integrating we obtain the share of clean athletes given by

$$1 - \theta_t = \left(\frac{c}{[1 + \alpha(1 + \theta_t)^\gamma] - [1 - \alpha \theta_t]^\gamma}\right)^{1/\gamma}.$$  

Setting $\theta_t = 0$ and solving for $c$ provides the cost threshold that eliminates doping as $c = (1 + \alpha)\gamma - 1$. For example, plugging in the parameters of Figure 5 and $\gamma = 10$ provides the estimate $c = 56.6$.

Recalling that for the basic model costs had just to exceed the power of drugs in improving rank ($c > \alpha$) to eliminate doping, and given $\alpha = 0.6$ from the example, we infer that the higher taste of victory lead to almost a one-hundred fold increase in the costs needed to eliminate doping. In conclusion, a strong taste for victory (a strong focus on winners) can be identified as a major impediment in the fight against doping.

6. Ability-dependent Costs of Doping

This Section reconsiders the basic model but gives up the simplifying assumption of constant costs of doping and assumes instead that costs depend on ability. There are certainly good reasons to assume that costs are lower for high-ability athletes, for example, because they have better contacts for drug acquisition or because drugs are sponsored by third parties (see footnote 17). A negative ability-cost correlation, however, would not change any results. In fact, it would make the conclusion of doping to be particular prevalent among the ablest athletes even more compelling. It is thus more interesting to explore the consequences of a positive correlation between ability and costs, an assumption which has the potential to reverse or qualify the results obtained so far. A positive ability-cost correlation could be motivated by the notion that athletes of high ability have possibly more to lose in terms of expected income when identified as a doper.

A positive linear correlation between ability and costs does not have the power to change any conclusions. To see this assume that costs of doping for athlete $i$ are $c + \eta A(i)$. Assumption 1, which
ensured that the power of drugs is strong enough for the phenomenon of doping to exist, is then replaced by $\alpha - \eta \geq c$ and thus, necessarily, $\alpha - \eta > 0$. Proceeding as for the basic model we obtain that athletes stay clean if their ability-susceptibility tuple lies below the threshold

$$A = \frac{c - \sigma \beta (S_t - \phi)}{\alpha - \eta}.$$ 

Since $\alpha - \eta > 0$ this condition is isomorph to condition (3). A linear ability-cost correlation (and thus, naturally, any concave association) is not capable to affect the results obtained so far. For that the ability-cost association has to be convex.

In the following we exemplarily focus on quadratic costs because they enable an explicit solution. Suppose the costs of doping for athlete $i$ are $c + \eta A(i)^2$ with $c > 0$ and $\eta > 0$. It is illustrative to consider for a moment socially-independent preferences ($\sigma(i) = 0$ for all $i$). Then, athletes stay clean if $\alpha A(i) < c + \eta A(i)^2$. The interesting case here emerges when the quadratic equation has two real solutions, which is the case for sufficiently strong power of drugs, i.e. for $\alpha^2 > 4c\eta$. In that case we get a split athletes' community. Athletes stay clean if

$$A(i) \leq \max(0, A_1) \text{ or } A(i) \geq \min(1, A_2)$$

with $A_1 \equiv \alpha/(2\eta)\sqrt{s}$ and $A_2 \equiv \alpha/(2\eta) + \sqrt{s}$, $s \equiv \alpha^2/(4\eta^2) - c/\eta$. With contrast to the model variants discussed so far, it are now athletes of intermediate ability who are mostly inclined to use drugs. Athletes at the lower end and at the upper end of the ability range stay clean if $A_1 > 0$ and $A_2 < 1$ respectively. Intuitively, athletes of low ability are not getting enough improvement of rank and utility out of using drugs whereas athletes of high ability refrain from doping because of the entailed high costs. In the following we focus on this case of a split athletes’ community because it is the only case that constitutes a qualitative modification of the basic model. This implies the following restriction on parameters.

**Assumption 2.** Performance-enhancing drugs are powerful enough for doping to exist if preferences were independent from social (dis-) approval of doping, i.e. $\alpha^2 > 4c\eta$. Costs are sufficiently strongly increasing in ability such that there exists at least one clean athlete of highest ability, i.e. $\eta > \alpha - c$.

Assumption 2 replaces Assumption 1. The first part of the assumption ensures that $A_1 > 0$, the second part ensures that $A_2 < 1$. Diagrammatically, the assumption implies that there exists
a closed interval along the ability line within which doping occurs, as shown in Figure 6. The incidence of doping is then obtained as $\theta_u = A_2 - A_1$. Taking the derivatives of $A_1$ and $A_2$ with respect to the relevant parameters verifies that a reduction of the power or drugs $\alpha$ and an increase of ability-independent costs $c$ compresses the doping interval at both ends and that an increase of ability-dependent costs $\eta$ compresses the interval (predominantly) by reducing the upper bound $A_2$. Naturally, athletes of high ability are more strongly affected by an increase of ability-dependent costs of doping.

Turning towards socially-dependent preferences, we see that, facing quadratic costs, athlete $i$ stays clean if $\alpha A(i) < c + \eta A(i)^2 - \sigma(i) [\beta S_t + \phi]$. Given the non-monotonous association between ability and drug use, it is more convenient for the analysis to swap axes of the ability-susceptibility space because the threshold can then be represented as a function $\sigma(A)$, which can be easily integrated to obtain the incidence of doping. This way, the threshold is given by

$$\sigma = \frac{A [\eta A - \alpha] + c}{\beta S_t - \phi}. \quad (12)$$

It is visualized in Figure 6. The strict ability split obtained for socially-independent preferences becomes more smooth under socially-dependent preferences, implying that on average (i.e. for given $\sigma$) athletes of intermediate ability are more inclined to use drugs.

As shown in Figure 6, the threshold (12) has two roots given by $A_1$ and $A_2$, i.e. by the boundaries of the doping range under socially-independent preference. In principle, as for the standard model, athletes with ability-susceptibility endowment below the threshold stay clean while those with an endowment above the threshold use drugs. Also similarly to the standard model, we distinguish three different cases, which materialize depending on the size of peer-group approval $S_t$.

In order to disentangle effects of social approval, it is helpful inspect the slope of (12).

$$\frac{\partial \sigma}{\partial A} = \frac{2\eta A - \alpha}{\beta S_t - \phi}.$$ 

Generally, the threshold exhibits an extremum at $A = \alpha/(2\eta)$. If peer-group approval is sufficiently high, $S_t > S_{low} \equiv \phi/\beta$, the extremum is a minimum, as in the left and central panel of Figure 6. It implies that for given ability the incentive to use drugs is higher among athletes of high-susceptibility to social approval. Similarly to the results from the basic model, we can further distinguish a case where subgroups of athletes refrain from doping completely. If $S_t < S_{high}^1 \equiv (c + \phi)/\beta$ then there exists a range of athletes of lowest ability who are not doping, $b_1 = \alpha/(2\eta) - \sqrt{x}$, $x \equiv$
\[ \frac{\alpha^2}{(4\eta^2)} - (c + \phi - \beta S_t) \equiv \frac{(c + \phi + \eta - \alpha)}{\beta} \]

On the other side of the ability spectrum, if \( S_t < S_{t,\text{high}} \equiv \frac{(c + \phi + \eta - \alpha)}{\beta} \), there exists a range of athletes of highest ability who are not doping, \( 1 - b_2 \equiv 1 - \frac{\alpha}{2\eta} - \sqrt{x} \). This case is shown in the central panel of Figure 6.

Figure 6: Doping Threshold for Ability-Dependent Costs of Doping

Left and central panel: \( S_t \) is sufficiently high, \( S_t > S_{t,\text{low}} \), such that all athletes of intermediate ability, \( A \in (A_1, A_2) \) use drugs. Central panel: \( S_t \) is sufficiently low such that all athletes of lowest ability stay clean \( (S_t < S_{t,\text{high}}) \) and such that all athletes of highest ability stay clean \( (S_t < S_{t,\text{high}}^2) \). Right panel: \( S_t \) is so low that some athletes of intermediate ability refrain from drug use as well \( (S_t < S_{t,\text{low}}) \). See text for definition of \( S_{t,\text{high}} \) and \( S_{t,\text{low}} \) and further explanations.

Finally, if \( S_t \) is even lower, \( S_t < S_{t,\text{low}} \), social approval turns into disapproval, the extremum becomes a maximum, and the threshold flips around in the \( A-\sigma \)–diagram. This case is shown in the panel on the right hand side of Figure 6. For given ability, it are now athletes who are largely immune against social approval who are mostly inclined to use drugs.

In order to obtain the share of doping athletes we proceed as before, integrate the area below the threshold, and solve for \( \theta \). This provides

\[
\theta = \begin{cases} 
B_2 - B_1 - \frac{1}{\beta S_t - \phi} \left[ \frac{\eta}{3} \left( A_1^3 - B_1^3 + B_2^3 - A_2^3 \right) - \frac{\eta}{2} \left( A_1^2 - B_1^2 + B_2^2 - A_2^2 \right) + c(A_1 - B_1 + B_2 - A_2) \right] & \text{for } S_t \geq S_{t,\text{low}} \equiv \frac{\phi}{\beta} \\
\frac{1}{\beta S_t - \phi} \left[ \frac{\eta}{3} \left( A_2^3 - A_1^3 \right) - \frac{\eta}{2} \left( A_2^2 - A_1^2 \right) + c(A_2 - A_1) \right] & \text{for } S_t \leq S_{t,\text{low}}.
\end{cases}
\]

where \( B_1 = \max \{ 0, \frac{\alpha}{2\eta} - \sqrt{x} \} \), \( B_2 = \min \{ 1, \frac{\alpha}{2\eta} + \sqrt{x} \} \), \( x \equiv \frac{\alpha^2}{4\eta^2} - (c + \phi - \beta S_t)/\eta \).

Figure 7 shows the effectiveness of anti-doping policy. For that purpose parameters were used that generate approximately the same initial situation as discussed for the basic model in Figure 3. Of course, parameter values had to change in order to generate an interior ability range of dopers, i.e. to fulfill Assumption 2. With parameter values specified below Figure 7 one gets an initial situation represented by the solid line, implying global stability of a doping culture \( \theta_{\text{high}} \).

The left hand side resumes the analysis of rising ability-independent costs of doping. Obviously
the cost increase has exactly the same effects as in the basic model although it are now the athletes of intermediate ability who are mostly inclined to use drugs. A medium-sized increase of \( c \) (from 0.4 to 0.5) generates multiple equilibria along the dashed line. It implies the conclusion that the athletes’ community remains at \( \theta_{\text{high}} \) unless it manages a sufficiently strong reduction of \( \theta \) below \( \theta_{\text{mid}} \) by a one-time collective action effort. The dotted line represents a scenario that effectively eliminates doping under socially-independent preference. Costs increase such that \( c \rightarrow \alpha^2/(4\eta) \), which is 0.55 according to the numerical specification of the example. As for the basic model, this cost increase is sufficient to initiate an escape from the doping culture and convergence towards \( \theta_{\text{low}} \).

The panel on the right side of Figure 7 shows effects of an increase of the new policy variable \( \eta \). The picture that emerges is very similar to the by now well-known one. A medium-size increase of ability-dependent costs (from 1.8 to 2.1) produces multiple equilibria but is of no avail to eliminate the doping culture \( \theta_{\text{high}} \). If cost increase further (to 2.3) the doping culture \( \theta_{\text{high}} \) ceases to exist and the community converges towards \( \theta_{\text{low}} \).

While the policy effects look very similar on the aggregate level in both panels, there are of course great differences on the individual level. While an increase of \( c \) (for example, the monetary fine to be paid if being identified as a doper) affects athletes of the entire ability spectrum, an increase of \( \eta \) (for example, the length of a ban from participation) discourages doping predominantly around the
upper boundary of the original drug users, i.e. among athletes of high (but not highest) ability.

In conclusion, imposing ability-dependent costs modifies the doping decision on the individual level but does not change results obtained on the aggregate level, i.e. results about the emergence and stability of doping cultures and the fight against them. A discussion of cases where doping is mainly a problem of athletes of intermediate ability is valuable as a check for robustness of results, the fact, however, that athletes of highest ability are actually caught doping time and again lets appear the cases discussed in Section 2 to 5 to be more relevant in practice. This seems to be particularly true in cycling since the introduction of EPO and own-blood doping. For example, almost all 23 riders disqualified in the events around Operacion Puerto were among the top-tour contenders, some of them former tour champions.

Intuitively, one would expect a particularly strong rise of $\eta$ through the introduction of retroactive testing and punishment (which opens the possibility to get ex post deprived of victories and prize money). Yet the first cases of retroactive tests in cycling have actually exposed high-ability riders as dopers (2008 tour stage winners Leonardo Piepoli and Stefan Schumacher and overall third Bernard Kohl using a then undetectable EPO-derivate) This indicates that the performance enhancing power of EPO and blood doping are just too strong, and that the basic model and, perhaps, the taste-of-victory variant are more appropriate to describe the doping situation in cycling.

7. Conclusion

This article should be seen as an essay in positive theory. It has not tried to develop normative arguments against doping. For societies were medical improvements of the natural endowments of beauty, powers of concentration, or happiness are socially approved it could indeed be not that easy to find a waterproof moral argument against medical improvements of natural talent in sports.\footnote{See, for example, Kayser et al. (2007) for a compilation of anti-anti-doping arguments.} Instead, the proposed theory has shown how community dynamics can move a sport into an equilibrium where a large majority is using drugs without getting much out of it ranking-wise. Not because of sudden moral qualms but because costly doping has become so inefficient, a situation is reached where it is in the self-interest of the majority of professional athletes to get rid of their doping culture.

It has been shown that a doping culture can be conceptualized as an equilibrium where a majority of athletes uses performance enhancing drugs and where the therewith generated peer-group approval
of doping makes the situation sustainable. Depending on the size of parameters capturing the individual and social costs and benefits of doping such an equilibrium appears in two variants. In one variant there exists simultaneously a locally stable equilibrium of very low incidence of doping, in the other variant such an equilibrium of low incidence of doping does not exist.

In the case of multiple equilibria, athletes could, in principle, release themselves from the equilibrium of high incidence of doping. This would require a strong collective action, i.e. a one-time drastic reduction of $\theta_t$, so that the system leaves the domain of attraction of the equilibrium of high incidence of doping. Generating such a collective action, however, may be too hard to be managed by athletes without external help. In this case, and in any case if the equilibrium of high doping incidence is unique, a change of the rules of the game is needed in order to initiate a movement towards low or absent doping.

With respect to the individual costs of doping the article has shown that a cost increase is effective in eliminating the equilibrium of high doping incidence only if it is sufficiently strong. Besides this perhaps obvious conclusion, it has been shown that similar effects can be expected by a sufficiently strong increase of stigma costs from being detected as a doper and from a reduction of group cohesion of the athletes community. The theory thus supports recently launched educational programs that are designed to reduced peer-group influence (e.g. WADA, 2009a, UCI, 2009). But the model also suggests that – with contrast to increasing individual costs – one cannot expect from programs addressing social influence to eliminate doping entirely.

An extension of the model has shown why rank loss aversion generated, for example, by qualification or disqualification marks set be the best (and presumably doped) athletes in competition generally increases the incidence of doping and may actually eliminate an equilibrium of low incidence of doping. It has also been shown that such rules reduce the power of anti-doping policies. In order to eliminate a doping culture the theory thus recommends to abandon or to reduce standards for participation that are set by the best athletes in a sport. Certainly, this is more easily achieved in some sports (stage disqualification cut offs at the Tour de France) than in others (qualification marks for the Olympic games).

A further modification of the model has demonstrated that conclusions are even more drastic

\[19\] For example, a collective action was tried by eight French and German teams at the Tour de France 2007. They formed the so-called “Movement for Credible Cycling”, subjected themselves to additional, voluntary anti-doping surveillance, and organized sit-down strikes against doping before the beginning of stages. Since, nevertheless, some riders of the “Collective” were caught doping, the idea was not successful.

\[20\] In cycling, since 2007 participants of the Tour de France had to sign a “code of ethics” agreeing to pay a year’s salary in addition to two years suspension. Obviously this change of rules was not enough to deter doping.
with respect to the disproportionate emphasis of winning (or, more generally finishing among the
top athletes). The larger the disproportionate utility received from finishing at the top the lower the
influence of policy to prevent doping within a subgroup of superheroes, i.e. among the anyway most
talented competitors in a sport. This has shown to be true with respect to both monetary costs and
social costs of doping. If a sport is situated in such a superhero-equilibrium the anti-doping strategy
of highest priority should be to downplay the role of winners and top finishers by, for example,
distributing money and tv appearances more equally among all ranks that professional athletes can
achieve.

Finally, it could be useful to think of cycling not as a special, particularly rotten sport but as
a precursor of the things to come in other disciplines. Because doping has been so effective in
increasing performance and because of the long history of doping in the sport and the thereby
generated peer-group approval, the social dynamics may have carried the cyclists’ community close
towards an equilibrium of particularly high incidence of doping. In the future, when gene doping
allows for pronounced improvements of performance in every kind of physical activity, other sports
will have the advantage to learn from the development of doping and anti-doing policies in cycling.
Appendix

Derivation of (4). The critical $\sigma$ above which all athletes dope is obtained by setting $A = 0$ in (3), which provides $\sigma = c / (\beta S_t - \phi) \equiv \bar{\sigma}$. For $\bar{\sigma}$ to exist within $[0, 1]$ it has to be smaller or equal to unity, requiring that $S_t \geq (c + \phi)/\beta \equiv S_{\text{high}}$. Likewise, when $S_t < \phi$, the critical level of $\sigma$ above which all athletes stay clean is obtained by setting $A = 1$ in (3), which provides $\sigma = (\alpha - c)/(\phi - \beta S_t) \equiv \bar{\sigma}$. For $\bar{\sigma}$ to exist within $[0, 1]$ it has to be smaller or equal to unity, requiring that $S_t \leq (c + \phi - \alpha)/\beta$.

Generally, integrating the area below the threshold (3) within the limit 0 and $x$ provides

$$\int_0^x \frac{c - \sigma(\beta S_t - \phi)}{\alpha} \, d\sigma = \frac{1}{\alpha} \left| c\sigma - \frac{\sigma^2}{2}(\beta S_t - \phi) \right|^x_0.$$ 

If $S_t > S_{\text{high}}$, integrate up to $x = \bar{\sigma}$ to obtain

$$1 - \theta = \frac{c^2}{2\alpha(\beta S_t - \phi)}.$$ 

Likewise for $S_{\text{low}} < S_t < S_{\text{high}}$, integrate up to $x = 1$ to obtain

$$1 - \theta = \frac{1}{\alpha} \left[ c - \frac{1}{2}(\beta S_t - \phi) \right].$$ 

The case $S_t \leq S_{\text{low}}$ is a little more involved. To obtain $1 - \theta$ integrate up to $x = \bar{\sigma}$ and add $(1 - \bar{\sigma})$, i.e. the share of athletes who stay clean irrespective of $\sigma$. This provides

$$1 - \theta = \frac{1}{\alpha} \left[ \frac{c(\alpha - c)}{\beta \phi - \beta S_t} + \frac{(\alpha - c)^2}{2(\phi - \beta S_t)} \right] + 1 - \frac{\alpha - c}{\beta S_t} = 1 - \frac{(\alpha - c)^2}{2\alpha(\phi - \beta S_t)}.$$ 

Collecting terms, we get (4).

Curve Discussion of (4). At $S_t = 0$ we have $\theta(0) = (\alpha - c)^2/(2\alpha \phi) > 0$. At $S_t = 1$ we have $\theta(1) = 1 - c^2/(2\alpha(\beta - \phi)) < 1$. Observe that $\theta(S_t)$ is continuous in $[0, 1]$. The first derivative is

$$\frac{\partial \theta(S_t)}{\partial S_t} = \begin{cases} \frac{c^2}{2\alpha(\beta S_t - \phi)^2} & \text{for } S_t \geq S_{\text{high}} \\ \frac{\beta}{2\alpha} & \text{for } S_{\text{high}} \geq S_t \geq S_{\text{low}} \\ \frac{(\alpha - c)^2}{2\alpha(\phi - \beta S_t)^2} & \text{for } S_t \leq S_{\text{low}}. \end{cases}$$

and thus everywhere positive. The second derivative is

$$\frac{\partial^2 \theta(S_t)}{\partial S_t^2} = \begin{cases} -\frac{c^2}{2\alpha(\beta S_t - \phi)^3} < 0 & \text{for } S_t \geq S_{\text{high}} \\ 0 & \text{for } S_{\text{high}} \geq S_t \geq S_{\text{low}} \\ \frac{(\alpha - c)^2}{2\alpha(\phi - \beta S_t)^3} > 0 & \text{for } S_t \leq S_{\text{low}}. \end{cases}$$

The curve is thus convex at its lower part, linear at its middle part, and concave at its upper part.

Proof of Proposition 2. To begin with consider possible equilibria along the convex part of the $\theta(S_t)$-curve, Setting $S_t = \theta$ in (4) when $S_t < S_{\text{low}}$ provides $\theta = (\alpha - c)^2/(2\alpha(\phi - \beta \theta))$. Solving for $\theta$ we get two solution candidates

$$\theta_{1,2} = \frac{\phi}{2\beta} \pm \sqrt{\frac{\phi^2}{4\beta^2} - \frac{(\alpha - c)^2}{2\alpha \beta}}$$

(A.1)
Existence of a solution requires that the radicand is non-negative, i.e. $\phi > a \equiv (\alpha - c)\sqrt{2\beta/\alpha}$. For being assumed as a solution along the convex part the equilibrium $\theta = S$ has to be smaller than the upper boundary of the convex segment $\theta(S_{\text{low}})$. Inserting $S_{\text{low}}$ into the convex segment of (4) we obtain the upper boundary $\theta(S_{\text{low}}) = (\alpha - c)/(2\alpha)$. For the smaller root to be feasible this implies that

$$\frac{\phi}{2\beta} - \sqrt{\frac{\phi^2}{4\beta^2} - \frac{(\alpha - c)^2}{2\alpha \beta}} < \frac{\alpha - c}{2\alpha} \Rightarrow (\alpha - c)\left(1 + \frac{\beta}{2\alpha}\right) < \phi.$$ 

Likewise we obtain for the larger root to be smaller than $\theta(S_{\text{low}})$ the condition that $(\alpha - c)(1 + \beta/(2\alpha)) > \phi$. Because the two conditions are mutually exclusive, there exists at most one equilibrium $\theta_{\text{low}}$. Since the $\theta(S_t)$ curve starts out above the 45 degree line, it cuts the 45 degree line first at the smaller root. If this equilibrium exists, then there exists no other equilibrium along the convex segment. This provides $\theta_{\text{low}}$ of Proposition 2.

The most convenient way to see that $\theta_{\text{low}} < \theta_u$ is to notice that maximum that $\theta_{\text{low}}$ can potentially assume is at the boundary of the convex segment, i.e. at $\theta(S_{\text{low}})$ and that

$$\theta(S_{\text{low}}) = \frac{\alpha - c}{2\alpha} < \frac{\alpha - c}{\alpha} \equiv \theta_u.$$

Next consider equilibria along the intermediate, linear part of the $\theta(S_t)$ curve. Because of linearity, there exists at most one equilibrium. Setting $S_t = \theta$ in (4) when $S_{\text{low}} < S_t < S_{\text{high}}$ and solving for $\theta$ provides

$$\theta_{\text{mid}} = \frac{\alpha - c - \phi/2}{\alpha - \beta/2}. \quad \text{(A.2)}$$

Note that the equilibrium is stable if the slope of the $\theta(S_t)$ curve is smaller than unity (i.e. flatter than the 45-degree line, which identifies equilibria). Inspecting (4) we obtain the slope along the linear part as $\beta/(2\alpha)$. Thus, for a slope smaller than unity, $\alpha > \beta/2$, implying a positive denominator in (A.2). Existence then requires that the numerator is positive as well implying $\alpha - c - \phi/2 < 0$. Because $\beta/2 < \alpha$ existence of a stable equilibrium necessarily requires

$$\frac{\beta - \phi}{2} < c. \quad \text{(A.3)}$$

For $\theta_{\text{mid}} > \theta_u$ we must have that

$$\frac{\alpha - c - \phi/2}{\alpha - \beta/2} > 1 - \frac{c}{\alpha} \iff \frac{(\alpha - c)}{\alpha} \beta > \phi.$$

In order to verify that this condition is indeed fulfilled begin with noting that $a \equiv (\alpha - c)\sqrt{2\beta/\alpha} > \phi$ for $\theta_{\text{mid}}$ to exist. Moreover, the slope of the linear part of $\theta(S_t)$ has to be smaller than unity, i.e., $\beta/(2\alpha) < 1$. From this follows

$$1 > \sqrt{\frac{\alpha}{2\beta}} \Rightarrow \frac{\beta}{\alpha} > \sqrt{\frac{2\beta}{\alpha}} \Rightarrow \frac{(\alpha - c)}{\alpha} \beta > (\alpha - c)\sqrt{\frac{2\beta}{\alpha}} > \phi$$

implying $\theta_{\text{mid}} > \theta_u$.

Finally consider equilibria along the upper, concave part of the $\theta(S_t)$ curve. Setting $S_t = \theta$ in (4)
when $S_t > S_{\text{high}}$ and solving for $\theta$ provides solution candidates

$$\theta_{3,4} = \frac{\beta + \phi}{2\beta} \pm \sqrt{\frac{(\beta - \phi)^2}{4\beta^2} - \frac{c^2}{2\alpha\beta}}.$$  \hspace{1cm} (A.4)

Again, existence of a solution requires a non-negative radicand, which requires that $\phi < b \equiv \beta - c\sqrt{2\beta/\alpha}$. Let $\theta_{\text{high}}$ denote the larger root and $\theta_{\text{mid}}$ the smaller root. For $\theta_{\text{mid}}$ to exist along the concave part of the $\theta(S_t)$-curve, it has to be larger than $S_{\text{high}}$, which means that

$$\frac{\beta + \phi}{2\beta} - \sqrt{\frac{(\beta - \phi)^2}{4\beta^2} - \frac{c^2}{2\alpha\beta}} > \frac{c + \phi}{\beta} \quad \Rightarrow \quad c \left(1 + \frac{\beta}{2\alpha}\right) < \frac{\beta - \phi}{2} \quad \Rightarrow \quad c < \frac{\beta - \phi}{2} \hspace{1cm} (A.5)$$

where the last condition follows necessarily from $\beta/(2\alpha) > 0$. Compare (A.5) with (A.3) to see that these conditions are mutually exclusive. Either the equilibrium $\theta_{\text{mid}}$ is assumed along the linear part or it is assumed along the concave part of the $\theta(S_t)$-curve. In conclusion there are at most three equilibria.

In order to verify that $\theta_{\text{high}} > 1/2$ begin with noting that $\theta_{\text{high}}$ is assumed along the concave segment of which the lower boundary is given by $S_{\text{high}}$. Thus $\theta_{\text{high}} \geq \theta(S_{\text{high}})$. Insert $S_{\text{high}}$ into (4) to obtain $\theta(S_{\text{high}}) = 1 - c/(2\alpha)$ which exceeds 1/2 because $1 > c/\alpha$.

We have now gathered all elements needed for the discussion of existence, uniqueness, and stability of equilibria of the $\theta(S_t)$ curve. Figure A shows all possible positions of the convex, linear, and concave segments of the curve with respect to the 45 degree line. The 45 degree line, along which $\theta_t = S_t$, identifies equilibria. Note from (4) that $\theta(0) > 0$ and $\theta(1) < 1$. Along the concave part there exist thus either one (1.b) or two (1.c) equilibria, or the convex part lies entirely below the 45 degree line (1.a). The linear part lies either entirely above (2.a) or below (2.b) the 45 degree line, or there exists a unique intersection, either from above (2.c) or from below (2.d). The convex part originates from positive $\theta(0)$ and intersects the 45 degree line either once (3.b) or not at all (3.a).

Fortunately, not all possible permutations of the segments are feasible. Indeed, there are “only” four qualitatively different cases. To see this, begin with noting from (4) that the $\theta(S_t)$ curve is everywhere continuous in the $(0, 1)$ interval implying smooth pasting of the individual segments.

To begin with, consider segment 1.a. This can be pasted smoothly only to segments 2.b or 2.c. Consider first the sequence 1.a–2.b. It can be pasted smoothly only to 3.b. The sequence 1.a-2.b-3.b establishes Case I of Proposition 2 visualized in Panel I of Figure 2. As shown above, for Case I to hold we must have that $\phi > a$ such that there exists an equilibrium along the lower convex part, and $\phi > b$ such that there exists no equilibrium along the concave part.

The sequence 1.a-2.c can be pasted smoothly only with segment 3.a. The emerging sequence 1.a-2.c-3.a establishes Case II of Proposition 2 visualized in Panel II of Figure 2. This case identifies a unique equilibrium $\theta_{\text{mid}}$ assumed along the linear part of the curve. For this case to occur there must be no equilibrium along the upper concave part, which – as shown above – requires $\phi > b$ and no equilibrium along the lower convex part of the curve, which requires $\phi < a$.

Next consider segment 1.b. It can be pasted smoothly only with segments 2.a and 2.d. Consider first the sequence 1.b–2.a. It can be pasted smoothly only with 3.a. The sequence 1.b-2.a-3.a constitutes Case IV of Proposition 2 visualized in Panel IV of Figure 2. It identifies a unique equilibrium $\theta_{\text{high}}$ assumed along the upper, concave segment of the $\theta(S_t)$ curve, As shown above, for
The figure shows the possible position of the upper concave part (first row), the intermediate, linear part (central row) and lower convex part (final row) of the $\theta(S_t)$ curve with respect to the 45 degree line.

In this case to hold we must have $b > \phi$, such that there exists an equilibrium along the concave part of the curve, and $a > \phi$ such that there exists no equilibrium along the convex part of the curve.

The sequence 1.b.-2.d. can be pasted smoothly only with segment 3.b. The emerging sequence 1.b.-2.d-3.b establishes Case III of Proposition 2 visualized in Panel III of Figure 2. Here we have three equilibria, which – as shown above – requires $\phi > a$ for $\theta_{low}$ to exist and $\phi < b$ for $\theta_{high}$ to exist.

Finally consider segment 1.c, i.e. the case where there exist two equilibria along the concave segment of the curve. As shown above this fact precludes that there exists another equilibrium along the linear part of the curve, Thus the only feasible solution to match 1.c smoothly is 2.b. The sequence 1.c-2.b can be pasted smoothly only with 3.b. The sequence 1.c-2.b-3.b identifies another variant of Case III, where we have three equilibria. As already shown above, we must have $\phi > a$ for $\theta_{low}$ to exist and $\phi < b$ for $\theta_{high}$ to exist.

Note that the social dynamics (5) lead to increasing $\theta$ when the $\theta(S_t)$ curve lies above the 45 degree line and to decreasing $\theta$ when it lies below such that the arrows of motion shown in Figure 2 arise and the results about stability follow immediately. This completes the proof of Proposition 2.
Derivation of (8). As for the basic model, the critical $\sigma$ above which all athletes dope is obtained by setting $A = 0$ in (7), which provides $\sigma = c/(\beta S_t - \phi) \equiv \bar{\sigma}$. For $\bar{\sigma}$ to exist within $[0, 1]$ it has to be smaller or equal to unity, requiring that $S_t \geq (c + \phi)/\beta \equiv S_{high}$. Likewise, when $S_t < \phi$, the critical level of $\sigma$ above which all athletes stay clean is obtained by setting $A = 1$ in (7), which provides $\sigma = (\alpha[1 + \theta_t(\lambda - 1)] - c)/(\phi - \beta S_t) \equiv \bar{\sigma}$. For $\bar{\sigma}$ to exist within $[0, 1]$ it has to be smaller or equal to unity, requiring that $S_t \leq \{\phi + c - \alpha[1 + \theta_t(\lambda - 1)]\}/\beta$.

Generally, integrating the area below the threshold (7) within the limit $0$ and $x$ provides

$$
\int_0^x \frac{c - \sigma(\beta S_t - \phi)}{\alpha[1 + \theta_t(\lambda - 1)]} d\sigma = \frac{1}{\alpha[1 + \theta_t(\lambda - 1)]} \left| c\sigma - \frac{\sigma^2}{2}(\beta S_t - \phi) \right|_0^x.
$$

If $S_t > S_{high}$, integrate up to $x = \bar{\sigma}$ to obtain

$$
1 - \theta = \frac{c^2}{2\alpha[1 + \theta_t(\lambda - 1)](\beta S_t - \phi)}.
$$

Likewise for $S_t$ in the medium range, integrate up to $x = 1$ to obtain

$$
1 - \theta = \frac{1}{\alpha[1 + \theta_t(\lambda - 1)]}\left[ c - \frac{1}{2}(\beta S_t - \phi) \right].
$$

For $S_t \leq S_{low}$ to obtain $1 - \theta$ we integrate up to $x = \bar{\sigma}$ and add $(1 - \bar{\sigma})$. This provides

$$
1 - \theta = \frac{1}{\alpha[1 + \theta_t(\lambda - 1)]}\left[ \frac{c(\alpha[1 + \theta_t(\lambda - 1)] - c)}{(\phi - \beta S_t)} + \frac{(\alpha[1 + \theta_t(\lambda - 1)] - c^2)}{2(\phi - \beta S_t)} \right] + \frac{\alpha[1 + \theta_t(\lambda - 1)] - c}{(\phi - \beta S_t)}
= 1 - \frac{(\alpha[1 + \theta_t(\lambda - 1)] - c^2)}{2\alpha[1 + \theta_t(\lambda - 1)](\phi - \beta S_t)}.
$$

Collecting terms, we get (8).

**Proof of Proposition 5.** The proof is easiest by noting that at any equilibrium $\theta_t = S_t$ and by applying the implicit function theorem with respect to $S_t$. Begin with taking the derivative with respect to $S_t$

$$
\frac{\partial G}{\partial S_t} = \begin{cases} \frac{2\alpha c^2}{2\alpha[1 + \theta_t(\lambda - 1)](\beta S_t - \phi)^2} & \text{for } S_t \geq S_{high} \\ \frac{2\alpha[1 + \theta_t(\lambda - 1)]}{2\alpha[1 + \theta_t(\lambda - 1)](\phi - \beta S_t)^2} & \text{for } S_{high} \geq S_t \geq S_{low} \\ \frac{2\alpha[1 + \theta_t(\lambda - 1)]}{2\alpha[1 + \theta_t(\lambda - 1)](\phi - \beta S_t)^2} & \text{for } S_t \leq S_{low}. \end{cases}
$$

and notice that it is everywhere positive. Next, take the derivative with respect to $\lambda$

$$
\frac{\partial G}{\partial \lambda} = \begin{cases} \frac{2\alpha c^2}{4\alpha^2(1 + \theta_t(\lambda - 1))^2(\phi - \beta S_t)} & \text{for } S_t \geq S_{high} \\ \left[ c - \frac{1}{2}(\beta S_t - \phi) \right] & \text{for } S_{high} \geq S_t \geq S_{low} \\ \frac{2\alpha[1 + \theta_t(\lambda - 1)]}{2\alpha[1 + \theta_t(\lambda - 1)](\phi - \beta S_t)^2} & \text{for } S_t \leq S_{low}. \end{cases}
$$

From this follows immediately that $\partial G/\partial \lambda > 0$ for $S_t \geq S_{high}$. For the intermediate range of $S_t$ note that here $S_t < S_{high}$, i.e.

$$
\frac{c + \phi}{\beta} \geq S_t \quad \Rightarrow \quad c \geq \beta S_t - \phi \quad \Rightarrow \quad c \geq \frac{1}{2}(\beta S_t - \phi).
$$

Thus $\partial G/\partial \lambda > 0$ within the intermediate range. For $S_t < S_{low}$, the derivative is positive if

$$
2\alpha \{\alpha[1 + \theta_t(\lambda - 1)] - c\} (1 + \theta_t(\lambda - 1) - \{\alpha[1 + \theta_t(\lambda - 1)] - c\}^2 > 0,
$$
i.e. if
\[ 2\alpha [1 + \theta_t(\lambda - 1)] - \alpha [1 + \theta_t(\lambda - 1)] + c = \alpha [1 + \theta_t(\lambda - 1)] + c > 0, \]
which is true. Thus \( \partial G/\partial \lambda > 0 \) everywhere. The final step combines the results, \( \partial S_t/\partial \lambda = - (\partial G/\partial \lambda)/(\partial G/\partial S_t) > 0 \).

The derivatives of \( \partial G/\partial c < 0, \partial G/\partial \beta > 0, \) and \( \partial G/\partial \phi < 0 \) can be immediately read off (8). Likewise \( \partial G/\partial \alpha > 0 \) for all \( S_t \geq S_{low}. \) For \( S_t \leq S_{low} \) note that the sign of the derivative of \( G \) with respect to \( \alpha \) is the same as
\[
\frac{(\alpha [1 + \theta(\lambda - 1)] - c)^2}{\alpha} \cdot \frac{\partial}{\partial \alpha} = \frac{1}{\alpha^2} \cdot (\alpha [\theta(\lambda - 1)] - c) \cdot (\alpha [\theta(\lambda - 1)] + c) > 0.
\]

Applying the implicit function formula \( \partial S_t/\partial x = - (\partial G/\partial x)/(\partial G/\partial S_t), \) \( x = \{\alpha, \beta, c, \phi\} \) completes the proof.

**Derivation of (11).** As in the derivation of (4) the critical \( \sigma \) above which all athletes dope is obtained by setting \( A = 0 \) in (10), which provides \( \sigma = c/(\beta S_t - \phi) \equiv \tilde{\sigma}. \) For \( \tilde{\sigma} \) to exist within [0, 1] it has to be smaller or equal to unity, requiring that \( S_t \geq (c + \phi)/\beta \equiv S_{high}. \) Likewise, when \( S_t < \phi, \) the critical level of \( \sigma \) above which all athletes stay clean is obtained by setting \( A = 1 \) in (10), which provides \( \sigma = (z_t^\gamma - c)(\phi - \beta S_t) \equiv \bar{\sigma}. \) For \( \bar{\sigma} \) to exist within [0, 1] it has to be smaller or equal to unity, requiring that \( S_t \leq (c + \phi - z_t^\gamma)/\beta. \)

Generally, integrating the area below the threshold (11) within the limit 0 and \( x \) provides
\[
\int_0^x \frac{[c - \sigma(\beta S_t - \phi)]^{1/\gamma}}{z_t} d\sigma = - \frac{1}{z_t} \frac{\gamma}{1 + \gamma} \frac{1}{\beta S_t - \phi} \left[ (c - \sigma(\beta S_t - \phi))^{1+1/\gamma} \right]_0^x.
\]

If \( S_t > S_{high}, \) integrate up to \( x = \tilde{\sigma} \) to obtain
\[
1 - \theta = \frac{1}{z_t} \frac{\gamma}{1 + \gamma} \frac{e^{1+1/\gamma}}{\beta S_t - \phi}.
\]
Likewise for \( S_t \) in the medium range, integrate up to \( x = 1 \) to obtain
\[
1 - \theta = \frac{\gamma \left( e^{1+1/\gamma} - (c + \phi - \beta S_t) \right)^{1+1/\gamma}}{(1 + \gamma)(\beta S_t - \phi) z_t}
\]

To obtain \( 1 - \theta \) for the case \( S_t \leq S_{low} \) integrate up to \( x = \bar{\sigma} \) and add \( (1 - \bar{\sigma}). \) This provides
\[
1 - \theta = - \frac{1}{z_t} \frac{\gamma}{1 + \gamma} \frac{1}{\beta S_t - \phi} \left\{ \left[ c + \frac{z_t^\gamma - c}{\phi - \beta S_t} (\phi - \beta S_t) \right]^{1+1/\gamma} - c^{1+1/\gamma} \right\} + 1 - \frac{z_t^\gamma - c}{\phi - \beta S_t}.
\]
Noting that \( (z_t^\gamma)^{(1+\gamma)/\gamma} = z_t^{1+\gamma} \) and \( z_t^{1+\gamma}/z_t = z_t^\gamma, \) this expression simplifies to
\[
\theta = \frac{1}{\beta S_t - \phi} \left\{ \frac{\gamma}{1 + \gamma} z_t^\gamma - \frac{e^{1+1/\gamma}}{z_t} \right\}.
\]
Collect terms to get (11).
References


