# Birth, Death, and Development: A Simple Unified Growth Theory

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**Abstract.** This study provides a unified growth theory to correctly predict the initially negative and subsequently positive relationship between child mortality and net reproduction observed in industrialized countries over the course of their demographic transitions. The model captures the intricate interplay between technological progress, mortality, fertility and economic growth in the transition from Malthusian stagnation to modern growth. It identifies a number of structural breaks over the course of development, suggesting a high degree of complexity regarding the relationships between various economic and demographic variables.

*Keywords:* Economic Growth, Mortality, Fertility, Structural Change, Industrial Revolution.

*JEL:* O11, O14, J10, J13.

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## 1. Introduction

In recent years, several so-called *unified growth theories* have been forwarded to try to motivate the historical shift from economic stagnation to modern growth. Following seminal work by Galor and Weil (2000), these include Boucekkine et al. (2002), Doepke (2004), Galor and Moav (2002), Galor and Mountford (2008), Jones (2001), Kögel and Prskawetz (2001), Lucas (2002), Strulik and Weisdorf (2008), Cervellati and Sunde (2005, 2007) and Tamura (2002). A part of this exercise consists of providing a micro foundation for the fundamental links between economic and demographic variables from pre-industrial times to the present day. One particular issue that scholars have been struggling with for a long time (although not only in the context of unified growth theory) is the impact of lower child mortality on fertility and net reproduction.[1] Most macroeconomic models can replicate the fact that lower death risk of children leads to fewer child births. However, since falling child mortality reduces the cost of surviving children, net reproduction in these models ends up increasing in response to lower child mortality, a contrast to the experience of most industrialized countries in the later part of their demographic transitions (Doepke 2005). This indicates that factors other then declining child mortality were responsible for the reduction in net reproduction observed in the western world over the last century.

This study provides a model where the relationship between child mortality and net reproduction is positive during early phases of development, but negative during later phases. Our result arises from combining two existing contributions to the literature. First, as in Strulik (2008), we assume that parents care not only about surviving offspring but also their nutritional status (i.e. child quality). For a given level of nutritional input, an exogenous decrease in child mortality (higher survival probability) leads to lower fertility because more children now survive. This lead parents to nourish their children better (a quantity-quality substitution effect). Building on the fact that nutrition played a key role in Britain's mortality transition (Harris 2004), and on ample evidence that malnutrition has severe effects on child mortality (see Rice et al., 2000, Pelletier et al., 2003, and Caulfield et al., 2004), this investment of parents in child nutrition further improves the offsprings' survival probability. Fertility, however, is also affected by the price of food. The interplay of mortality and food prices will thus determine the paths

[1] Net reproduction measures the number of offspring (normally women) living through to the end of their fertile age.

of fertility and net reproduction. Specifically, lower mortality goes together with rising fertility when the effect of falling food prices on fertility dominates the quantity-quality substitution effect. By contrast, lower mortality appears together with lower fertility when quantity-quality substitution effect dominates that of falling food prices.

Second, as in Weisdorf (2008), we show that the price of food, measured in terms of non-food goods, responds to the relative rate of growth of total factor productivity (TFP) in agriculture and industry. Specifically, TFP growth in agriculture reduces the price of food, whereas TFP growth in industry has the opposite effect. In the end, therefore, the effects of falling child mortality on fertility depend on the preferences of parents, as well as on technology advancements in agriculture and industry. As will become apparent below, this creates a unified growth theory that correctly predicts the relationship between child mortality and net reproduction over the course of the demographic transition, while at the same time capturing the intricate interplay between technological progress, mortality, fertility and income per capita in the process from stagnation to growth. In addition, this is the first unified growth theory to rightly replicate an increase in total food expenditure as income rises. At the same time, however, the model also complies with Engel's Law, which says that that the *share* of food expenditures to the total income falls as income rises. Finally, by calibrating the model, we detect several structural breaks in the relationship between various variable over the very long run, implying that empirical scholars will have a hard time identifying the correlation between economic and demographic variables.

The paper continues as follows. Section 2 provides a brief introduction to the stylized facts for Western Europe (particularly England) regarding the evolution of mortality, fertility and net reproduction over the long run. It also gives a summary of the theoretical contributions that our work compares to. Section 3 details the theoretical framework that we propose, and Section 4 explores the its balanced growth dynamics. Finally, section 5 calibrates the model to analyze its adjustment dynamics, and Section 6 concludes.

## 2. Empirical evidence and the related literature

In most Western Europe countries, the demographic transition occurred in the later half of the 19th century.[2] After a peak in the 1870s, birth rates dropped roughly one-third over the subsequent 50 years. In England, total fertility rates declined by close to 50 percent, from nearly five children per women in 1875 to 2.4 children by 1920. Crude birth rates followed a similar pattern, declining by 44 percent from 36 per thousand inhabitants in 1875 to 20 in 1920 (Figure 1).

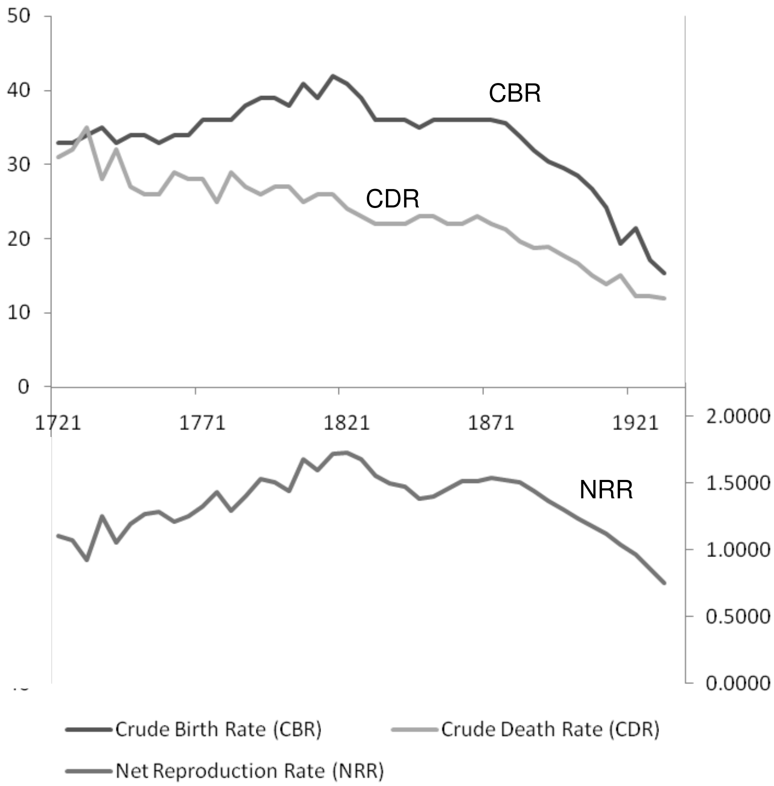
With the exception of France and the US, substantial mortality decline preceded the fall in fertility. In England, mortality rates began to fall roughly one and a half century prior to the drop in fertility. During early phases of England's so-called *mortality revolution*, it was primarily lower child mortality that gave momentum to the drop in death rates. As the drop in child mortality appeared before fertility began to decline, falling mortality initially gave rise to an increase in net reproduction (Figure 1). After the onset of the demographic transition, however, the relationship changes. From then on, falling birth rates went hand in hand with declining rates of net reproduction. Once the demographic transition was running its course, therefore, falling child mortality was outpaced by the decline in fertility, as reflected in a reduction in the net rate of reproduction (Figure 1).

It follows that a unified growth theory, which wants to correctly capture the long-run evolution of mortality, fertility and growth, must account not only for the increase of net reproduction and the spike in birth rates reported shortly prior to the onset of the demographic transition. It must also be able to predict the decrease of both fertility and net reproduction that appears once the demographic transition runs it course (Figure 1).

Economic research on the relationship between child mortality and fertility goes back at least to Becker and Barro (1988, 1989). In the Barro-Becker model, parents are altruistic towards surviving offspring, and child mortality rates affect their birth decisions to the extent that they affect the costs of surviving offspring. Since lower child mortality reduces the costs of surviving offspring, this lead to higher net reproduction, a contrast to what was observed after the onset of the demographic transition. Barro-Becker type models, therefore, seek other explanations for the fertility decline then falling mortality (Doepke, 2005). Among recent studies, where

[2] For a more detailed description of the Western Europe's demographic patterns, see Galor (2005). Here, we follow the Galor's terminology according to which only the drop in fertility (and not mortality) constitutes the demographic transition.

Figure 1: Crude Birth Rates, Crude Death rates,
and Net Reproduction Rates for England, 1721-1931




Sources: Wrigley and Schofield (1984), Wrigley (1969) and Reher (2004).

mortality plays a role in the process of development, a shift from child quality to child quantity is thus generated through parents' investments in education and human capital accumulation (Azarnert 2006; Ehrlich and Kim 2005; Kalemli-Ozcan, Ryder, and Weil 2000; Soares 2005).

A refinement of these models, a shift from exogenous to endogenous mortality has been invoked to capture the long run trends in economic and demographic variables (Doepke 2005; Jones 2001; Kalemli-Ozcan 2002; Lagerlöf 2003; Weisdorf 2004). Jones (2001), Lagerlöf (2003) and Weisdorf (2004) compare directly to our work in that they analyze the effect of mortality on fertility in the context of unified growth theory. Remarkably, they all share the common feature that falling death rates have no impact on parents' fertility decision. In Jones (2001), parents' preferences imply that the elasticity of substitution between consumption and children is always greater than one, an assumption that ultimately generates a drop in fertility as income grows. In Lagerlöf

(2005) and Weisdorf (2004), the decline of fertility is a result of human capital accumulation and a parental trade off between child quantity and quality.

The current paper is most closely related to a recent contribution by Cervellati and Sunde (2007) that also investigates the complex interaction between mortality and fertility in the transition from stagnation to growth. They focus on the interaction between education and adult longevity as the main driver for the transition, emphasizing the skill premium as the important relative price during transition. Their approach is thus complementary to ours in that it highlights the role of relative prices for development. As in previous unified growth models, however, Cervellati and Sunde have human capital accumulation at the heart of their theory. This makes the present study the first unified growth model to endogenize mortality, fertility and growth, while not relying on human capital accumulation as the driving force behind the demographic transition.

## 3. The Model

The model presented below largely follows the unified growth theory proposed by Strulik and Weisdorf (2008), but with additional elements from Strulik (2008) and Weisdorf (2008).

3.1. **Fertility, mortality, and net reproduction.** We consider a two-period overlapping generations economy with children and adults. Let $L_t$ denote the number of adults in period $t$, and $n_t$ the number of births per adult.[3] The birth rate (sometimes referred to as *total* fertility) is determined endogenously below. Furthermore, let the variable $\pi_t \in [0, 1]$ measure a child's survival probability, which is synonymous to the fraction of children, born in period $t$, that are still alive at period $t + 1$. It thus follows that the net reproduction rate – i.e. the number of offspring living through their fertile age– is given by $\pi_t n_t$. Hence, change in the size of the labor force (i.e. the adult population) between any two periods is given by

$$L_{t+1} = \pi_t n_t L_t. \tag{1}$$

Following Strulik (2008), we operate with two types of child survival probabilities: an extrinsic and an intrinsic survival rate. The extrinsic rate, denoted $\bar{\pi}_t \in [0, 1]$, is exogenous to a parent, but is affected by general-purpose hygiene or health improving factors, such as sewerage, water toilets, central heating, clinical devices, vaccines, pharmaceuticals and medical knowledge in

[3]To keep the model tractable, we assume that $n_t$ is continuous, and that reproduction is asexual.

general. We take advances in industrial knowledge, measured by $M_t$, to be a good proxy for this. Specifically, it is assumed that $\bar{\pi}_t = \bar{\pi}(M_t)$, where $\bar{\pi}_M > 0$ with $\lim_{M_t \to \infty} \bar{\pi}_t = a < 1$. The variable $M_t$, and thus $\bar{\pi}_t$, are determined endogenously below.

By contrast to the extrinsic child survival probability, the intrinsic survival rate is affected by parents' nutritional investment in their offspring. We allow for diminishing marginal productivity to nutritional investments, so that an additional unit of nutrition is more effectively hedging against child mortality at low levels of extrinsic survival rates than at high ones. More specifically, we assume that the overall child survival probability (intrinsic as well as extrinsic) is given by

$$\pi_t = \bar{\pi}_t + [1 - \bar{\pi}_t] \cdot \lambda \cdot h_t, \qquad \lambda > 0, \tag{2}$$

where $\lambda$ is a productivity parameter, and with $h_t$ measuring the nutritional status of an offspring, the level of which is determined endogenously below.[4]

3.2. **Preferences and optimization.** Adult individuals maximize utility, which they derive from three sources: surviving offspring, $\pi_t n_t$, the nutritional status of offspring, $h_t$, and manufactured goods consumption, measured by $m_t$. We assume that preferences are described by a utility function in which the elasticity of marginal utility is higher for $h_t$ and $n_t$ than for $q_t$. This means that in times of crises parents try to smooth fertility and nutritional status of their offsprings relatively stronger than consumption of manufactured goods. The simplest utility function that captures such a 'hierarchy of needs' is of quasi-linear form.

$$u_t = m_t + \beta \log(h_t) + \gamma \log(\pi_t n_t), \quad (\beta, \gamma) > 0. \tag{3}$$

Similar to Andreoni (1989) and Becker (1960), the parameter $\beta$ measures the extent to which parents care about the nutritional status of their offspring. It is assumed that $\gamma > \beta$, which simply means that parents, who do not give birth, will not allocate income to child nutrition.

One important implication of (3), which follows from the specific form of the preferences, is that the income-elasticity of the demand for children and their nutritional status is zero. Instead, as will become apparent below, the number of children that parents choose to have, as

[4]We furthermore assume that $a \leq \beta/\gamma$. This ensures, as will become obvious below – when (6) is inserted in (2) – that $\pi \in (0, 1)$.

well as their nutritional status are determined by the costs of nutrition, as well as their survival probability.[5]

To make the model tractable, suppose that nutritional goods are demanded only during childhood and some of it then stored for adulthood.[6] The price of manufactured goods is set to one, so that we can let $p_t$ denote the price of one unit of nutrition, measured in terms of manufactured goods. Since each offspring consumes $h_t$ units of nutrition, the total costs of having $n_t$ children, measured in terms of manufactured goods, is $p_t h_t n_t$. The budget constraint of an individual adult therefore reads

$$w_t = p_t h_t n_t + m_t, \tag{4}$$

where $w_t$, which will be determined below, is the income of a representative adult individual, also measured in terms of manufactured goods.

The optimization problem of an adult individual involves maximizing (3) subject to (2) and (4). The solutions, i.e. the optimal number of births and nutritional input of children, are given by

$$n_t = \frac{\beta\gamma\lambda(1-\bar{\pi}_t)}{p_t\bar{\pi}_t(\gamma-\beta)}, \tag{5}$$

$$h_t = \frac{(\gamma-\beta)\bar{\pi}_t}{\beta\lambda(1-\bar{\pi}_t)}. \tag{6}$$

Based on (5) and (6) the following should be noted. First, the price-effect on the demand for children, as expected, is negative. Second, due to the specific form of the preference function, there is no direct in income-effect on the demand for children. However, as will become apparent below, an *indirect* income-effect will enter through the price of nutrition ($p_t$). Third, it follows from (5) that an improvement of the child survival probability reduces fertility ($\partial n_t/\partial\bar{\pi}_t < 0$), meaning that lower child mortality reduces birth rates, a conclusion perfectly consistent with evidence (Galloway et al., 1998). Fourth, equation (6) reveals that a higher extrinsic probability of child survival leads to a higher nutritional status of the offspring ($\partial h_t/\partial\bar{\pi}_t > 0$). This follows from quantity-quality substitution. As more children survive, parents reduce fertility, and put more emphasis on nutrition. Better nutrition in turn improves the intrinsic probability of child

[5]In Strulik and Weisdorf (2008), we demonstrate that the results obtained below are preserved also when using a more general iso-elastic utility function, as long as the elasticity of marginal utility for children is higher than for manufactured goods.

[6]It will not affect the qualitative nature of the results, if, instead, the individual's nutritional demand where to be divided over two periods.

survival, i.e. a feedback mechanism generates bi-directional causality: from better (extrinsic) child survival to better nutrition to better (intrinsic) child survival.

By combining (5) and (6) with (2), we get the net rate of reproduction, which is given by

$$\pi_t n_t = \bar{\pi}_t n_t + (1 - \bar{\pi}_t)\lambda h_t n_t = \frac{\lambda \gamma^2 (1 - \bar{\pi}_t)}{p_t(\gamma - \beta)} \tag{7}$$

It follows that a higher child survival probability—corresponding to lower child mortality—reduces net fertility, an empirical regularity that previous contributions have not been able to capture (see the discussion in Doepke, 2005).

3.3. **Production.** In this subsection, we largely follow the outline in Strulik and Weisdorf (2008). We consider a dual-sector economy with agriculture and industry. In both sectors, new technology arises from learning-by-doing. More specifically, output, as well as new knowledge, occurs according to the following production functions:

$$Y_t^A = \mu A_t^{\varepsilon} (L_t^A)^{\alpha} = A_{t+1} - A_t, \quad 0 < \alpha, \varepsilon < 1 \tag{8}$$

$$Y_t^M = \delta M_t^{\phi} L_t^M = M_{t+1} - M_t, \quad 0 < \phi < 1. \tag{9}$$

The variable $A_t$ measures TFP in agriculture, whereas $M_t$ measures TFP in industry (manufacturing). We allow for diminishing returns to new knowledge by assuming that $0 < \varepsilon, \phi < 1$. Agricultural production is subject to constant returns to labor and land. Land is assumed to be in fixed supply, and the total amount is normalized to one. With $0 < \alpha < 1$ there is thus diminishing returns to labor in agriculture. Industrial production, by contrast, is subject to constant returns to labor, implying that land is not an important factor in industrial production.[7] As is standard in the related literature, we abstract throughout from the use of physical capital in production.

3.4. **Equilibrium.** The variables $L_t^A$ and $L_t^M$ measure total labor input in agriculture and industry, respectively. Added together, they equal the size of the labour force, i.e.

$$L_t^A + L_t^M = L_t. \tag{10}$$

[7]Qualitatively identical results can be obtained also with diminishing returns to labor in industry (see Strulik and Weisdorf 2008).

The share of total labor devoted to agriculture, $L_t^A/L_t$, is determined by the market equilibrium condition for nutritional (i.e. agricultural) goods. This condition says that the total supply of nutrition, $Y_t^A$, equals total demand, which—given that each child demands $h_t$ units of food—is $h_t n_t L_t$. Using (8), the market equilibrium condition for nutrition thus implies that the fraction of workers engaged in agriculture is given by

$$\theta_t \equiv \frac{L_t^A}{L_t} = \left(\frac{h_t n_t L_t^{1-\alpha}}{\mu A_t^{\varepsilon}}\right)^{\frac{1}{\alpha}}. \tag{11}$$

Note that agricultural TFP growth releases labor from agriculture, whereas population growth and higher levels of child nutrition has the opposite effect.

Suppose that there are no property rights over land, meaning that the land rent is zero, and thus that a representative adult individual receives the average product of the sector in which it is employed. The labor market equilibrium condition then implies that the real price of nutrition adjusts, so that farmers and manufacturers earn the same income, i.e. so that $w_t = p_t Y_t^A / L_t^A = Y_t^M / L_t^M$. By the use of (5)-(6) and (7)-(11), this means that the price of one unit of nutrition, measured in terms of manufactured goods, is given by

$$p_t = \frac{\left(\delta M_t^{\phi}\right)^{\alpha} (\gamma L_t)^{1-\alpha}}{\mu A_t^{\varepsilon}}. \tag{12}$$

It follows that the real price of nutrition increases with TFP growth in industry, as well as with the size of the population, whereas TFP growth in agriculture has the opposite effect.

Inserting (12) into (7) gives us the net reproduction rate in a general equilibrium, which reads

$$\pi n_t = \frac{\mu A_t^{\varepsilon}}{\left(\delta M_t^{\phi}\right)^{\alpha} (\gamma L_t)^{1-\alpha}} \cdot \frac{\gamma^2 \lambda (1-\bar{\pi}_t)}{(\gamma - \beta)}. \tag{13}$$

Note how (13) captures the results found in Strulik (2008) and Weisdorf (2008). The first term includes the negative effect on net reproduction of higher (real) prices of nutrition as per Weisdorf (2008). The second term contains the negative effect on net reproduction of lower child mortality, as described by Strulik (2008).

## 4. Balanced and Unbalanced Growth in the Long Run

In the following, we explore the balanced growth dynamics of the model. Along a balanced growth path, all variables are constant or grow at constant rates. Let a balanced growth rate of a variable $x$ be denoted by $g^x$ (to be identified by a missing time index). According to (8), the gross rate of TFP growth in agriculture is $g_t^A = (A_{t+1} - A_t)/A_t = \mu(L^A{}_t)^\alpha / A_t^{1-\epsilon}$. Along a balanced growth path, the left hand side is constant by definition, so that the right hand side must be constant as well. Furthermore, the share of labor in agriculture must be constant, implying that $L^A$ grows at the same rate as $L$. Thus, constant TFP growth in agriculture requires that

$$1 + g^A = (1 + g^L)^{\alpha/(1-\epsilon)}. \tag{14}$$

Similarly, we get from (9) that a constant rate of growth of TFP in industry requires that

$$1 + g^M = (1 + g^L)^{1/(1-\phi)}. \tag{15}$$

Combining (13) with (14) and (15), the gross rate of growth of net reproduction can then be written as

$$1 + g_{t+1}^L = \frac{\pi_t n_t}{\pi_{t+1} n_{t+1}} = \frac{1 - \bar{\pi}_{t+1}}{1 - \bar{\pi}_t} \cdot \frac{(1 + g_t^A)^\epsilon (1 + g_t^L)^\alpha}{(1 + g_t^M)^{\alpha\phi}}. \tag{16}$$

Along a balanced growth path, the level of TFP in industry is either constant, or is growing at a constant rate. In either case, $\bar{\pi}$ will eventually assume a constant value, meaning that, along a balanced growth path, the first term on the right-hand side is equal to one. Using this information, and inserting (14) and (15) into (16), we find that the equilibrium law of motion for population growth is given by

$$1 + g_{t+1}^L = (1 + g_t^L)^\eta, \qquad \eta \equiv \alpha + \frac{\alpha\epsilon}{1 - \epsilon} - \frac{\phi\alpha}{1 - \phi}. \tag{17}$$

Along a balanced growth path, the population level grows at constant rate, meaning that $g_{t+1}^L = g_t^L = g^L$. This leaves two possibilities for balanced growth. Either there is no population growth ($g^L = 0$) or – assuming the knife-edge condition that $\eta = 1$ – the population level is growing or shrinking at a constant rate. However, it follows from (14) that $|\eta| < 1$ is required for stability reasons. Therefore, growth on the knife-edge not only demands a very specific parameter constellation; it also implies that the economy starts off *on* a balanced growth path (with suitable initial values) and remains there forever. This essentially eliminates the possibility of having balanced growth together with population growth. The implication—that there is no population growth on a balanced growth path—means that there is also no TFP growth in steady state (as can be verified by looking at (14) and (15)).[8] This conclusion is summarized in the following proposition.

Proposition 1. *There exists a unique balanced growth path with zero population growth and zero (exponential) economic growth. A sufficiently small knowledge elasticity in agriculture,*

$$\epsilon < \frac{1-\phi-\alpha+2\alpha\phi}{1-\phi+\alpha\phi}, \tag{18}$$

*prevents unbalanced growth in the long-run.*

*Proof.* The proof is found in the Appendix.[9] □

## 5. Adjustment Dynamics and Calibration

The aim of this section is to see whether the model can replicate the stylized development pattern observed among industrialized countries, from their pre-industrial era to the present-day and beyond.[10] Therefore, in the following, we explore the model's adjustment dynamics towards balanced growth.

In brief, the most interesting adjustment path can be described as follows. Suppose we start off with an economy in which the population level is relatively small; the share of labor employed in agriculture is relatively high; and the level of income per capita is relatively close to subsistence. Furthermore, suppose that birth rates, as well as rates of child mortality, are relatively high, meaning that the net rate of reproduction is close to that of replacement. Roughly speaking, these are the characteristics of a pre-industrial, agricultural society.

As explained in the model section, there are economies-of-scale to population. Since the initial population level is low, learning-by-doing effects, to begin with, are relatively modest. Hence, TFP growth in agriculture is slow, yet faster than in industry, where labor resources, and thus learning-by-doing effects, are even smaller.

TFP growth in agriculture has two effects on development. On the one hand, because it releases labor from agriculture, agricultural TFP growth increases the share of labor allocated to industrial activities. On the other hand, higher TFP growth in agriculture *relative to industry*

[8] In principle, this result is identical to that found in Strulik and Weisdorf (2008).

[9] Note that the term 'unbalanced growth' refers to an equilibrium growth path along which growth rates are exploding or imploding (and should not be confused with 'unstable', i.e. off-equilibrium growth.

[10] For a detailed description of the development course of industrialized countries, see Galor (2005).

makes nutrition, and therefore children, relatively less expensive. According to (5), this raises fertility, which tends to increase the net reproduction rate.

At the same time, with economies-of-scale at work in both agriculture and industry, the transfer of labor out of agriculture gradually speeds up TFP growth in industry. As industrial knowledge gains momentum, extrinsic child survival begins to increase. This leads parents to invest in more nutrition per child, which further improves the survival probability of offspring through a reduction in intrinsic child mortality. As follows from (13), this drop in mortality lowers fertility, which tends to slow down the net reproduction rate.

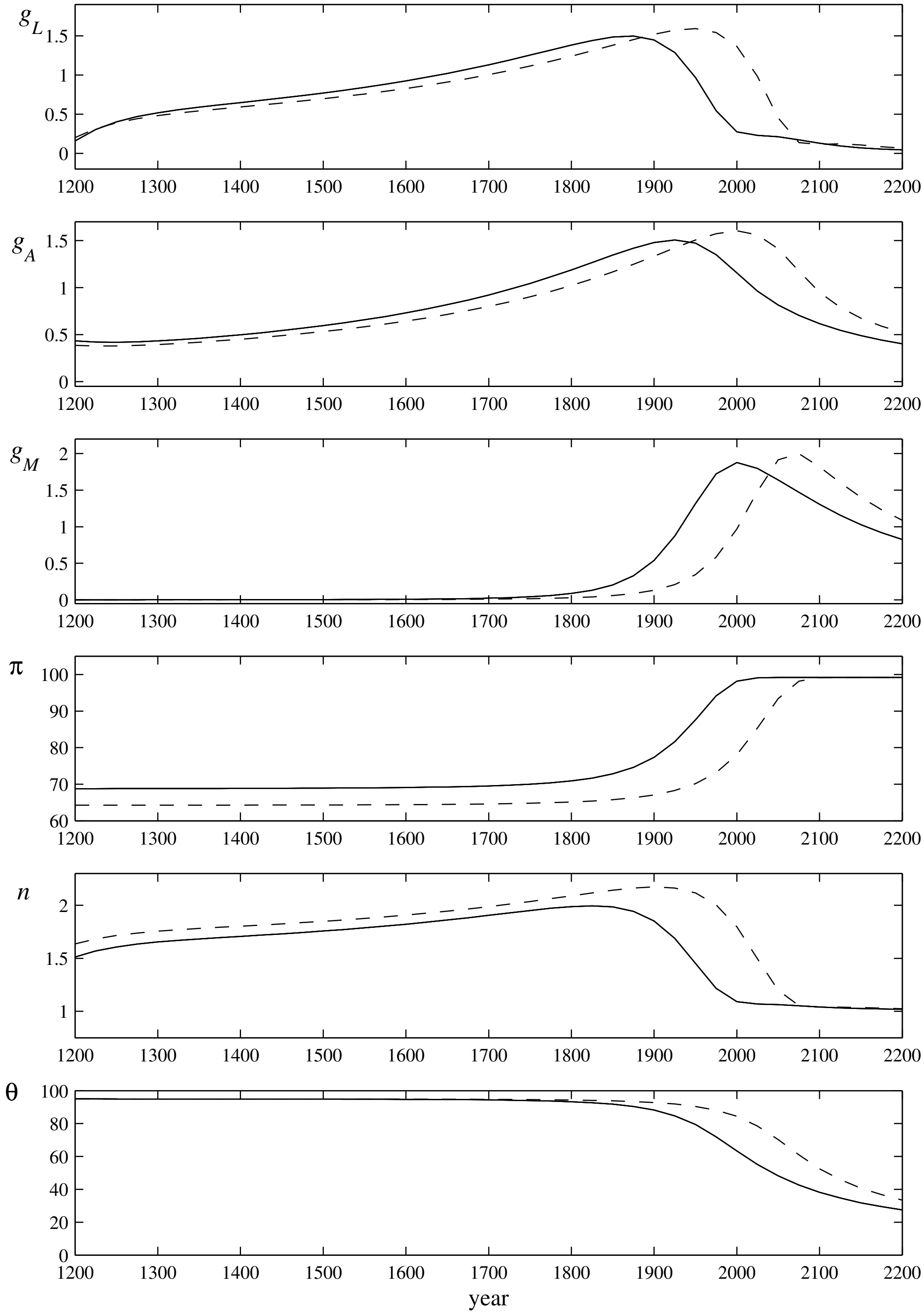
When advances in industrial knowledge, and thus child mortality decline, are relatively slow, the 'cheaper nutrition' effect dominates the 'mortality decline' effect. Hence, during early stages of development, the net reproduction rate goes up. In this period, therefore, declining child mortality is accompanied by rising rates of birth and net reproduction.

Since the transfer of labor out of agriculture gradually accelerates TFP growth in industry, the industrial sector's TFP growth rate eventually surpasses that of agriculture. Henceforth, the price of nutrition gradually increases, and children thus become relatively more expensive. All else equal, this leads to lower birth rates and therefore falling net reproduction. At the same time, advances in industrial knowledge creates further decline in extrinsic child mortality, leading to more investment in children's nutritional status, and therefore to lower intrinsic mortality. Hence, by contrast to previous periods, declining child mortality is now accompanied by falling rates of birth and net reproduction.

Eventually, the child survival rate reaches its maximum. Due to rising prices of nutrition, however, birth rates are still falling. Sooner or later, therefore, the net rate of reproduction reaches that of replacement, and population growth ultimately (and endogenously) comes to a halt.

5.1. **Calibration.** In order to get a more vivid picture of the adjustment dynamics, we now calibrate the model. Parameter values are chosen, so that the peak of the demographic transition matches that of 19th-century England, and so that the maximum rate of industrial TFP growth (and subsequent slowdown) appears in the late 20th century. Specifically, the rate of net reproduction reaches a peak of 1.5 percent per year in 1875, and industrial TFP growth begins to slow down around 1975, after a maximum of 1.5 percent per year.

Figure 2: Long-Run Dynamics




Solid lines: benchmark case, dashed lines: higher higher extrinsic mortality: $b = 0.15$ and less initial endowment $L_0$ and $A$=0, otherwise benchmark parameters. From top to bottom the diagrams show population growth, productivity growth in agriculture, productivity growth in manufacturing, child survival rate, fertility rate, and the labor share in agriculture.

For comparative purposes, we use as many parameter values as possible from the benchmark run calibrated in Strulik and Weisdorf (2008). Hence, the following parameters are chosen: $\alpha = 0.7$, $\epsilon = 0.45$, $\phi = 0.3$, $\mu = 0.55$, $\delta = 2.5$, and $\gamma = 3.4$. Start values are $\theta_0 = L_0^A/L_0 = 0.95$, $L = 0.066$ and $A_0 = 0.027$. The value for $M_0$ is obtained endogenously, and is given by $(\gamma/(\delta\theta_0))^{1/\phi}$. Extrinsic child mortality is parameterized by the logistic function $\bar{\pi}_t = a(1-\mathrm{e}^{-bM_t})$. The parameters $a$, $b$, $\lambda$ and $\beta$ are chosen, so that the child survival probability is 70 percent in the High Middle Ages, and its maximum is set just below one hundred percent. The parameter values are thus $a = 0.7$, $b = 0.17$, $\beta = 2.38$, and $\lambda = 1.25$. For better readability of the results, one generation is set to 25 years, or approximately the length of the fecundity period.

Figure 2 shows the calibrated adjustment path. The period analyzed runs from year 1200 to year 2200. Solid lines show the path of the benchmark economy; dashed lines concern an alternative economy to be discussed further below. Initially, almost all labor is allocated to agriculture. In line with numbers provided by Galor (2005), industrial TFP growth is almost absent during early stages of development. Productivity growth in agriculture is around 0.4 percent per year, supporting a population growth rate of 0.15 percent per year.

Increasing knowledge in agriculture, which to begin with manifesting itself in a slowly decreasing price of nutrition, translates almost entirely into population growth, meaning that standards of living are hardly affected by technological progress. A slow growth of population slowly furthers agricultural development, and little by little agricultural TFP growth builds up to reach 0.85 percent per year by the mid-18th century. By then, agriculture TFP growth makes possible a substantial transfer of labor into industry, causing an upsurge in industrial TFP growth. This substantiates significant decline in extrinsic mortality, leading parents to increase their spending on child nutrition. With decreasing rates of extrinsic and intrinsic mortality, child survival probability is on a fast rise.

Despite the increase in industrial TFP growth, its rate during the 19th century is still lower than that of agriculture. As a result, the price of nutrition continues to fall, causing further increase in fertility. This increase, however, is gradually being eaten up by the negative effect of falling mortality on fertility. What is more, around 1950 industrial TFP growth exceeds that of agriculture and the price of nutrition starts to increase. At this point, falling child mortality and higher costs of nutrition work in the same direction, leading to substantial fertility decline.

By the end of the 20th century, the demographic transition is almost complete. The child survival probability assumes its maximum value close to one hundred percent, and net reproduction is close to its replacement level. TFP growth in industry reaches its peak around 1975, and then begins to slow down. This decline, however, is a gradual process, leaving enough momentum for industrial TFP growth to exceed one percent per year far into the 22st century.

Note how the introduction of endogenous child mortality modifies the results obtained in Strulik and Weisdorf (2008). In the latter study, the peak of population growth coincides with that of fertility, which in turn takes place after agricultural TFP growth is surpassed by TFP growth in industry. Here, by contrast, the breakup of the population growth rate into a fertility and a mortality rate permits us to track down separately the occurrence of important economic and demographic events. For example, in the present calibration exercise, fertility peaks around 1825, whereas population growth reaches its maximum in 1875. Agricultural TFP growth peaks around 1925, whereas industrial TFP growth arrives at its summit close to year 2000. What is more, consistent with empirical evidence (with the exception of France), falling child mortality in the present framework precedes the drop in fertility. The current model thus correctly predicts the positive relationship between child mortality and net reproduction observed during most industrialized countries' demographic transitions, a feature that seems to be missing in previous contributions.

Turning to the dashed lines of Figure 2, the model permits us to implicitly analyze the effect of differences in geographic or climatic conditions on the timing of a country's industrial revolution and its demographic transition. The dashed lines show the adjustment dynamics for an otherwise identical economy, except for the fact that the parameter $b$ is now lower (0.15 instead of 0.17) and that the economy starts off with less knowledge ($A_0 = 0.017$ instead of 0.027) and less population density ($L_0 = 0.0038$ instead of 0.0066). The lower value of $b$ implies that, for any given level of industrial knowledge, the extrinsic child mortality rate is higher than in the benchmark case. As pointed out by Strulik (2008), countries have higher extrinsic child mortality the closer to the equator they are situated. Hence, the dashed line of Figure 2 would represent a country closer to the equator than England (the solid line of Figure 2).

The initially higher child mortality causes the demographic transition to initiate about one century later than in the benchmark economy. Higher initial mortality leads not only to a higher fertility maximum before the demographic transition sets in, but also generates a higher maximum net reproduction rate. However, as noted by Bloom *et al.* (2001), there is a demographic dividend to be gained from such a setback. As is also demonstrated in Figure 3, once a backward economy eventually reach into its demographic transition, it very quickly catches up in terms of TFP growth. In any case: geographical or climatic conditions that affects a countries' child survival probability may ultimately explain why some countries industrialize later than others.[11]

The model also generates some interesting non-linearities along the adjustment path analyzed above. Based on Figure 3, which shows some of the relevant non-monotonous relationships (for the benchmark economy), the following observations can be made. First, during early stages of development, the model predicts a positive correlation between child survival probability and fertility, while, during later stages, the sign of the correlation is reversed (panel 1). Specifically, lower mortality goes together with rising fertility when the effect of falling food prices on fertility dominates the quantity-quality substitution effect. By contrast, lower mortality appears together with lower fertility when quantity-quality substitution effect dominates that of falling food prices.

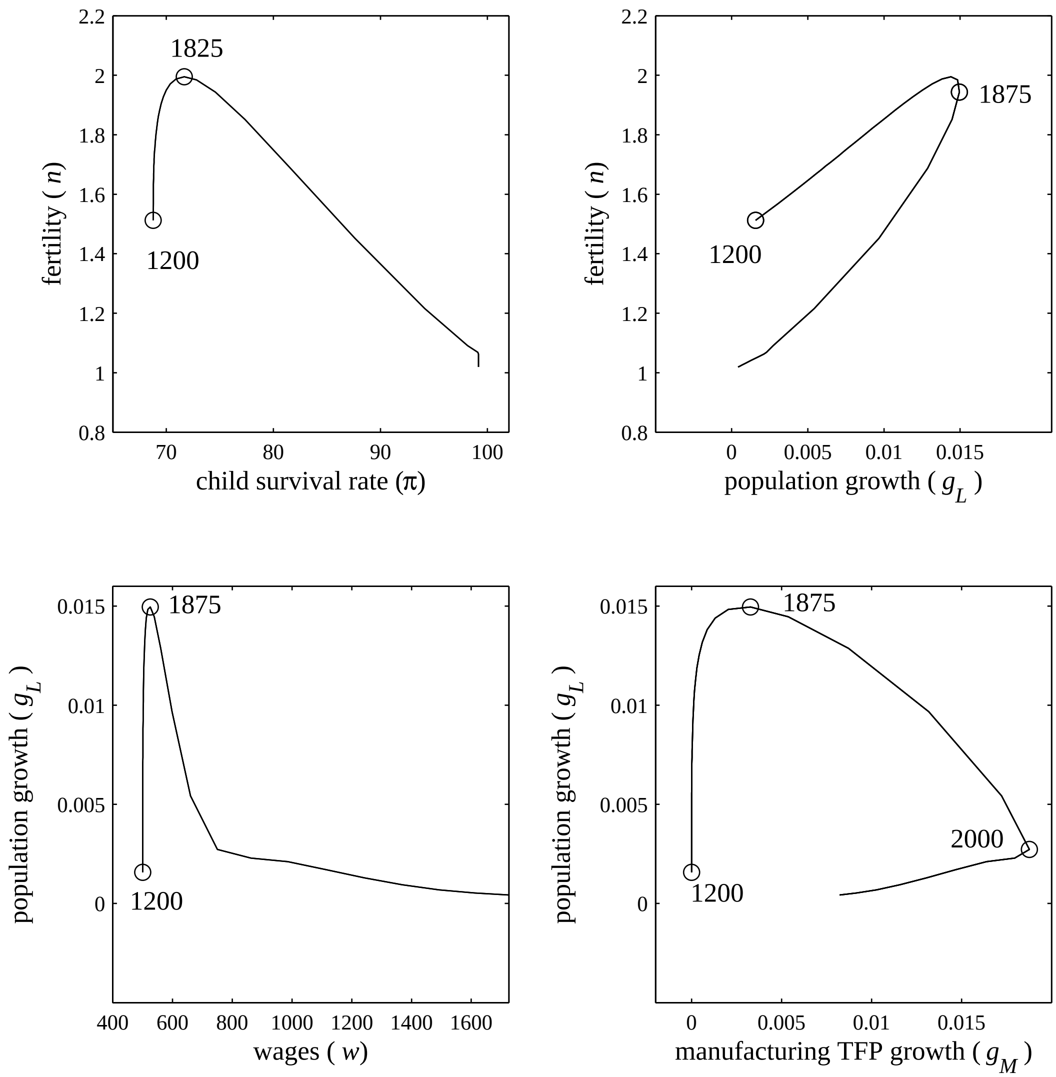
Second, due to a strong preventive-check mechanism during early phases of development, there is almost no correlation between wages and population growth in the beginning of the analyzed period. At later stages, however, such as during the demographic transition, there is a strong negative relationship between the two, followed by a weaker negative correlation after the ending of the transition (panel 2).

Third, the overall relationship between birth rates (total fertility) and net reproduction (net fertility) is a slightly positive one (panel 3). That is, during early stages of development, birth rates and net reproduction are both rising, while, during late stages, both are falling. For a short period in between, however, the birth rate is declining while net reproduction still grows.

Finally, the relationship between net reproduction and industrial TFP growth forms an almost perfect orbit (panel 4). During early stages of development, there is a positive relationship between TFP growth and growth of population. However, during the 'industrial revolution', i.e. the sharp rise in industrial TFP growth rates, the sign of the correlation turns negative. In the

[11]However, as regards present-day African countries that now (at least in principle) have access to most hygiene and health improving medical technology, these do not have to await advances in industry to help their child survival rates improve. Imports of medical technologies, vaccines, etc, may lead to a steep growth of population long before development in their industrial sectors set in. Therefore, since there is no advancement in industry to depress fertility though an increase in the real costs of nutrition, the prospects for fertility decline (i.e. a demographic transition) may be far away.

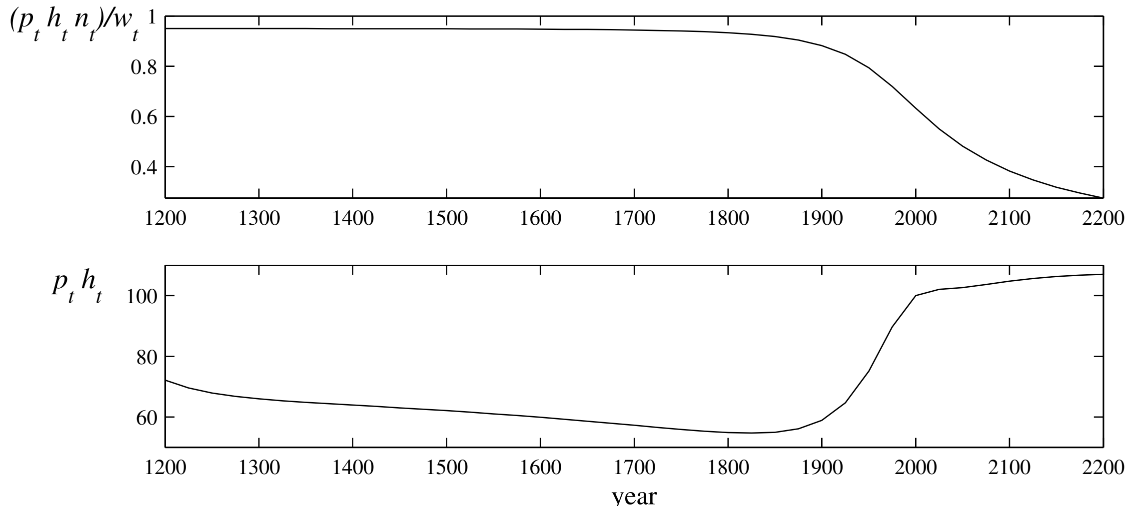
Figure 3: Correlations Along the Transition: 1200 to 2200




post-industrial period, i.e. after the peak in industrial TFP growth, the sign turns positive once again, as rates of productivity and population growth are both decreasing. The non-monotonic relationships depicted in Figure 3 should give some ideas as to why empirical studies are having a hard time reaching consensus regarding the correlation between economic and demographic variables (Brander and Dowrick, 1994).

A final observation to be made based on the model's predictions concerns its consistency with Engel's Law. Engel's Law states that nutrition expenditure's share of income, measured by $(p_t n_t h_t)/w_t$, will decline along with the process of growth. This is illustrated in Figure 4, which also shows that nutritional expenditures per child, measured by $p_t h_t$, gradually drop until the

Figure 4: Engel's Law and Expenditure per Child




mid-19th century, then increase steeply throughout the 20th century, after which they continue to increase, but at a somewhat slower pace.

## 6. Conclusion

This study provides the first unified growth model to endogenize mortality, fertility and growth, while not relying on human capital accumulation to generate a demographic transition. This result is based on the fundament ideas that parents care not only about surviving offspring, but also about their offspring's nutritional status, and that the price of nutritional goods, on which parents' fertility decision depends, responds to structural transformation in productive sectors. One of the major implications of the study, as demonstrated by the use of calibration, is the predicted non-monotonic correlations between various economic and demographic variables, a heads-up for empirical scholars who try to make sense of such relationships.

As is true for the related framework found in Strulik and Weisdorf (2008), the current model is robust to three important modifications. First, constant returns-to-labor in industrial production can be replaced by diminishing returns without affecting the model's qualitative conclusions. Second, the current preference function implies that the income-effect on the demand for children is zero. It is possibly to show, however, that adding a positive (and direct) effect of income on children will not affect the qualitative results of the model. Finally, the long-run predictions of the model—that there is no economic growth in steady state—can be modified without damage to the qualitative results if we permit a positive rate of growth of population on the balanced growth path. All three modifications, however, come at a cost to simplicity.

**Appendix: Proof of Proposition 1** While a *balanced* growth path involves stagnant levels of population and income, an *unbalanced* growth path, characterized by imploding or exploding growth, may in principle exist. In the following, we explore the two cases of unbalanced growth, starting with the case of imploding growth. Imploding growth implies perpetually negative population growth, i.e. $n_t$ is smaller than one and $L_t$ is decreasing. It is easy to see that imploding growth is not an option since $g_t^A$ and $g_t^M$ are bound to be non-negative. There is no forgetting-by-doing. With $\lim_{L\to 0} g^A = 0$ and $\lim_{L\to 0} g^M = 0$, we have $\lim_{L\to 0} n = const./L^{1-\alpha}$ from (11). As $L_t$ converges to zero, $n_t$ goes to infinity. A contradiction to the initial assumption of $n_t$ being smaller than one. There is no imploding growth. Intuitively, decreasing marginal returns of labor in agriculture ($\alpha < 1$) prevent implosion. As population size decreases agricultural productivity goes up and prices go down so that fertility and thus next period's population increases.

Explosive growth, on the other hand, cannot be ruled out if $\eta > 1$, In this case the relative price of food ultimately goes to zero and fertility to infinity. With growing population growth, productivity growth in both sectors grows hyper-exponentially until the economy reaches infinite fertility in finite time. We can solve the stability condition $\eta < 1$ for the critical $\epsilon$. A sufficient condition for stability of balanced growth is (18). Thus, the learning elasticity in agriculture must not be too large. Otherwise agricultural productivity rises so steeply with population growth that it can sustain further falling food and triggers further population growth. Inspection of (14) shows that the critical $\epsilon$ is decreasing in $\alpha$ and increasing in $\phi$. Intuitively, the larger the counterbalancing forces of limited land (i.e. the lower $\alpha$) and of learning in the manufacturing sector (the larger $\phi$) are, the higher can learning in the agricultural sector be without leading to explosion. The following proposition summarizes the considerations made above about balanced and unbalanced growth.

Note that $\epsilon < \epsilon_{crit}$ is a *sufficient* condition of stability since it holds for any path along which there is constant population growth. It could be relaxed for the only existing balanced growth path according to which, as shown, population growth is not only constant but also zero. For an intuition of the result inspect (16) again and imagine adjustment dynamics along which an "agricultural revolution" occurs before an "industrial revolution", i.e. a path along which $g^A$ is – because of decreasing returns – already declining whereas $g^M$ is still on the rise. The fact that $g^A$ and thus $gL$ are faster approaching to zero than $g^M$ relaxes the stability condition obtained from analytical considerations, which implicitly assume that all growth rates are in the neighborhood of a balanced growth path.

## References


Andreoni, J., 1989, Giving with impure altruism: Applications to charity and Ricardian equivalence, *Journal of Political Economy* 97, 1447-1458.

Azarnert, L.V., 2006, Child mortality, fertility, and human capital accumulation, *Journal of Population Economics* 19, 285-297.

Becker, G.S., 1960, An economic analysis of fertility. In: National Bureau of Economic Research (ed), *Demographic and Economic Change in Developed Countries*, Princeton University Press, Princeton, 209-231.

Becker, G.S. and R.J. Barro. 1988, A Reformulation of the theory of fertility, *Quarterly Journal of Economics* 103, 1-25.

Becker, G.S. and R.J. Barro, 1989, Fertility choice in a model of economic growth, *Econometrica* 57, 481-501.

Bernanke, B.S. and R.S. Guerkaynak, 2001, Is growth exogenous? Taking Mankiw, Romer, and Weil seriously, *NBER Macroeconomics Annual 16*, 11-57.

Bloom, D.E., D. Canning, and J. Sevilla, 2001, Economic growth and the demographic transition, *NBER Working Paper* 8685.

Boucekkine, R., D.de la Croix and O. Licandro, 2002, Vintage human capital, demographic trends, and endogenous growth, *Journal of Economic Theory* 104, 340-375

Brander, J.A., and S. Dowrick, 1994, The role of fertility and population in economic growth, *Journal of Population Economics* 7, 1-25.

Caulfield, L.E., M. de Onis, M. Bloessner, and R.E. Black, 2004, Undernutrition as an underlying cause of child deaths associated with diarrhea, pneumonia, malaria, and measles, *American Journal of Clinical Nutrition* 80, 193-198.

Cervellati, M., and Sunde, U., 2005, Human capital formation, life expectancy, and the process of Development, *American Economic Review* 95, 1653-1672.

Cervellati, M., and Sunde, U., 2007, Human capital, mortality and fertility: A unified theory of the economic and demographic transition, IZA Discussion Paper 2905.

Cleland, J., 2001, The effects of improved survival on fertility: A re-assessment, *Population and Development Review* 27 (Suppl.) 60-92.

Dasgupta, P., 1993, *An Inquiry into Well-being and Destitution*, Oxford University Press, Oxford.

Doepke, M., 2004, Accounting for fertility decline during the transition to growth. *Journal of Economic Growth* 9: 347–383

Doepke, M., 2005, Child mortality and fertility decline: Does the Barro-Becker model fit the facts?, *Journal of Population Economics* 18, 337-366.

Ehrlich, I. and Y. Kim, 2005, Endogenous fertility, mortality and economic growth: Can a Malthusian framework account for the conflicting historical trends in population?, *Journal of Asian Economics* 16. 789-806.

Galloway, P.R., Lee, R.D., and Hammel, E.A., 1998, Infant mortality and the fertility transition: Macro evidence form Europe and new findings from Prussia, in: Montgomery, M.R. and Cohen, B. (eds.), *From Death to Birth: Mortality Decline and Reproductive Change*, National Academic Press, Washington D.C.

Galor, O., 2005, From stagnation to growth: unified growth theory, in: P. Aghion and S. Durlauf, *Handbook of Economic Growth* Vol 1A, Amsterdam, North-Holland.

Galor, O. and O. Moav, 2002, Natural selection and the origin of economic growth, *Quarterly Journal of Economics* 117, 1133-1191.

Galor, O. and A. Mountford , 2008, Trading population for productivity: Theory and evidence, *Review of Economic Studies* 75, forthcoming.

Galor, O. and D.N. Weil, 2000, Population,technology and growth: From the Malthusian tegime to the demographic transition and beyond, *American Economic Review* 110, 806-828

Grigg, D., 1994, Food expenditure and economic development, *Geo Journal* 33, 377-382.

Harris, B., 2004, Public health, nutrition, and the decline of mortality: The McKeown thesis revisited, *Social History of Medicine* 17, 379-407.

Jones, C.I., 2001, Was an industrial revolution inevitable? Economic growth over the very long run, *Advances in Macroeconomics* 1, 1-43.

Kalemli-Ozcan, S., H.E. Ryder and D. N. Weil, 2000, Mortality decline, human capital investment, and economic growth, *Journal of Development Economics* 62,1-23.

Kalemli-Ozcan, S., 2002, Does the mortality decline promote economic growth, *Journal of Economic Growth* 7: 411-439.

Kögel, T. and Prskawetz, A., 2001, Agricultural productivity growth and escape from the Malthusian trap, *Journal of Economic Growth* 6, 337-357.

Lagerlof, N., 2003, From Malthus to modern growth: The three regimes revisite, *International Economic Review* 44, 755-777.

Lucas, R.E., 2002, *The Industrial Revolution: Past and Future*, Cambridge: Harvard University Press.

McKeown, T., 1976, *The Modern Rise of Population*, Academic Press, New York.

Pelletier, D.L., E.A. Frongillo, and J.P. Habicht, 2003, Epidemiologic evidence for a potentiating effect of malnutrition on child mortality, *American Journal of Public Health* 83, 1130-1133.

Reher, D.S., 2004, The demographic transition revisited as a global process, *Population, Space and Place* 10, 19-41.

Rice, A.L., L. Sacco, A. Hyder, R.E. Black, 2000, Malnutrition as an underlying cause of childhood deaths associated with infectious diseases in developing countries, *Bulletin of the World Health Organization* 78, 1207-1221.

Soares, R.R., 2005, Mortality reductions, educational attainment, and fertility choice, *American Economic Review* 95, 580-601.

Strulik, H., 2008, Geography, health, and the pace of demo-economic development, *Journal of Development Economics* 86, 61-75.

Strulik, H. and Weisdorf, J., 2008, Population, food, and knowledge: A simple unified growth theory, *Journal of Economic Growth* 13, 169-194.

Weisdorf, J. L., 2004, From stagnation to growth: revisiting three historical regimes, *Journal of Population Economics* 17, 455-472.

Weisdorf, J.L., 2008, Malthus revisited: Fertility decision making based on quasi-linear preferences, *Economics Letters* 99: 127-130.

Wrigley, E.A. and R.S. Schofield (1981), *The Population History of England 1541-1871: A Reconstruction*, Cambridge: Harvard University Press.

Wrigley, E.A. (1969), *Population and History*, London: World University Library.