The Open Method of Coordination (OMC) as an Evolutionary Learning Process*

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Abstract

We interpret the Open Method of Coordination (OMC), recently adopted by the EU as a mode of governance in the area of social policy and other fields, as an imitative learning dynamics of the type considered in evolutionary game theory. The best-practise feature and the iterative design of the OMC correspond to the behavioral rule "imitate the best." In a redistribution game with utilitarian governments and mobile welfare beneficiaries, we compare the outcomes of imitative behavior (long-run evolutionary equilibrium), decentralized best-response behavior (Nash equilibrium), and coordinated policies. The main result is that the OMC allows policy coordination on a strict subset of the set of Nash equilibria, favoring in particular coordination on *intermediate* values of the policy instrument.

Keywords: Open Method of Coordination, Finite-population Evolutionarily Stable Strategy, Imitation, Mobility, Redistribution.

JEL Classification: H77, H75, C73.

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1 Introduction

Over the last fifteen years, a new mode of governance has emerged within the European Union (EU). Since the European Council summit in Lisbon (March 2000), it has been coined the Open Method of Coordination (OMC). Initially designed for, and applied in, social policy (social inclusion, health care, pensions and long-term care), various European Councils approved of the extension of the OMC to a broad spectrum of policy areas, encompassing, e.g., immigration, technology and environment.

The Presidency Conclusions of the Lisbon European Council[1] define the OMC as "a means of spreading best practice and achieving greater convergence towards the main EU goals." The OMC is best described as an iterative process of mutual learning and imitation among governments.[2]

Roughly, the OMC proceeds as follows (see Figure 1 for a sketch): Having agreed on EU-wide common objectives and indicators, EU member states individually design and implement their national policies. After a certain period, these national strategies are jointly evaluated and compared within the EU. Best practices are identified and member states are encouraged (but not forced) to adopt them. The process is then iterated.

The OMC induces member states to systematically compare themselves to one another in terms of their (relative) policy performance. It promotes the imitation of successful policies, thus aiming at policy convergence (Trubek and Mosher, 2001). Diversity, though, is not disallowed (Zeitlin, 2005; Daly, 2007). The OMC is a soft-law method, leaving to member states control of their policies (Pochet, 2005), thus keeping agency costs and losses in national sovereignty minimal (Borrás and Jacobsson, 2004). The rationale behind the OMC is the hope that the quality of policy decisions improves and that policy-learning through benchmarking is enhanced.

In this paper, we provide a theoretical model of the OMC and an assessment of its performance in a social policy (redistribution) setting. Following the design depicted in

[1] Available at http://europa.eu.int/council/off/conclu/mar2000/index.htm

[2] While the OMC still lacks a unique and precise definition, there seems to be a consensus among its commentators that "learning" and "imitation of best practise" are core ingredients of the method. Detailed information on the OMC in the area of social policy in the EU is available at http://ec.europa.eu/employment_social/spsi/the_process_en.htm.

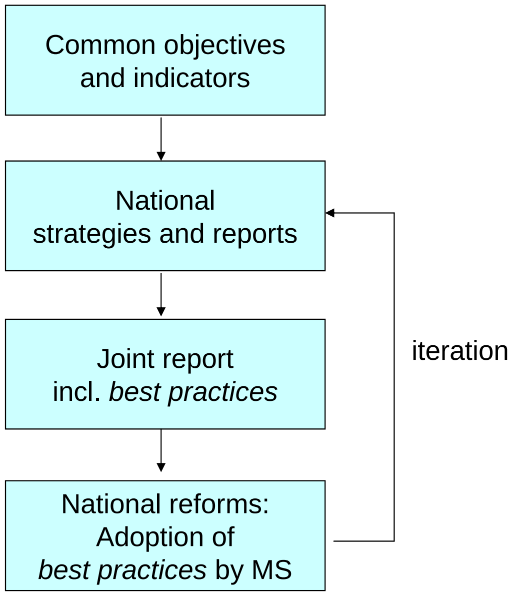

Figure 1: The iterative OMC process.

Figure 1, we model the OMC as an iterative process with an emphasis on mimicking best practices that, at the same time, allows for country-specific deviations. The resulting political process exhibits evolutionary learning with imitation and experimentation. This allows us to employ concepts and results from evolutionary game theory in the analysis of the OMC. Specifically, we argue that the OMC leads to the emergence of *evolutionarily stable strategies (ESS)*. An ESS is a strategy which, once chosen by all players, cannot be invaded by any competing alternative strategy. ESS are also the outcome of imitative learning. For an iterated process like the OMC with its emphasis on copying best practices evolutionary stability, thus, appears to be the appropriate solution concept.

In its most common usage, evolutionary game theory considers large populations of players who are randomly matched to recurrently play some game. In that case, ESS are always Nash equilibrium strategies.[3] For the application in this paper, however, the number of players (say, the now 27 governments of the EU member states) is far from infinite. Therefore, finite-population results have to be applied. With a small number of players,

[3]Moreover, they are asymptotically stable in the replicator dynamics (see, e.g., Weibull, 1995). The replicator dynamics, in turn, can be justified on the basis of imitation of other randomly observed, successful strategies (Björnerstedt and Weibull, 1996). In this sense, a central contribution of evolutionary game theory has been to show that boundedly rational behavior, like imitation, does not preclude the emergence of Nash equilibrium and, thus, of rational outcomes in the aggregate.

ESS is *not* necessarily a refinement of Nash equilibrium. Rather, ESS can be interpreted as Nash equilibrium strategies for a game defined in relative (rather than absolute) payoffs (see Schaffer, 1988). This makes the concept suitable for the analysis of the OMC, where policy adoption is recommended on the basis of governments' relative performance. The tension between finite-population ESS and Nash equilibrium will not show up in our model. In Section 3 we will show that all ESS of our model are indeed Nash equilibria of the game.[4]

We do want to stress the importance of the strategic effects arising in a setting with a finite number of players. In our analysis we compare the ESS resulting from the OMC to Nash equilibria and co-operative solutions of the policy competition game. These two solution concepts represent the traditional modes of governance in the EU: they are the outcomes when, respectively, policies remain fully decentralized (fiscal competition) or are fully coordinated (integration or community method). In general, all these concepts differ in the presence of payoff externalities (in our model: fiscal externalities). It therefore makes sense to put the OMC into an explicitly game-theoretic context, even though the literature on the OMC hardly ever mentions strategic interdependencies as a relevant impact factor on the method's performance.[5] Our main result will be that the ESS coming out of the OMC are indeed a subset of the set of Nash equilibria. However, we will find no general logical nexus between ESS and efficiency.

More specifically, we embed the OMC into a standard game of redistribution from rich to poor in a multi-country setting with labour mobility. As a stylized example of social policy, this application seems reasonably close to the contexts for which the OMC was originally designed. In our model, the poor beneficiaries of transfers are internationally mobile and settle where the welfare state is most generous. Governments are inequality-averse utilitarians and, thus, have a preference for redistribution. With decentralization, transfer policies are best responses to the policies of other jurisdictions. The decentral-

[4]The relation between Nash equilibrium and ESS for finite populations has been explored by Ania (2008) and Guse et al. (2008) and is still not fully understood.

[5]A few exceptions should be mentioned: Pestieau (2005) and Coelli et al. (2008) relate the OMC to yardstick competition and its idea that information spill-overs would enable citizens to compare the performance of their governments with that of goverments elsewhere and then to punish and reward politicians. However, no formal analysis is provided. Büchs (2008) informally analyzes the OMC as a "two-level" (more precisely: two-stage) game where governments first agree on objectives and then implement policies to meet these objectives.

ized redistribution game has a large set of Nash equilibria. With the OMC, however, governments adopt imitative behavioral rules or, which turns out to be equivalent, choose policies that perform best *relative* to what other governments do. Comparing the resulting ESS with the Nash equilibria of the decentralized redistribution game, we find in Section 3 that the former are a strict subset of the latter. Hence, in our model the OMC cannot achieve anything that could not also be obtained via a decentralized approach. However, the OMC avoids some extreme outcomes that are possible under decentralization.

As our main contribution, we provide in Section 4 a dynamic approach that reflects the iterative and imitative gist of the OMC as illustrated in Figure 1. For commonly agreed-upon objectives (represented in the model by a social welfare function), governments choose policies mimicking what was observed to be successful in previous periods; this captures the idea of learning from other's experience. The "open" nature of the OMC process is modeled as experimentation: there is no binding commitment to adopt best practices and countries are free to implement policies as they wish. We show that such a process of imitation and experimentation indeed converges to an ESS. Not all ESS, however, are equally robust to experimentation. In particular, long-run equilibria come from the "medium" range of ESS, resulting in a convergence towards moderate redistribution policies where transfers to the poor are neither extremely low nor overly generous. Empirically, a trend of convergence of social policies and the absence of a race-to-the-bottom have been observed for OMC participants by Coelli et al. (2008). This fits well to our theoretical observations.

To summarize, our paper makes the following points. Models of learning and evolution in games provide a suitable framework for an analysis of the OMC. Evolutionary stability captures both the static (relative performance) and the dynamic features (learning process) of the OMC. The imitative process of the OMC converges and settles at intermediate transfer levels. However, we will illustrate how the OMC can result in underprovision as well as in overprovision of redistribution.

The rest of this paper is organized as follows. Section 2 sets up a game of decentralized redistribution. Section 3 derives Nash equilibria and ESS for that game (static analysis). Section 4 presents a formal analysis of the imitation dynamics induced by the OMC. Section 5 discusses efficiency issues and some basic extensions of the model. Section 6 concludes. All proofs are relegated to the Appendix.

2 The Model

2.1 Mobility and redistribution

There are $n \geq 2$ identical countries that form an integrated economic area with free mobility. Countries, indexed by $i = 1, \ldots, n$, decide on whether and, if so, to what degree to engage in redistribution among their residents. In each country there is one very rich and immobile resident earning w_R; this normalization to one rich per country is innocuous in our framework. A large population of poor individuals, each earning $w_P < w_R$, can benefit from redistribution. Poor individuals are perfectly mobile and decide where to establish their residence based on the generosity of social policy. Let $\nu \geq n$ be the total size of the population of poor individuals in the economic area. Each individual inelastically supplies one unit of labor. Thus, labor supply and basic earnings do not depend on social policy; this guarantees that the total size of the population of individuals affected by redistribution is constant.

We denote by ℓ_i (with $0 \leq \ell_i \leq \nu$) the amount of mobile poor living in country i. Redistribution from rich to poor is organized as follows. Each country i implements a non-negative lump-sum transfer, s_i, payable to each poor within its jurisdiction and financed by a lump-sum tax t_i on its rich resident. Government budgets are required to balance; i.e., the sum of transfers equals the amount of revenues raised:

$$s_i \cdot \ell_i = t_i.$$

With such a redistribution scheme, consumption levels of the poor and the rich residing in country i, respectively, amount to

$$c_i^P = w_P + s_i \quad \text{and} \quad c_i^R = w_R - t_i = w_R - \ell_i \cdot s_i.$$

The set of possible subsidies is restricted to $S = [0, w_R]$. We henceforth write $\mathbf{s} = (s_1, \ldots, s_n) \in S^n$ for vectors of redistributive policies. It is convenient to use the notation $\mathbf{s} = (s_i | \, \mathbf{s}_{-i})$, where s_i is the subsidy chosen by country i and (with some abuse of notation) $\mathbf{s}_{-i}$ is the vector of subsidies chosen by countries other than country i or any permutation thereof. Finally, denote $\overline{s}_i = \max\{\mathbf{s}_{-i}\}$; for any given i, $\overline{s}_i$ is the maximum subsidy chosen by any country other than i.

Individuals care only about their consumption. Thus, mobile individuals establish their residence in the country with the most generous redistribution policy. Given $\mathbf{s} = (s_i | \, \mathbf{s}_{-i})$, denote by $M_i(\mathbf{s}_{-i}) = \{j \neq i | s_j = \overline{s}_i\}$ the set of countries offering the highest subsidy when we exclude i and let $m_i(\mathbf{s}_{-i}) = |M_i(\mathbf{s}_{-i})|$ be its cardinality. Given a vector $\mathbf{s}$ of subsidies, we denote the distribution of mobile poor by $(\ell_1(\mathbf{s}), \ldots, \ell_n(\mathbf{s}))$, where $\ell_i(\mathbf{s})$ denotes the amount of poor residing in country i. Assume that whenever two countries, i and j, choose the same transfer level, they attract the same amount of poor; i.e.

$$s_i = s_j \quad \Longrightarrow \quad \ell_i(\mathbf{s}) = \ell_j(\mathbf{s}).$$

As the mobile poor settle only in the most generous countries, their distribution across countries follows the pattern in (1):

$$\ell_i(\mathbf{s}) = \ell(s_i | \ \mathbf{s}_{-i}) := \begin{cases} 0 & \text{if } s_i < \overline{s}_i \\ \frac{\nu}{1+m_i(\mathbf{s}_{-i})} & \text{if } s_i = \overline{s}_i \\ \nu & \text{if } s_i > \overline{s}_i. \end{cases} \tag{1}$$

Clearly, $0 \leq \ell_i(\mathbf{s}) \leq \nu$ and $\sum_{i=1}^{n} \ell_i(\mathbf{s}) = \nu$. Two observations about (1) will become important later on. First, the fraction of the poor residing in country i is invariant to permutations of other countries' subsidies. Second, so expressed, $\ell(s_i | \ \mathbf{s}_{-i})$ is also the amount of poor that would reside in *any* country (not only i) choosing $s = s_i$ when all other countries choose subsidies according to $\mathbf{s}_{-i}$.

We postulate our model in terms of governments choosing subsidies that attract mobile poor. This is in line with models by Wildasin (1991, 1994), Cremer and Pestieau (2003), and many others. Alternatively, we could have chosen to make poor individuals immobile, let rich individuals be mobile, and governments choose taxes instead of subsidies. This would not change the essence of our analysis.[6] A potentially more critical modeling choice is the assumption that *all* poor are mobile. In Section 5 we discuss the robustness of our results to the introduction of some immobile poor. Due to costless mobility, migration responses in our model are extremely sensitive and discontinuous: a slight change in transfers might cause a complete reshuffling of the population in the economic area. This assumption, which is similarly made in other papers (see, e.g., Cremer and Pestieau, 2003; Kolmar, 2007), gives our approach a Bertrand-type flavor. In a companion paper, Ania and Wagener (2009) consider a model with smooth migration flows.

[6]It is well-known, however, that results for decentralized redistribution change if *both* tax payers and welfare recipients are mobile. See e.g. Leite-Monteiro (1997).

2.2 Policy objectives

At least since Mansoorian and Myers (1997), it is well-known that government objective functions play an important role in decentralized redistribution games where population sizes are endogenous (also see Cremer and Pestieau, 2004). We consider here utilitarian governments that evaluate individuals' utility derived from consumption by some utility function, $u(c)$, which is twice continuously differentiable and such that $u'(c) > 0 > u''(c)$ for all $c \geq 0$. We assume that $u(w_P) \geq 0$, and there exists $K \geq 0$ such that $u(0) < -K$.

The government of any country $i = 1, \dots, n$ assesses different policies by comparing the sum of the utilities of those currently living in i under such policies; i.e.

$$\pi_i(\mathbf{s}) = \ell_i(\mathbf{s}) \cdot u(c_i^P) + u(c_i^R) = \ell_i(\mathbf{s}) \cdot u(w_P + s_i) + u(w_R - \ell_i(\mathbf{s}) \cdot s_i). \tag{2}$$

The fact that $\ell_i(\mathbf{s})$ is invariant to permutations of other countries' subsidies allows to write payoffs also as:

$$\pi_i(\mathbf{s}) = \pi(s_i|\, \mathbf{s}_{-i}) = \ell(s_i|\, \mathbf{s}_{-i}) \cdot u(w_P + s_i) + u(w_R - \ell(s_i|\, \mathbf{s}_{-i}) \cdot s_i), \tag{3}$$

where now $\pi(s_i|\, \mathbf{s}_{-i})$ is the payoff to *any* country choosing $s = s_i$ when *all other* countries choose subsidies according to the vector $\mathbf{s}_{-i}$.

If all governments set identical transfers, the poor will be equally distributed across countries (i.e., $\ell_i(\mathbf{s}) = \nu/n$). Then the optimal subsidy is given by[7]

$$\begin{aligned} s^0 \quad &:= \quad \arg\max_{s \in S} \left\{ \frac{\nu}{n} \cdot u(w_P + s) + u(w_R - \nu/n \cdot s) \right\} \\ &= \quad \frac{w_R - w_P}{1 + \nu/n}. \end{aligned} \tag{4}$$

We refer to s^0 as the *efficient symmetric* solution; transfers s^0 lead to an egalitarian income distribution.

The objective function (2) is called *generalized utilitarianism.* In settings with variable population sizes, it is one out of many utilitarian-type social welfare functions (Blackorby et al., 2009). A serious flaw of generalized utilitarianism is that it gives rise to the so

[7]To see this, first observe that s^0 is strictly positive since the objective function is strictly increasing at $s = 0$ by strict concavity of $u(c)$ and the fact that $w_P < w_R$. Moreover, s^0 must satisfy the first order condition $u'(w_P + s^0) = u'(w_R - \nu/n \cdot s^0)$ which gives the expression in the second line of (4); clearly $s^0 < n/\nu \cdot w_R$, so that the rich is not completely expropriated in the symmetric efficient allocation.

called *repugnant conclusion* (Parfit, 1982; Blackorby et al., 2009) – for every population of arbitrary well-offs, there exists another, suitably larger population of paupers such that utilitarians will strictly prefer the latter to the former. This substitutability of population size for quality of life is ethically questionable. Obviously, the set of Nash equilibria, and our results, depend crucially on the choice of objectives. Our focus here is on the workings of the OMC *given a particular type of objective function,* which is meant to reflect the common objectives and target indicators that member states agreed upon.

3 Static Analysis: Nash equilibria vs. ESS

The model presented in Section 2 defines a game where the players (countries $i = 1, \dots, n$) simultaneously choose subsidies out of a common strategy set given by the feasible set of subsidies $S = [0, w_R]$. Migration decisions as summarized by expression (1) determine the payoffs $\pi_i : S^n \to \mathbb{R}$ which are given by expression (2). The game is symmetric, since payoffs can be written as $\pi_i(\mathbf{s}) = \pi(s_i|\ \mathbf{s}_{-i})$ shown in (3). Payoffs do not depend on the players' names and are invariant to permutations of other players' strategies.

Before proceeding with the analysis, let us recall here the definitions of a *symmetric* Nash equilibrium and a *finite-population* evolutionarily stable strategy and shortly comment on the difference between the two concepts. By focusing directly on symmetric equilibria we can write both definitions using the same notation.[8]

Definition 1 *A strategy s^N is played in a* symmetric Nash equilibrium *if*

$$\pi(s^N|\ s^N, s^N, \dots, s^N) \geq \pi(s|\ s^N, s^N, \dots, s^N) \qquad \textit{for all } s \in S.$$

A strategy $s^E \in S$ is said to be an evolutionarily stable strategy *(ESS) if*

$$\pi(s^E|\ s, s^E, \dots, s^E) \geq \pi(s|\ s^E, s^E, \dots, s^E) \qquad \textit{for all } s \in S.$$

[8]Schaffer (1988) gives a definition of evolutionary stability for a finite population of N individuals who are randomly matched to play an n-person game. We take here Schaffer's definition for the case of $n = N$. See also Vega-Redondo (1996, pp. 31-33) for a discussion of this concept. See Crawford (1991) and Tanaka (2000) for closely related concepts. See Nowak et al. (2004) for a recent dynamic concept of evolutionary stability for finite populations.

We say that a Nash equilibrium or an ESS is strict *if the corresponding inequality holds strictly for all $s' \neq s$.*

In a Nash equilibrium no player would strictly benefit from a deviation, given what other players are doing. In an evolutionarily stable profile no player would be able to gain a strict *relative* advantage by deviating. Note that for a Nash equilibrium we compare the deviator's payoffs before and after deviation. In an evolutionarily stable profile, instead, we compare payoffs to the deviator, choosing s, with payoffs to the non-deviators, choosing s^E, *after* a unilateral deviation. For this reason, when the population is finite and each player has a non-negligible impact on the payoffs of all other players, it may pay in relative terms to deviate from a Nash equilibrium, if the loss imposed on non-deviators is bigger than the loss suffered by the deviators themselves. This is referred to as *spiteful* behavior (Hamilton, 1970).[9]

Before we characterize the Nash equilibria and the ESS of the game, let us introduce the following family of auxiliary functions.

$$f(k,s) = \frac{\nu}{k} \cdot u(w_P + s) + u\left(w_R - \frac{\nu}{k} \cdot s\right)$$

where $k \in \{1, \ldots, n\}$. The value of $f(k,s)$ can be interpreted as the payoff to any of k countries equally sharing all the poor at subsidy level s, of course provided they attract the poor with that subsidy (i.e. s is currently the maximum subsidy). Note that $f(k,0) \geq u(w_R)$. By the strict concavity of $u(c)$ and since $w_P < w_R$, we get that $f(k,s)$ is strictly increasing at $s = 0$. Moreover, for every k, $f(k,s)$ is strictly concave in s. Let

$$s^*(k) = \arg\max_{s \geq 0} f(k,s).$$

The properties of f guarantee that $s^*(k)$ is strictly positive for all k and it must satisfy the first order condition

$$u'(w_P + s^*(k)) = u'(w_R - \nu/k \cdot s^*(k)),$$

which yields

$$s^*(k) = \frac{w_R - w_P}{1 + \frac{\nu}{k}}.$$

[9]Such considerations would not play a role in a continuum population, since each player has a negligible impact on the payoffs of others in that case. It is well known that ESS are always Nash equilibrium strategies in a continuum population. See, e.g., Weibull (1995, p. 36).

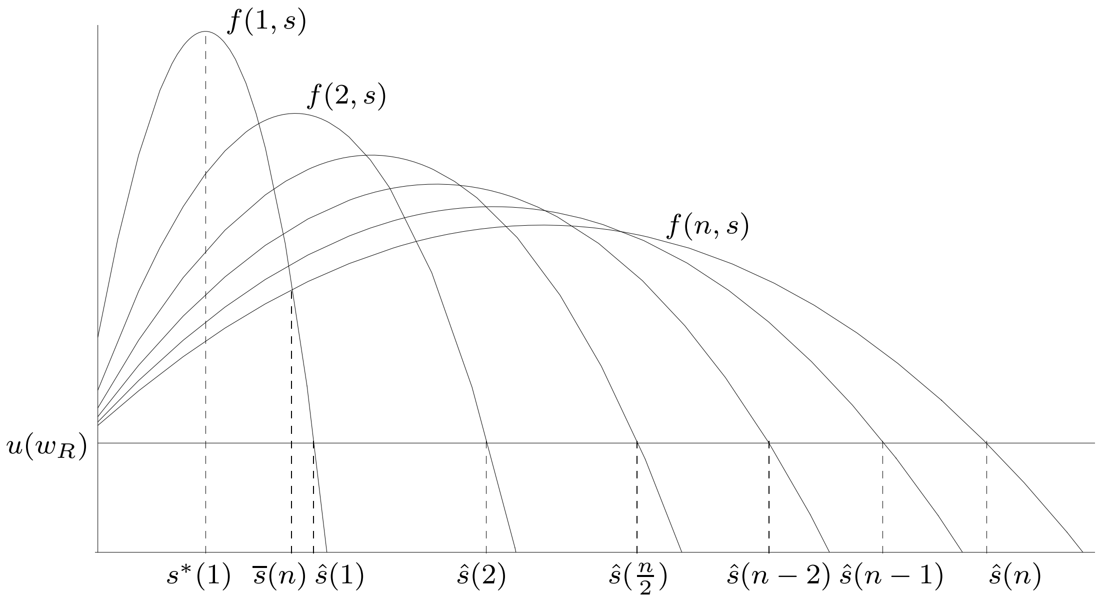

Figure 2: Properties of auxiliary welfare functions $f(k, s)$.

Given that k countries share the burden of redistribution, they maximize social welfare by choosing $s = s^*(k)$. Clearly, $0 < s^*(k) < k/\nu \cdot w_R$ is increasing in k; i.e., social policy should optimally be more generous as more countries engage in redistribution.

Given any $k \in \{1, \ldots, n\}$, define $\widehat{s}(k)$ as the strictly positive value of s that solves

$$f(k, \widehat{s}(k)) = u(w_R),$$

and $\bar{s}(k)$ as the strictly positive value that solves

$$f(k, \bar{s}(k)) = f(1, \bar{s}(k)).$$

At $\widehat{s}(k)$, a country is indifferent between paying transfers $\widehat{s}(k)$ to ν/k poor and not attracting any poor at all. At $\bar{s}(k)$, a country is indifferent between paying transfers $\bar{s}(k)$ to ν/k poor and, at the same transfer, hosting all poor. In Appendix A we show that the family of functions $\{f(k, s)\}_{k=1,\ldots,n}$ and the corresponding values of $\widehat{s}(k)$ and $\bar{s}(k)$ have the properties depicted in Figure 2.[10] In particular, we show that

$$s^*(1) < \bar{s}(2) < \bar{s}(3) < \ldots < \bar{s}(n) < \widehat{s}(1) < \widehat{s}(2) < \ldots < \widehat{s}(n). \quad (5)$$

[10]Our game has a structure akin to that of an oligopolistic market of the type analyzed by Dastidar (1995), where firms have decreasing returns to scale and compete in prices.

This observation allows an easy characterization of Nash equilibria and ESS. We first show that our game has a large set of symmetric, pure-strategy Nash equilibria:

Proposition 1 *Under generalized utilitarianism the set of pure-strategy Nash equilibria is given by*

$$\Sigma^N = \{\mathbf{s} = (s, \ldots, s) \mid \bar{s}(n) \leq s \leq \widehat{s}(n)\}.$$

Observe that transfer levels at Nash equilibria are quite generous; even overprovision of transfers (i.e., values of s^N larger than $s^0 = s^*(n)$) is possible in a Nash equilibrium. This is in contrast with the widespread fear of an erosion of the welfare state due to migration pressures (Cremer and Pestieau, 2004). It is a consequence of the government objective which entails a strong preference for large population sizes (recall the *repugnant conclusion*). Hence, the widely feared demise of the welfare state does not occur in our model (see Secton 5 for modifications).

Our next proposition characterizes the set of ESS. It shows that ESS are a strict subset of the set of Nash equilibrium strategies found in Proposition 1:

Proposition 2 *Under generalized utilitarianism the set of ESS is the interval*

$$S^E = [\widehat{s}(1), \widehat{s}(n-1)].$$

Proposition 2 conveys that whatever can be achieved at an ESS could also be achieved through decentralization at a Nash equilibrium. The set of ESS, however, is a strict subset of the set of Nash equilibria, precluding some extremely low and extremely high subsidies that can be rationalized as a Nash equilibrium. In particular, for $s \in [\bar{s}(n), \widehat{s}(1))$, subsidies are still too low and a single country could still achieve a relative advantage through more redistribution even if it attracted all poor. The situation with $s \in (\widehat{s}(n-1), \widehat{s}(n)]$ is also too unstable; if a single country were to lower its subsidy, all others would be left with too high subsidies given the number of countries sharing the burden of redistribution.

It is worth pointing out special properties of the two extreme values in the interval S^E. Starting at the symmetric profile where all countries set $s = \widehat{s}(n-1)$, a deviation downwards to some $s' < s = \widehat{s}(n-1)$ would result in a relocation of all poor among the remaining $n-1$ non-deviating countries; their payoff, however, would be exactly $u(w_R)$

by definition of $\widehat{s}(n-1)$ and the deviator would have no strict disadvantage. This cannot happen with any other ESS in the interval S^E — from any other ESS a deviator would suffer a strict disadvantage. All ESS in the interval are strict except the upper boundary $\widehat{s}(n-1)$, which is ESS, but it is not strict. This will make a subtle difference in the dynamics analysis below.

4 The OMC as a Dynamic Imitative Process

In the present section we come to what we consider is the spirit of the OMC. We now explicitly take a dynamic approach, allowing countries to observe each others' subsidies and welfare levels and to make sequential decisions based on this information. We assume that countries tend to adopt subsidy levels associated with the highest welfare levels currently observed. Occasionally, countries can experiment with random subsidies. However, such experiments are followed by other countries only when they prove to be successful compared to other currently observed subsidy levels and will not persist otherwise. The model intends to capture the main features of the OMC as stated by the European Commission (see Figure 1 again). In our model, the (symmetric) welfare function stands for the commonly agreed-upon objectives; subsidies are the policy instrument and each period welfare levels constitute the target indicator that is reported by each country. Our imitation dynamics intends to capture the iterative loop by which countries learn from each other's experience. Finally, experimentation captures the open nature of the process; namely, the most successful policies observed and the recommendations of the Commission are not binding – countries are allowed to adopt other policies based on their own motivations to do so, which may range from mistakes to national political interests. We now proceed to introduce and analyze the dynamic model.

The analysis is applied to a discretized version of the model presented in Section 3. Specifically, we assume that countries choose subsidies from a finite set $\Gamma \subset S$. For simplicity of exposition, we assume that Γ contains the values $\widehat{s}(k)$ for all $k = 1, \ldots, n$.[11] The state space of the process is Γ^n, the state at $t = 1, 2, \ldots$ is given by the vector of subsidies chosen by all countries at t denoted

$$\mathbf{s}(t) = (s_1(t), \ldots, s_n(t)) .$$

[11] Note that this assumption may preclude Γ from being a regular grid.

Subsidies at t determine welfare levels given by the vector

$$\pi(\mathbf{s}(t)) = (\pi_1(\mathbf{s}(t)), \ldots, \pi_n(\mathbf{s}(t))),$$

where $\pi_i(\mathbf{s}(t))$ is the welfare attained by country i in state $\mathbf{s}(t)$ and is defined as in expression (2) in Section 3. In any period $t = 1, 2, \ldots$ all countries observe $\mathbf{s}(t)$ and the vector $\pi(\mathbf{s}(t))$. Given any $\mathbf{s}(t)$, define the set

$$B(\mathbf{s}(t)) = \{s \in \Gamma \mid s = s_i(t) \text{ for some } i \text{ and } \pi_i(\mathbf{s}(t)) \geq \pi_j(\mathbf{s}(t)) \text{ for all } j\}.$$

The set $B(\mathbf{s}(t))$ contains all subsidy levels that have earned highest welfare in period t.

At the end of every t each country has a probability $0 < \lambda < 1$ of revising its subsidy for the next period. In doing so, it chooses any $s \in B(\mathbf{s}(t))$ with positive probability. With probability $1 - \lambda$ this kind of imitative revision does not take place; this may reflect some inertia in observing the system or in revising subsidies, due for example to restrictions in the administrative or political decision-making process that are left out of our model. Regardless of the results of imitation or inertia, at any point in time, each country has probability $0 \leq \varepsilon < 1$ to experiment with any subsidy $s \in \Gamma$ at random. Both, revision and experimentation opportunities, are drawn independently across countries. Given λ and ε, let $P^{\lambda,\varepsilon}_{\mathbf{s},\mathbf{s}'}$ be the probability of a direct transition of the process from state $\mathbf{s}$ to state $\mathbf{s}'$. This defines a Markov process with transition probability matrix given by

$$P^{\lambda,\varepsilon} = \left(P^{\lambda,\varepsilon}_{\mathbf{s},\mathbf{s}'}\right)_{\mathbf{s},\mathbf{s}' \in \Gamma^n}.$$

We refer to the process with $\varepsilon = 0$ as the *unperturbed imitation dynamics*. A state $\mathbf{s}$ that is reached with positive probability with $P^{\lambda,0}$ but cannot be abandoned without experimentation (i.e. $P^{\lambda,0}_{\mathbf{ss}'} = 0$ for all $\mathbf{s}' \neq \mathbf{s}$) is called an *absorbing* state of the unperturbed dynamics. An obvious property of imitation is that it leads to *monomorphic* states – in our setup, states where all countries choose the same subsidy level. Define

$$M = \{\mathbf{s} \in \Gamma^n \mid \mathbf{s} = (s, \ldots, s),\ s \in \Gamma\}.$$

From any $\mathbf{s}(t) \notin M$ there is positive probability that all countries revise their subsidies in the same period and that they all choose the same $s \in B(\mathbf{s}(t))$, reaching a monomorphic state. At such a monomorphic state with an identical subsidy for all countries, the set of best-performing strategies B simply consists of that subsidy, so imitation alone cannot take the system out of a monomorphic state. This gives us

Lemma 1 *The set M of monomorphic states is the set of absorbing states of the unperturbed imitation dynamics $P^{\lambda,0}$.*

With experimentation, i.e., for $\varepsilon > 0$, however, there is positive probability to exit every state and the model presents permanent randomness. The following features of the model make the Markov process well behaved. First, there is positive probability (no matter how small) that any subset of countries experiment simultaneously and that they experiment with any subsidy so that $P^{\lambda,\varepsilon}_{\mathbf{ss}'} > 0$ for all $\mathbf{s}, \mathbf{s}' \in \Gamma^n$ and the process is irreducible. Moreover, at any given period there is also positive probability that no country revises its subsidy due to inertia ($\lambda, \varepsilon < 1$) and the process stays at the same state for one period; i.e., $P^{\lambda,\varepsilon}_{\mathbf{ss}} > 0$ for all $\mathbf{s}$. This implies that the Markov process is aperiodic. For every λ and ε, this guarantees convergence to a unique invariant distribution denoted

$$\mu^{\lambda,\varepsilon} = \left(\mu^{\lambda,\varepsilon}(\mathbf{s})\right)_{\mathbf{s}\in\Gamma^n},$$

which satisfies $\mu^{\lambda,\varepsilon} = \mu^{\lambda,\varepsilon} P^{\lambda,\varepsilon}$, where $\mu^{\lambda,\varepsilon}(\mathbf{s})$ gives both, the probability that the system is at any state $\mathbf{s} \in \Gamma^n$ in the long run as well as the average frequency with which the process visits any state $\mathbf{s} \in \Gamma^n$ along any sample path.

Following the literature on stochastic evolutionary learning models, the analysis in this paper focusses on the *limit* invariant distribution $\mu^{\lambda} = \lim_{\varepsilon\to 0} \mu^{\lambda,\varepsilon}$. States with positive probability in μ^{λ} are called *stochastically stable* (alternatively, *long-run equilibria*). These are the states that we would observe almost always as the probability of experimentation approaches zero. This distribution exists, and it is a fundamental result in this literature that it only gives positive probability to absorbing sets of the unperturbed dynamics – in our case singleton monomorphic states. Henceforth, we focus the analysis on the set M from Lemma 1 – only monomorphic states are candidates to be observed in the long run as the probability of experimentation becomes small.[12] The formal proofs are relegated to Appendix C. Here, we provide an intuitive explanation of our main result which appears in Proposition 4 at the end of this section.

Define by

$$E = \{\mathbf{s} \in M \mid \widehat{s}(1) \leq s < \widehat{s}(n-1)\}$$

[12]The seminal papers in this literature are Foster and Young (1990), Young (1993), and Kandori et al. (1993). Ellison (2000) gives an alternative characterization of stochastically stable states using the concepts of radius and (modified) coradius. Our analysis relates also to Nöldeke and Samuelson (1993) and Samuelson (1994) and their concepts of *adjacent* states and *mutation-connected component.*

the subset of monomorphic states in which all countries choose a subsidy that is strict ESS; E corresponds essentially to the interval S^E from Proposition 2 above. From a state in E, a single country experimenting with a different subsidy would perform strictly worse and will not be followed by imitation. I.e., states in E cannot be abandoned after a single experimentation. All other states, however, can be abandoned after a single experiment. It is always possible to do this in favor of some subsidy in the interval S^E. To see this, recall that $\widehat{s}(k)$ is the subsidy at which k countries sharing all the poor would attain social welfare exactly equal to $u(w_R)$; for lower (higher) subsidies their welfare would be strictly higher (lower) than $u(w_R)$. Now suppose the process is currently at a monomorphic state with $s < \widehat{s}(1)$ and consider a deviation to $\widehat{s}(1)$. The deviating country will be the only one actively performing redistribution but will attain welfare level exactly equal to $u(w_R)$. Both the deviator and non-deviators are equally successful. Thus, at the next opportunity for revision there is a positive probability to follow the deviator by imitation. The case of high subsidies is even stronger. Suppose the process is currently at a state where all countries set subsidy $s > \widehat{s}(n-1)$. If a single country lowers its subsidy (e.g. to any subsidy in the interval S^E), the welfare of the remaining $n-1$ non-deviating countries will fall below $u(w_R)$ and the deviator will be followed. Finally, note that $\widehat{s}(n-1)$ is excluded from E. Starting at the monomorphic state where all countries choose $s = \widehat{s}(n-1)$, if a single country lowers its subsidy (discontinuing its redistribution policy), both the deviator and the non-deviators are equally successful and there is positive probability that the deviator will be followed. These arguments show that E can be reached from outside if a single country experiments with a subsidy that is a strict ESS, but E cannot be abandoned with a single experiment. This makes it more likely that the process moves into the set E than that it moves out.[13] Recall from Section 3 that the interval S^E is itself a subset of the set of strategies played in any Nash equilibrium. Our prediction, summarized in the following proposition, is that we always end up in E and thus always in the set of Nash equilibria.

Proposition 3 *If* $\mathbf{s}^*$ *is stochastically stable, then* $\mathbf{s}^* \in E \subset \Sigma^N$.

By definition of strict ESS, single experiments will not be enough to move the process from one state to another within the set E. Yet it is possible to abandon states in E after the simultaneous deviation of several experimenting countries. We will argue that the number of simultaneous experiments needed to disturb states in E increases as we

[13]In the terminology of Ellison (2000), the set E has coradius 1 and radius strictly larger than 1. Thus, stochastically stable states must be contained in E.

move towards intermediate levels of the subsidy.[14] To see this, we partition the set E into subintervals

$$E^k = \{\mathbf{s} \in M \mid \widehat{s}(k-1) \leq s < \widehat{s}(k)\}, \qquad k = 2, \ldots, n-1.$$

States in E^k are *monomorphic* with subsidies in the interval $[\widehat{s}(k-1), \widehat{s}(k))$. It is also convenient to introduce some notation for monomorphic states with subsidy equal to $\widehat{s}(k)$, which must often be treated separately in our analysis. Let

$$\mathbf{s}^k = (\widehat{s}(k), \ldots, \widehat{s}(k)).$$

Except for the lower bound of the interval, these subsidies could be profitably sustained with at least k countries, where *profitably* means here with welfare higher than or equal to $u(w_R)$. Instead, if less than k countries actively engaging in redistribution set a subsidy in $(\widehat{s}(k-1), \widehat{s}(k))$, welfare will fall below $u(w_R)$. Although in our partition state $\mathbf{s}^k$ is an element of E^{k+1}, the same applies to subsidy $\widehat{s}(k)$.

- *Moving from lower to higher subsidies.*

  Notice that the process will move away from state $\mathbf{s}^1$ and any other state in E^2 if *two* countries experiment with the same higher subsidy out of the interval $(\widehat{s}(1), \widehat{s}(2)]$. In particular, it is possible to go from any state in E^2 to any other state in E^2 with a strictly higher subsidy, *or* to reach state $\mathbf{s}^2$ in E^3, with two simultaneous experiments. Analogously, the process will move away from any state in E^3 to any other state in E^3 with higher subsidy or to state $\mathbf{s}^3$ if *three* countries simultaneously experiment with the same higher subsidy in $(\widehat{s}(2), \widehat{s}(3)]$, and so on. In general, given any pair $\mathbf{s}, \mathbf{s}' \in E^k$ with $s < s'$, state $\mathbf{s}'$ can be reached from $\mathbf{s}$ if k countries simultaneously experiment with s'. Moreover, state $\mathbf{s}^k$ can always be reached from *a*ny monomorphic state with lower subsidy if k countries simultaneously experiment with $\widehat{s}(k)$. Coordinating on higher subsidies requires more and more countries as k increases and, hence, *transitions upwards* become less likely the higher k.

[14]Underlying this is the fact that strategies in the interval S^E display different levels of so-called m-*stability* (see Schaffer, 1988). A strategy is m-stable if it is robust to the simultaneous deviation of m players to the same alternative strategy. A strategy is called globally stable if it is m-stable for all $m \in [1, n)$. Note that it is enough that $n - m$ players experiment with a m-stable strategy to move the system to that strategy, since players choosing the m-stable strategy perform better.

- *Moving from higher to lower subsidies.*

  Since migration takes place to countries with highest subsidies, experimenting with lower subsidies means giving up redistribution policy and losing all poor. Experimentation with lower subsidies can thus only be successful if the non-deviators are left with welfare lower than *or equal* to $u(w_R)$. All subsidy levels associated to states in E^{n-1} can be sustained by $n-1$ countries with welfare strictly higher than $u(w_R)$. Therefore, we would need at least *two* countries simultaneously lowering their subsidies (in this case experimentation does not necessarily have to be with the same subsidy) in order to move away from states in E^{n-1}. Analogously, we would need *three* countries simultaneously lowering their subsidies to leave states in E^{k-2}, and so on. In general, the process will move *downwards* from any state in E^k if $n-k+1$ countries lower their subsidies. Moving from more to less generous redistribution policies, that is from higher to lower subsidies, requires more and more simultaneously experimenting countries the lower the starting subsidy is, i.e.for lower k. States with high subsidies are more likely to be disturbed in favor of lower subsidies. At states with lower subsidies it becomes less likely to continue with further lowering subsidies.

These findings can be roughly summarized as follows. From states in E^k with low k the process is more likely to move upwards; for high k the process is more likely to move downwards. Detailed accounting of these transition probabilities has to be done carefully depending on whether n is odd or even and, in particular, for values of k around $n/2$. This is done in the proof in Appendix C. In Figure 3 we illustrate the case of n odd – the relevant case for the current EU-27.

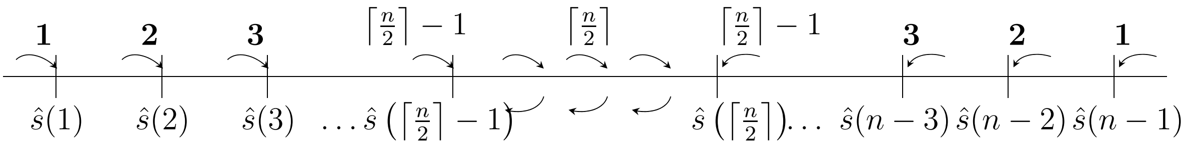

Figure 3: Most likely transition paths. The case of n odd.

The central interval for the case of n odd is $E^{\lceil \frac{n}{2} \rceil}$. As Figure 3 shows, the lower and the upper bounds of this interval can be reached from outside with $\lceil \frac{n}{2} \rceil - 1$ simultaneous experiments. Within $E^{\lceil \frac{n}{2} \rceil}$ it is then possible to move upwards with exactly $\lceil \frac{n}{2} \rceil$ experiments up to the state $\mathbf{s}^{\lceil \frac{n}{2} \rceil} \in E^{\lceil \frac{n}{2} \rceil + 1}$ which itself can be disturbed downwards with $\lceil \frac{n}{2} \rceil - 1$ experiments (e.g., with $n = 5$ we need two deviations downwards to exit $\mathbf{s}^3$). Moving downwards from any state in $E^{\lceil \frac{n}{2} \rceil}$, including its lower bound, requires $\lceil \frac{n}{2} \rceil$ experiments (with $n = 5$ we need three deviations downwards to exit $\mathbf{s}^2$). The process thus moves into $E^{\lceil \frac{n}{2} \rceil}$ with $\lceil \frac{n}{2} \rceil - 1$ experiments and out of it with $\lceil \frac{n}{2} \rceil$ experiments. Within the set we need the same experiments to move up and down. A typical path of this process is likely to move into $E^{\lceil \frac{n}{2} \rceil}$ eventually and then stay bouncing up and down in this set.[15] Hence,

Proposition 4 *The set of stochastically stable states is*

i. $E^{\lceil \frac{n}{2} \rceil}$ *if* n *is odd.*

ii. $E^{\frac{n}{2}} \cup \{s^{\frac{n}{2}}\}$ *if* n *is even;*

The main difference between the cases of odd and even n in Proposition 4 is that the upper bound of the interval, $s^{\frac{n}{2}}$, can be included in the prediction in the case of n even. This is because, in that case, $n/2$ experiments are enough to reach $\mathbf{s}^{\frac{n}{2}}$ but also to exit it downwards.

Proposition 4 shows that the most likely outcomes of the OMC applied to decentralized redistribution lie in the intermediate range of the ESS. Intuitively, low subsidy levels can be easily destabilized by a small number of countries coordinating with their experiments on the same higher subsidy; high subsidy levels become *unprofitable* if only a small number of countries cut their subsidies. Only the intermediate subsidies, which correspond to intermediate values of the ESS are robust to such small-group experimentation.

The learning process of the OMC, thus, eschews ESS with very low or overly generous support for the poor. *A fortiori*, since ESS are a subset of the set of Nash equilibria in our model, the OMC also avoids potentially extreme states that could emerge in the

[15]The proof in Appendix C is done by showing that states $\mathbf{s} \in E^{\lceil \frac{n}{2} \rceil}$ have minimum cost trees (see e.g. Theorem 1 in Samuelson (1994)). The illustration above shows that, for the case of n odd, we could alternatively use the radius-coradius theorem of Ellison (2000). However, we would need to resort to *tree surgery* to show that all states in $E^{\lceil \frac{n}{2} \rceil}$ are stochastically stable. Also for even n there are many states in the two central intervals with radius equal to their modified coradius. For these reasons and because the set $E^{\lceil \frac{n}{2} \rceil}$ resembles what Samuelson (1994) calls a *mutation-connected component* it turns out to be more convenient to compute transition costs.

traditional setting of decentralized choice of redistribution policies. In that sense, the learning process in the OMC can be viewed as a moderating device.

5 Discussion of the results

5.1 Efficiency

Given that, in general, an important motivation for policy coordination is the avoidance of externality-induced efficiency failures of decentralization, it seems natural to ask whether the learning process of the OMC indeed leads to efficient policy outcomes. Formally, we have to check for the relationship between the symmetric efficient solution, s^0, and the set of stochastically stable states characterized in Proposition 4. Intuitively, $s^0 = s^*(n)$ maximizes the joint welfare function $f(n, s)$, whereas the values of $\widehat{s}\left(\left\lceil \frac{n}{2} \right\rceil - 1\right)$ and $\widehat{s}\left(\left\lceil \frac{n}{2} \right\rceil\right)$, relevant to define $E^{\left\lceil \frac{n}{2} \right\rceil}$, refer to properties of the functions $f(\frac{n-1}{2}, s)$ and $f(\frac{n}{2}, s)$. Technically, there is no good reason why s^0 should be related to our prediction in the set $E^{\left\lceil \frac{n}{2} \right\rceil}$. Indeed, it is possible to construct examples where the efficient outcome is in our prediction. In general, however, the predictions of our model entail subsidies which could be higher or lower than the efficient outcome. This is illustrated in Example 1 bellow. It should still be stressed that the convergence to "intermediate" subsidy levels precludes the more extreme outcomes that could emerge under decentralization in a Nash equilibrium.

Example 1 *Consider a utility function of the form $u(x) = \sqrt{x} - 1$. Take $w_P = 1$ and assume that $w_R \leq 4$. We have that*

$$\widehat{s}(k) = \frac{4\left(\sqrt{w_R} - 1\right)\left(\frac{\nu}{k} + \sqrt{w_R}\right)}{\left(\frac{\nu}{k} + 1\right)^2}$$

Take $n = 10$, in which case our prediction is always the interval $(\widehat{s}(4), \widehat{s}(5))$. Table 1 summarizes our results for different values of w_R and ν. We see that the efficient outcome, s^0, may always be lower than $\widehat{s}(4)$, in which case we would predict inefficiently high redistribution; it may be always higher that $\widehat{s}(5)$, in which case we would predict inefficiently low redistribution; finally, the efficient outcome may some times be contained in our prediction.

w_R	ν	s^0	$\widehat{s}(4)$	$\widehat{s}(5)$
2	12	$\frac{5}{11}$	$\frac{2\sqrt{2}-1}{4}$	
2	20	$\frac{1}{3}$	$\frac{4\sqrt{2}-3}{9}$	$\frac{4(3\sqrt{2}-2)}{25}$
4	20	1		$\frac{24}{25}$

Table 1: Efficiency results

5.2 Mobility of the poor

Let us now briefly discuss the implications of our assumption on perfect mobility of the poor. We will argue that our qualitative results are robust to the introduction of a small fraction of immobile poor in each country. Consider a small variation of our model, where all countries have some fixed amount β of immobile (native) poor. The parameter β measures the relative mass of immobile poor to rich in any given country. Symmetry is preserved if β is the same for all countries. Assume $\nu > n\beta$; that is the total amount of immobile poor is smaller than the amount of mobile poor. All other features of our model are as before. The total amount of poor living in country i is now given by $\widetilde{\ell}_i(\mathbf{s}) := \beta + \ell_i(\mathbf{s})$, where $\ell_i(\mathbf{s})$ is determined as in (1). Governments are still assumed to adhere to generalized utilitarianism. Thus, payoffs can be written as in expression (2) by replacing $\ell_i(\mathbf{s})$ with $\widetilde{\ell}_i(\mathbf{s})$. Countries now always have an incentive to actively pursue some transfer policy, at least for their native poor. If no mobile poor moves to country i; i.e., if $\tilde{\ell}_i = \beta$, its government would set the transfer to maximize the following welfare function

$$g(\beta, s) = \beta \cdot u(w_P + s) + u(w_R - \beta s).$$

Note that $g(\beta, s)$ is a strictly concave function of s with $g(\beta, 0) > u(w_R)$ and $g'(\beta, 0) > 0$. The optimal transfer in the absence of mobile poor is then given by

$$\widetilde{s}^0 = \frac{w_R - w_P}{1 + \beta}.$$

Our assumptions guarantee that $g(\beta, s)$ has the same properties as our auxiliary functions $f(k, s)$, derived in Appendix A. Thus, for $\beta < \nu/n$, we have that $\widetilde{s}^0 > s^*(n) = s^0$. Intuitively, when the fraction of immobile poor is not too large, optimal transfer policy can be more generous when a country only has to provide social policy for the native immobile. Moreover, define $\widetilde{s}$ as the value of the subsidy that satisfies $g(\beta, \widetilde{s}) = u(w_R)$. This is the analogous to our previous $\widehat{s}(k)$, which we proved to be strictly increasing in k.

Again, since $\beta < \nu/n$, we have that $\widetilde{s} > \widehat{s}(n)$. An intuitive picture can be obtained from Figure 2 by replacing the horizontal line at the value $u(w_R)$ by a humped g curve with the same features as the f curves and cutting through $u(w_R)$ further to the right. The points at which $g(\beta, s)$ cuts trough $f(1, k)$ and $f(n-1, s)$ will now be the boundaries of the ESS interval. Note finally that $g(\beta, \widehat{s}(n-1)) > u(w_R) = f(n-1, \widehat{s}(n-1))$; analogously, $g(\beta, \widehat{s}(1)) > u(w_R) = f(1, \widehat{s}(1))$. This indicates that the bounds of the ESS interval will be lower than for the case without immobile poor. All other elements of the analysis are as before.

5.3 Average utilitarianism

Finally, we turn our attention to the welfare function. Obviously, payoffs and, thus, the equilibria of our game depend crucially on the choice of welfare function (Mansoorian and Myers, 1997). We discussed at the end of Section 2 that *generalized utilitarianism*, given in expression (2), as a government objective has the drawback of allowing for the repugnant conclusion. With decentralized redistribution, this strong predilection for large population sizes leads to quite generous subsidies to the mobile poor – an observation that is at odds with the widespread fear of a decline of the welfare state in the presence of labor mobility. There exist social welfare functions that avoid the repugnant conclusion (for a survey, see Blackorby et al., 2009). One alternative that has captured some attention is *average utilitarianism.* As the name suggests, government payoffs given by

$$\pi_i^{AU}(\mathbf{s}) = \frac{1}{\ell_i(\mathbf{s}) + 1} \cdot \left[\ell_i(\mathbf{s}) \cdot u(w_P + s_i) + u(w_R - \ell_i(\mathbf{s}) \cdot s_i)\right]. \tag{6}$$

It can be easily checked that the *symmetric efficient* solution under average utilitarianism coincides with the one obtained for generalized utilitarianism, s^0, given by expression (4). However, average utilitarianism gives a large welfare weight to well-off people, providing strong incentives to cut back transfers to the poor. This actually results in a remarkable efficiency failure both in a Nash equilibrium and in an ESS.

To see this, note first that for $\ell_i = 0$, we have $\pi_i^{AU} = u(w_R)$. Denote $\lambda_i = \frac{\ell_i}{1+\ell_i}$. By strict monotonicity and strict concavity of u we have that, for any $\ell_i > 0$,

$$\pi_i^{AU} = \lambda_i u(w_P + s_i) + (1 - \lambda_i) u(w_R - \ell_i s_i) <$$
$$u(\lambda_i(w_P + s_i) + (1 - \lambda_i)(w_R - \ell_i s_i)) = u\left(\frac{w_R + \ell_i w_P}{1 + \ell_i}\right) < u(w_R). \tag{7}$$

This implies that at any profile with a subset of countries sharing the burden of redistribution at some strictly positive subsidy level, there is an incentive to cut down transfers (leading to $\ell_i = 0$). The only Nash equilibrium will have all countries setting $s_i = 0$. Expression (7) also shows that $s = 0$ is the only ESS of the game, since at any symmetric profile with $s > 0$ a relative advantage can be obtained by cutting down s; alternatively, starting at the symmetric profile with $s_i = 0$ for all i, any increase in s results in a relative disadvantage. Clearly, the imitative process of the OMC will also not deliver any improvement over the inefficient Nash equilibrium.

6 Conclusions

We propose to analyse the Open Method of Coordination (OMC), which the EU has adopted since its Lisbon Summit in 2000 for social policy and elsewhere, as a dynamic stochastic learning process of the type studied in evolutionary game theory. The OMC is based on the idea that, for certain commonly agreed policy objectives, national policies emerge from a process where governments compare themselves to one another in terms of policy performance, learn from each other, and imitate what they perceive as best practices. If convergence occurs under the OMC, then not due to express legislation but by the force of example.

We formalize and explore the workings of OMC for the particular case of income redistribution in an integrated economic area with perfectly mobile social welfare beneficiaries. Our main observation is that the OMC strongly favors coordination on a subset of Nash equilibria. In a dynamic interpretation, the OMC results in a powerful equilibrium refinement. In particular, intermediate values of subsidies that can be sustained by coordination of approximately half of the countries are the most likely ones to be observed in the long run.

To our knowledge, this is the first paper that provides a formal, game theoretic analysis of the OMC. Both opponents and advocates of the OMC will, with good reason, argue that our stylized analysis ignores many of the OMC's advantages (e.g., the higher degree of legitimacy), defines away a number of problems (e.g., the definition and measurement of performance indicators, communication procedures etc.) and discusses the OMC in an artificial setting (decentralized redistribution) to which it may not at all be suited.

Notwithstanding these concerns, our analysis entails important messages for policy-makers and mechanism designers in the EU where welfare and redistribution policies are still in the domain of national governments. The hope that the OMC will "recalibrate" European welfare states (Ferrera et al., 2000) seems justified. On a first pass, the OMC does indeed provide a successful way to attain policy coordination and to avoid extreme, undesirable outcomes. This appears to be in line with preliminary empirical evidence compiled in Coelli et al. (2008).

A Properties of the payoff function

We show here the main properties of the family of functions $\{f(k,s)\}_{k=1,\dots,n}$ that are used in the proofs of our results. Recall

$$f(k,s) = \frac{\nu}{k} \cdot u(w_P + s) + u(w_R - \nu/k \cdot s) \qquad k = 1,\dots,n.$$

Notice $f(k,0) \geq u(w_R)$ and $f_s(k,0) = \nu/k \cdot (u'(w_P) - u'(w_R)) > 0$, since $u'' < 0$ and $w_P < w_R$ implies $u'(w_P) > u'(w_R)$. Moreover, $f_{ss}(k,s) < 0$.[16]

Recall $s^*(k) = \arg\max_{s\geq 0} f(k,s) = \frac{w_R - w_P}{1+\frac{\nu}{k}}$ satisfies

$$f_s(k, s^*(k)) = \frac{\nu}{k}\left(u'(w_P + s^*(k)) - u'\left(w_R - \nu/k \cdot s^*(k)\right)\right) = 0.$$

Clearly, $s^*(k) < \frac{k}{\nu} w_R$ and $s^*(k)$ is strictly increasing with k.

Given $k \in \{1,\dots,n\}$, let $\widehat{s}(k)$ be a strictly positive value of s such that $f(k,\widehat{s}(k)) = u(w_R)$. The properties of f imply that $f(k,s) > u(w_R)$ for all $s \in (0, s^*(k)]$. By definition of $s^*(k)$ and $f_{ss} < 0$, f is strictly decreasing for all $s > s^*(k)$. Moreover, for $s = \frac{k}{\nu} w_R$ we have that

$$f(k, \frac{k}{\nu} w_R) = \frac{\nu}{k} u(w_P + \frac{k}{\nu} w_R) + u(0) < u(w_R)$$

for $u(0)$ sufficiently low. In particular, $\frac{\nu}{k} u(w_P + \frac{k}{\nu} w_R)$ is strictly decreasing with k.[17] Let

[16]We adopt the conventional notation $f_i(k,s) = \frac{\partial f(k,s)}{\partial i}$ and $f_{ij}(k,s) = \frac{\partial f_i(k,s)}{\partial j \partial i}$ with $i, j = k, s$.

[17]Define $F(x) = \nu/x \cdot u(w_P + x/\nu \cdot w_R)$ with $x \in \mathbb{R}_{++}$. We have that

$$F'(x) = -\frac{\nu}{x^2} u(w_P + \frac{x}{\nu} w_R) + \frac{w_R}{x} u'(w_P + \frac{x}{\nu} w_R) < 0,$$

$u(0) < -K := u(w_R) - \nu \cdot u(w_P + 1/\nu \cdot w_R)$ to obtain the desired inequality. This implies existence and uniqueness of $\widehat{s}(k)$ for all k and it shows that $s^*(k) < \widehat{s}(k) < \frac{k}{\nu} w_R$. Finally, we have that $\widehat{s}(k') > \widehat{s}(k)$ for all $k, k' \in \{1, \ldots, n\}$ and $k' > k$. To see this, note that, by definition, $\widehat{s}(k)$ satisfies

$$\Gamma(k, s) := \frac{k}{\nu} \cdot \left[u(w_R) - u\left(w_R - \frac{\nu}{k} s \right) \right] = u(w_P + s). \tag{8}$$

The right hand side of expressions (8) is strictly increasing and strictly concave with s and it equals $u(w_P)$ for $s = 0$. For any k, $\Gamma(k, 0) = 0 \leq u(w_P)$ and $\Gamma(k, s)$ is strictly increasing and strictly convex in s with

$$\Gamma_s(k, s) \quad = \quad u'\left(w_R - \frac{\nu}{k} s \right) > 0.$$

Furthermore, $u'' < 0$ implies that $\Gamma_s(k', s) < \Gamma_s(k, s)$ for all $s > 0$ and $k' > k$; i.e. the Γ functions become flatter with s as we increase k. Thus, $\Gamma(k, s) > \Gamma(k', s)$ for $s > 0$ and $k' > k$. It follows that $\widehat{s}(k') > \widehat{s}(k)$ for all $k, k' \in \{1, \ldots, n\}$ and $k' > k$.

Given $k \in \{1, \ldots, n\}$, let $\bar{s}(k)$ be a strictly positive s such that $f(k, \bar{s}(k)) = f(1, \bar{s}(k))$. We now proceed to show existence and uniqueness of $\bar{s}(k)$. To this purpose define

$$\begin{aligned} \Delta(k, s) \quad &:= \quad f(1, s) - f(k, s) \\ &= \quad \nu \left(1 - \frac{1}{k} \right) \cdot u(w_P + s) + u(w_R - \nu s) - u\left(w_R - \frac{\nu}{k} s \right). \end{aligned} \tag{9}$$

Clearly, $\Delta(k, 0) \geq 0$. Moreover, strict concavity of u implies that[18]

$$\Delta(k, s) > \nu \left(1 - \frac{1}{k} \right) \{ u(w_P + s) - s \cdot u'(w_R - \nu \cdot s) \}. \tag{10}$$

For $s \leq s^*(1)$ we have $w_P + s \leq w_R - \nu \cdot s$ and, by $u'' < 0$, then $u'(w_P + s) \geq u'(w_R - \nu \cdot s)$. For $s \leq s^*(1)$, we can then replace the right hand side of expression (10) by the following smaller expression

$$\Delta(k, s) > \nu \left(1 - \frac{1}{k} \right) \{ u(w_P + s) - s \cdot u'(w_P + s) \} > \nu \left(1 - \frac{1}{k} \right) u(w_P) \geq 0. \tag{11}$$

since, by strict concavity of $u(c)$, $u(w_P + y) - y u'(w_P + y) > u(w_P) \geq 0$ for all $y > 0$.

[18]By strict concavity

$$u(w_R - \nu/k \cdot s) < u(w_R - \nu \cdot s) + \left(1 - \frac{1}{k} \right) \nu \cdot s \cdot u'(w_R - \nu \cdot s).$$

The second inequality in (11) follows again from strict concavity of u (cf. Footnote 17). This shows that $\Delta(k,s) > 0$ for $s \in (0, s^*(1)]$ and thus, if $\bar{s}(k)$ exists, we must have $\bar{s}(k) > s^*(1)$. Furthermore,

$$\begin{aligned}
\Delta_s(k,s) &= \nu\left(1-\frac{1}{k}\right)\cdot u'(w_P+s) - \nu\cdot u'(w_R-\nu s) + \frac{\nu}{k}\cdot u'\left(w_R-\frac{\nu}{k}s\right) \\
&= \nu\left(1-\frac{1}{k}\right)\cdot\left[u'(w_P+s)-u'(w_R-\nu s)\right] + \frac{\nu}{k}\cdot\left[u'\left(w_R-\frac{\nu}{k}s\right)-u'(w_R-\nu s)\right] \\
&< \nu\left(1-\frac{1}{k}\right)\cdot\left[u'(w_P+s)-u'(w_R-\nu s)\right].
\end{aligned}$$

The inequality follows from $u'\left(w_R-\frac{\nu}{k}s\right)-u'(w_R-\nu s) < 0$ by $u'' < 0$. Recall that $u'(w_P+s)-u'(w_R-\nu s) \leq 0$ for $s \geq s^*(1)$. Thus, $\Delta_s(k,s) < 0$ for all $s \geq s^*(1)$. Finally, take $s = \widehat{s}(1)$. We know that $s^*(1) < \widehat{s}(1) < \widehat{s}(k)$ for $k > 1$ and we have that

$$\Delta(k,\widehat{s}(1)) = f(1,\widehat{s}(1)) - f(k,\widehat{s}(1)) = u(w_R) - f(k,\widehat{s}(1)) < 0. \tag{12}$$

Existence and uniqueness of $\bar{s}(k)$ for all $k > 1$ follows. Furthermore, (11) and (12) also imply that $s^*(1) < \bar{s}(k) < \widehat{s}(1)$ for all k. Finally, re-writing (9), we have that $\bar{s}(k)$ must satisfy

$$\Omega(k,s) := \frac{k}{\nu(k-1)}\cdot\left[u\left(w_R-\frac{\nu}{k}s\right)-u(w_R-\nu s)\right] = u(w_P+s).$$

It is easy to check that strict concavity of u implies $\Omega(k,s)$ is strictly decreasing with k and, thus, $\bar{s}(k)$ increases with k.

B Proofs of Section 3

Proof of Proposition 1. We proceed in two steps.

Step 1: *All Nash equilibria are symmetric.*

Consider any non-symmetric profile $(s_1,\ldots,s_n)$ with $s_i \neq s_j$ for some $i \neq j$. Without loss of generality suppose $s_i < s_j$. Country i attracts currently no poor and gets payoff $u(w_R)$. The payoff to all countries currently choosing maximum subsidy can be expressed as $f(m_i(\mathbf{s}_{-i}),\bar{s}_i)$, where $\bar{s} := \bar{s}_i$ is the current maximum subsidy and $m := m_i(\mathbf{s}_{-i})$ is the number of countries currently choosing maximum subsidy. If $\bar{s} > \widehat{s}(m)$, then $f(m,\bar{s}) < u(w_R)$. This cannot be an equilibrium, since any of the countries currently choosing maximum subsidy could strictly increase its payoffs by lowering the subsidy, thus attracting no poor. If, alternatively, $\bar{s} \leq \widehat{s}(m)$, then $f(m, \bar{s}) \geq u(w_R)$. Since $\widehat{s}(m) < \widehat{s}(m+1)$, we have that $f(m+1, \bar{s}) > u(w_R)$. Thus, if country i would deviate from s_i and choose $\bar{s}$, it would strictly improve its payoffs.

Step 2: *Characterization of Nash equilibria*

Consider any symmetric profile $\mathbf{s} = (s, \ldots, s)$. Payoffs to all countries at $\mathbf{s}$ can be expressed as $f(n, s)$.

(i) Suppose $s < \bar{s}(n)$. Then, by definition of $\bar{s}(n)$, $f(1, s) > f(n, s)$; i.e. a single country alone offering subsidy s would attain strictly greater payoffs. By continuity, there exists s' such that $s < s' < \bar{s}(n)$ where $f(1, s') > f(n, s)$. Thus, there are incentives to deviate.

(ii) Suppose $s > \widehat{s}(n)$. Then, by definition of $\widehat{s}(n)$, $f(n, s) < u(w_R)$; i.e. the subsidy is too generous and these countries would be better off by attracting no poor. Any country could strictly improve by choosing $s' < s$.

(iii) Suppose now that $\bar{s}(n) \leq s \leq \widehat{s}(n)$. By definition of $\bar{s}(n)$ we have $f(n, s) \geq f(1, s)$ and by definition of $\widehat{s}(n)$ we have $f(n, s) \geq u(w_R)$. Any country reducing the subsidy would get payoff $u(w_R)$ with no strict improvement. Any country increasing the subsidy to $s' > s$ would get payoff $f(1, s')$. Recall $\bar{s}(n) > s^*(1)$ and, thus, $f(1, s') < f(1, s) \leq f(n, s)$. Therefore, there are no incentives to deviate.

It follows from (i)-(iii) that $[\bar{s}(n), \widehat{s}(n)]$ is the interval of equilibrium subsidies. ■

Proof of Proposition 2. Consider any symmetric profile $\mathbf{s} = (s, \ldots, s)$. Suppose a single country deviates to some subsidy $s' \neq s$. The relevant payoffs to characterize ESS are now the payoffs obtained *after* deviation.

(i) Suppose first $s < \widehat{s}(1)$. A deviation upwards with $s < s' < \widehat{s}(1)$ gives payoff $f(1, s') > u(w_R)$ for the deviator while all others get $u(w_R)$ after deviation. Thus, it is possible to obtain a strict relative advantage and $s < \widehat{s}(1)$ is not ESS.

(ii) Consider now $s > \widehat{s}(n-1)$. A deviation downwards to any $s' < s$ gives the deviating country payoff $u(w_R)$. After deviation, non-deviators get payoff $f(n-1, s) < u(w_R)$

by definition of $\widehat{s}(n-1)$. Thus, the deviator has a strict advantage and $s > \widehat{s}(n-1)$ is not ESS.

(iii) Let now $s \in [\widehat{s}(1), \widehat{s}(n-1)]$. Deviations to $s' < s \leq \widehat{s}(n-1)$ will earn payoff $u(w_R)$ while those countries sticking to s obtain $f(n-1, s) \geq u(w_R)$. Deviations to $s' > s \geq \widehat{s}(1)$ will earn the deviator a payoff $f(1, s') < u(w_R)$ while all others get $u(w_R)$. Thus, it is not possible to attain a strict relative advantage through deviation.

It follows from (i)-(iii) that only $s \in [\widehat{s}(1), \widehat{s}(n-1)]$ are ESS. ∎

C Proofs of Section 4

Proof of Lemma 1. From any $\mathbf{s}(t) \notin M$ there is positive probability that all countries revise their subsidies at the same time and that they all choose the same $s \in B(\mathbf{s}(t))$, reaching a monomorphic state. At a monomorphic state $\mathbf{s}(t) = (s, \ldots, s) \in M$ we have that $B(\mathbf{s}(t)) = \{s\}$, so imitation alone cannot take the system out of a monomorphic state. ∎

The support of the limit invariant distribution is contained in the set M of absorbing monomorphic states (see, e.g., Samuelson, 1994, Theorem 1). We thus restrict the *mutation-counting* analysis to states $\mathbf{s} \in M$. Let $\mathbf{s} \in M$ be an absorbing state of $P^{\lambda,0}$. The *basin of attraction* of $\mathbf{s}$ is the set of states from which there is positive probability that the unperturbed imitation dynamics moves the process to $\mathbf{s}$ in a finite number of periods. Given $P^{\lambda,\varepsilon}$, an $\mathbf{s}$-tree on Γ^n, denoted h, is a collection of ordered pairs $(\mathbf{s}', \mathbf{s}'')$, or *arrows* from $\mathbf{s}'$ to $\mathbf{s}''$, such that:

i.) For each $\mathbf{s}' \in \Gamma^n$, there is at most one arrow from $\mathbf{s}'$ to any other $\mathbf{s}'' \in \Gamma^n$.

ii.) For each $\mathbf{s}' \in M \setminus \{s\}$, there is a sequence of pairs $\{(\mathbf{s}', \mathbf{s}_1), (\mathbf{s}_1, \mathbf{s}_2), \ldots, (\mathbf{s}_m, \mathbf{s}'')\}$ with $\mathbf{s}''$ in the basin of attraction of $\mathbf{s}$.

We denote $\mathcal{H}(\mathbf{s})$ the set of all possible $\mathbf{s}$-trees. Let $\mathbf{s}', \mathbf{s}'' \in \Gamma^n$ and define the *cost* of an arrow from $\mathbf{s}'$ to $\mathbf{s}''$, denoted $c(\mathbf{s}', \mathbf{s}'')$, as the minimum number of experiments needed to get from $\mathbf{s}'$ to $\mathbf{s}''$ in the following sense. Let $d(\mathbf{s}', \mathbf{r})$ be the number of coordinates which differ between $\mathbf{s}'$ and any $\mathbf{r} \in \Gamma^n$. Let $P(\mathbf{s}'')$ be the basin of attraction of state $\mathbf{s}''$. Then

$$c(\mathbf{s}', \mathbf{s}'') = \min_{\mathbf{r} \in P(\mathbf{s}'')} d(\mathbf{s}', \mathbf{r})$$

The cost of a sequence of pairs and the cost of an $\mathbf{s}$-tree can be obtained as the sum of costs of all pairs in the sequence, respectively all pairs in the $\mathbf{s}$-tree. Denote by

$$C(\mathbf{s}) = \min_{h \in \mathcal{H}(\mathbf{s})} \sum_{(\mathbf{s}', \mathbf{s}'') \in h} c(\mathbf{s}', \mathbf{s}'')$$

the cost of a minimum-cost $\mathbf{s}$-tree. An absorbing state $\mathbf{s}^*$ is stochastically stable (i.e., $\mathbf{s}^*$ is in the support of the limit invariant distribution) if and only if $\mathbf{s}^*$ solves $\min_{\mathbf{s} \in M} C(\mathbf{s})$; i.e. stochastically stable states have minimum-cost trees.

Proof of Proposition 3. Consider the set

$$E = \{\mathbf{s} \in M \mid \widehat{s}(1) \leq s < \widehat{s}(n-1)\}.$$

Denote $\mathbf{s}^k = (\widehat{s}(k), \ldots, \widehat{s}(k))$ with $k = 1, \ldots, n$. Let us first argue that for all $\mathbf{s} \notin E$, there exists $\mathbf{s}' \in E$ such that $C(\mathbf{s}') < C(\mathbf{s})$ and, thus, $\mathbf{s} \notin E$ cannot be stochastically stable. Let $\mathbf{s}(t) = (s, \ldots, s) \in M$ be state at which the process starts in period t and call $\mathbf{s}'(t+1) \in \Gamma^n$ the resulting state when one of the countries deviates to subsidy $s' \in \Gamma$ in period $t+1$ while the remaining $n-1$ countries still choose s. Consider the following cases separately:

(i) Suppose $s < \widehat{s}(1)$ and $s' = \widehat{s}(1)$. The deviating country attracts all poor and gets payoff $u(w_R)$ by definition of $\widehat{s}(1)$. Non-deviating countries with lower subsidies attract no poor and also get $u(w_R)$. It follows that $B(\mathbf{s}'(t+1)) = \{s, s'\}$.

(ii) Suppose $s \geq \widehat{s}(n-1)$ and $s' \in [\widehat{s}(1), \widehat{s}(n-1)) \cap \Gamma$. The deviating country with lower subsidy attracts no poor and gets payoff $u(w_R)$. The remaining $n-1$ non-deviating countries must now share all the poor at $s \geq \widehat{s}(n-1)$ which results in payoff $f(n-1, s) \leq u(w_R)$ by definition of $\widehat{s}(n-1)$. It follows that $s' \in B(\mathbf{s}'(t+1))$.

Both in (i) and (ii) there is positive probability that all countries revise their subsidies and choose s' at the end of $t+1$.

(iii) Suppose $\widehat{s}(1) \leq s < \widehat{s}(n-1)$. If $s' < s$ (deviation downwards), the deviating country gets $u(w_R)$, while the remaining $n-1$ non-deviating countries with $s < \widehat{s}(n-1)$ get payoffs $f(n-1, s) > u(w_R)$ by definition of $\widehat{s}(n-1)$. Alternatively, if $s' > s$ (deviation upwards), the deviating country with $s' > s \geq \widehat{s}(1)$ attracts all poor and gets payoffs $f(1, s') < u(w_R)$, while the non-deviating countries get $u(w_R)$. In both cases $s' \notin B(\mathbf{s}'(t+1))$.

It follows from (i) and (ii) that one experimenting country is enough to move the process from any $\mathbf{s} \notin E$ to some $\mathbf{s}' \in E$. By (iii), however, one experimenting country alone is not enough to exit states $\mathbf{s} \in E$. Now take any $\mathbf{s} \in M$ such that $s < \widehat{s}(1)$ and consider any $\mathbf{s}$-tree of minimum cost $C(\mathbf{s})$. We can now construct an $\mathbf{s}^1$-tree of minimum cost in the following way: connect $\mathbf{s}$ to $\mathbf{s}^1$ at cost $c(\mathbf{s}, \mathbf{s}^1) = 1$ as described in (i) and remove the arrow starting at $\mathbf{s}^1$; by (iii) we have that $c(\mathbf{s}^1, \mathbf{s}') > 1$ for all $\mathbf{s}'$ so that we have reduced the total cost of the tree and thus $C(\mathbf{s}) > C(\mathbf{s}^1)$. We can now proceed analogously with any $\mathbf{s} \in M$ such that $s \geq \widehat{s}(n-1)$ and any $\mathbf{s}' \in E$; by (ii) and (iii) $C(\mathbf{s}') < C(\mathbf{s})$. ■

Proof of Proposition 4. Define the following partition of the set E

$$E^k = \{\mathbf{s} \in M \mid \widehat{s}(k-1) \leq s < \widehat{s}(k)\} \qquad k = 2, \ldots, n-1.$$

States in E^k are *monomorphic.* Note, however, that all these subsidies could be profitably sustained with k countries, where *profitably* means here with welfare higher than or equal to $u(w_R)$. If less than k countries actively engaging in redistribution set a subsidy in $(\widehat{s}(k-1), \widehat{s}(k))$, welfare will fall strictly below $u(w_R)$. Although in our partition state $\mathbf{s}^k$ is an element of E^{k+1}, the same applies to subsidy $\widehat{s}(k)$. Analogously, state $\mathbf{s}^{k-1} \in E^k$ but subsidy $\widehat{s}(k-1)$ can be sustained with at least $k-1$ countries. Denote $I(n)$ the set of *central states* given as follows:

(i) If n is odd, $I(n) = E^{\lceil \frac{n}{2} \rceil}$

(ii) If n is even, $I(n) = E^{\frac{n}{2}} \cup \{\mathbf{s}^{\frac{n}{2}}\}$

We now proceed to prove the proposition in three steps. First, we compute the cost of reaching (exiting) any state in E from (to) a lower and from a higher state. Then, argueing as in the proof of Proposition 3, we show that for any state which does not belong to the set $I(n)$ of central states we can find some state in $I(n)$ with strictly lower cost trees. This shows that states which are not central cannot be stochastically stable. Finally, we will argue that all states in $I(n)$ have minimum-cost trees of equal costs. It follows that all states in $I(n)$ are stochastically stable.

Step 1: *Transitions upwards are increasingly costly depending on the state to be reached. Transition downwards are decreasingly costly depending on the state we want to exit.*

Let the process start at $\mathbf{s}(t) \in E^k$ in period t and denote $\mathbf{s}'(t+1) \in \Gamma^n$ the resulting state after deviation. Consider the following cases separately:

(i) Suppose $\mathbf{s}'(t+1)$ has k' countries choosing $s' > s$ with $\widehat{s}(k'-1) < s' \leq \widehat{s}(k')$ and $k' \geq k$. Deviating countries get payoff $f(k', s') \geq u(w_R)$ by definition of $\widehat{s}(k')$. The remaining non-deviating countries get $u(w_R)$. It follows that $s' \in B(\mathbf{s}'(t+1))$.

(ii) Suppose $\mathbf{s}'(t+1)$ has $n-k+1$ deviating countries choosing $s' < s$. Deviating countries get payoff $u(w_R)$. The remaining $k-1$ non-deviating countries get payoff $f(k-1, s) \leq u(w_R)$. It follows that $s' \in B(\mathbf{s}'(t+1))$.

Both in (i) and (ii) there is positive probability that all countries revise their subsidies and choose s' at the end of $t+1$.

(iii) If $\widetilde{k} < k'$ countries coordinate on $s' > s$ with $s' \in (\widehat{s}(k'-1), \widehat{s}(k')]$, their payoff will be $f(\widetilde{k}, s') < u(w_R)$, implying $B(\mathbf{s}'(t+1)) = \{s\}$, so that the process will not exit $\mathbf{s}(t)$. Moreover, since $\widehat{s}(k')$ increases with k', the minimum number of experiments needed to exit $\mathbf{s}(t) \in E^k$ upwards is $k' = k$ for $s < s' \leq \widehat{s}(k)$. Analogously, less than $n-k+1$ countries are not enough to exit states in E^k downwards.

It follows from (i)–(iii) that for any pair $\mathbf{s}, \mathbf{s}' \in E$ we have:

(A) $c(\mathbf{s}, \mathbf{s}') = k$ if $s' > s$ and $\widehat{s}(k-1) < s' \leq \widehat{s}(k)$.

(B) $c(\mathbf{s}, \mathbf{s}') = n-k+1$ if $s' < s$ and $\mathbf{s} \in E^k$.

Therefore, the cost of moving upwards to a state $\mathbf{s}' \in E^{k'} \setminus \mathbf{s}^{k'-1}$ increases with k'; i.e. it is higher the higher the state we want to reach. Instead the cost of moving downwards from state $\mathbf{s} \in E^k$ decreases with k; i.e. it is lower the higher the state we intend to leave.

Step 2: *Central states have lower cost trees.*

Here we proceed as in the proof of Proposition 3. There will be a slight difference for the cases of odd and even n.

We want to show that for all $\mathbf{s} \in E \setminus I(n)$, there exists $\mathbf{s}' \in I(n)$ such that $C(\mathbf{s}') < C(\mathbf{s})$. Therefore, states $\mathbf{s} \notin I(n)$ cannot be stochastically stable.

Case 2.1. Suppose that n is odd.

- *States to the left of $I(n)$.* Take any $\mathbf{s} \in E^k$ with $k \leq \lceil \frac{n}{2} \rceil - 1$. Let h be an $\mathbf{s}$-tree of minimum cost $C(\mathbf{s})$. Note that (A) and (B) above imply that the cheapest way to connect state $\mathbf{s}^{\lceil \frac{n}{2} \rceil - 1}$ to h is with a direct transition to the basin of attraction of $\mathbf{s}$ at cost $\lceil \frac{n}{2} \rceil$. On the other hand, by (A) we can connect $\mathbf{s}$ directly to the basin of attraction of $\mathbf{s}^{\lceil \frac{n}{2} \rceil - 1}$ at cost $\lceil \frac{n}{2} \rceil - 1$. We can now remove from h the transition from $\mathbf{s}^{\lceil \frac{n}{2} \rceil - 1}$ to $\mathbf{s}$ saving $\lceil \frac{n}{2} \rceil$ and we can add the opposite transition at cost $\lceil \frac{n}{2} \rceil - 1$. The result is an $\mathbf{s}^{\lceil \frac{n}{2} \rceil - 1}$-tree with cost strictly lower than $C(\mathbf{s})$, implying that $\mathbf{s}$ cannot be stochastically stable.

- *States to the right of $I(n)$.* Take now any $\mathbf{s} \in E^k$ with $k > \lceil \frac{n}{2} \rceil$. Let h be an $\mathbf{s}$-tree of minimum cost $C(\mathbf{s})$. Consider $\mathbf{s}' \in E^{\lceil \frac{n}{2} \rceil}$, then $s > s'$. By (A), the cheapest way to connect $\mathbf{s}'$ to h is with a direct transition to the basin of attraction of $\mathbf{s}$ at cost higher than or equal to $\lceil \frac{n}{2} \rceil$.[19] On the other hand, by (B) we can connect $\mathbf{s}$ to $\mathbf{s}'$ at cost $n - k + 1 < \lceil \frac{n}{2} \rceil$. We can now remove from h the transition from $\mathbf{s}'$ to $\mathbf{s}$ saving at least $\lceil \frac{n}{2} \rceil$ and we can add the opposite transition adding cost strictly lower than $\lceil \frac{n}{2} \rceil$. The result is an $\mathbf{s}'$-tree with cost strictly lower than $C(\mathbf{s})$, implying that $\mathbf{s}$ cannot be stochastically stable.

Case 2.2. The case of even n can be proved analogously with the only difference that: states to the left of $I(n)$ can be connected to $\mathbf{s}^{\frac{n}{2}-1} \in I(n)$ at cost $\frac{n}{2} - 1$ while the opposite transition costs $\frac{n}{2} + 1$; states to the right of $I(n)$ can be connected to any $\mathbf{s} \in I(n)$, in particular also to $\mathbf{s}^{\frac{n}{2}}$, at cost $\frac{n}{2}$ while exiting $I(n)$ to the right costs at least $\frac{n}{2} + 1$.

Step 3: *All $\mathbf{s}$-trees for states in $I(n)$ have the same costs.*

Suppose n is odd. Take $\mathbf{s}, \mathbf{s}' \in I(n) = E^{\lceil \frac{n}{2} \rceil}$ with $\mathbf{s}' \neq \mathbf{s}$. Notice we can always reach $\mathbf{s}$

[19]Note state $\mathbf{s}^{\lceil \frac{n}{2} \rceil} \in E^{\lceil \frac{n}{2} \rceil + 1}$ can be reached at cost exactly $\lceil \frac{n}{2} \rceil$.

from $\mathbf{s}'$ and vice-versa with a direct transition at cost $\lceil \frac{n}{2} \rceil$. Suppose h is an $\mathbf{s}$-tree of minimum cost $C(\mathbf{s})$. We can construct an $\mathbf{s}'$-tree as we did in Step 2, but the total cost of the tree will not change. Thus $C(\mathbf{s}') \leq C(\mathbf{s})$. In fact, there is no way to reduce the total cost and $C(\mathbf{s}') = C(\mathbf{s})$ because there is no way to move up or down within $E^{\lceil \frac{n}{2} \rceil}$ with less than $\lceil \frac{n}{2} \rceil$ experiments and we have shown so far that moving out of $I(n)$ is even more costly.

Suppose n is even. Take $\mathbf{s}, \mathbf{s}' \in I(n) = E^{\frac{n}{2}} \cup \{\mathbf{s}^{\frac{n}{2}}\}$. Assume without loss of generality that $s < s'$. Suppose h is an $\mathbf{s}$-tree of minimum cost $C(\mathbf{s})$. We can now remove from h the arrow starting at $\mathbf{s}'$ which must have cost at least $\frac{n}{2}$ and connect $\mathbf{s}$ with a direct transition to the basin of attraction of $\mathbf{s}'$ with $\frac{n}{2}$ (this is possible because $s < s'$). This results in an $\mathbf{s}'$-tree of cost no higher than $C(\mathbf{s})$. Thus, $C(\mathbf{s}') \leq C(\mathbf{s})$. Suppose now h' is an $\mathbf{s}'$-tree of minimum cost $C(\mathbf{s}')$. Remove from h' the arrows starting at $\mathbf{s}$ and at $\mathbf{s}^{\frac{n}{2}}$, at cost at least $\frac{n}{2}$ each; connect $\mathbf{s}'$ with a direct transition to the basin of attraction of $\mathbf{s}^{\frac{n}{2}}$ and connect $\mathbf{s}^{\frac{n}{2}}$ to the basin of attraction of $\mathbf{s}$, both with $\frac{n}{2}$ experiments. This results in an $\mathbf{s}$-tree of cost no higher than $C(\mathbf{s}')$. Thus, $C(\mathbf{s}) \leq C(\mathbf{s}')$. It follows that $C(\mathbf{s}) = C(\mathbf{s}')$ for any pair $\mathbf{s}, \mathbf{s}' \in I(n)$. ■

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