# The Determinants of Subsistence Income in a Malthusian World

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**Abstract.** This study constructs a simple, two-sector Malthusian model with agriculture and industry, and use it to identify the determinants of subsistence income. We make standard assumptions about preferences and production technology, but by contrast to existing studies we assume that children and other consumption goods are gross substitutes. Consistent with the traditional Malthusian model, we find that productivity growth in agriculture has no effect on subsistence income. More importantly, we also find that subsistence income increases, not just with the death rate as has recently been demonstrated in the literature, but also with productivity in manufacturing.

*Keywords:* Malthusian Model, Subsistence Income.

*JEL:* J13, N10, O11.

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## 1. Introduction

Subsistence economies are often characterized by Malthusian population dynamics. In a Malthusian world, higher income causes more births and fewer deaths, and so raises the level of population. This effect, however, only operates temporarily: due of diminishing returns to labour in production, the growth of population gradually drives down income. This, then, leads to fewer births and more deaths, until the population eventually ceases to grow, and income stagnates at the level of subsistence.

The terminology 'subsistence income', however, can lead to the confused notion that in a Malthusian economy people live on the verge of starvation. Even in the mid-seventeenth century—a time when England's population was constant and income, therefore, at the level of subsistence—the wage of the poorest workers (unskilled agricultural laborers) was well above the biological minimum of about 1,500 calories a day. This led Clark (2007) to conclude that, 'preindustrial societies, while they were subsistence economies, were not typically starvation economies' (*ibid.*, p. 23).[1] It remains to be explained, therefore, how subsistence income, defined as income in a Malthusian equilibrium, i.e. where the population is constant, can vary across time and space.

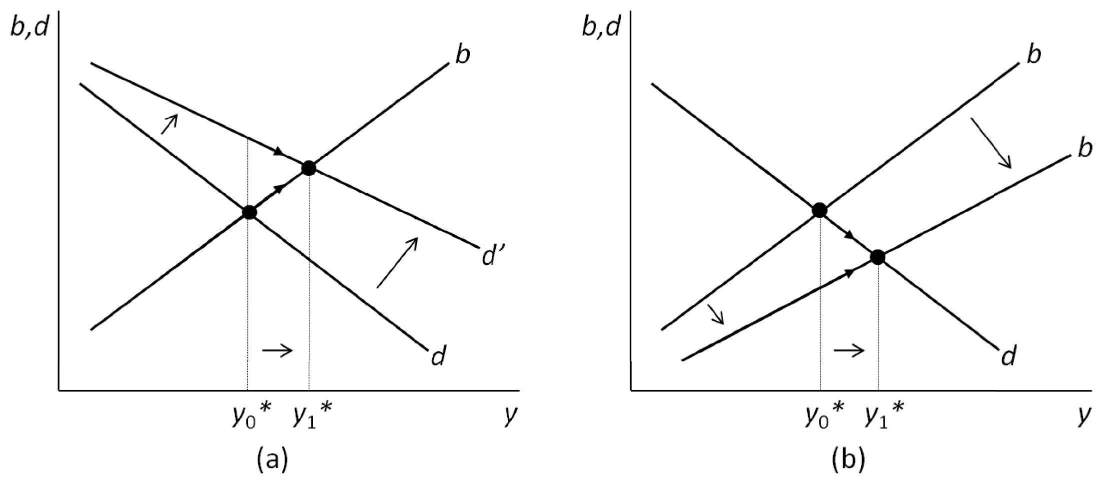
In this note, we construct a simple, two-sector Malthusian model with agriculture and industry, and use it to identify the determinants of subsistence income. We arrive at the conventional conclusion that productivity growth in agriculture has no effect on subsistence income. However, we also show that subsistence income rises, not only with the death rate as recently emphasized by Voigtländer and Voth (2008), but also with productivity growth in industry.

Previous attempts to predict the determinants of subsistence income in a Malthusian economy are easily understood through the use of Figure 1. Following Malthus (1798), a change in income has a dual effect on population growth. On the one hand, lower income reduces the marriage rate, leading therefore to fewer births. This is know as the 'preventive checks' hypothesis, and explains the upward-sloping birth schedule in Figure 1. On the other hand, lower income raises the death rate, as captured by the so-called 'positive checks' hypothesis, which is reflected in the downward-sloping death schedule in Figure 1. As is evident from the illustration, the intersection of the birth and the death schedules determines the income of subsistence, denoted $y^*$. This is

[1] Recent work by Ashraf and Galor (2008) provides empirical support of the idea that pre-industrial economies displayed Malthusian population dynamics.

characterized as the level of income at which the size of population remains constant over time. It follows from the illustration that shifts in the position of the birth and death schedules are responsible for variations in subsistence income.

Figure 1: The Effect of Subsistence Income of Shifts in Deaths (a) and Births (b)




Existing studies focus mainly on the effects of changes in deaths on subsistence income. More specifically, Clark (2007) highlights the benign effect of higher death rates on living standards, and Voigtländer and Voth (2008) use this to draw a link between European wars and deceases and the sharp rise of European urbanization rates, as well as its growing income per capita.

The main mechanics of Voigtländer and Voth's (2008) work is well-captured by Figure 1(a). Suppose the economy starts off when income is at $y_0^*$. At this income level, births equal deaths, so the population level remains constant, and the income is at subsistence by construction. An upward shift in the death schedule (higher deaths at any given income) means that deaths momentarily exceed births. The population thus starts to shrink, and with diminishing returns to labour in production, this gradually raises the level of income. In turn, births now rise and deaths fall (movements along the birth and deaths curves) until the two meet again—this time at a higher level of subsistence income ($y_1^*$) and a lower, fixed level of population.

In parallel with the results in Voigtländer and Voth (2008), the point we make in this paper is that changes in the birth schedule have similar effects on urbanization rates and income per capita as those of changes in deaths. Our main argument is that a shift in the costs of foods, and therefore children, relative to the costs of other goods, affects the position (or, more specifically, the slope) of the birth schedule. Similar to shifts in the position of the death schedule, shifts in the birth schedule impact on the intersection point between the birth schedule and the death schedule, ultimately affecting the level of subsistence income. This is captured by Figure 1(b).

In effect, the current note demonstrates how advances in industrial productivity reduce the price of manufactured goods, thereby increasing the relative price of food, and thus the costs of raising children. If children are ordinary goods and gross substitutes of manufactured goods, then parents respond to a higher cost of children by reducing births. This, for any given level of income, make the birth schedule less steep, leading ultimately to a higher level of subsistence income.

In the following, we first describe the model, and then point to the determinants of subsistence income, and their effects on the rate of urbanization.

## 2. The Model

Let $b_t$ denote the number of births per adult, and $d$ the fraction of those dying before adulthood.[2] The number of surviving children per adult is thus $n_t = b_t(1 - d)$. Parents derive utility from the number of surviving children and from consumption of manufactured goods $m_t$. Each child born costs one unit of food, and the price of food is denoted $p_{A,t}$.

Parents divide their income between children and manufactured goods, so that the budget constraint of a parent reads

$$w_t = p_t b_t + m_t \tag{1}$$

where $w_t$ is parental income and $p_t \equiv p_{A,t}/p_{M,t}$ the relative price of food. The latter is also known as agricultural terms of trade. Note that income and the relative price of food are both measured in units of the manufactured goods.

### 2.1. Preferences.

The results obtained below rely on the crucial assumption that parents consider children and manufactured goods to be gross substitutes, i.e. that the cross-price elasticity between the two goods is positive. Most related studies rely on a Cobb-Douglas type utility function, but such preferences involve a cross-price elasticity between the two goods of zero. A

[2] As in Voigtländer and Voth (2008), the death risk could be inversely related to income per capita. However, for the point we wish to make this is not a necessary assumption, and so for tractability reasons we consider the death risk to be exogenous throughout.

simple and tractable way in which to allow children and manufactured goods to be gross substitutes, however, is by assuming that parents maximize a so-called *quasi-linear* utility function.[3] This could be given by

$$u_t = m_t + \gamma \ln n_t. \tag{2}$$

By maximizing (2) subject to the budget constraint given by (1), the first-order condition tells us that the number of births, and surviving children, per adult in optimum is

$$b_t = \gamma/p_t \qquad \Rightarrow \qquad n_t = (1-d)\gamma/p_t, \quad \gamma > 0, \tag{3}$$

where $\gamma$ denotes the relative weight of children in utility. Note that gross substitutability between children and manufactured goods is represented by the negative effect of the *relative* price of food on the demand for children.

2.2. **Production.** Consistent with the existing literature, suppose that the agricultural sector's output is subject to constant returns to land and labour, and that land is fixed and its amount set to unity. Furthermore, industrial output is subject to constant returns to labour, so that the total output of each sector is given by

$$Y_{A,t} = \Omega_A L_{A,t}^{\alpha}, \quad \alpha \in (0,1) \tag{4}$$

$$Y_{M,t} = \Omega_M L_{M,t}. \tag{5}$$

where $\Omega_i$ is total factor productivity in sector $i \in \{A, M\}$, with subscript $A$ referring to agricultural produce, and subscript $M$ to that of manufacturing. The variable $L_i$ captures the number of workers employed in sector $i \in \{A, M\}$. The fraction of labour allocated to agriculture and industry, respectively, is determined endogenously below.

2.3. **Labor Market Equilibrium.** Suppose that land rents are zero; that there is free labour mobility; and that each sector is characterized by perfect competition. This means that the workers of each sector are paid according to their average product, i.e.

$$w_t = p_t \frac{Y_{A,t}}{L_{A,t}} = \frac{Y_{M,t}}{L_{M,t}}. \tag{6}$$

[3]While a CES (constant elasticity of substitution) utility function would permit any sort of substitutability between children and manufactured goods, assuming such preferences would seriously complicate matters, and prevent us from reaching the tractable closed-form solutions we obtain below.

Full employment implies that

$$L_t = L_{A,t} + L_{M,t}. \tag{7}$$

2.4. **Food Market Equilibrium.** Suppose that, over the course of a lifetime, each individual consumes a fixed quantity of foods (or calories) measured by $\eta \equiv 1$.[4] For tractability reasons, food is demanded only during childhood and some of it stored for adulthood.[5] The fact that each individual demands a fixed amount of calories implies that, as income rises, people allocate a growing share of their income to manufactured goods (and vice versa). This is a main implication of Engel's Law.

By equating total food demand to total food supply, the latter being given by (4), the food market equilibrium condition implies that

$$b_t L_t = \Omega_A L_{A,t}^{\alpha}. \tag{8}$$

2.5. **Population Dynamics.** Finally, it follows from the demographic components described above that change in the evolution of the labour force is given by

$$L_{t+1} = n_t L_t = b_t \left(1 - d_t\right) L_t. \tag{9}$$

Equation (9) completes the model.

## 3. Analysis

In the following, we derive the closed-form solutions for a number of variables relevant for analyzing a Malthusian equilibrium, and ultimately subsistence income. The variables include fertility, agricultural terms of trade, the share of labour employed in agriculture, and subsistence income per capita. First, we compute the variables in a static equilibrium, and then turn to the Malthusian equilibrium, in which the population level remains constant over time.

We begin by rewriting (8) to obtain the share of labour employed in agriculture, which we denote $l_A \equiv L_A/L$. This is given by

$$l_A = \left(\frac{\gamma L_t^{1-\alpha}}{\Omega_A p_t}\right)^{1/\alpha}. \tag{10}$$

[4] It will not affect the qualitative nature of the results, if we allow children to consume more food goods as their parents receive more income. For such a construction, see Strulik and Weisdorf (2008).

[5] It will not affect the qualitative nature of the results, if, instead, individual food demand is divided over two periods. Such a construction, however, severely complicates matters.

Equation (10) shows that the fraction of workers in agriculture increases with the size of the labour force, but decreases with agricultural productivity, as well as agricultural terms of trade.

Next, inserting (3), (4), (5) and (10) into (6), the market-clearing agricultural terms of trade are given by

$$p_t = \frac{\Omega_M^{\alpha} (\gamma L_t)^{1-\alpha}}{\Omega_A}, \tag{11}$$

It follows that agricultural terms of trade increase with industrial productivity and labour, but decrease with agricultural productivity.

Income per capita, measured in units of food goods, is obtained by dividing $w_t$ by (11), and is given by

$$y \equiv \frac{w}{p} = \Omega_A \left( \frac{\Omega_M}{\gamma L_t} \right)^{1-\alpha}. \tag{12}$$

We have that income increases with productivity in both sectors, but decrease with labour.

Further, inserting (11) into (3), we obtain the birth rate which is given by

$$b_t = \left( \frac{\gamma}{\Omega_M} \right)^{\alpha} \frac{\Omega_A}{L_t^{1-\alpha}} = \frac{\gamma}{\Omega_M} \frac{w_t}{p_t} = \frac{\gamma}{\Omega_M} y_t. \tag{13}$$

It follows that births—opposite to agricultural terms of trade—decreases with industrial productivity and labour, but increases with agricultural productivity.

For the purpose of understanding how the model relates to Figure 1, note that births (by the use of (6) and (8)) can be expressed as a function of income per capita, measured in units of food goods. Hence, it follows from (13) that the slope of the birth schedule is inversely related to the level of productivity in industry, which is the main point of the current study.

While the variables described in equations (10)-(12) are all static equilibrium variables, our main interest is to identify the determinants of income per capita in a Malthusian equilibrium, i.e. in a situation where the population level remains constant over time, and where income, therefore, is at the level of subsistence. We know that a constant population implies that $L_{t+1} = L_t$, and hence from (9) that $b_t = 1/(1-d)$ in a Malthusian equilibrium. Inserting (13) into (9), we can then describe the law of motion of population as

$$L_{t+1} = (1-d)\gamma^{\alpha}\Omega_A L_t^{\alpha} \equiv f(L_t), \tag{14}$$

and solving for the population level in a Malthusian equilibrium (denoted by an asterisk), we find that

$$L^{*}=\left((1-d)\,\Omega_{A}\left(\frac{\gamma}{\Omega_{M}}\right)^{\alpha}\right)^{\frac{1}{1-\alpha}}. \tag{15}$$

This leads us to conclude the following.

PROPOSITION 1. *The two-sector Malthusian model has a unique, globally stable equilibrium (a steady-state) in which the population size is given by (15).*

*Proof.* Stability of the steady state follows from the fact that $f(L_t)$, defined in (14), is a concave function that intersects the $L_{t+1}=L_t$ identity-line in the positive quadrant of a phase diagram exactly at $L^*$, with $f(L_t)>L_t$ for $L_t<L^*$, and vice versa for $L_t>L^*$. □

Equation (15) permits us to compute the level of subsistence income per capita, using (12) and (15), as to get

$$y^{*}=\frac{\Omega_{M}}{\gamma(1-d)}. \tag{16}$$

Based on (15)and (16), then, the following can be observed.

PROPOSITION 2. *The two-sector Malthusian model predicts that: (i) higher agricultural productivity leads to a higher steady state population level, but has no effect on subsistence income; (ii) higher death rates lead to a lower steady state population level and a higher subsistence income; and (iii) higher manufacturing productivity leads to a lower steady state population level and a higher subsistence income.*

*Proof.* The Proof follows directly from observing equations (15) and (16). □

Starting with part (i) of Proposition 2, this is the conventional result of the standardized Malthusian model. Namely that productivity growth in agriculture is eventually 'eaten up' by a larger population, so that, in the long run, this sustains a larger population, but has no effect on subsistence income. Accordingly, variations in agricultural productivity cannot account for variations in subsistence income across time and space.

Turning to part (ii) of Proposition 2, this captures the benign effect of higher death rates on living standards, as highlighted by Clark (2007), and discussed at length by Voigtländer and Voth (2008). Evidently, variation in death impacts on the level of subsistence income, and so may explain why some societies characterized by Malthusian population dynamics are richer than others.

The main contribution of this study, part (iii) of Proposition 2 points to yet another reason why subsistence incomes may vary across time and space. It follows from the Proposition that advances in industrial productivity have a permanent and benevolent impact on standards of living. The intuition is this: productivity growth in industry increases agricultural terms of trade, which, via the labour market equilibrium condition, reduces the price of manufactured goods relative to those of agriculture. In turn, this raises the costs of raising children. Now, if children are ordinary goods, as well as a gross substitutes of manufactured goods (both are captured by the quasi-linear utility function in (2)), then parents respond to higher industrial productivity by lowering births. Speaking in terms of Figure 1(b), this corresponds to a reduction in the slope of the birth schedule, causing its intersection with the death schedule to take place at a higher level of subsistence income.

Similar to Voigtländer and Voth (2008), we can compute the share of labour devoted to industry as an indirect measurement of the rate of urbanization. Combining (10), (11) and (15), it follows that the fraction of the labour force allocated to industry is

$$l_M = 1 - l_A = 1 - \left( \frac{\gamma^\alpha}{\Omega_M} \right)^{1/\alpha}.$$

While Voigtländer and Voth (2008) find that higher death rates are a stimulus to urbanization, in our study urbanization is driven by advances in the productivity in industry.

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