Abstract. Can a large-scale deficit spending program speed up recovery after recession? To answer that question we calibrate a standard neoclassical growth model with US data and assume that an exogenous shock has driven aggregate output far below steady-state level. We calibrate the model such that a permanent increase of government expenditure is effective in raising output. We then show that a “fiscal stimulus”, i.e. a temporary increase of government expenditure is not only ineffective but detrimental. Even before the spending program expires, aggregate output is lower than it could be without fiscal stimulus. We show the generality of this result w.r.t. size and persistence of the shock, size of the government spending multiplier, and the scale and duration of the stimulus program. Using a phase diagram we provide the economic intuition for our unpleasant finding and explain why, generally, private capital stock reaches its lowest level when a deficit spending program expires. We also show how an accompanying temporary cut of capital income taxes helps to prevent the negative repercussion of deficit spending on economic recovery.

Keywords: deficit spending, government spending multiplier, economic recovery, economic growth.

JEL: E60, H30, H50, O40

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1. Introduction

The current economic recession has resuscitated a debate which not long ago seemed to have been settled among mainstream economists, a debate about the need for and the capability of deficit spending. The conclusion from the year 1997 that “there is now widespread agreement that countercyclical discretionary fiscal policy is neither desirable nor political feasible” (Eichenbaum, 1997, p. 236) is less clear in the year 2009 and many economists have uttered second thoughts about the usefulness of “fiscal stimulus”. The U.S. government has launched a huge spending program, which the economic advisors of the President predict to be highly effective in speeding up economic recovery (Romer and Bernstein, 2009). Many other governments around the world have displayed similar fiscal activities and passed large-scale deficit spending programs. These observations have motivated the present research where we reinvestigate the consequences of deficit spending in dynamic general equilibrium using a calibrated version of the neoclassical growth model.

In our endeavour we are “standing on the shoulders of giants”. An incomplete list of studies of fiscal policy in the neoclassical growth model includes Judd (1985), Greenwood and Huffman (1991), Christiano and Eichenbaum (1992), Aiyagari et al. (1992), Cooley and Hansen (1992), Baxter and King (1993), McGrattan (1994), Ludvigson (1996), Burnside et al. (2004), and Traband and Uhlig (2007). While most of this literature investigates fiscal policy changes that are either permanent or follow a stochastic law we focus on a temporary deficit spending program which is pre-announced and known by the economic agents who, in particular, correctly anticipate its expiry.

From the relative small literature on temporary fiscal policy change in dynamic general equilibrium, namely Judd (1985), Ludvigson (1996) and Baxter and King (1993), we deviate by focussing on off-steady-state behavior and by trying to calibrate the model according to the current state of the US economy and the current deficit spending program. While the literature so far investigated impulse responses to fiscal policy change for economies assumed to rest at steady-state initially, we investigate whether deficit spending can speed up economic recovery of an economy that is initially situated far below its steady-state.

We calibrate the economy such that the long-run multiplier of a permanent increase of government expenditure is positive and lies roughly in the middle of the values that have been
suggested in empirical and applied literature. We then show that a temporary increase of government purchases is ineffective in speeding up economic recovery. After a straw fire of initially higher output the multiplier becomes negative and GDP falls below the laissez faire path before deficit spending expires.

We show robustness of this result with respect to the severity and length of the recession, the size and duration of deficit spending, and the size of the long-run spending multiplier. Moreover, we are able to derive some general, qualitative conclusions. Because we set up our model in continuous time we can not only derive impulse responses (as in the related real business cycle literature) but draw qualitative conclusions with help of a phase diagram. From phase diagram analysis we infer that generally the private capital stock at the time when a deficit spending expires is lower than under laissez faire, i.e. without deficit spending. The phase diagram also turns out to be useful in explaining the economic rationale behind our findings.

In order to let the economy without deficit spending recover as predicted by Romer and Bernstein (2009) we calibrate a particular productivity shock that returns to steady-state with a particular adjustment speed. Here, naturally the question occurs whether a negative productivity shock can sufficiently well approximate the situation of the U.S. economy, now and in the near future. Although we agree that the causes of the current slump of output and its expected slow recovery are mainly financial frictions, we think we can nevertheless represent the aggregate consequences of these frictions by a productivity shock. This procedure may appear odd to a reader trained in the Keynesian tradition. But it is in fact supported by recent business cycle research. Chari et al. (2007) set up a detailed economy with input-financing frictions that lead to inefficient utilization of capital and labor and show that an economy in which the technology is constant but input financing frictions vary over time is observationally equivalent to a neoclassical growth model under productivity shocks, i.e. it displays the same aggregate properties. Since we are not interested here in the causes and cure of financial frictions as such but in macroeconomic consequences of off-steady-state deficit spending, we are confident that taking our shortcut modelling by a productivity shock is sufficiently justified by Chari et al.’s equivalence result.

Many journalists and scholars have compared the current economic recession with the Great Depression. This is probably so because the fall of output has been unusually far below steady-state, a fact which is difficult to reconcile with the size of the variance of productivity shocks
calibrated in the real business cycle literature (e.g. King and Rebelo, 1999). This observation relates the present investigation to a series of recent articles applying the neoclassical growth model to the study of the Great Depression.\(^1\) Imposing the historically observed times series for TFP this literature shows that the neoclassical growth model manages to explain 85 percent of the movement of GDP in the years 1929 to 1933. In order to assess these results properly it is important to conceptualize TFP broadly: “It is the efficiency with which inputs are used in producing output and can be thought of as the price of the composite input in terms of the composite output good.” (Kehoe and Prescott, 2002, p. 5).

Productivity shocks are less successful in motivating the long duration of the Great Depression over the period 1933 to 1939. The second phase of the Great Depression is more successfully explained by labor market frictions, or “labor wedges”, representing “New Deal” competition and labor market policies (see e.g. Cole and Ohanian, 2002, Chari et al., 2002). For the 2008-9 recession, however, we assume that it does not trigger an extra labor wedge. This assumption is in line with the prediction that economic recovery from the current recession, notwithstanding that it is expected to be slower than from an ordinary business cycle downswing, will be much faster than it was for the Great Depression.

The remainder of the paper is organized as follows. In the next section we set up the neoclassical growth model with endogenous labor supply and in Section 3 we calibrate it with U.S. data such that the economy follows CBO’s prediction for the near future under laissez faire, i.e. without deficit spending program. We then specify a deficit spending program that approximates government purchases planned under the American Recovery and Reinvestment Act (ARRA). In Section 4 we develop our main result that a deficit spending program will boost economic recovery only in its initial phase and slow down recovery before spending expires. We show effects of the deficit spending package on other macroeconomic aggregates and provide intuition and a qualitative discussion of the result with help of a phase diagram. In Section 5 we present robustness checks with respect to size and persistence of the recession, scale and duration of the stimulus program, and size of the long-run government spending multiplier. In Section 6 we suggest a “remedy”. We show that a sufficiently strong cut of capital income taxes, accompanying the deficit spending program, amplifies recovery in the initial period and prevents the eventual fall below the laissez faire path. Section 6 concludes.

2. The Model

Consider the neoclassical growth model with endogenous labor supply stated in continuous time. The representative household maximizes intertemporal utility,

\[
\max_{c,\ell} \int_0^{\infty} \left( \log(c) + \beta \frac{(1 - \ell)^{1 - \gamma}}{1 - \gamma} + \xi \frac{G^{1 - \eta}}{1 - \eta} \right) e^{-\rho t} dt,
\]  

(1)

where \( c \) denotes per-capita consumption of private goods, \( G \) consumption of public goods (government purchases), and \( \ell \) labor supply, \( \rho \) is the time preference rate, \( 1/\gamma \) the elasticity of intertemporal substitution for leisure, and \( 1/\eta \) the elasticity of intertemporal substitution for consumption of public goods. We follow Ludvigson (1996) and assume power utility for leisure. This nests two special cases. For \( \gamma = 0 \) utility is linear in leisure as suggested by the indivisible labor model of Hansen (1985) and Rogerson (1988). For \( \gamma = 1 \) utility is logarithmic in leisure as frequently assumed in the RBC literature. Here we only require that \( \gamma \geq 0 \). This renders a degree of freedom, which we exploit in the calibration of the model.

The household’s budget constraint is given by

\[
\dot{k} = (1 - \tau_w)w\ell + (1 - \tau_k)rk - (1 + \tau_c)c + T
\]  

(2)

where \( k \) denotes the household’s asset holdings, \( w \) the wage rate, and \( r \) the interest rate. Transfers received from the government are denoted by \( T \), \( \tau_w \) is the labor income tax, \( \tau_k \) the capital income tax, and \( \tau_c \) the consumption tax.

From the first order conditions we obtain consumption growth according to the Ramsey rule (3) and the optimal trade-off between consumption and leisure (4).

\[
\frac{\dot{c}}{c} = (1 - \tau_a)r - \rho \quad \text{and} \quad c = \frac{(1 - \tau_w)(1 - \ell)^\gamma w}{\beta(1 + \tau_c)}.
\]  

(3)  

(4)

Facing aggregate productivity \( A \), firms produce output according to the Cobb-Douglas technology \( y = A k^\alpha \ell^{1 - \alpha} \) under perfect competition. Hence, factor prices equal their marginal product \( r = \alpha A k^{\alpha - 1} \ell^{1 - \alpha} - \delta \), \( w = (1 - \alpha)Ak^\alpha \ell^{-\alpha} \).

The government collects taxes on labor income, capital income, and consumption. Tax revenues are spent on income transfers to households, \( T \), and on purchases of public goods, \( G \).
We abstain from explicitly modelling government expenditure financed by debt. Instead, we assume that the Ricardian equivalence proposition applies such that the path of government debt necessary to balance the current budget is represented by a time series of lump sum income transfers. The government budget constraint is thus given by (5).

\[ \tau_ww\ell + \tau_krk + \tau_c = T + G. \] (5)

Given an equilibrium on capital and goods markets, aggregate capital evolves according to

\[ \dot{k} = y - c - \delta k - G. \] (6)

Total factor productivity \( A \) follows a first-order autoregressive process. Initially, at time 0 an exogenous shock has driven \( A \) down to \( A(0) \). From then on, factor productivity converges towards its steady state value \( A^* \) at constant rate \( \sigma \).

\[ A(t) = A^* + A_0 e^{-\sigma t}. \] (7)

As motivated in the Introduction we take the productivity shock as a simplifying “shortcut” to the modelling of frictions in financing the input bill of firms.

Besides the general formulation of labor supply and the setup in continuous time we thus re-investigate the standard neoclassical growth model augmented with variable labor supply, in other words a deterministic version of the standard RBC model.\(^2\) For simplicity we furthermore ignore exogenous time trends of productivity and population growth. The model’s variables can be interpreted as detrended and the deviation from equilibrium as deviation from steady-state growth path.

The continuous time setup may appear unfamiliar to the RBC expert but is, of course, also not affecting the results. We use continuous time for two reasons. It alleviates a straightforward application of an exact numerical solution method and it allows an intuitive explanation of adjustment dynamics using a phase diagram.

While the model is standard, innovation lies in our assumption about the specific point of time at which a change of government behavior occurs. So far, at least to our knowledge, the impulse-response analysis of fiscal policy in the RBC literature focussed exclusively on cases where the economy was assumed to rest at a steady-state initially, i.e. at the time when the

\(^2\)See King et al. (1988), King and Rebelo (1999), or Prescott (2006) for a review.
change of policy occurs. Here we investigate instead a change of government behavior when the
economy is far off its steady-state position. This allows us to address a new question within the
neoclassical setup, a question that has so far been extensively discussed in the short-run oriented
“Keynesian” literature: how helpful is deficit spending in shortening the time of recovery after
severe recession?

3. Calibration

We calibrate the model with US data in line with the existing RBC literature on fiscal pol-
icy and government expenditure. Parameter values for tax rates, government share, capital
share, depreciation rate, and time preference are taken from Traband and Uhlig’s (2007) re-
cent calibration of the RBC model with US data. In line with standard RBC modelling (King
and Rebelo, 1999) we set \( \rho \) such that the real interest equals 6 percent per year and we set
\( \beta \) such that households in equilibrium supply a quarter of their time on the labor market. At
the pre-shock steady-state the government purchases 20 percent of output, i.e. \( g \equiv G/y = 0.2. \)
Altogether these values imply an investment rate of 23 percent. Table 1 summarizes the setup
of parameters.

Initial factor productivity \( A_0 \) is calibrated such that the resulting output gap matches the
fall of GDP by about 10 percent below steady-state as predicted for the U.S. in year 2009 by
the Congressional Budget Office (CBO, 2009d). The rate of recovery, \( \sigma \), is calibrated such that
the duration of economic recovery matches the pattern of GDP expected by the CBO in their
baseline case (excluding the impact of the American Recovery and Reinvestment Act, ARRA).
In particular, we match the CBO’s prediction that half of the original GDP gap is closed after
two years (i.e. in year 2011) by setting \( \sigma = 0.4 \). In Section 5 we provide sensitivity analysis with
respect to the size of initial shock and the laissez-faire speed of recovery.

Table 1. Parameter Values

<table>
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<tr>
<th>( \alpha )</th>
<th>( \gamma )</th>
<th>( \delta )</th>
<th>( \rho )</th>
<th>( \ell^* )</th>
<th>( g )</th>
<th>( \tau_a )</th>
<th>( \tau_w )</th>
<th>( \tau_c )</th>
<th>( A^* )</th>
<th>( A_0 )</th>
<th>( \sigma )</th>
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<td>0.36</td>
<td>1.5</td>
<td>0.06</td>
<td>0.037</td>
<td>0.25</td>
<td>0.2</td>
<td>0.37</td>
<td>0.26</td>
<td>0.05</td>
<td>1</td>
<td>-0.066</td>
<td>0.4</td>
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Of course, quantitative results will depend crucially on the assumption about the effectivity
of government expenditure, i.e. the assumed magnitude of the long-run spending multiplier.
Unfortunately the literature has not yet converged towards a generally accepted value for the
multiplier. We try to meet the involved uncertainty by defining a reasonable benchmark case
and by providing sensitivity analysis. For the benchmark case we assume a multiplier of 0.89 (which results for $\gamma = 1.5$), a value that lies in the middle of Barro’s (1981) estimate of 0.62 and Baxter and King’s (1993) estimate of 1.16. A value of 0.89 lies also roughly in between Blanchard and Perotti’s (2002) estimates for the long-run multiplier of deficit spending (of 0.97 and 0.66 under the assumption of “deterministic trend” and “stochastic trend”, respectively). However we have found in the literature also suggestions of much lower long-run multipliers of zero and below (e.g. Mountford and Uhlig, 2008).

President Obama’s administration, on the other hand, imposes a much higher long-run multiplier of 1.55 on their projections of future GDP with and without fiscal stimulus (Romer and Bernstein, 2009, CBO, 2009). In Section 5 we thus provide a robustness check with respect to the size of the multiplier. In particular, we show that a larger multiplier causes a larger boost of GDP during an initial phase in which the economy seems to recover faster but does not overturn the result that deficit spending is harmful for economic recovery in the medium and long-run.

The model produces a long-run positive impact of government expansion by a wealth effect. Households adjust to the lower disposable income generated by less transfers (more purchases of government bonds, higher expected future taxes) by supplying more labor. This in turn makes capital more productive and increases investment and, in the long-run, aggregate capital stock. While the wealth effect is mostly known from the RBC literature it may be useful to note that it is not a particularity of the neoclassical approach. Models in the New-Keynesian tradition usually rely on the wealth effect as well when they are capable to generate a positive long-run multiplier.

We use the degree of freedom stemming from our power-utility modelling of labor supply to calibrate the strength of the wealth effect such that the model in its benchmark version produces a spending multiplier of 0.89. This leads to the estimate of $\gamma = 1.5$. In Section 5 we provide a robustness check with respect to the government spending multiplier, which serves – by construction – also as a sensitivity analysis for the underlying elasticity of labor supply.

Finally, we have to specify deficit spending. We define as “laissez-faire” the policy that keeps total purchases $G$ constant. This assumption is in contrast to the standard RBC modelling where laissez faire means that the government maintains a constant GDP share of purchases $g = G/y$. Holding $G$ constant instead, adds more realism to our model since it implies – in line with CBO’s prediction – that the GDP share of government purchases rises after a negative
shock of GDP already under laissez faire, i.e. without the additional expenditure triggered by the shock.\footnote{We have checked that, qualitatively, all our main findings remain unchanged if “laissez faire” is defined by a constant GDP share of government purchases.}

To calibrate additional expenditure we take CBO’s “Cost Estimate for Conference Agreement for H.R.1” (CBO, 2009c). We approximate the fact that over 90 percent of extra government purchases according to ARRA are planned for the years 2009-2013 by modelling a five year deficit spending program. Following Romer and Bernstein (2009) we assume that 60 percent of the increase of transfers to states and localities are used for purchases of goods and services. Taken together with planned federal purchases we then infer for the 5 year period an average increase of total annual purchases by 60.7 billion, implying an average increase of the GDP share of government purchases by 0.52 percentage points, i.e. $\Delta g = \Delta G/Y = 0.0052$. In Section 5 we provide a sensitivity analysis with respect to size and duration of the spending plan.

For the computation of adjustment dynamics we apply the relaxation algorithm, which is in detail described in Trimborn et al. (2008). The method does not require linearization around the steady-state or any other transformation of the original dynamic system. It provides the exact solution (up to a user-specified error) irrespective of the deviation from steady-state and is thus a reliable tool for the investigation of big shocks which drive an economy far away from its steady-state. In Strulik and Trimborn (2009) we show more generally how the method can be used to get exact impulse responses for pre-announced and temporary policy changes. One particularly useful feature of the method is that impulse responses of temporary policy changes can be retraced by phase diagram analysis, which allows for a straightforward and intuitive economic explanation of the numerical results.

4. Results

In order to assess the effects of a temporary fiscal expansion on economic recovery properly we compare it with two alternative scenarios: recovery without government intervention ($\Delta G = 0$), and recovery when the increase of government spending is permanent. Because the long-run spending multiplier is assumed to be positive, we impose that a permanent fiscal expansion speeds up recovery. This has, of course, no normative implications because the effect on welfare could be positive or negative depending on how valuable the additionally provided public goods...
are (i.e. how large $\xi$ and $\eta$ are in eq. (1)) compared to leisure and consumption of private goods, which are both caused to decrease by fiscal expansion.

However, if an increase of government purchases would be welfare enhancing in the long-run, it should indeed be made permanent. The fact that (most of) ARRA’s additional expenditures are planned to be temporary indicates that the extra spending is thought to be purely instrumental with respect to the speed of economic recovery. The question then arises whether “fiscal stimulus” works, i.e. whether the presence of a positive long-run spending multiplier is sufficient to expect a positive short-run multiplier.

Figure 1 shows adjustment dynamics for several variables of interest during economic recovery. Dotted lines represent the baseline scenario of no policy intervention, subsequently called “laissez-faire”, solid lines show dynamics when the increase of government purchases is permanent, and dashed lines represent recovery under temporary spending program. As motivated above, we consider in our baseline case a collapse of GDP of 10 percentage points at time 0, after which the gap to steady-state closes with a half life of 2 years under laissez-faire. The $A$ panel at the lower right corner in Figure 1 shows the time path of productivity that produces this behavior. It is assumed to hold for all three cases.

To begin with, consider the laissez-faire case (dotted lines). Due to the negative shock consumption and investment fall below their steady-state positions. Consumption smoothing behavior causes – in line with findings from business cycle research – a much stronger effect of the shock on investment than on consumption. The severe drop of investment, which is predicted to fall by 7.5 percentage points, i.e. by 42 percent, results in a prolonged decline of capital stock, which reaches a trough between year 4 and year 5 after the shock. Lower factor productivity and less investment causes employment to fall by 5.4 percentage points initially, not too far away from CBO’s estimate. The fact that the government maintain its pre-shock purchases of goods implies, together with the output fall, an increase of the government share of GDP from 0.2 to 0.23. Since tax revenue declines the shock entails an increasing government deficit already under laissez-faire.

Next consider adjustment dynamics when public spending increases (possibly triggered by the expected prolonged stay of investment and employment below steady-state level under laissez-faire). In our benchmark scenario we assume that the government reacts to the shock with a
Dotted lines: laissez faire policy, solid lines: permanent increases of government share, dashed lines: temporary increase of government share for 5 years. Capital ($k$), consumption ($c$), employment ($\ell$), government expenditure ($G$), and productivity ($A$) are measured relative to their pre-shock (steady-state) position. The investment rate is measured in percent.

Because the long-run multiplier has been assumed to be positive, the permanent policy effectively raises employment, triggers investment, and speeds up the recovery process. The driving mechanism is the wealth effect: expecting lower permanent income because of lower transfers (higher government debt) households supply more labor, more employment raises capital productivity, which causes higher investment. With permanently higher $k$ and $\ell$ the economy does not only converge towards a permanently higher level of GDP. Also, the time of recovery from the shock is substantially reduced.
One might hope that a temporary increase of government expenditure could deliver the short-run benefits (faster recovery) without implicating the long-run costs (lower consumption and leisure). Unfortunately, the model outcome (reflected by dashed adjustment paths) does not support this conjecture. As expected, fiscal stimulus effectively raises employment, which falls only by 5 percent instead of 5.4 percent under laissez-faire. The temporary policy, however, also drives private investment below the laissez-faire path. As a consequence capital stock falls much harder than under laissez-faire.

Adjustment dynamics for GDP result from the interaction of investment and employment in the following way. During an initial period, immediately after deficit spending becomes operative, the positive effect on GDP through employment dominates. The negative effect through falling investment is not yet effective because capital stock is a state variable, i.e. it cannot jump and adjusts slowly over time. After some time, however, when employment has already converged far towards its steady-state, the effect of dampened and delayed investment becomes dominating and not only capital stock but also GDP falls below the laissez-faire path.

**Figure 2: Deviation of GDP from laissez-faire (left) and multiplier of government expenditure (right)**

![GDP and multiplier](image_url)

Dotted lines: laissez faire policy, solid lines: permanent increases of government share, dashed lines: temporary increase of government share for 5 years.

Figure 2 demonstrates – in two diagrammatic variants – that this is indeed the case. The negative effect from delayed investment eventually deteriorates GDP and turns fiscal stimulus from government purchases into a flash in the pan. The panel on the left hand side shows percentage deviation from laissez-faire adjustment of GDP given permanently or temporarily higher government expenditure. While the permanent expenditure program increases GDP
permanently, the fiscal stimulus program speeds up recovery only initially. GDP is predicted to be about 0.2 percent above laissez-faire initially with subsequently decreasing lead. After about 3 years, at a time when the economy has closed about 60 percent of the initial gap, and when deficit spending is still running for 2 more years, the negative investment effect becomes dominating and GDP falls below its laissez-faire level. The greatest distance from laissez-faire is reached at the time when deficit spending is terminated, after which the “stimulated” economy converges towards the laissez-faire path from below, catching up the loss as time goes to infinity.

The panel on the right hand side of Figure 2 shows the implied multiplier of government spending, $\Delta Y/\Delta G$. The multiplier is smaller for the temporary policy than for the permanent one, continuously declining, and turning negative before fiscal stimulus expires.

**Figure 3: Phase Diagram: Permanent and Temporary Rise of Government Purchases**

The black vector field is associated with the initial equilibrium A and the grey vector field is associated with the equilibrium assumed under higher government expenditure C. Note that the grey vector field applies always if the fiscal expansion $\Delta g$ is permanent. For temporary policy it applies when fiscal stimulus is active. After fiscal stimulus expired the black vector field applies again.

An economic intuition for the generality of the result can be obtained with help of a phase diagram. For that purpose we keep productivity $A$ constant such that the economy is represented by a two-dimensional system of differential equations (3) and (6) with labor supply according to
(4) and production $y = A k^\alpha \ell^{1-\alpha}$. Adjustment dynamics can be represented in two-dimensional $c - k$ space. The steady-state is where the $\dot{c} = 0$ isocline and the $\dot{k} = 0$ isocline intersect. The $\dot{c} = 0$ isocline is the vertical line at $k^*$ where the net interest rate $(1 - \tau_k) r(k^*, \ell(k^*))$ equals the time preference rate $\rho$. The $\dot{k} = 0$ isocline is obtained by setting (6) to zero and solving for $c$. The curve is increasing when the concave part stemming from the neoclassical production function is dominating and falling when the linear part stemming from depreciation is dominating. There exists a unique intersection of the isoclines and the resulting arrows of motion identify the equilibrium as a saddlepoint. The unique adjustment path after a change of government behavior is given by the movement along the stable manifold towards the steady-state.\(^4\)

It is helpful to first inspect adjustment dynamics after a permanent increase of government expenditure. Higher $G$ reduces households’ claims on output, $y - G$, and the $\dot{k} = 0$ isocline moves down. Because both consumption and leisure are superior goods, households react on their permanently lower income (the negative wealth effect) by consuming less and supplying more labor. Higher labor supply increases the marginal product of capital and causes the $\dot{c} = 0$ isocline to shift to the right. Adjustment dynamics after fiscal expansion are given by an instantaneous drop of private consumption, after which both capital and consumption increase along the stable manifold towards the new equilibrium. The economy moves from A to C via B. Since we have imposed a long-run government spending multiplier larger than zero but smaller than one, the economy moves toward a state where capital (and employment and GDP) are higher than before and consumption of private goods is lower.

When government spending is only temporarily higher, it remains of course true that increasing expenditure lowers present value of households’ income. The drop of permanent income is now just smaller than under the permanent policy. As a consequence the temporary policy causes leisure and consumption to fall less than under permanent policy. The economy jumps from A to D initially. The magnitude of the jump is not arbitrary but uniquely determined by the fact that the economy must have reached the stable manifold leading to the “old” equilibrium A when fiscal stimulus expires (in our benchmark case after 5 years). The underlying

\(^4\)A detailed derivation of the phase diagram for the neoclassical growth model is contained in many textbooks on economic growth, for example, Barro and Sala-i-Martin (2004). Here, we adapt the diagrammatic analysis for temporary policy change. See Summers (1981) and Abel (1982) for a related analysis of temporary policy in partial equilibrium.
saddlepoint dynamics during the interim period, i.e. when the temporary policy has shifted the isoclines down and to the right and when the grey vector field applies, forces a movement from D to E, from which onwards the economy travels along the stable manifold towards A.

To understand adjustment dynamics note that there can only be one jump of household wealth and thus just one jump of consumption and employment, namely at the moment when the economy is unexpectedly hit by the output fall and the triggered deficit spending plan. Afterwards there are – by construction – no further shocks and no-arbitrage rules out any further jump. On a more formal level, the first order condition requires that the shadow price of capital, $\lambda = (1 + \tau_c)/c$, is continuous and thus consumption is continuous. Because there cannot be any further jumps, and because any trajectory not starting on the stable manifold will never arrive at the steady-state, the economy has to be on the manifold toward A when deficit spending expires and the “pre-shock” saddlepoint dynamics apply again. Altogether, this identifies uniquely the ADEA adjustment path.

Note that the result obtains generally, regardless of magnitude and duration of the deficit spending program. We can thus generally conclude that for any temporary deficit spending program, capital stock reaches its low at the time when fiscal stimulus expires and has to be below steady-state (the laissez-faire solution) at all times, i.e. before and after expiry of fiscal stimulus.

Because capital stock is permanently lower (reaching the laissez faire path only as time goes to infinity), increasing labor supply triggered by deficit spending cannot unfold its expansive power on output. After an initial boost during which both employment and wages are above the laissez-faire path, employment converges towards its pre-shock steady-state whereas wages are below laissez-faire because labor productivity is low due to the “missing” capital stock.

The result that capital stock is decreasing when deficit spending is active and increasing after it expired is explained by the individuals’ desire to smooth their loss of consumption over time. Trying to keep consumption smooth, individuals reduce investment when the government claims additional shares of GDP and expand investment afterwards. The change of household behavior at the time of expiry is also visible in the panel for the investment rate $I/Y$ in Figure 1. Investment falls short of the laissez-faire path during deficit spending and exceeds it afterwards. But it fails to make up for the initial loss of capital in finite time. In summary, the ineffectivity of the deficit spending program is produced by the correct expectation that a temporary expansion
of government purchases cannot lead to permanently higher employment but that it does lead to permanently lower disposable income.

**Figure 4: Government Deficit (left) and Interest Rate (right)**

![Graph showing government deficit and interest rate](image)

Dotted lines: laissez-faire policy, solid lines: permanent increase of government share, dashed lines: temporary increase of government share for 5 years. Deficit is the difference between government outlays and tax revenue normalizing the pre-shock deficit to zero.

Figure 4 shows on the left hand side adjustment dynamics for the government deficit aggregated from time 0 to time \( t \). In order to extract the effects caused by the shock we have normalized the pre-shock deficit to zero. We see that the recession causes already an increase of the deficit under laissez-faire because of decreasing tax revenue. In the first year after the shock, for example, the deficit increases by 2.7 percent of GDP. Impulse responses of the temporary policy (dashed lines) closely follow the path for the permanent policy until its termination. When deficit spending expires in year 5, the government has accumulated a total (extra) deficit of 10 percent of GDP.

The panel on the right hand side of Figure 4 shows the development of the real interest rate. The interest rate lies below steady-state during the first phase after the shock because then the effect through lower total factor productivity is dominating. During the second phase the interest rate adjust towards steady-state from above because capital stock adjusts more slowly than employment and a falling capital labor share becomes the dominating effect. It is interesting to see that government purchases cause almost no crowding out of investment through the Keynesian mechanism: interest rates are – in line with results from business cycles research – almost invariant to government policy. Crowding out of investment takes place almost exclusively through consumption smoothing and the wealth effect.
5. Robustness Checks

We have already argued with help of the phase diagram that our main result that the multiplier of a temporary deficit spending program turns negative before the deficit spending expires, is of general nature rather than an artefact of the particular shock and the particular government reaction assumed in the benchmark calibration. To further validate this claim and to assess the quantitative impact of alternative assumptions we now present a sensitivity analysis of our result with respect to the duration of the deficit spending program, the size of the government spending multiplier, the scale of the spending program, and the severity of the recession and its persistence.

We begin with investigating the impact of the duration of the spending program. For that purpose we assume that the same total increase of government outlays is stretched over an alternative number of years. A longer spread of the stimulus thus reduces extra expenditure per year. To be specific, we take total outlays from our benchmark case (where they were spent over 5 years) and assume that they are spent alternatively over 3 years, implying $\Delta g = 0.0087$ annually or over 8 years, implying $\Delta g = 0.0033$.

**Figure 5: Robustness w.r.t. Timing of Deficit Spending (left) and Magnitude of the Multiplier (right)**

The panel on the left hand side of Figure 5 shows that our result is robust against the assumed length of the deficit spending period. For comparison, the solid line reiterates deviation from laissez-faire GDP for our benchmark policy from Section 4. The dashed line shows the implied

![Diagram](image-url)
deviation when total extra outlays are spent over 3 years while the dotted line shows the deviation for an 8-year spending program. As already theoretically derived from phase diagram analysis, the capital stock is lowest at the moment when the fiscal stimulus expires. Here we see that this fact translates one-to-one into the result that the fall of output below laissez-faire is lowest at the time of expiry of deficit spending. This observation in turn implies that the moment at which the associated government spending multiplier changes its sign from positive to negative occurs in any case before deficit spending expires.

If deficit spending is performed over a shorter period of time, the triggered initial increase of GDP is stronger and of shorter duration, and the fall below laissez-faire is harder. The opposite is true for a prolonged spending program. Regardless of the duration of the stimulus, the deviation from laissez faire after expiry follows almost the same path. The result reflects the fact that the induced loss of permanent income depends mainly on total extra government purchases, which have been assumed to be the same in all three cases.

The panel on the right hand side of Figure 5 shows deviation from laissez-faire for alternative assumptions of the long-run multiplier of government spending. Again, the solid line reiterates the benchmark case. The dashed line reflect the outcome when $\gamma = 5$. In that case the implied Frisch elasticity is $3/5$, closer to the micro estimates (e.g. Pencavel, 1986). The weaker response of employment implies a reduction of the long-run multiplier from 0.89 to 0.54. Dotted lines show the result for the case of $\gamma = 1$, which is frequently investigated in the RBC literature. Then, the implied elasticity of labor supply becomes 3 and the long-run multiplier of government spending is 0.97. Finally, dashed-dotted lines reflect the case of $\gamma = 0$, i.e. the case when labor enters linearly in utility. This assumption gets theoretical foundation from the indivisible labor approach and has been very popular in the RBC literature as well. In this case the long-run multiplier assumes its maximum (of 1.22).

In all cases our main finding that GDP falls below laissez faire before deficit spending (at year 5) is robust against the magnitude of the multiplier. Naturally, the model produces more optimistic predictions when the multiplier is higher: the positive effect through higher employment is stronger such that the initial impact is larger and the fall below laissez faire is softer. The negative effect through delayed investment, however, is never overturned. On the other hand, if the implied labor supply elasticity is closer to what microeconometricians suggest and the long-run multiplier is smaller, the fall below laissez faire happens earlier and is harder.
The panel on left hand side of Figure 6 demonstrates robustness with respect to the scale of the stimulus program. Again, the solid line reiterates adjustment dynamics of the benchmark model. Dashed and dotted lines show adjustment dynamics when the increase of government purchases is much higher than scheduled under ARRA ($\Delta G$ rises such that the GDP share $\Delta g$ increases by 1 or 2 percentage points, respectively). As expected, a larger deficit spending program produces faster recovery initially because of its higher effect on employment. But it produces also a harder fall of the economy during the second spending phase when the negative
repercussions through reduced and delayed investment become dominating. As a rule, we observe that economic recovery in the medium and long-run is the slower the larger the scale of deficit spending.

On the right hand side of Figure 6 we investigate alternative assumptions about the severity of the recession. The solid line reiterates the benchmark case and dashed and dotted lines show adjustment dynamics when the initial fall of output is assumed to be (much) harder, i.e. of 15 or 20 percent below steady-state level. We see that the relative deviation from laissez faire is almost invariant to the assumed severity of the recession.

Finally, Figure 7 shows robustness of the result with respect to the assumed duration of the recession. Solid lines show adjustment dynamics for the benchmark case of $\sigma = 0.4$, i.e. for a half-life of recovery of 2 years. Dotted lines show adjustment dynamics when recovery is assumed to be much faster with a half life of half a year ($\sigma = 1.5$) and dashed lines show results for a much slower recovery with a half life of 12 years ($\sigma = 0.1$). As shown in the panel on the right hand side the relative deviation from laissez faire is almost invariant to the assumed length of the recession. In any case the government spending multiplier turns negative before the deficit spending program expires. The focus on relative performance compared to laissez faire hides, of course, the fact that the assumed length of the recession has severe implications on macroeconomic behavior in absolute terms. This is revealed, by way of example, on the right hand side of Figure 7 for the evolution of the government deficit.


The fact that deficit spending slows down recovery in the medium- and long-run because of its unintended effect on dampened and delayed investment suggests a natural countermeasure: a fiscal incentive to invest. The obvious fiscal channel to foster investment in our simple model is the capital income tax rate. In this section we thus investigate whether an accompanying cut of the capital income tax can prevent the multiplier of deficit spending to turn negative and how large such a tax cut has to be.

Before we show results for the combined fiscal policy package, it is instructive to investigate separately economic recovery with and without reduction of the capital tax rate. Figure 8 shows, analogously to Figure 1, recovery of macroeconomic aggregates from recession under laissez faire
(dotted lines) and under a permanent (solid lines) and temporary (dashed lines) cut of the capital income tax of 2.5 percentage points, i.e. for a reduction of $\tau_k$ from 0.37 to 0.345.

Of course, a permanent reduction of distortionary taxation is very effective not only in raising consumption and investment in the long-run but also in preventing capital stock to fall much below its pre-shock steady-state level. More interestingly, we see that the temporary cut is effective as well. It pushes investment above laissez faire and slows down the fall of capital stock. We also observe a positive feedback effect (through higher labor productivity) on employment.

Thus, even if a permanent cut of $\tau_k$ is undesirable for (political) reasons outside the model, a temporary cut could be desirable in order to prevent the negative repercussions from deficit spending.
Figure 9 shows the implication on the speed of recovery of GDP relative to laissez faire. Dashed lines reiterate adjustment dynamics under (ARRA-) deficit spending from our benchmark model and solid lines show adjustment dynamics under a successful combined policy of deficit spending and capital tax cutting. It turned out that the minimum tax cut preventing deficit spending to slow down recovery is 2.5 percentage points, i.e. the policy that we have just discussed in isolation in Figure 8. The spending-cum-tax-cut policy preserves the positive initial impact of deficit spending and prevents output to fall below laissez faire in the medium-run. Actually, the economy performs much better under the combined policy already in the short-run. Because the investment effect of the tax cut prevents capital stock to fall much below steady-state, the employment effect of deficit spending can unfold its expansive force more powerfully. The price is, of course, a higher deficit caused by fiscal stimulus. The panel on the right hand side of Figure 9 shows that the deficit accumulated when the stimulus expires increases from 7 to 11.5 percent.

7. Conclusion

In this paper we have calibrated a standard neoclassical growth model with US data and investigated whether deficit spending can speed up economic recovery after a deep recession. We have tried to approximate the laissez-faire recovery path of GDP predicted by CBO and the path of government purchases implied by the American Recovery and Reinvestment Act. We have
demonstrated that deficit spending is a flash in the pan. Before the spending program expires, aggregate output is lower than it could be without fiscal stimulus. This unpleasant finding results from the fact that economic agents know that a temporary expenditure program has no lasting effect on employment but lasting effect on disposable income. We have demonstrated the generality of this result by providing economic intuition from phase diagram analysis and by performing robustness checks with respect to size and persistence of the shock, size of the multiplier, and the scale and duration of deficit spending.

We have also shown how an accompanying temporary cut of the capital income tax helps to prevent the negative repercussion of deficit spending on economic recovery. Of course, just cutting the capital income tax would be the first best policy with respect to GDP recovery. If (for political reasons) the government nevertheless prefers deficit spending, an accompanying cut of the capital tax appears as a reasonable second best policy.

We could have added more quantitative realism to the model by adding more delays and frictions in the New Keynesian tradition, e.g. stickiness of wages and prices. Yet we have sacrificed detail for generality of results, generality which we gain from the feature that our small scale model is accessible to phase diagram analysis. As argued in the introduction sacrificing detail for generality is supported by recent business cycle research (Chari et al., 2007). We are thus confident that adding more detail would not change our main result. Integrating more frictions would possibly enlarge the initially positive impact of fiscal stimulus. The mechanism that produces a negative multiplier before the spending program terminates, however, would be still operative. The negative effect on investment and capital stock results irrespective of the adjustment speed of employment and is generated “only” by the interplay between households’ loss of disposable income and need for consumption smoothing.

We would like to close with two qualifying remarks. Firstly, we could have given government purchases a bigger chance to boost investment by introducing elements beyond the standard neoclassical growth model (and beyond standard Keynesian reasoning). For example, we could have assumed that (large parts of the) additional government spending is productivity enhancing. But then, the question arises why these expenditures are not planned to be permanent and why, in the first place, they have not been made at times before the recession when tax revenues to finance them were more plentiful.
Secondly, this article has not argued against a larger scale of government purchases as such, be they permanent or temporary. We are not advocating laissez faire. Temporarily increasing government spending could be very sensible in order to reduce individual economic hardship after a severe shock and may even raise aggregate welfare. We have just tried to explain why we expect that government purchases will not speed up the process of recovery after recession, at least not in the medium and long run when its repercussions on investment become dominating.
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