A New Simple Test Against Spurious Long Memory Using Temporal Aggregation

Heri Kuswanto[1]

Institute of Statistics, School of Economics and Management, Leibniz Universität Hannover
Königsworther Platz 1, 30167 Hannover, Germany

Discussion Paper No. 425
August 2009

Abstract

We have developed a new test against spurious long memory based on the invariance of long memory parameter to aggregation. By using the local Whittle estimator, the statistic takes the supremum among combinations of paired aggregated series. Simulations show that the test performs good in finite sample sizes, and is able to distinguish long memory from spurious processes with excellent power. Moreover, the empirical application gives further evidence that the observed long memory in German stock returns is spurious.

Keywords: Local-Whittle method, Spurious long memory, Change point, Aggregation

1 Introduction

Let x_t be a linear long memory process characterized mainly by the following condition

$$\rho_k \sim C_\rho(k)k^{2d-1}, \quad \textbf{as } k \to \infty \tag{1}$$

for $d \in (0, 0.5)$. We consider an aggregated long memory process defined as

$$y_t = \sum_{j=0}^{m-1} x_{mt-j} = \sum_{j=0}^{m-1} B^j x_{mt} \tag{2}$$

[1]Email: kuswanto@statistik.uni-hannover.de

where B is backshift operator and m denotes the aggregation level. Chambers (1998), Man and Tiao (2001) and Souza (2008) show that if x_t satisfies (1) with $d < 0.5$, then its aggregation process y_t also satisfies (1) with the same fractional integration order d. This condition implies invariance of the memory parameter to aggregation.

Spurious long memory can arise in many cases. Especially in stock market data, it has been still highly debated whether the observed long memory is real or spurious phenomena. Many studies found long memory in the volatility of stock returns (Hiemstra and Jones (1997), Henry (2002), Tolvi (2003) and among others). Lobato and Savin (1998) and the references therein discuss the real and spurious long memory properties of stock market data. They investigated the major causes of spurious long memory, such as aggregation, nonstationarity and regime switching. By using LM type test of Lobato and Robinson (2003), they estimated the memory parameter and tested the significance of the parameter to conclude whether the observed memory is real or spurious. However, it is well known that several processes are able to create spurious long memory by generating a certain degree of fractional integration (see Granger and Ding (1996), Granger and Teräsvirta (1999), Kuswanto and Sibbertsen (2008) and among others). Therefore, developing a test which is able to distinguish long memory from the spurious processes is still of interest, which may lead to the proper model choice.

The fact that the memory parameter does not change with aggregation can be used as a means to distinguish long memory from the spurious processes. Ohanissian et al. (2008) estimates the memory parameter across several aggregation levels and proposes Wald type test to distinguish these two phenomena. They show that the test is able to detect the spurious process under alternative with very considerable power. Their results are based on the simulation study by examining very large number of observation, meaning that it has good performance for high frequency data and our initial study shows that the test losses the power significantly under finite sample size. They uses GPH estimator of Geweke and Porter-Hudak (1983) method to estimate the memory parameter and the theoretical properties of the test has been well investigated. However, Teles et al. (1999) proved that using GPH estimator of aggregated series for testing long memory has very serious consequences on the power of the test which may lead to the wrong conclusion, especially using bandwidth frequency $T^{0.5}$.

In this paper, we propose a new test against spurious long memory based on the invariance principle, in line with the basic idea of Ohanissian et al. (2008). Our test is developed by calculating the statistic value for every pair of aggregation level and taking the maximum among the values, the same concept with testing change in the memory parameter of Beran and Terrin (1996). We estimate the memory parameter by semi-parametric local Whittle maximum likelihood instead of GPH estimator. This estimation method has been proved to have smallest bias estimate with minimum standard deviation (Souza (2007)).

This paper is organized as follows. Section 2 discusses the main result including the proposed test and its asymptotic distribution. Section 3 presents the performance of the test in finite sample size. The empirical application , ie. the case of German stock returns is given in section 4. The proof is given in appendix.

2 Main Result

A stationary ARFIMA(p, d, q) process x_t has the following representation:

$$\phi(B)(1-B)^d x_t = \theta(B)\epsilon_t \quad t = 1, ..., N \tag{3}$$

where B is the backshift operator, $\phi(B)$ and $\theta(B)$ are the AR and MA polynomials respectively and ϵ_t is a white noise process with variance σ_ϵ^2. The spectral density of (3) satisfies

$$f_x(\omega) = C_f(\omega)|\omega|^{-2d} \;\; \textbf{as} \;\; \omega \to 0 \tag{4}$$

We aggregate the process x_t by a level of aggregation m following (2), with $m = 2, ..., M$. Under the aggregated series y_t, the series length becomes $n = N/m$. Note that $m = 1$ corresponds to the original series x_t. The spectral density of y_t with memory parameter d satisfies

$$f_y(\lambda) \sim m^{2d+1} C_{f_x}(\omega)|\lambda|^{-2d}, \;\; \textbf{as} \; \lambda \to 0 \tag{5}$$

where $\lambda = 2\pi jm/N = \omega m$ and the periodogram of y_t is given by

$$I_{y^{(m)}}(\lambda_j) = \frac{1}{2\pi n}\left|\sum_{j=1}^{n}(y_j - \bar{y}) \exp^{ij\lambda_j}\right|^2, \;\; \bar{y} = \sum_{j=1}^{n} y_j/n \tag{6}$$

Our statistic is constructed based on the semi-parametric local Whittle estimator proposed by Robinson Robinson (1995). Let us consider the Gaussian objective function for original series x_t:

$$Q(G,d) = \frac{1}{l}\sum_{j=1}^{l}\left[\log(G\omega_j^{-2d}) + \frac{\omega_j^{2d}}{G} I_x(\omega_j)\right] \tag{7}$$

by which discrete averaging is evaluated over a small bandwidth frequency $l < N$. As G can be estimated by $\hat{G} = \frac{1}{l}\sum_1^l \omega_j^{2d} I_x(\omega_j)$, then the memory parameter d can be estimated by minimizing the following objective function

$$\mathcal{Q}(d) = \log\left(\frac{1}{l}\sum_1^l \omega_j^{2d} I_x(\omega_j)\right) - 2d\frac{1}{l}\sum_1^l \log\omega_j \tag{8}$$

Souza (2007) discusses consistency of the estimator for aggregated series. It is worthwhile to summarize it as follows. Under the following regularity conditions:

1. $f(\omega) \sim G_0\omega^{-2d}$ as $\omega \to 0+$ where $G_0 \in (0, \infty)$, $-0.5 < \Delta_1 \leq d \leq \Delta_2 < 0.5$.
2. $f(\omega)$ is differentiable near the origin such that

$$\frac{d}{d\omega}\log f(\omega) = O(\omega^{-1}) \; as \; \omega \to 0+$$

3.

$$x_t - \mathbf{E}[x_0] = \sum_{j=0}^{\infty} \alpha_j \epsilon_{t-j}, \quad \sum_{j=0}^{\infty} \alpha_j^2 < \infty$$

where $\mathbf{E}(\epsilon_t|F_{t-1}) = 0, \mathbf{E}(\epsilon_t^2|F_{t-1}) = 1 \ a.s., \ t = 0, \pm 1, ...$, in which F_t is the σ-field of events generated by $\epsilon_s, s \leq t$, and there exists a random variable ϵ_t such that $\mathbf{E}(\epsilon_t^2) < \infty$ and for all $\eta > 0$ and some $K > 0, P(|\epsilon_t| > \eta) \leq KP(|\epsilon_t| > \eta)$.

4. As $N \to \infty, \frac{1}{l} + \frac{l}{N} \to 0$
5. $f(\omega)$ is bounded above, $f'(\omega)$ exists and is finite in the vicinity of the non-zero Nyquist frequencies.
6. $f(\omega) \sim G_0 \omega^{-2d}(1 + O(\omega^{\beta}))$ as $\omega \to 0+$ for some $\beta \in (0, 2]$ where $G_0 \in (0, \infty)$ and $-0.5 < \Delta_1 \leq d \leq \Delta_2 < 0.5$.
7. $\alpha(\omega)$ is differentiable near the origin such that

$$\frac{d}{d\omega}\alpha(\omega) = O\left(\frac{|\alpha(\omega)|}{\omega}\right), \ \ as \ \ \omega \to 0+$$

where $\alpha(\omega) = \sum_{j=0}^{\infty} \alpha_j e^{ij\omega}$

8. Condition 3 holds and $\mathbf{E}(\epsilon_t^3|F_{t-1}) = \mu_3, a.s., \mathbf{E}(\epsilon_t^4) = \mu_4, t = 0, \pm 1, ...$ for finite constant μ_3 and μ_4.
9. There exists a β satisfying Condition 6 such that

$$\frac{1}{l} + \frac{l^{1+2\beta}(\log l)^2}{N^{2\beta}} \to 0, \quad \mathbf{as} \ \ N \to \infty$$

If condition (1) to (5) hold for x_t, then it builds the consistency of the local Whittle estimator for aggregated time series y_t. Also, if condition (5) to (9) hold for x_t, then the local Whittle estimator for y_t is asymptotically normal such that

$$\sqrt{l}(\hat{d} - d) \xrightarrow{D} N(0, 1/4) \tag{9}$$

The readers are referred to Souza (2007) for the proof and the details of these conditions.

Now, consider two objective functions for two aggregated series $y^{(m_1)}$ and $y^{(m_2)}$ as follows:

$$\begin{aligned} \mathcal{Q}(n_1, d) &= \log\left(\frac{1}{l}\sum_{1}^{l} \lambda_j^{2d} I_{y^{(m_1)}}(\lambda_j)\right) - 2d\frac{1}{l}\sum_{1}^{l}\log\lambda_j \\ \mathcal{Q}(n_2, d) &= \log\left(\frac{1}{l}\sum_{1}^{l} \lambda_j^{2d} I_{y^{(m_2)}}(\lambda_j)\right) - 2d\frac{1}{l}\sum_{1}^{l}\log\lambda_j \end{aligned}$$

where $\mathcal{Q}(n_1, d)$ and $\mathcal{Q}(n_2, d)$ denote the objective function of the aggregated series y_t with level m_1 and m_2 respectively. From this, the local Whittle estimator $\hat{d}$ is defined by

$$\hat{d}^{(m_1)} = \mathbf{argmin}\ \mathcal{Q}(n_1; \hat{d}), \quad \hat{d}^{(m_2)} = \mathbf{argmin}\ \mathcal{Q}(n_2; \hat{d})$$

We will test the constancy of the estimated memory parameter among several aggregation levels to prove the invariance principle of the memory parameter to aggregation. The null hypothesis we attempt to test is that

$$\mathbf{H_0} : d^{(m_1)} = d^{(m_2)} = ... = d^{(m_M)}$$

The alternative hypothesis is therefore defined as any violation of the equalities in $\mathbf{H_0}$, i.e at least one pair of aggregated levels, m_i and m_j, $d^{(m_i)} \neq d^{(m_j)}$ where $i \neq j$.

In this paper, the idea of the test is similar to testing change in long memory parameter (see Beran and Terrin (1996), Horväth and Shao (1999), Lee and Lee (2007)). To test the constancy of the long memory parameter between two aggregated levels $\{m_1 \neq m_2\}$, we propose the following statistic

$$z_{m_1,m_2} = \sqrt{n_1 + n_2} \left\{ \frac{n_1 n_2}{(n_1 + n_2)^2} \right\} \left(\hat{d}^{(m_1)} - \hat{d}^{(m_2)} \right).$$

The calculation of z_{m_1,m_2} involves two levels of aggregated series for all combinations of the paired m. It means that for any choice of M aggregation level, we have ${}_MC_2$ values of z. In this case, M is chosen such that the aggregated series can still be used for estimating the long memory parameter. The supremum value is proposed as the statistical test. Therefore, to test the constancy of parameter d among several aggregation levels, we suggest the statistic

$$\chi_n = \max_{1 \leq i,j \leq M} |z_{m_i,m_j}|, \quad i \neq j.$$

Let a_k is the coefficient of moving average representation of y_t defined as $y_t = \sum_{i=0}^{\infty} a_i \epsilon_{t-i}$, $b_k = \frac{1}{4\pi^2} \int_{-\pi}^{\pi} e^{ik\lambda} f^{-1}(\lambda, d) d\lambda$ and $c_k = b_0 a_k + 2 \sum_{i=1}^{k} b_i a_{k-i}$. The asymptotic distribution of the proposed test statistic is given in the following theorem.

Theorem 1:
Assume $0 < d < 0.5$ and the condition (5), (6), (7), (8) and (9) are satisfied, then by the asymptotic normality of $\hat{d}$ we have for $m_1 \neq m_2$

$$z_{m_1,m_2} \xrightarrow{D} \sigma B(t)$$

in $\mathbf{D}[0, 1]$ as $T \to \infty$ where $T = n_1 + n_2$, and $B(t), 0 \leq t \leq 1$ is a Brownian bridge with $\xrightarrow{D}$ denotes convergence in distribution. Hence, the statistic χ_n converges to

$$\chi_n \xrightarrow{D} \sigma \sup_{0 \leq t \leq 1} |B(t)|, \quad i \neq j$$

and the variance σ^2 is given by

$$\sigma^2 = \mathbf{E}(\epsilon_0^4 - \sigma_\epsilon^4)\left(\sum_{j=0}^{\infty} a_j c_j\right)^2 + \sigma_\epsilon^2 \sum_{i=1}^{\infty}\left(\sum_{j=0}^{\infty}\{a_j c_{j+i} + c_j a_{j+i}\}\right)^2$$

Proof: see appendix

From the theorem above, we reject the null hypothesis for large values of χ_n. In principle, it is possible to generate the critical values from a sequence of Brownian bridge $B(t)$ and variance σ^2 as written in the theorem. However, it seems that σ^2 has a very complicated form which leads to some difficulties. To avoid this, the critical values will be determined by using the simulated sampling distribution of χ_n.

3 Simulation

This section carries out simulation studies to obtain the critical values, as well as to assess the test performance in finite sample. As we pointed out above, the critical values are obtained by using the simulated sampling distribution of $\max_{1\leq i,j\leq M}|z_{m_i,m_j}|$. It is done by generating 50000 sample sizes and 10000 replications. The aggregation levels are set to be $m = 2, 3, 4, 6, 8, 12$, which are commonly used in empirical applications (Teles et al. (1999), (2008)). In the latter work, they studied the effect of the use of aggregate time series on the Dickey-Fuller test for unit root and a new unit root test based on aggregate time series was developed. In brief, the procedure to obtain the critical values can be described as follows:

- we generate the process under null hypothesis with sample size of 50000
- we apply the test statistic with each setting of aggregation level to obtain the statistic z_{m_1,m_2} as well as χ_n
- we do above steps 10000 times and therefore, we have 10000 values of χ_n.
- we tabulate the sampling distribution of χ_n to determine the quantile of the sampling distribution as the critical values. Having this sampling distribution, we do not need to generate σ and $B(t)$ since we have that $\chi_n \xrightarrow{D} \sigma \sup_{0\leq t\leq 1}|B(t)|$.

It is easy implemented procedure and commonly applied to simulate the critical value of tests whose nonstandard asymptotic distribution such as unit root Dickey Fuller test and the extensions.

Table 1: Quantile of the asymptotic distribution

d	sign. level	Aggregation level (m)					
		2	3	4	6	8	12
0.1	90%	0.4542	0.5549	0.6030	0.6872	0.7586	0.7588
	95%	0.5258	0.6456	0.7133	0.7740	0.8248	0.8407
	99%	0.7098	0.8253	0.8708	0.9034	0.9518	1.0595
0.2	90%	0.5095	0.6351	0.6579	0.7232	0.7454	0.7780
	95%	0.5909	0.7203	0.7650	0.8108	0.8454	0.8516
	99%	0.8253	0.8708	0.9177	0.9756	0.9784	0.9967
0.3	90%	0.5164	0.6572	0.7160	0.7298	0.7802	0.7953
	95%	0.6347	0.7617	0.8025	0.8354	0.8723	0.8849
	99%	0.8195	0.9282	0.9832	1.0579	1.0718	1.0534
0.4	90%	0.6111	0.6981	0.7530	0.8083	0.8226	0.8390
	95%	0.7037	0.8213	0.8495	0.8982	0.9517	0.9179
	99%	0.8732	0.9930	1.0313	1.0782	1.0995	1.0796

Table 1 provides the critical values of the test for $d = 0.1, 0.2, 0.3, 0.4$. We see that the critical value increases with d and m through the constant σ in theorem 2.1.

Size experiment is done by evaluating the performance of the test in finite sample size. In this case, we generate 1000 time series of 5000 sample sizes. The rejection rate is calculated based on the critical values in table 1. The data generating process (DGP) is pure stationary long memory with a certain degree of fractional integration $d = 0.1, 0.2, 0.3, 0.4$. Therefore, the DGP does not account for short dependencies ϕ and θ written in (3). The model can be rewritten as

$$(1 - B)^d x_t = \epsilon_t \quad t = 1, ..., N.$$

Table 2 presents mean and standard deviation of the estimated long memory parameter for several aggregation levels. It is useful to assess the performance of the local Whittle estimator.

Table 2: Invariance of memory parameter to aggregation

d	Aggregation level (m)					
	2	3	4	6	8	12
0.1	0.1002 (0.0209)	0.1053 (0.0253)	0.1016 (0.0262)	0.1021 (0.0377)	0.1045 (0.0377)	0.0999 (0.047)
0.2	0.2061 (0.0227)	0.2030 (0.0275)	0.2070 (0.0296)	0.2007 (0.0372)	0.2112 (0.0412)	0.2131 (0.0490)
0.3	0.3058 (0.0254)	0.3084 (0.0283)	0.3106 (0.0309)	0.3138 (0.0370)	0.3155 (0.0370)	0.3160 (0.0400)
0.4	0.4088 (0.0220)	0.4143 (0.0244)	0.4135 (0.0308)	0.4163 (0.0336)	0.4160 (0.0405)	0.4238 (0.0527)

Note: The Data Generating Process (DGP) is ARFIMA(0,d,0)

As expected, the estimated memory parameters are very close to the original value. For instance, under ARFIMA(0,0.1,0), the estimated memory parameters range from 0.0999 to

0.1053. Also, under DGP ARFIMA(0,0.2,0), the estimated memory parameters range from 0.2007 to 0.2131 and so for ARFIMA(0,0.3,0) and ARFIMA(0,0.4,0). It indicates that the local Whittle estimator is a good approximation for our test. In line with Souza (2003), the standard deviation of the estimated memory parameter increases with the aggregation level. The following table presents the result of size experiment.

Table 3: Size experiment

d	nom. size	Aggregation level (m)					
		2	3	4	6	8	12
0.1	0.05	0.059	0.042	0.041	0.049	0.053	0.044
	0.1	0.106	0.086	0.103	0.086	0.093	0.096
0.2	0.05	0.058	0.037	0.052	0.043	0.057	0.051
	0.1	0.090	0.088	0.098	0.078	0.103	0.085
0.3	0.05	0.055	0.032	0.047	0.054	0.056	0.045
	0.1	0.101	0.088	0.070	0.097	0.101	0.078
0.4	0.05	0.042	0.054	0.050	0.046	0.048	0.046
	0.1	0.090	0.101	0.092	0.087	0.098	0.094

Note: The Data Generating Process (DGP) is ARFIMA(0,d,0)

From table 3, it is obvious that the rejection rate is very close to the nominal value although some values indicate size distortion, meaning that the test is correctly sized under the null of long memory process.

The power experiment is carried out by generating several processes which are able create spurious long memory, ie. Markov switching, STOP-BREAK and random level shift process. These models can be described as follows:

- Markov-switching process

$$x_t = \begin{cases} \phi_1 x_{t-1} + \epsilon_t & \text{if } s_t = 1 \\ \phi_2 x_{t-1} + \epsilon_t & \text{if } s_t = 2 \end{cases}$$

  with $\epsilon_t \sim N(0,1)$, s_t is state of the Markov process with the state transition probability p_{00} and p_{11}.

- STOP-BREAK process

$$x_t = \mu_t + \epsilon_t, \quad \mu_t = \mu_{t-1} + \frac{\epsilon_{t-1}^2}{\gamma + \epsilon_{t-1}^2}\epsilon_{t-1}$$

  with $\epsilon_t \sim N(0,1)$.

- Stationary random level shift process

$$x_t = \mu_t + \epsilon_t, \quad \mu_t = (1 - j_t)\mu_{t-1} + j_t\varepsilon_t$$

  with j_t is IID Bernoulli(p), $\varepsilon_t \sim$ iidN$(0, \sigma_{\varepsilon_t}^2)$ and $\epsilon_t \sim$ iidN$(0, \sigma_{\epsilon_t}^2)$.

- Nonstationary random level shift process

$$x_t = \mu_t + \epsilon_t, \quad \mu_t = \mu_{t-1} + j_t \varepsilon_t$$

with j_t is IID Bernoulli(p), ε_t and ϵ_t are defined as in the stationary random level shift process.

These models are strong candidates which can easily mislead the properties of long memory (Granger and Ding (1996), Diebold and Ineoue (2001), Granger and Hyung (2004), Sibbertsen (2004b), Banarje and Urga (2005), etc). We call them as model 1, model 2, model 3 and model 4 respectively hereafter. Basically, they are short memory processes with zero integration order. Therefore, any degree of fractional integration more than zero observed from these processes are spurious results. For each model, the considered parameters as well as the result of power experiment can be seen in table 4, 5, 6 and 7.

In this part, we generate data with two different sample sizes, $N = 2000$ and $N = 5000$ with 1000 replications. Note that for $N = 2000$, it is considered a very small sample in practice, especially in the context of volatility modeling. Meanwhile, $N = 5000$ is a reasonable sample size for this case. Moreover, aggregating 5000 sample size with level of 12 results on big enough sample required to estimate the memory parameter. In the table, we present mean value of the fractional integration order obtained from 5000 sample sizes. Smaller bias is observed for smaller sample size. However, we omit the results for the reason of space.

Table 4: Power experiment

Model 1									
	$p_{00} = p_{11} = 0.90$ $\phi_1 = -\phi_2 = 0.8$			$p_{00} = p_{11} = 0.90$ $\phi_1 = -\phi_2 = 0.5$			$p_{00} = p_{11} = 0.90$ $\epsilon_1 = N(1,1), \epsilon_2 = N(-1,1)$		
m	mean(d)	reject freq.		mean(d)	reject freq.		mean(d)	reject freq.	
		N=2000	N=5000		N=2000	N=5000		N=2000	N=5000
1	0.3470 (0.0251)	-	-	0.1031 (0.0193)	-	-	0.3281 (0.0154)	-	-
2	0.2115 (0.0319)	0.989	1.000	0.0450 (0.0225)	0.799	0.949	0.2712 (0.0211)	0.207	0.878
3	0.1567 (0.0318)	0.994	1.000	0.0366 (0.0271)	0.666	0.940	0.2419 (0.0285)	0.432	0.995
4	0.1178 (0.0376)	0.998	1.000	0.0253 (0.0306)	0.731	0.940	0.1759 (0.0340)	0.680	0.995
6	0.0988 (0.0370)	0.999	1.000	0.0200 (0.0359)	0.722	0.901	0.1299 (0.0337)	0.795	1.000
8	0.0610 (0.0455)	1.000	1.000	0.0120 (0.0407)	0.653	0.870	0.0943 (0.0437)	0.808	1.000
12	0.0411 (0.0495)	1.000	1.000	0.0072 (0.0518)	0.567	0.854	0.0679 (0.0456)	0.852	1.000

Note: The third model specification has parameter $\phi_1 = -\phi_2 = 0$

Table 5: Power experiment

Model 2									
		$\gamma = 180$			$\gamma = 90$			$\gamma = 40$	
m	mean(d)	reject freq.		mean(d)	reject freq.		mean(d)	reject freq.	
		N=2000	N=5000		N=2000	N=5000		N=2000	N=5000
1	0.2290 (0.0587)	-	-	0.3409 (0.0571)	-	-	0.4709 (0.0554)	-	-
2	0.2842 (0.0590)	0.608	0.989	0.4055 (0.0655)	0.809	0.985	0.5660 (0.0645)	0.999	1.000
3	0.3353 (0.0658)	0.794	1.000	0.4589 (0.0667)	0.928	0.996	0.6276 (0.0696)	0.998	1.000
4	0.3577 (0.0712)	0.735	1.000	0.5025 (0.0713)	0.957	0.995	0.6708 (0.0738)	1.000	1.000
6	0.4005 (0.0831)	0.779	1.000	0.5586 (0.07613)	0.985	1.000	0.7407 (0.0689)	1.000	1.000
8	0.4458 (0.0781)	0.823	1.000	0.6003 (0.0797)	0.987	1.000	0.7815 (0.0602)	1.000	1.000
12	0.5011 (0.0865)	0.828	1.000	0.6817 (0.0852)	0.987	1.000	0.8417 (0.0569)	1.000	1.000

Table 6: Power experiment

Model 3									
		$p = 0.001$			$p = 0.01$			$p = 0.1$	
m	mean(d)	reject freq.		mean(d)	reject freq.		mean(d)	reject freq.	
		N=2000	N=5000		N=2000	N=5000		N=2000	N=5000
1	0.2596 (0.0919)	-	-	0.4931 (0.0738)	-	-	0.6581 (0.2208)	-	-
2	0.3370 (0.1134)	0.553	0.951	0.5845 (0.0777)	0.955	1.000	0.7238 (0.2788)	-	-
3	0.3747 (0.1223)	0.625	0.965	0.6419 (0.0899)	0.986	1.000	0.8070 (0.2468)	-	-
4	0.4047 (0.1275)	0.647	0.963	0.6881 (0.0940)	0.985	1.000	0.8022 (0.2980)	-	-
6	0.4606 (0.1596)	0.664	0.968	0.7563 (0.0925)	0.992	1.000	0.8770 (0.2527)	-	-
8	0.4926 (0.1659)	0.668	0.978	0.8106 (0.09004)	0.983	1.000	0.8797 (0.2782)	-	-
12	0.5976 (0.1617)	0.634	0.981	0.8554 (0.1097)	0.990	1.000	0.8747 (0.3131)	-	-

Table 7: Power experiment

Model 4									
	$p = 0.001$			$p = 0.01$			$p = 0.1$		
m	mean(d)	reject freq.		mean(d)	reject freq.		mean(d)	reject freq.	
		N=2000	N=5000		N=2000	N=5000		N=2000	N=5000
1	0.2802 (0.0911)	-	-	0.4927 (0.0681)	-	-	0.7185 (0.0567)	-	-
2	0.3374 (0.1109)	0.553	0.941	0.5950 (0.0723)	0.963	1.000	0.8266 (0.0486)	-	-
3	0.3875 (0.1048)	0.560	0.964	0.6496 (0.0712)	0.996	1.000	0.8806 (0.0407)	-	-
4	0.4064 (0.1277)	0.618	0.972	0.7110 (0.0713)	0.999	1.000	0.9124 (0.0361)	-	-
6	0.4587 (0.1445)	0.683	0.973	0.7656 (0.0692)	1.000	1.000	0.9483 (0.0347)	-	-
8	0.5258 (0.1254)	0.640	0.975	0.8118 (0.0664)	1.000	1.000	0.9665 (0.0368)	-	-
12	0.5757 (0.1398)	0.626	0.980	0.8744 (0.0607)	1.000	1.000	0.9827 (0.0470)	-	-

Dealing with the ability of the processes to resemble long memory, we see that all data generating processes are able to generate fractional integration orders which lie in long memory range. It can be seen from the mean values of the long memory parameter under $m = 1$, which corresponds to the original series. Therefore, the examined parameters are correctly specified. However, the point of consideration in this paper is not focused on whether the models are able to create spurious long memory or not, since it has been proved in the aforementioned references. Through the power experiment, we assess the behavior of the estimated memory parameter to aggregation and the ability of our test to specify these models into their class, which is spurious long memory. Since our test involves a pair of aggregation levels, thus we cannot obtain any value for $m = 1$. We denote it with "-" in the table.

Let us consider Markov switching processes in table 4. The choice of the transition probabilities mainly refers to previous works which found that the higher the transition probability p_{ii}, the longer the process is expected to remain in state i and the process becomes more persistent. Under this condition, the process will easily be confused with long memory (see Kuswanto and Sibbertsen (2007, 2008) for intensive simulation results). The first two parameter settings in model 1 are general Markov switching processes and the last is Markov switching with iid regimes (MS-IID) and therefore, $\phi_1 = -\phi_2 = 0$. From table 4, under the defined parameter settings, the test is able to specify the Markov switching processes as spurious long memory process with high power. Only two cases have power lower than 0.5. The power increases with sample size and shows no monotonic tendency regarding the level of aggregation. However, we can see that most cases have higher power with higher aggregation level.

Now, we discuss the results for model 2. The STOP-BREAK model was introduced by Engle and Smith [?]. Similar results as Markov switching are observed for this case. Under the three different parameter settings defined in table 5, the test is able to detect the model as spurious long memory with satisfying power, both in small and medium sample size. Especially when $N = 5000$, the power reaches almost one for all cases. For random level shift processes, either stationary or nonstationary, the test also performs very well. Under small probability of Bernoulli distribution, the estimated fractional integration parameters are biased toward stationary long memory. For $p = 0.1$, the memory parameter is biased toward nonstationary long memory. It indicates that higher probability leads to a more persistent process. Since our test is derived under stationary long memory condition, therefore, this case (nonstationary long memory with $d \geq 0.5$) is out of consideration and the power of the test cannot be presented. The considered random level shift processes in this paper were firstly introduced by Chen and Tiao (1990). Further conditions about the possibility of these models to resemble long memory has been clearly investigated by Breidt and Hsu (2002).

Our results in this experiment are consistent with the test proposed by Ohanissian et al. (2008). Their test is also able to distinguish long memory from the spurious processes with extremely high power by setting $N = 610304$. Since Ohanissian et al. (2008) uses Wald type test, it is well known that the test will tend to have full power for infinite sample size. However as we pointed out before, their test looses the power significantly in finite sample size. Therefore, our test fills this gap by having good performance in finite sample size.

4 Empirical Application

The dataset used in this study consists of daily absolute and squared returns for 9 German stock price series, listed in the DAX30. The examined cases are Allianz, BASF, BAYER, BMW, Commerz Bank, Continental, Deutsche Bank, Siemens and Volskwagen (VW) spanning from the period of January 1973 to December 2007. Therefore we have 9132 observations for each stock. Several previous studies have considered German stock returns and found long memory in the considered cases (Sibbertsen (2004a), Gurgul and Wojtowiczh (2006)), based on the fact that several estimation procedures such as GPH, Whittle estimator or Wavelet estimator give a fractional integration order within long memory interval. Again, it becomes crucial since several processes are able to create spurious long memory by having a certain degree of fractional integration as discussed in the previous section. Hassler and Olivares (2007) independently study the daily absolute returns of the German stock price index DAX and found a significant break in mean, which might be one source of the spurious long memory.

Figure 1 and 2 depict the autocorrelation function (ACF) of absolute and squared returns of the considered stocks respectively. We plot the autocorrelations up to 300 lags. The figures show that the autocorrelations of both absolute and squared returns are strongly correlated until long lags. They decay slowly with hyperbolic rates and show the property of long memory process. Again, having this property does not provide enough evidence that the processes are long memory. Kuswanto and Sibbertsen (2008) demonstrate that several nonlinear processes under specific parameter settings may produce the same feature of autocorrelation function as long memory. This similarity holds also for the spectrum of both processes. Therefore, using only this information may lead to the wrong conclusion.

We apply our new test as a formal procedure to detect whether long memory observed in the

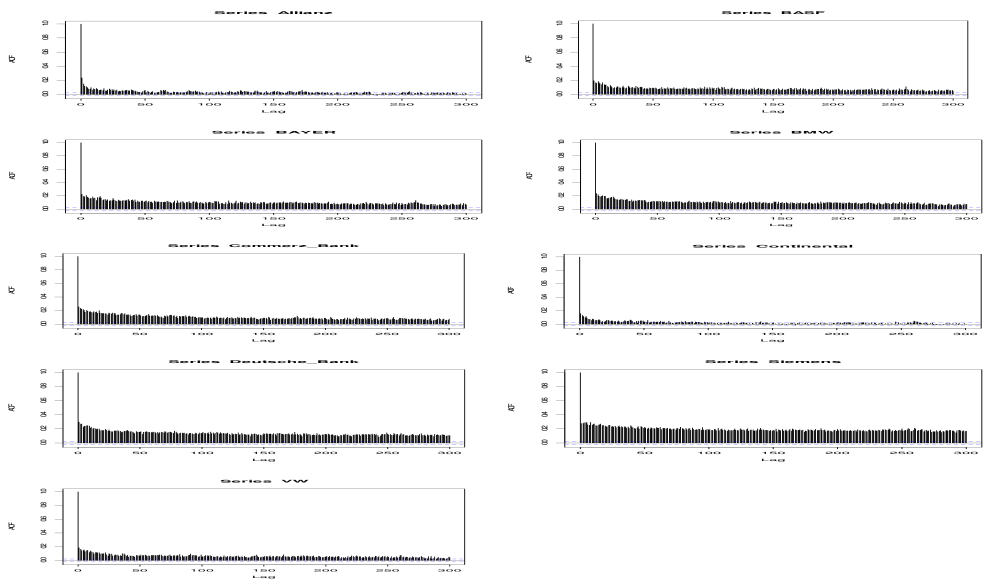

Figure 1: ACF plot of absolute returns

German stocks is real or spurious. The results of the test are presented in table 8 and table 9, for absolute and squared returns respectively. In the tables, we provide the estimated long memory parameter of the aggregated series under several aggregated levels m. The value in the last column is statistic χ_n obtained from applying the test with $m = 12$. This choice is based on the simulations which suggest that the test tends to have more power for high aggregation level. Table 8 presents the results of the test for absolute returns.

Table 8: Test for absolute returns

stock	m							$\lvert\lambda_n\rvert$
	1	2	3	4	6	8	12	
Allianz	0.1959	0.2170	0.2363	0.2426	0.2587	0.2883	0.3272	1.0166*
BASF	0.2365	0.2945	0.3201	0.3201	0.2982	0.3070	0.3475	1.7279*
BAYER	0.2491	0.2880	0.3373	0.3640	0.3872	0.3963	0.4189	1.9809*
BMW	0.2437	0.3015	0.3569	0.3730	0.3894	0.3942	0.4050	2.3434*
Commerz Bank	0.2705	0.3142	0.3534	0.3795	0.3982	0.4335	0.4806	1.8642*
Continental	0.2060	0.2280	0.2460	0.2455	0.2499	0.2763	0.3068	0.8276**
Deutsche Bank	0.2701	0.3398	0.3936	0.3986	0.3966	0.3898	0.4367	2.5551*
Siemens	0.2951	0.3480	0.3766	0.3404	0.4323	0.4709	0.5167	2.3127*
VW	0.2278	0.2829	0.3097	0.3473	0.3440	0.3582	0.3623	2.0418*

The * and ** sign represent significance under 5% and 10% level respectively

From the table, by 5% level of significance the test rejects almost all cases, except for Continental. Since we have under the alternative hypothesis that there is a violation to the invariant condition of the estimated memory parameters, then to reject the null hypothesis means

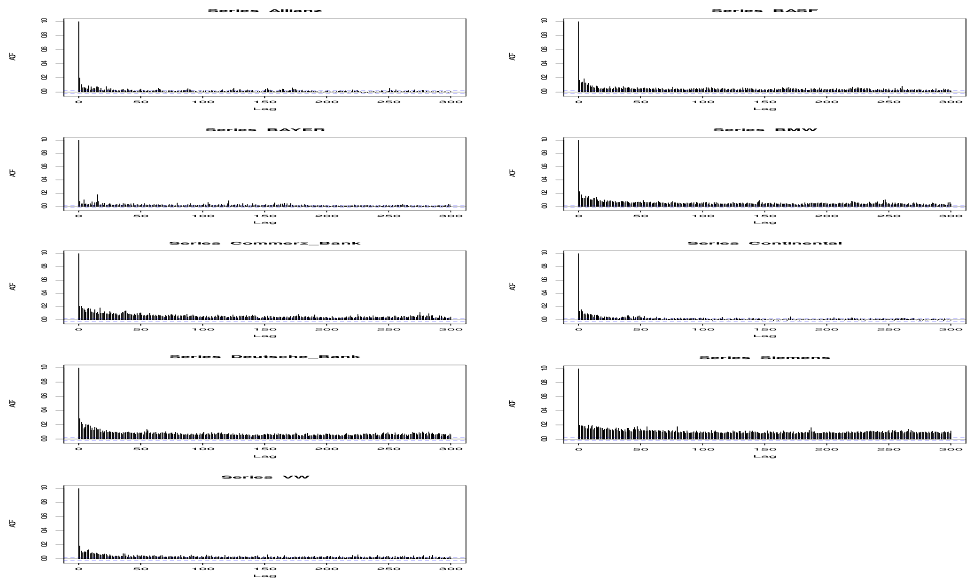

Figure 2: ACF plot of squared returns

that the observed long memory is spurious. Continental is the only case which seems to have real long memory. It is quiet natural if we look at the d values under several aggregation levels, they are very close to each other. For this, we are only able to reject the null of long memory by 10% level of significance. Now we analyze the results for squared returns, which are given in the following table

Table 9: Test for squared returns

stock	m							$\lvert\lambda_n\rvert$
	1	2	3	4	6	8	12	
Allianz	0.1470	0.1713	0.2020	0.2244	0.2467	0.2675	0.2763	1.6631*
BASF	0.2378	0.2673	0.2783	0.2629	0.2319	0.2389	0.2739	0.8362**
BAYER	0.1422	0.1501	0.1912	0.2911	0.3087	0.2995	0.2601	2.4222*
BMW	0.1994	0.2486	0.3128	0.3268	0.3227	0.3290	0.3191	2.3460*
Commerz Bank	0.2385	0.3029	0.3362	0.3622	0.3646	0.3801	0.4076	2.1151*
Continental	0.2028	0.2290	0.2615	0.2646	0.2555	0.2762	0.3037	1.2861*
Deutsche Bank	0.2326	0.3109	0.3698	0.3631	0.3505	0.3281	0.3399	2.8387*
Siemens	0.2469	0.2842	0.3215	0.3812	0.4020	0.4202	0.4285	2.2959*
VW	0.1757	0.2454	0.2724	0.3049	0.2987	0.2991	0.2968	2.2086*

The * and ** sign represent significance under 5% and 10% level respectively

In line with the result for absolute returns, the test rejects the null of real long memory. By 5% level of significance, it fails to reject the null only for BASF case. Therefore, we may say that long memory observed in most of the German stock returns is spurious process, both in absolute and squared returns. The existence of this spurious process could be the result of non-stationarity, regime switching, mean shift, aggregation, etc. These results thus give new evidence about the behavior of German stock returns dealing with long memory.

5 Conclusion

This paper contributes to the literature on spurious long memory tests by providing a simple procedure to detect the spurious long memory based on the invariance principle of the estimated memory parameter under several aggregation levels. The test performs well in finite sample size. The empirical application gives evidence of spurious long memory in the absolute and squared German stock returns.

6 Appendix

This session gives the proof of theorem 1. We start the proof by showing that the following holds

$$\mathcal{Q}(n) - \sigma W(n) = O(n^{1/2-\varepsilon}) \;\; a.s \qquad << A1 >>$$

where $\{W(t), 0 \leq t < \infty\}$ is a Wiener process and $\varepsilon > 0$. By theorem 1.1 of Horväth and Shao (1999), condition $<< A1 >>$ is satisfied if we can show that there exists $\varsigma > 0$, $\tau > 0$, $\vartheta > 0$ satisfying $\varsigma + \tau > 1/2$ and $\vartheta + 2\varsigma > 1$, such that

$$(i).\ a_k = O(|k|^{-\frac{1}{2}-\varsigma}), \;\; (ii).\ b_k = O(|k|^{-\frac{1}{2}-\vartheta}), \;\; (iii).\ c_k = O(|k|^{-\frac{1}{2}-\tau}) \tag{10}$$

where a_k, b_k, c_k are defined in the previous section.

Suppose that the original series x_t has the following infinite moving average representation:

$$x_t = \sum_{i=1}^{\infty} \alpha_i \epsilon_{t-i} \tag{11}$$

where ϵ_t is mean zero independent, identical distributed random variable and having variance σ_ϵ^2. Now, equation (2) can be written as

$$y_t = \sum_{j=0}^{m-1} B^j \sum_{i=1}^{\infty} \alpha_i \epsilon_{t-i} \tag{12}$$

$$= \sum_{i=0}^{\infty} a_i \epsilon_{t-i} \tag{13}$$

where $a_i = \displaystyle\sum_{j=i-m+1}^{i} \alpha_j$ and $\alpha_j = 0$ for $j < 0$. Before we examine (i), we firstly need to show that a_i converges in mean square. This condition has been previously examined by Teles et al. (1999). Nevertheless, let us describe it in brief here since it is very important for the test.

Let us define

$$(1+B)^d = \sum_{j=0}^{\infty} \varphi_j B^j \tag{14}$$

where $\varphi_j = \left(\begin{array}{c} d \\ j \end{array} \right) = \dfrac{\Gamma(d+1)}{\Gamma(j+1)\Gamma(d-j+1)}$ and satisfies $\varphi_j \sim \dfrac{\Gamma(d+1)}{2\pi}(1)^{-(j-d-1/2)} j^{-(d+1)}$ as $j \to \infty$. From the definition of aggregated long memory y_t in (2) and the theorem 1 of Teles et al. (1999),

$$\begin{aligned} (1+B+...+B^{m-1})^d &= \prod_{j=1}^{m-1} (1+\zeta_j B)^d && (15) \\ &= \prod_{j=1}^{m-1} \left[\sum_{k=0}^{\infty} \varphi_k \zeta_j^k B^k \right] && (16) \end{aligned}$$

therefore, for $d > -0.5$, $\prod_{j=1}^{m-1} \left[\sum_{k=0}^{\infty} |\varphi_k \zeta_j^k|^2 \right] < \infty$ and this implies that $\sum_{j=0}^{\infty} a_j^2 < \infty$, which is the basic condition allowing to develop test statistic by using aggregated long memory. Moreover, from equation (6), it implies

$$a_k \sim C(k) k^{2d-1} \tag{17}$$

as $k \to \infty$ for some C slowly varying at infinity.

To examine (ii), let us define $b_k = \frac{1}{4\pi^2} \int_{-\pi}^{\pi} e^{ik\lambda} f^{-1}(\lambda, d) d\lambda$ and assume that $f(\lambda, d)$ and $f^{-1}(\lambda, d)$ are continuous at all λ and d (Tsay and Chan (2005)) such that

$$\frac{\partial f^{-1}(\lambda, d)}{\partial d} \approx O(|\lambda|^{-2d}) \tag{18}$$

Recall the covariance of y_t as follow

$$\mathbf{E} y_j y_k = \sigma_x^2 \rho(j-k) = \sigma_x^2 \int_{-\pi}^{\pi} e^{i(j-k)\lambda} d\lambda \tag{19}$$

Define a Teoplitz matrix $\mathcal{R}_{n\mathbf{x}n}$ with the j,k-th entry $\rho(j-k)$ and a matrix $\mathcal{A}_{n\mathbf{x}n}$ with the j,k-th entry b_{j-k}. Then, by condition (1) and Parseval relation, $\mathcal{A}_{n\mathbf{x}n}$ can be defined as an inverse of the covariance matrix $\mathcal{R}_{n\mathbf{x}n}$ (Fox and Taqqu (1986), Bleher (1981)) written as

$$\mathcal{R}\left(\frac{1}{4\pi^2} f^{-1}(\lambda, d) \right) \tag{20}$$

By this relation, we intend to get the asymptotic of b_k. Furthermore, by proposition 1 of Souza (2008), the autocovariance of y_t is given by

$$\gamma_y(k) \sim m^2 \sigma_x^2 C_\rho(k) k^{2d-1} + O(k^{2d-3}), \quad \textbf{as} \;\; k \to \infty \tag{21}$$

From this, it is sufficient to show that

$$\gamma_y(k) \sim C(k)|k|^{2d-1}, \quad \textbf{as} \ \ k \to \infty \tag{22}$$

and therefore for $0 < \delta < 1/2 - d$

$$|b_k| = O(|k|^{\delta - 1}), \quad \textbf{as} \ \ k \to \infty \tag{23}$$

Further details about the autocovariance function of y_t, the readers are referred to Souza (2008).

From (17) and (23), it is sufficient to have as $n \to \infty$,

$$\begin{aligned} |c_k| &= O(C(k)k^{2d-1}) + O(C(k)k^{2d-1+\delta})\beta(\delta, d) && (24) \\ &= O(C(k)k^{2d-1+\delta}) && (25) \end{aligned}$$

where $\beta(\delta, d)$ is beta function defined as $\beta(\delta, d) = \displaystyle\int_0^1 y^{\delta-1}(1-y)^{2d-1}dy$.

Now, the condition $<< A1 >>$ is satisfied and we can define a sequence of Brownian bridges $B_n(t), 0 \leq t \leq 1$ such that

$$\max_{0 \leq s_1, s_2 \leq 1} T^{1/2} s_1 s_2 \left| \left\{ \frac{\mathcal{Q}(n_1, d)}{n_1} - \frac{\mathcal{Q}(n_2, d)}{n_2} \right\} \right| \xrightarrow{D} \sup_{0 \leq t \leq 1} \sigma |B(t)| \tag{26}$$

and

$$\sup_{0 \leq s_1, s_2 \leq 1} |T^{1/2} s_1 s_2 \{\hat{d}^{(m_1)} - \hat{d}^{(m_2)}\} - \sigma B_n(t)| = O_p(T^{-1/2}) \tag{27}$$

with $s_1 = \dfrac{n_1}{n_1 + n_2}$, $s_2 = \dfrac{n_2}{n_1 + n_2}$ and $T = n_1 + n_2$ and theorem 1 is proved.

References

[1] Banarjee, A. and Urga, G., (2005), "Modelling structural break, long memory and stock market volatility: an overview", *Journal of Econometrics*, 129, 1-34.

[2] Beran, J. and Terrin, N., (1996), "Testing for a change of the long memory parameter", *Biometrika*, 83(3), 627-638.

[3] Bleher, M., (1981), "Inversion of Teoplz matrices", *Trans. Moscow Math. Soc. 1981*, 2, 201-229.

[4] Breidt, F.J. and Hsu, N-J., (2002), "A class of nearly long memory time series models", *International Journal of Forecasting*, 18, 265-281.

[5] Chambers, M.J., (1998), "Long memory and aggregation in macroeconomic time series", *International Economic Review*, 39(4), 1053-1072.

[6] Chen, C. and Tiao, G.C., (1990), "Random level-shift time series models, ARIMA approximations, and level-shift detection", *Journal of Business and Economic Statistics*, 8, 83-97.

[7] Engle, R. and Smith, A., (1999), "Stochastic permanent breaks", *Review of Economics and Statistics*, 81, 553-574.

[8] Fox, R. and Taqqu, M.S., (1987), "Central limit theorem for quadratic forms in random variables having long-range dependence", *Probab. Th. Rel. Fields*, 74, 213-240.

[9] Dieblod, F.X., and Inoue, A., (2001), "Long memory and regime switching", *Journal of Econometrics*, 105 , 131-159.

[10] Geweke, J. and S. Porter-Hudak, (1983), "The estimation and application of long memory time series models", *Journal of Time Series Analysis*, 4, 221-238.

[11] Granger, C.J.W. and Ding, Z., (1996), "Varieties of long memory models", *Journal of Econometrics*, 73. 61-77.

[12] Granger, C.J.W and Hyung, N., (2004), "Occasional structural breaks and long memory with an application to the S&P 500 absolute stock returns", *Journal of Empirical Finance*, 11(3). 399-421.

[13] Granger, C.W.J. and Terasvirta, T., (1999), "A simple nonlinear time series model with misleading linear properties", *Economics Letters*, 62(2), 161-165.

[14] Grugul, H. and Wojtowicz, T., (2006), "Long memory on the German stock exchange", *ECzech Journal of Economics and Finance*, 56(09-10), 447-468.

[15] Hassler, U. and Olivares, M., (2007), "Long memory and structural change: new evidence from German stock market returns", Working Paper.

[16] Henry, O.T., (2002), "Long memory in stock returns: some international evidence", *Applied Financial Economics*, 12, 725-729.

[17] Hiemstra, C. and Jones, J.D., (1997), "Another look at long memory in common stock returns", *Journal of Empirical Finance*, 4, 373-401.

[18] Horväth, L. and Shao, Qi-M., (1999), "Limit theorems for quadratic forms with applications to Whittle's estimate", *The Annals of Applied Probability*, 9(1), 146-187.

[19] Kuswanto, H. and Sibbertsen, P., (2007), "Can we distinguish between common nonlinear time series models and long memory?", *Working paper of Department of Economic and Management, Hannover University,* **380** .

[20] Kuswanto, H. and Sibbertsen, P., (2008), "A study on "spurious long memory in nonlinear time series models", *Applied Mathematical Science*, 2(55), 2713-2734.

[21] Lee, T. and Lee, S., (2007), "Test for parameter change in linear processes based on Whittle's estimator", *Communication in Statistics-Theory and Methods*, 36, 2129-2141.

[22] Lobato, I.N and Savin, N., (1998), "Real and spurious long-memory properties of stock-market data", *Journal of Business & Economic Statistics*, 16, 261-268.

[23] Lobato, I.N and Robinson, P.M, (2003), "A nonparametric test for I(0)", *Review of Economic Studies*, 65(3), 475-495.

[24] Man, K.S. and Tiao, G.C., (2006), "Aggregation effect and forecasting temporal aggregates of long memory processes", *International Journal of Forecasting*, 22(2), 267-281.

[25] Ohanissian, A., Russell, J.R. and Tsay, R.S., (2008), "True or spurious long memory? a new test", *Journal of Business & Economic Statistics*, 26(2), 161-175.

[26] Robinson, P.M., (1995), "Gaussian semiparametric estimation of long range dependence", *Annals of Statistics*, 23, 1630-1661.

[27] Sibbertsen, P., (2004a), "Long-Memory in Volatilities of German Stock Returns", *Empirical Economics*, 29, 477-488.

[28] Sibbertsen, P., (2004b), Long-memory versus structural change: An overview, *Statistical Papers*, 45, 465 - 515.

[29] Souza, L.R., (2007), "Temporal aggregation and bandwidth selection in estimating long memory", *Journal of Time Series Analysis*, 28(5), 701-722.

[30] Souza, L.R., (2008), "Spectral properties of temporally aggregated long memory processes", *Brazilian Journal of Probability and Statistics*, 22(2), 135-155.

[31] Teles, P., Wei, W.W.S. and Crato, N., (1999), "The use of aggregate series in testing for long memory", *Bulletin of International Statistical Institute*, 22nd Session Book 3, 341-342.

[32] Teles, P., Wei, W.W.S. and Hodges, E.M., (2008), "Testing unit root based on aggregate time series", *Communication in Statistics-Theory and Methods*, 37, 365-590.

[33] Tolvi, J., (2003), "Long memory in a small stock market", *Economics Bulletin*, 7(3), 1-13.

[34] Tsai, H. and Chan, K.S., (2005), "Quasi maximum likelihood estimation for a class of continuous time-long memory processes", *Journal of Time Series Analysis*, 26(5), 691-713.