

Patience and Prosperity

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Abstract. This paper introduces wealth-dependent time preference into a simple model of endogenous growth. The model generates adjustment dynamics in line with the historical facts on savings and economic growth in Europe from the High Middle Ages to today. Along a virtuous cycle of development more wealth leads to more patience, which leads to more savings and even higher wealth. Savings rates and income growth rates are thus jointly increasing during the process of development until they converge towards constants along a balanced growth path. During the transition to modern growth an economy in which the association of wealth and patience is stronger overtakes an otherwise identical economy and generates temporarily diverging growth rates. It is shown how wealth-dependent time preference can explain the existence of a locally stable poverty trap as well as the phenomenon of simultaneously falling interest rates and rising growth rates, and the phenomenon of overshooting of savings rates during transition towards balanced growth.

Keywords: economic growth, savings, time preference, poverty trap, moral consequences of economic growth.

JEL: O11, O41, D90, P48.

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Poverty bears down heavily on all portions of a man's expected life. But it increases the want for immediate income even more than it increases the want for future income.

(Irving Fisher)

He that can have patience, can have what he will.

(Benjamin Franklin)

1. INTRODUCTION

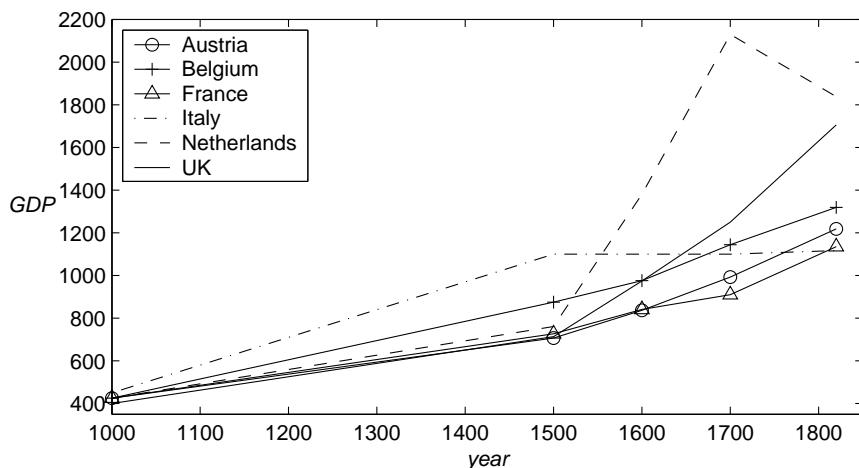
This article proposes a simple preference-driven theory of economic growth. It establishes a savings channel through which the character trait of patience operates simultaneously as cause and consequence of economic growth.

Ever since the work of Barro (1991) and of Mankiw, Romer and Weil (1992) empirical studies have found a strong correlation between savings rates and economic growth rates. While standard growth theory usually assumes savings as a cause of economic growth, there is now ample evidence for the reverse causality, i.e. the view of high savings rates as a consequence of high economic growth (see Edwards, 1996, Loayza et al., 2000, and the discussion of the literature in Carroll et al., 2000).

Savings rates are higher for richer people across individuals (Dynan et al, 2004) and for richer countries across countries (Loayza et al., 2000). Within countries, savings rates usually rise with economic development. In England, for example, investment rates have been estimated as 3-6% in 1688 (Dean and Cole), around 8% in 1761-70, and around 14% for 1791-1800 (Feinstein, 1981). In the year 2000 savings rates were above 25 percent in some of the world's richest countries (Heston et al., 2002). Likewise the historical evolution of economic growth rates has been gradual. From the Maddison (2001) data we compute the annual rate of GDP per capita growth for England as 0.0% from year 1 to 1000, 0.1% from 1000 to 1500, 0.2% from 1500 to 1700. In the 19th century growth gets momentum. It is computed as 0.3% from 1700 to 1820, 1.3% from 1820 to 1870, 1.0% from 1870 to 1913, 1.2 % from 1913 to 1960, and 2.1% from 1960 to 2000. Standard models of economic growth have problems with getting the gradual evolution of savings and growth rates right. For reasonable parameterization they predict too high savings and too high growth at low levels of income (Barro and Sala-i-Martin, 2004).

In order to take the empirical regularities into account we augment the well-known linear (Ak) growth model with endogenous, wealth-dependent preferences. The model exactly reverses the standard mechanism of development as established by neoclassical growth theory. Neoclassical growth theory assumes a constant rate of time preference rate (ρ) and produces the result that – starting from below steady-state – capital productivity decreases along the path of economic development. Consequently, it predicts that the rate of economic growth decreases with the level of development. With contrast, the theory proposed here assumes that capital productivity (A) is constant and produces the result that the rate of time preference adjusts from above as the economy develops, i.e. that people become more patient when they get richer. Consequently, and in line with the historical evidence, the model implies that the savings rate and the rate of economic growth increase with the level of development.¹

FIGURE 1. GDP per capita in Europe Year 1000-1820 (according to Maddison, 2003)



While the process of gradual take off was roughly the same everywhere in Western Europe (and the Western offshoots) there are also important differences discernable across countries. In particular there was a European “reversal of fortune”. Output in Italy, France, and Belgium, was higher than in England and the Netherlands until the 16th century while the opposite was true after the 17th century (see Figure 1). The present model explains the overtaking by cultural differences in the association between wealth and patience. When the development process gets momentum it is further amplified through increasing savings rates in countries where

¹ The proposed theory is not the only extension of the linear growth model that produces reasonable adjustment dynamics. The conclusion compares with the main alternatives, subsistence consumption (Steger, 2000, Strulik, 2009) and habit formation (Carroll et al., 2000).

the association between wealth and patience is stronger.² This way, small differences in time preferences can generate temporarily large cross-country differences in economic performance.

The rate of time preference is not a natural constant. As stated by Obstfeld (1990) “Mathematical convenience rather than innate plausibility has always been the main rationale for assuming time-additive preferences in economic modeling.” While this convenience is certainly welcome and justified for many research questions, it is particularly limiting when the focus of investigation is on the evolution of savings rates over time. As a result of this insight there exists already a literature of endogenous time preference to which the present paper relates. Starting with Uzawa (1968) important contributions have been made by Epstein and Hynes (1983), Obstfeld (1990), Drugeon (1996), Palivos et. al. (1997), and Das (2003).

The literature so far focussed mostly on consumption-dependent time preference within the neoclassical growth framework. A major and somehow counter-intuitive result is that, in general, stability of a steady-state requires a positive relationship between the time preference rate and the level of consumption.³ With contrast to the so far available literature the present article focusses on a negative association between time preference rate and wealth of the representative consumer and on the implied transitional dynamics towards balanced growth.

Recently, Schumacher (2009) integrated wealth-dependent time preference into the neoclassical growth model and demonstrated the possibility of multiple equilibria of stagnation. His article shares probably the most with the present one, in particular with respect to extension of the present model in Section 7. The main difference there is that the model of the present paper generates multiple equilibria of stagnation and perpetual growth. Moreover, using wealth-dependent preferences in order to explain the slow transition towards balanced growth, the secular rise (an possibly, later on, fall) of the savings rate during growth, and the process of catch-up and overtaking of countries during industrialization are all unique to the present paper.

Becker and Mulligan (1997) have proposed a theory of endogenous time preference according to which individuals can invest resources in order to make future pleasures less remote by imagining them today. Their model produces as a result that richer individuals are more patient and may serve as a micro-foundation for a positive association between wealth and time preference.

²For example, these countries will be more open towards doctrines that reward savings vis a vis current consumption (the Protestant belief system, Weber, 1904).

³Das (2003) has shown that stability of dynamic equilibrium can be obtained if the rate of return to capital falls faster than the rate of patience.

Empirically it has been found that the time preference rate decreases with income and wealth in panel data analysis (Lawrence, 1991, Samwick, 1998, Ventura, 2003) and large-scale experiments (Dohmen et al., 2009). Unfortunately, many empirical studies on intertemporal consumption behavior impose a constant time-preference rate. In that case we expect the fact of a positive association between wealth and the time preference rate to be reflected in the estimated intertemporal elasticity of substitution. Studies that control for wealth effects have indeed found that the intertemporal elasticity of substitution is higher for richer individuals (e.g. Atkeson and Ogaki, 1996, Guvenen, 2006).

Wealth-dependent time preference may be explained – besides Becker and Mulligan’s rational choice approach – as a result of a positive association between wealth and institutions, i.e. through the risk of expropriation or between wealth and health, i.e. through life expectancy (Pritchett and Summers, 1996). Both channels, either indirectly or directly, suggest that people are more patient when they populate countries with lower diversity of disease species, i.e. countries of high latitude (Masters and McMillan, 2001, Olsson and Hibbs, 2005). While not denying a direct impact of institutions or health on growth the present paper focusses on the indirect effect that operates through patience on savings and predicts that, *ceteris paribus*, countries at high latitude take off earlier from stagnation to growth (industrialize earlier) than countries in the tropics. The extended model (Section 6 and 7) can furthermore produce the result that an economy populated by impatient agents stagnates while an otherwise identical economy of more patient agents converges towards balanced growth.

In his book on the evolutionary aspects of long-run economic development Clark (2007) observes that people became substantially more patient before the onset of the industrial revolution. He discards improvements of institutions and life-expectancy for being not sufficient to having provoked such a substantial drop of time preference and provides instead an evolutionary argument based on Rogers (1994). According to this view the institutionally stable agrarian societies of the Middle Ages created a selective pressure towards lower time preference. More patient people were wealthier which allowed them to have more surviving children and to bequeath more wealth to them, a fact that amplified the genetic transmission of patience.⁴ Doepke and Zilibotti (2008) argue in favor of a related yet conceptually different mechanism. There, patience is transmitted through cultural evolution. Patient and thus economically successful

⁴Galor and Michalopoulos (2006) have developed an evolutionary model of entrepreneurial spirit based on risk aversion.

parents invest more resources into transferring the character trait of patience to their offspring. Using cultural evolution Doepke and Zilibotti are able to explain why an exogenous change of production technology (an industrial revolution) induced an endogenous change of fortune of different social classes.

The theory proposed in the present article is much less involved than the story of Clark or the model of Doepke and Zilibotti. It can, nevertheless, be related to their work. For that purpose it is shown in the Appendix that the infinitely living representative agent can be understood as an approximation of altruistically linked generations of finite life. The modeling of wealth-dependent time-preference then implies that more wealthy parents discount utility of their offspring less strongly and thus bequeath more. This, in turn, will induce their offspring to put even less emphasis on own present consumption and further amplify the intergenerational transmission of wealth. In other words, future generations will be richer and will appreciate more economic growth, i.e, consumption increases experienced by their descendants.⁵ This way, the present theory explains the industrial revolution as an endogenous event that occurred when the interaction between patience and growth attained its highest momentum. The endogenous economic transition from stagnation to balanced growth is shared with unified growth theory (Galor, 2005). However, demographic evolution, a major ingredient of unified growth theory, is not investigated here, a shortcoming that will be addressed in the Conclusions.

The present work is also related to Friedman (2005) who argues (verbally) that people behave morally better when they have “a sense of getting ahead”. With respect to one character trait the present theory is able to prove Friedman’s claim. It will be shown that from the first order conditions for optimal savings behavior follows an unambiguously positive effect of economic growth on patience.

2. THE BASIC MODEL

Consider an economy populated by a mass one of individuals consuming c and holding wealth k . For now we assume that production operates a linear technology, $y = Ak$, where A denotes productivity (net of depreciation). Later on we introduce a generalized form of the production function. The constant productivity of k implies that there is potential for accumulation-driven

⁵Such an intergenerational interpretation is supported by Dynan et al. (2004) who find that richer people save more when young but do not dissave more in old age. The evidence thus rejects the simple life cycle model and supports altruistically linked generations, which are here in a simplifying manner modeled as an infinitely living representative agent.

growth if A exceeds the time preference rate. As usual the model becomes more realistic when we interpret k broadly consisting of physical and human capital. In order to be brief we refer to k in the following as “capital” and to its accumulation as “saving”. The capital stock thus evolves according to

$$\dot{k} = Ak - c. \quad (1)$$

As usual individuals maximize discounted intertemporal utility from consumption. In deviation from the standard model the discount rate depends positively on wealth. As individuals get richer they become more patient.

$$\max_c V_0 = \int_0^{\infty} u(c) e^{-\int_0^t \rho(k(v)) dv} dt \quad (2)$$

with $\rho(k) > 0$, $\rho'(k) < 0$, $\rho''(k) > 0$ and $\lim_{k \rightarrow \infty} \rho(k) = \bar{\rho} \geq 0$, which is a formal representation of the idea that individuals become more patient as they get wealthier and that the time preference rate approaches a constant from above as wealth goes to infinity.

Since we are considering economic dynamics over the very long-run, the appropriate notion of the setup is as a continuous time approximation of an infinite series of altruistically linked generations. In order to emphasize this point, the Appendix 1 shows how problem (1) – (2) can be derived from the expansion of a two-period model of altruistically linked individuals with finite life. As will become clear below, the setup in continuous time is useful in order to develop an elegant representation of the economy that facilitates the analysis of stability of equilibria and transition dynamics.

Instantaneous utility is assumed to be iso-elastic, $u(c) = c^{1-\theta}/(1-\theta)$, $\theta \neq 1$. Besides the positive correlation between wealth and time preference the model is thus identical to the well-known textbook version of the Ak growth model (see, for example Barro and Sala-i-Martin (2004)). In fact, the textbook version is included here as the limiting case when $\rho(k) = \bar{\rho}$. In order to allow for positive long-run growth we assume $A > \bar{\rho}$.

For comparison it is helpful to recall some characteristic features of the standard model. It implies a unique long-run equilibrium which is assumed independently from initial endowments and it predicts a constant savings rate and a constant growth rate $g_c = (A - \bar{\rho})/\theta$ at all levels of development (for all k). With contrast, according to the standard *neoclassical* growth model the growth rate $g_c = [r(k) - \bar{\rho}]/\theta$ is at its highest level initially, when wealth k is smallest. With ongoing wealth accumulation, the growth rate is continuously falling because of decreasing

returns on capital, $r'(k) < 0$. Thus both, the standard Ak model and the neoclassical growth model cannot explain the historical adjustment dynamics sketched in the Introduction, i.e. the gradual take off with increasing growth rates over time. It is intuitive to see how the present modification of the model gets the historical adjustment dynamics right by holding capital productivity A constant and showing that $\rho(k)$ decreases with increasing wealth accumulation such that growth increases with ongoing wealth accumulation. Later on, in Section 5, it will be shown that this result is robust against the introduction of the neoclassical mechanism of $r'(k) < 0$. For now we focus on the case of constant capital productivity.

In order to solve the problem of maximizing (2) with respect to (1) we parameterize the time integral and define $q \equiv \int_0^t \rho(k(v))dv$ such that $dq/dt = \rho(k)$. The time-transformed problem reads

$$\max_c \int_0^\infty \frac{u(c)}{\rho(k)} e^{-q} dq \quad s.t. \quad \frac{dk}{dq} = \frac{Ak - c}{\rho(k)}.$$

Inserting the isoelastic $u(c)$ form, the current-value Hamiltonian reads $H = c^{1-\theta} / [(1-\theta)\rho(k)] + \mu(Ak - c) / \rho(k)$ with costate variable μ . The optimal solution fulfils the first order conditions

$$\frac{\partial H}{\partial c} = \frac{c^{-\theta}}{\rho} - \frac{\mu}{\rho} = 0 \tag{3a}$$

$$\frac{\partial H}{\partial k} = \frac{c^{1-\theta}}{1-\theta} \left(\frac{-\rho'}{\rho^2} \right) + \mu \frac{A\rho(k) - \rho'(Ak - c)}{\rho^2} = \mu - \frac{d\mu}{dq} \tag{3b}$$

and the transversality condition $\lim_{q \rightarrow \infty} \mu k e^{-q} = 0$.

In solving for optimal consumption we begin with reintroducing actual time by substituting $dq = \rho dt$. Multiplying (3b) with ρ , subdividing by $\mu = c^{-\theta}$, and inserting $\dot{\mu}/\mu = -\theta\dot{c}/c$ obtained from log-differentiating (3a) with respect to time, we derive the growth rate of consumption (4).

$$\frac{\dot{c}}{c} = \frac{1}{\theta} \left[A - \rho - \frac{\rho'}{\rho} \frac{c}{1-\theta} - \frac{\rho'}{\rho} (Ak - c) \right]. \tag{4}$$

For $k \rightarrow \infty$ and thus $\rho \rightarrow \bar{\rho} > 0$, $\rho' \rightarrow 0$, the result collapses to the familiar Ramsey rule $\dot{c}/c = (A - \bar{\rho})/\theta$. Compared with the standard model we get two additional effects of wealth-dependent time preference on growth: a level effect and a growth effect.

The level effect is reflected by the term $(-\rho'/\rho)c/(1-\theta) = (-\rho'/\rho)u(c)/u'(c)$. Since $\rho' < 0$ the term is negative if $u(c) < 0$. The level effect is reminiscent of the earlier literature on endogenous time preference where $u(c) < 0$ is frequently assumed in order to generate stability (see e.g. Obstfeld, 1990). Here, the level effect is irrelevant for stability and convergence towards balanced

growth, as will be shown below. For the case of $\theta > 1$ utility is indeed always negative with a bliss point at zero such that the level effect vanishes with economic growth. If $\theta < 1$ the level effect is positive and increasing with economic growth through the $u(c)/u'(c)$ term but decreasing with growth through the $(-\rho'/\rho)$ term because ρ' vanishes with wealth accumulation. It will be shown below that this interaction can generate an interesting overshooting behavior of the savings rate along the transition towards balanced growth.

To see that the last term in (4) reflects a growth effect on patience recall that $\dot{k} = Ak - c$. The growth effect is unambiguously positive. Living in a world of high growth makes people, *ceteris paribus* more patient. As a consequence they choose a steeper profile for consumption growth (\dot{c}/c is higher) implying that they save more. The growth channel implies that patience can be simultaneously conceptualized as a cause and consequence of economic growth. The model thus supports the observation that higher growth causes higher savings. It relates also to Friedman's (2005) "Moral consequences of Economic Growth". When people have a sense of "getting ahead" they appreciate more potential future gains of foregone consumption and are inclined to save more today, a behavior that further amplifies economic growth.

The fact that $\dot{\rho} = \rho'\dot{k}$ allows to write the partial differential equation (4) as an ordinary differential equation as in (5).

$$\frac{\dot{c}}{c} = \frac{1}{\theta} \left[A - \rho - \frac{\dot{\rho}}{\rho} \frac{c}{1 - \theta} \frac{1}{Ak - c} - \frac{\dot{\rho}}{\rho} \right]. \quad (5)$$

In order to get an explicit solution we introduce a simple functional form for the evolution of time preference.

$$\rho(k) = \bar{\rho} + \rho_0 k^{-\eta} \quad \Rightarrow \quad \dot{\rho} = (\rho - \bar{\rho}) \frac{\dot{k}}{k}. \quad (6)$$

The parameter $\eta > 0$ controls the speed at which time preference converges towards $\bar{\rho}$ when the economy is growing. Note that ongoing wealth accumulation has no effect on time preference once ρ has reached $\bar{\rho}$. This feature will cause balanced growth. Since there exists a natural lower bound for time preference at zero, the condition for balanced growth is a natural one.⁶

For equilibrium analysis let x denote the consumption capital ratio, $x \equiv c/k$. Behavior of the economy can then be conveniently discussed in the (ρ, x) space where x is the quasi-control variable, and ρ is the quasi-state variable because time preference, tied down by the state of the

⁶Evolutionary economists argue that the natural lower bound of time preference supported by nature is around 0.02, see, for example, Rogers(1994).

economy k , cannot jump instantaneously. The reduced form of the economy reads

$$\dot{\rho} = -\eta(\rho - \bar{\rho})(A - x). \quad (7a)$$

$$\dot{x} = \frac{x}{\theta} \left[A - \rho + (\rho - \bar{\rho}) \frac{\eta}{\rho} \left(A - x + \frac{x}{1 - \theta} \right) \right] - (A - x)x. \quad (7b)$$

In order to derive (7a) insert (1) and the definition of x into (6). In order to derive (7b) note that the consumption capital ratio grows at rate $\dot{x}/x = \dot{c}/c - \dot{k}/k$ and insert eqs. (5), (1), and (6). Given an initial state $k(0)$ and thus $\rho(0)$, the evolution of the economy is completely described by the trajectory of (7a) and (7b) that fulfills the first order conditions and the transversality condition.

3. EQUILIBRIUM ANALYSIS

PROPOSITION 1. *There exists a unique steady-steady-state of stagnation at*

$$x = x^* = A \quad (8)$$

$$\rho = \rho^* = \frac{A + \phi}{2} + \sqrt{\frac{(A + \phi)^2}{4} - \phi\bar{\rho}}, \quad \phi \equiv \eta A / (1 - \theta).$$

Proof. Inspect (7a) to see that ρ is constant when $x = x^* = A$ implying $c^* = Ak^*$. Insert $x = A$ into (7b) and solve for ρ where $\dot{x} = 0$ to obtain the roots $\rho_1 = (A + \phi)/2 - \sqrt{(A + \phi)^2/4 - \phi\bar{\rho}}$ and $\rho_2 = (A + \phi)/2 + \sqrt{(A + \phi)^2/4 - \phi\bar{\rho}}$. For existence of a real solution $(A + \phi)^2/4 > \phi\bar{\rho}$, i.e. $A^2 + 2A\phi + \phi^2 > 4\phi\bar{\rho}$. Since $A > \bar{\rho}$ a sufficient condition for this to hold is $A^2 + 2A\phi + \phi^2 > 4\phi A$, which can be written as $(A - \phi)^2 > 0$ and is thus fulfilled. Since the minimum ρ that can be reached with non-negative k is $\bar{\rho}$, at least one solution has to be larger than $\bar{\rho}$. From $\rho_2 > \bar{\rho}$ we get $\sqrt{(A + \phi)^2/4 - \phi\bar{\rho}} > \bar{\rho} - (A + \phi)/2$, which can be written as $A - \bar{\rho} > 0$ and is thus fulfilled. For uniqueness the other root has to be unfeasible, i.e. $\rho_1 < \bar{\rho}$. Proceeding as above this can be written as $A - \bar{\rho} > 0$ and is thus fulfilled. The unique solution is at $\rho^* = \rho_2$. From $x = A$ follows $c = Ak$ and thus $\dot{k} = 0$, i.e. stagnation. \square

PROPOSITION 2. *If $\bar{\rho}/A > 1 - \theta$, then there exists a unique balanced growth path along which the economy grows at rate $(A - \bar{\rho})/\theta$ and where*

$$x = x^{**} = \frac{(\theta - 1)}{\theta} A + \frac{\bar{\rho}}{\theta} \quad (9)$$

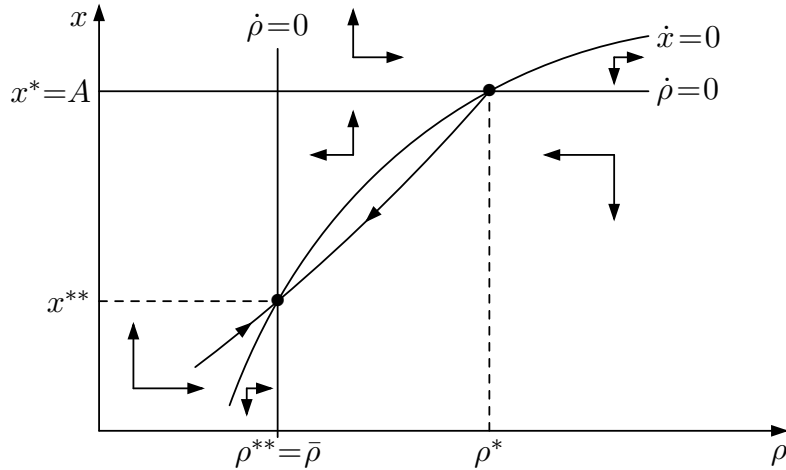
$$\rho = \rho^{**} = \bar{\rho}.$$

Proof. Inspect (7a) to see that ρ is constant when $\rho = \bar{\rho}$. Insert this information into (7b) and solve for x where $\dot{x} = 0$ in order to obtain the unique x^{**} . Verify that x^{**} is positive for $\bar{\rho}/A > 1 - \theta$. Insert $x = x^{**}$ into $\dot{k}/k = A - x$ to get the balanced growth rate. \square

The condition for balanced growth to exist is the same as for the standard Ak -model. It requires that utility is bounded such that x^{**} assumes a constant positive value (cf. Barro and Sala-i-Martin, 2004). Naturally, since ρ converges towards a constant, the balanced growth path is identical with the one obtained for the standard Ak growth model. The value added of the present model does not lie in its steady-state characteristics but in the new transitional dynamics.

PROPOSITION 3. *The equilibrium of stagnation is unstable and the balanced growth path is a saddlepoint. The optimal solution consists of a unique path along which the economy converges towards the balanced growth. During the transition the time preference rate decreases and the savings rate increases.*

Figure 2: Phase Diagram



Proof. In order to analyze transitional dynamics and stability of equilibria within a phase diagram infer from (7a) that there are two loci where $\dot{\rho} = 0$. In a (ρ, x) - diagram, the equilibrium of stagnation is found along the horizontal isocline where $x = A$. The equilibrium of balanced growth is found along the vertical isocline where $\rho = \bar{\rho}$. Inspect (7a) to conclude that $\dot{\rho} < 0$ in the area to the right of the $\bar{\rho}$ -line and below the A -line. The opposite is true if x is above

A and when ρ is below $\bar{\rho}$. Next compare x^* and x^{**} and conclude that $x^{**} < x^*$. From this follows that the $\dot{x} = 0$ -curve has a negative slope. Its intersection with the horizontal $\dot{\rho} = 0$ -isocline identifies x^* and the intersection with the vertical $\dot{\rho} = 0$ -isocline identifies x^{**} . Compute $\partial(\dot{x}/x)/\partial x = 1 + \eta(\rho - \bar{\rho})/(1 - \theta)$ from (7b) and conclude that this expression is positive for $\theta < 1$. In order to obtain the arrows of motion for $\theta > 1$ rewrite (7b) as

$$\theta \frac{\dot{x}}{x} = \underbrace{A(1 - \theta) + \theta x - \rho}_b + \underbrace{[A(1 - \theta) + \theta x]}_d \cdot \underbrace{\frac{(\rho - \bar{\rho})\eta}{(1 - \theta)\rho}}_{<0}.$$

Observe that $b < d$. Next note that at $x = A$, $d = A > 0$ and thus for $\theta > 1$, $b > 0$ for $\dot{x} = 0$ at x^* . On the other hand, at $x = x^{**}$, $b = 0$ and thus $d > 0$. Conclude $d > b \geq 0$ in the relevant range of x . Finally note that the derivative $\partial(\dot{x}/x)/\partial x$ obtained along the $\dot{x} = 0$ curve can be written as $1 - b/d$, which is positive since $d > b \geq 0$. Thus the arrows of motion point towards lower x below the $\dot{x} = 0$ curve and towards higher x above.

Inspect the resulting field of arrows of motion in Figure 1 to conclude that the equilibrium (x^*, ρ^*) is unstable and the equilibrium (x^{**}, ρ^{**}) is a saddlepoint. Suppose we start an economy at the equilibrium of stagnation and perturb it a little. The only feasible path is then to follow the stable arm of the saddlepoint towards (x^{**}, ρ^{**}) . Any other path leads either to $x = 0$ and thus zero consumption in finite time or to ever expanding x and thus zero capital stock in finite time. The paths leading to $k > 0$ but $c = 0$ and thus $\mu = \infty$ in finite time entail a violation of the transversality condition. Intuitively, savings react suboptimally strong on improving wealth. The paths leading to $k = 0$ in finite time entail a jump of c to zero, a violation of the first order condition (requiring μ to be continuous). Intuitively, people save too little in order to sustain survival in the long-run.

The savings rate is defined as $s = 1 - c/y$, i.e. $s = 1 - x/A$. It is thus rising along the stable manifold that leads from stagnation towards balanced growth. \square

For an intuitive assessment of adjustment dynamics consider an economy that is initially very close to the steady-state of stagnation. In this situation capital stock and production are small and the time preference rate is large. Almost all income is consumed implying that c is close to Ak . This in turn means that there are almost no (net) savings and \dot{k} is close to zero. Having only a relatively short time-window of observations at hand one would conclude that the economy stagnates. At glacier speed, however, capital is accumulated and people become more patient

with rising wealth. More patience leads to a higher savings rate (lower x) and further rising capital (wealth). Sooner or later economic growth gets momentum and the economy travels with rising savings rates towards the balanced growth path. During the transition both the savings rate and the growth rate are perpetually rising. The feature that produces this realistic outcome is wealth-dependent time preference. Higher economic growth makes people wealthier and thus more patient, which in turn triggers more savings and leads to even higher subsequent growth.

4. THE SLOW TRANSITION TOWARDS MODERN GROWTH: A CALIBRATION STUDY

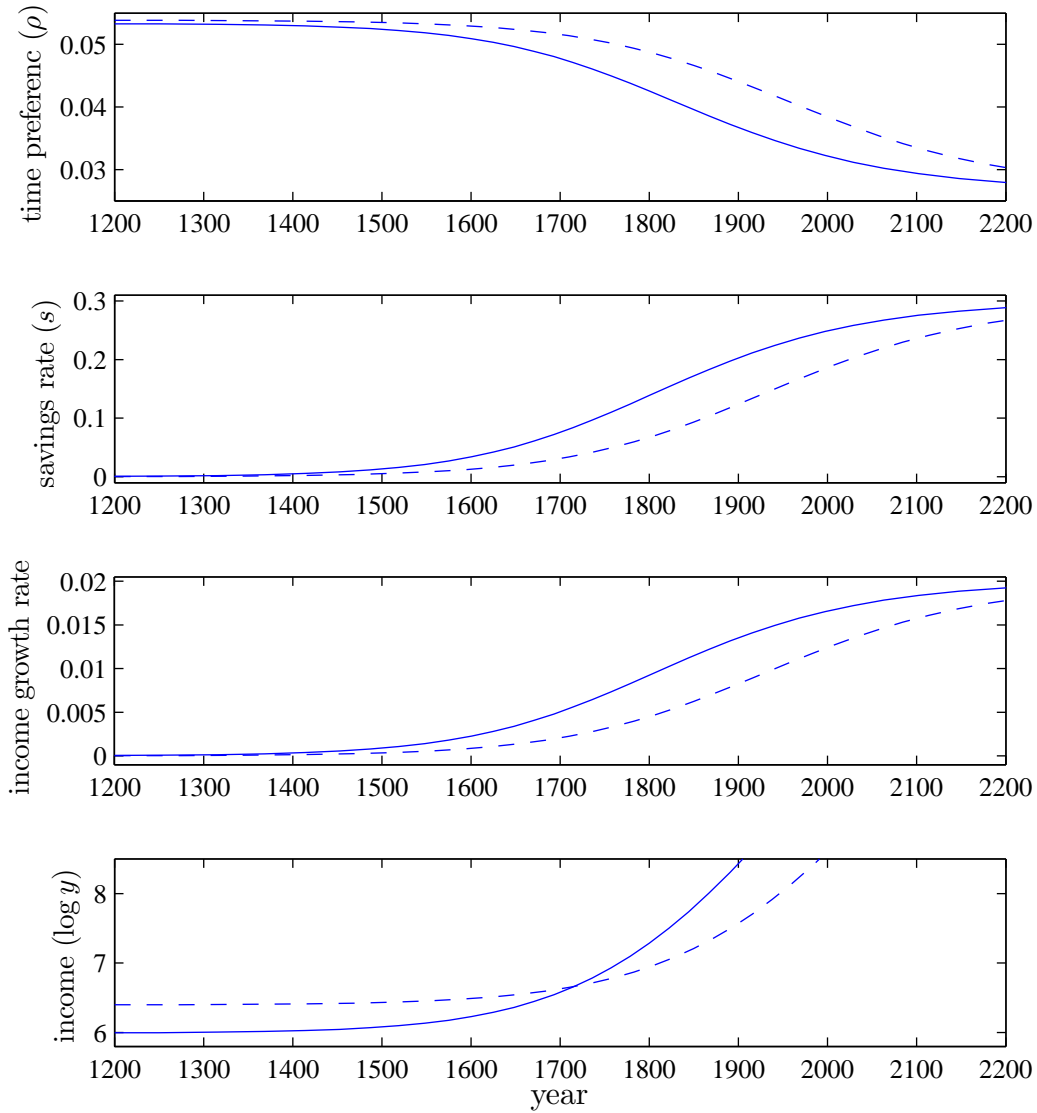
We begin with determining the balanced growth path. The savings rate, $s = 1 - c/y$, along the balanced growth path is $s^{**} = 1 - x^{**}/A$ and the implied growth rate is $(A - \bar{\rho})/\theta$. For the calibration we assume that the balanced growth rate is 2 percent annually and the consumption share of income is 70 percent, implying a savings rate of 0.3. Imposing $\theta = 2$, a frequently used value in calibration exercises, provides the estimates $A = 0.0667$ and $\bar{\rho} = 0.0267$. A real rate of return on capital around 7 percent accords well with the average real return on the stock market for the last century and has been used in other calibration studies as well (e.g. Jones and Williams, 2000).

The remaining parameter, η , can be used to control transitional dynamics. Roughly speaking, η determines how strongly patience reacts on changes of wealth. Here, the value of η is adjusted such that the dynamic system generates approximately the historical evolution of savings and income as observed for England, i.e. such that an economy that starts with almost zero growth in the High Middle Ages reaches its highest acceleration of growth in the 19th century. This leads to the estimate of $\eta = 0.4$.

In order to extract the path from stagnation towards balanced growth we employ the method of backward integration. The integration starts when the economy is arbitrarily close to the steady-state of growth $(\bar{\rho}, x^{**})$ and solves the dynamic system (7a) and (7b) backwards until it comes close to the steady-state of stagnation (ρ^*, A) . The calculation is terminated when $\rho = 0.999\rho^*$. A reversion of time finally renders the adjustment trajectories of the actual economy. The method solves the non-linearized system up to an arbitrarily small error, see Brunner and Strulik (2002) for details.

Figure 2 shows adjustment dynamics. For better comparison with the real data, time has been normalized such that $t = 1800$ when $s = 0.14$ (according to Feinstein, 1981) and income

FIGURE 2. Transitional Dynamics During the 2nd Millennium



Parameters $A = 0.0667$, $\theta = 2$, $\bar{\rho} = 0.0267$. Solid lines: $\eta = 0.4$ (“Model-England”). Dashed lines: $\eta = 0.38$ (“Model-Italy”).

has been normalized such that $y = 400$ when $t = 1200$ (according to Maddison, 2001). Let us first consider the benchmark case reflected by solid lines. The adjustment process towards modern growth is characterized by little growth in the middle ages. In line with the historical evidence, growth rates are below 0.005 percent before year 1500 and about 0.2 percent around the year 1600. Afterwards, the economy visibly takes off.

While the take-off is visible, compared with modern standards the rate of economic growth is nevertheless small during this period. In line with the empirical observations the model produces

a growth rate of about 0.5 percent around 1700, of about 1 percent around 1800 and of about 1.3 percent in 1900. In the early 19th century, when historically the industrial revolution takes place, the model produces the highest momentum, i.e. the *the rates of change* of growth, patience, and savings are the highest. In accordance with the Maddison data, by the year 1900 income has increased to 4492 ($\log(4492) = 8.41$) starting from 400 in the year 1200 ($\log(400) = 5.99$). In the late 19th century the acceleration of growth begins to lose momentum while the savings rate continues to rise. At the end of the 20th century the growth rate and savings rate are close to their long-run steady-state levels.

The duration of the transition and the qualitative behavior of adjustment dynamics depends sensitively on the assumed strength of the wealth–patience association. This can be demonstrated exemplarily by an alternative scenario in which $\eta = 0.38$ (instead of 0.40). All other parameters are taken from the basic run, implying that the alternative economy arrives eventually at the same balanced growth path as Model-England.

The fact that η is smaller for the alternative economy implies that its citizens are a little less patient not only along the transition path but also, in particular, close to the equilibrium of stagnation. Formally, ρ^* is higher at stagnation. This implies that the propensity to consume is somewhat higher in the less patient economy, i.e. $c'(k)$ is larger at all levels of k . Because $c''(k) < 0$ and $c/k = A$ at the steady-state of stagnation, it then follows that citizens of the alternative economy have to be somewhat wealthier in the neighborhood of the stagnation. In order to keep the analogy to the historical development of countries, we call the alternative, less patient economy “Model-Italy”.

The implied adjustment dynamics are shown by dashed lines in Figure 3. Assuming that both economies are close to stagnation in the middle ages, Model-Italy, because of the mechanism just explained, starts out with higher income per capita. However, in the 16th century when patience begins to rise visibly in Model-England, it stays almost constant for another century in Model-Italy. The character trait of being more patient and thrifty enables Model-England to overtake Model-Italy’s income per capita in the late 17th century, in line with the historical observation (see Figure 1).

In other words, take-off and industrial revolution are delayed in less patient Model-Italy. Levels of patience, savings, and growth that are reached by Model-England in 1800 are reached by Model-Italy about a century later. During the 19th century, the period of the industrial

revolution, the gap of growth rates between Model-England and Model-Italy gets largest and it closes slowly afterwards. Actually the model underestimates the catching up process of Model-Italy. Since it has been shown that the model predicts the evolution of income per capita for England quite precisely, this occurs because the model underestimates growth of Model-Italy during the 20th century. For the year 2000 it predicts income per capita of about 5000, a value that was actually reached in 1957 (according to Maddison, 2003), indicating that other, neglected mechanisms are increasingly at work that limit the predictive power of the simple model for modern economies (e.g. globalization, international capital flows).

5. DECLINING SAVINGS RATES IN THE 20TH CENTURY

The compound determination of consumption growth in (4) allows for an alternative, or perhaps complementing, but certainly new explanation for the secular fall of savings rates observed over the last decades in some developed countries, notably the U.S. and the U.K. This fact has puzzled economists for quite a while. So far as explanations have been suggested wealth changes, the changing age distribution, financial innovations and intergenerational fiscal transfers. Parker (1999) finds that financial innovation can explain about a third of the decline of savings and wealth changes about a fifth, implying that there exists so far no encompassing explanation of the phenomenon.

Inspection of (4) has shown that the standard effect through net interest rates (the first term $A - \rho$) and the growth effect (the third term, given by $-\rho'/\rho(Ak - c)$) are both increasing in a growing economy and – taken for itself – suggesting that consumption growth is unambiguously increasing during economic development. For $\theta > 1$ the level effect reflected by the second term ($-\rho'/\rho c/(1 - \theta)$) is negative and in absolute magnitude declining with economic development (increasing level of consumption). It thus operates in the same direction as the other two effects.

For $\theta < 1$, however, the second term is *positive* and declining with economic development. Intuitively, and analogously to the mechanism in the standard model, a low θ implies a high elasticity of intertemporal substitution. This means that people care relatively little about current consumption, even when its level is yet low, and are willing to save relatively more for the future. It implies also that for a growing economy the level effect operates in the reverse direction compared to the net interest effect and the growth effect. It opens the possibility of overshooting behavior of the consumption growth rate and the savings rate.

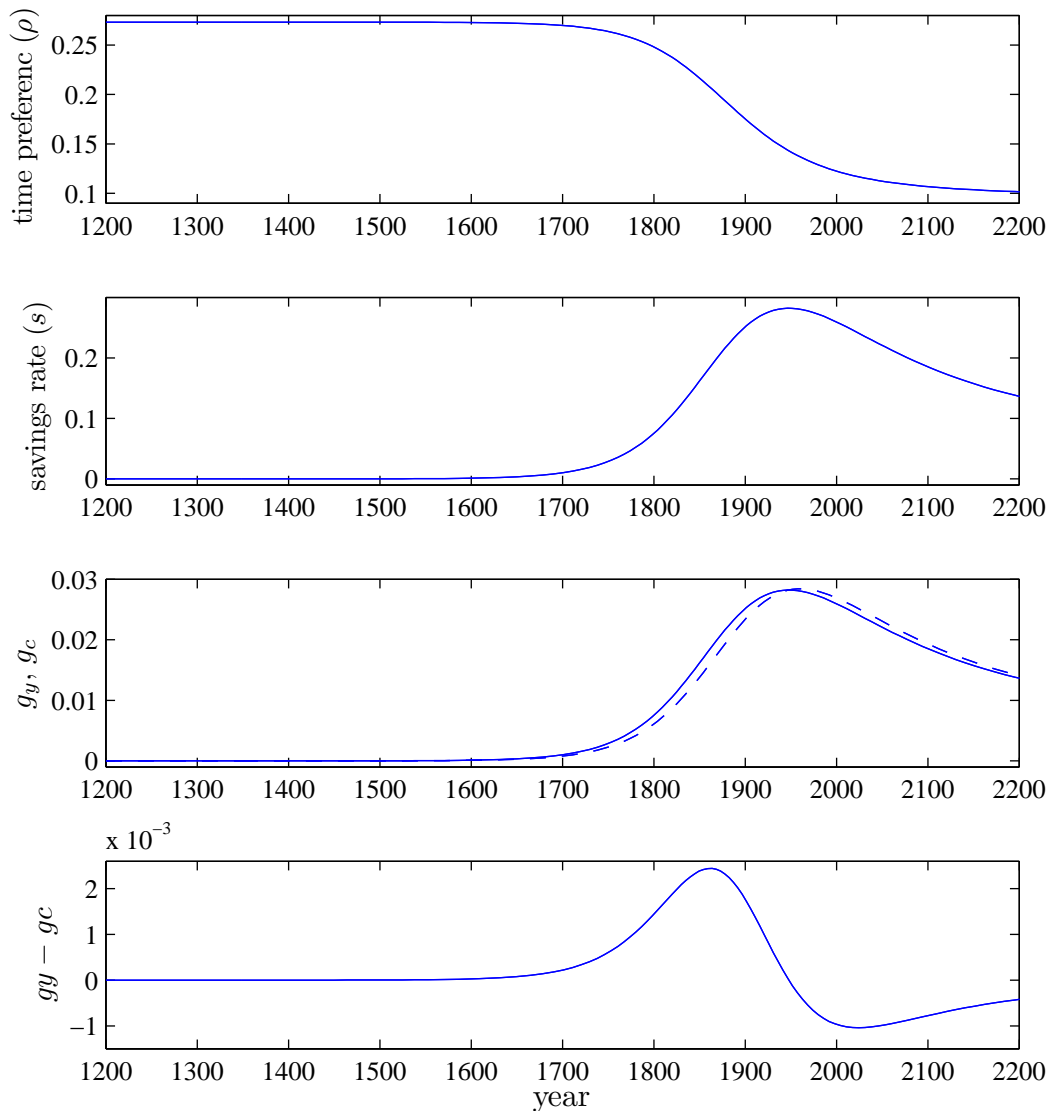
At first sight the case of $\theta < 1$ might seem to be unrealistic since econometric studies (for example Atkeson and Ogaki, 1997) as well as macroeconomic considerations (Barro and Sala-i-Martin, 2004) suggest that the elasticity of intertemporal substitution is quite small, implying that θ is large and certainly above one. This conclusion, however, has been derived under the assumption that the time preference rate is constant. As will be shown in this section (and later on in Section 6 for the neoclassical case of diminishing returns of capita), $\theta < 1$ is compatible with slow consumption growth in a reasonable order of magnitude if the time preference rate varies endogenously.

Figure 3 shows the evolution of an economy with savings rate overshooting. For that purpose we have to assume lower balanced growth rates for s and g_c . Otherwise the model would predict unreasonably high savings and growth along the transition during the overshooting phase. Here we assume of $s^* = 0.005$ and $g_c^* = 0.005$. Keeping $\eta = 0.4$ from the basic run, a plausible hump is generated for $\theta = 0.85$. These values imply $A = 0.1$ and $\bar{\rho} = 0.096$.

The model is now capable to explain a peak of the savings rate during the 1950s. Afterwards the savings starts to decline. The implied consumption share of income rises by about 4 percentage points during the second half of the 20th century (underestimating somewhat the real increase, which was about 6 percentage points, see Parker, 1999). The third panel shows the implied growth rates of income (solid lines) and consumption (dashed lines). Both growth rates approach their maximum in the 1950s. In line with the empirical observation the model thus produces the result that after the 1950s both the savings rate *and* the growth rate of consumption decline. In other words, the savings rate declines because the growth rate of income declines more strongly than the growth rate of consumption.

The fourth panel tries to visualize this fact more precisely by showing the difference between income growth and consumption growth. It reveals another interesting result. The difference between income growth and consumption growth was largest around 1850, during the heyday of the industrial revolution. At that time the absolute size of growth rates was moderate from today's perspective. Growth rates of income and consumption reach their maximum about a century later, in the 1950s. Afterwards income growth declines more strongly than consumption, a trend that is again reverted in the early 21st century such that the growth rate differential ($g_y - g_c$) approaches its steady-state value of zero from below.

FIGURE 3. Overshooting Savings Rate



Parameters $A = 0.1$, $\theta = 0.85$, $\eta = 0.4$, $\bar{\rho} = 0.096$. The third panel shows the income growth rate g_y (solid lines) and the consumption growth rate g_c (dashed lines). The fourth panel shows the difference $g_y - g_c$.

6. THE GENERAL CASE

While the model so far manages to generate reasonable paths for savings and growth it fails to predict the downward trend of interest rates documented by Clark (2007). This shortcoming is an inevitable yet unwanted side-effect of the linear production function, which provides a forever constant return to capital. In order to get the interest rate dynamics right we generalize towards a Jones-Manuelli (1990) type of production function. The notation follows largely the exposition of this model in Barro and Sala-i-Martin (2004). Production is assumed now to

consist of a neoclassical part with diminishing returns and – as before – of a linear part with constant returns, i.e. $y = Ak + Bk^\alpha \ell^{1-\alpha}$ where ℓ is labor input. The implied interest rate and wage rate are $r = A + \alpha Bk^{\alpha-1} \ell^{1-\alpha}$ and $w = (1 - \alpha)Bk^\alpha \ell^{-\alpha}$. The budget constraint for households is given by $\dot{k} = rk + w\ell - c$.

If we imagine that manufacturing operates with constant returns to scale with respect to the accumulable factors (physical and human capital) while agriculture is subject to diminishing returns because of limiting land, then the setup can be interpreted as a toy model of industrialization. In this sense the indicator $\lambda \equiv Bk^\alpha/y$ measures the relative contribution of the diminishing-returns part (i.e. agriculture) to GDP.⁷

Assuming that households supply one unit of labor inelastically, maximizing (2) w.r.t. $\dot{k} = rk + w\ell - c$, and solving the problem as proposed for the benchmark model leads to consumption growth according to the following Ramsey rule.

$$\frac{\dot{c}}{c} = \frac{1}{\theta} \left[r - \rho - \frac{\rho'}{\rho} \frac{c}{1-\theta} - \frac{\rho'}{\rho} (rk + w - c) \right].$$

Inserting equilibrium factor prices we obtain consumption growth (10), which replaces (4).

$$\frac{\dot{c}}{c} = \frac{1}{\theta} \left[A + \alpha Bk^{\alpha-1} - \rho - \frac{\rho'}{\rho} \frac{c}{1-\theta} - \frac{\rho'}{\rho} (Ak + Bk^\alpha - c) \right]. \quad (10)$$

For $\rho = \bar{\rho}$ this expression collapses to the Ramsey rule derived from the standard Jones-Manuelli setup (as discussed in Barro and Sala-i-Martin, 2004).

Inserting factor prices into the budget constraint we get the equation of motion (11), which replaces (1).

$$\dot{k} = Ak + Bk^\alpha - c. \quad (11)$$

In order to transform (10) – (11) into a system of ordinary differential equations, we again consider the particular form (6) for $\rho(k)$ and introduce the auxiliary variable $z \equiv k^{\alpha-1}$. The economy can then be represented by the three-dimensional system (12).

$$\dot{\rho} = -\eta(\rho - \bar{\rho})(A + Bz - x). \quad (12a)$$

$$\dot{x} = \frac{x}{\theta} \left[A + \alpha Bz - \rho + \eta(\rho - \bar{\rho}) \left(A + Bz + \frac{\theta x}{1-\theta} \right) \right] - (A + Bz - x)x. \quad (12b)$$

⁷This toy-model approach is only justified because industrialization in the sense of structural change is not the focus of the present paper. See, for example, Strulik and Weisdorf (2008) for a proper model on structural change during the industrial revolution.

$$\dot{z} = (\alpha - 1)(A + Bz - x)z. \quad (12c)$$

The ordinary Jones–Manuelli model is included as the special case of $\rho = \bar{\rho}$ for all k .

Because of the diminishing influence of the non-linear part of capital productivity, i.e. $z \rightarrow 0$ in a growing economy, the long-run growth path is, of course, the same as for the simple Ak growth model. At the steady-state of stagnation $x = A + Bz$. Using this in (12b) evaluated at a steady-state we get

$$0 = A\rho + \alpha Bz\rho - \rho^2 + \eta(\rho - \bar{\rho})\frac{A + Bz}{1 - \theta}. \quad (13)$$

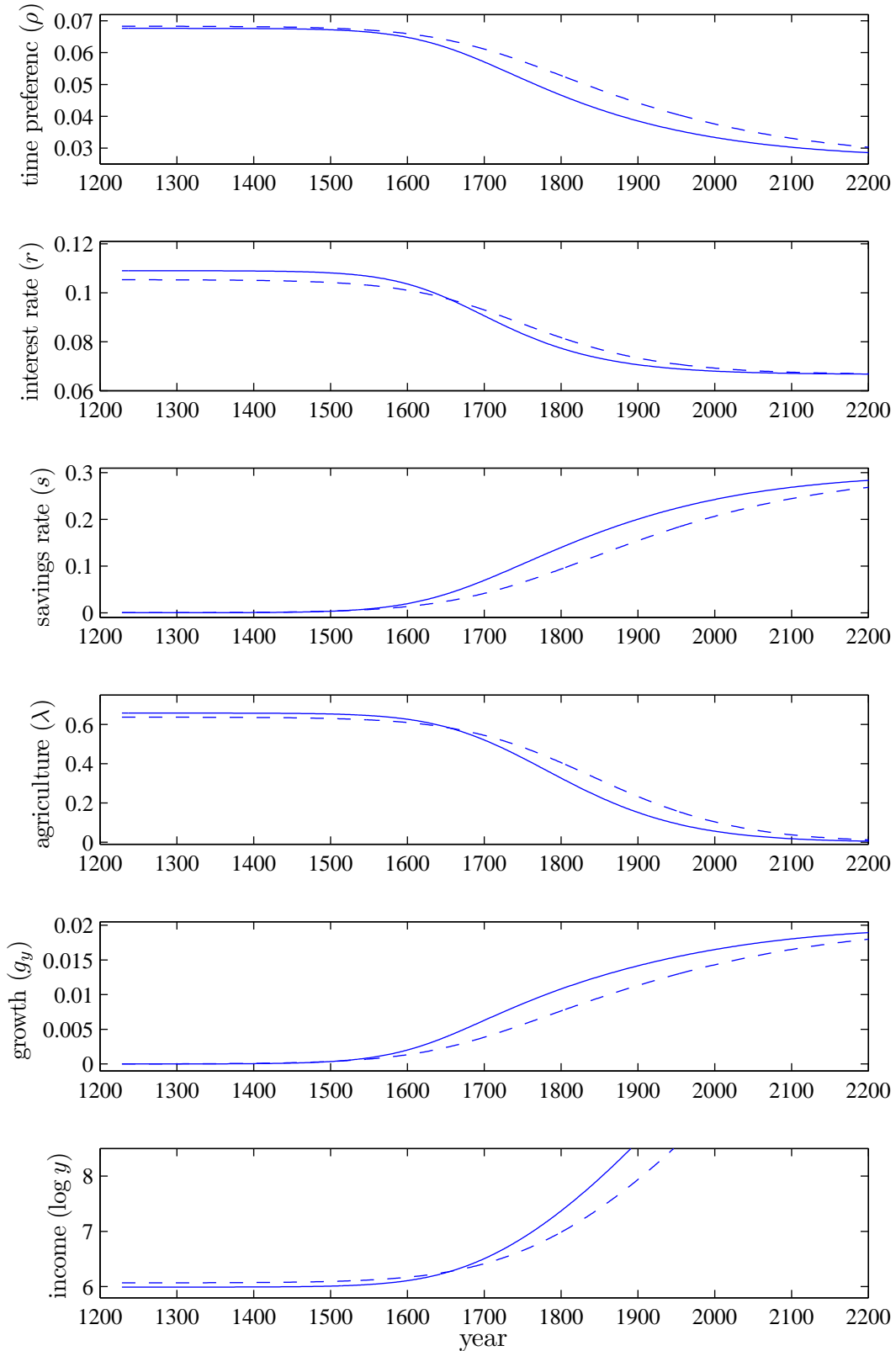
Inserting $\rho = \bar{\rho} + \rho_0 k^{-\eta}$ and $z = k^{\alpha-1}$ the equation can be solved numerically for k^* . From k^* we can infer z^* and ρ^* . Employing once again the method of backward integration the system (12) is solved by starting it arbitrarily close to the balanced growth path until it arrives at the steady-state of stagnation. Since both, ρ and z are state variables, adjustment dynamics evolve along a two-dimensional stable manifold. In order to approximate the unique optimal trajectory on the manifold the initial deviation from the balanced growth path is adjusted until the trajectory computed by backward integration ends at (ρ^*, z^*) , see Brunner and Strulik (2002) for details.

For the calibration we set $\alpha = 1/3$ and $B = 1$. The values of η and ρ_0 are chosen such that the adjustment speed is approximately the same as obtained for the solution of the basic model (under the Ak technology) and such that the initial interest rate (at the steady-state of stagnation) equals 10.9 percent as documented by Clark (2007) for England in the 13th century. This provides the estimates $\eta = 0.35$ and $\rho_0 = 0.1198$. The other parameters are kept from the last section so that the economy, again, converges towards a steady-state growth rate of 2 percent and a savings rate of 30 percent.

Adjustment dynamics for the benchmark case are shown by solid lines in Figure 4. While adjustment dynamics for ρ , x , and s are very similar to the simpler Ak case we additionally observe falling interest rates from historically high values to around 7 percent today ($r = A$ along the balanced growth path). Also, around time when income visibly takes off in the 18th century the downward trend of λ , our measure for the importance of agriculture, experiences the greatest momentum.

Dotted lines reflect again an alternative specification of an economy populated by somewhat less patient agents ($\eta = 0.33$ for Model-Italy). As explained above, around the steady-state of stagnation, capital stock must be (slightly) higher in the less patient economy because of

FIGURE 4. Patience and Prosperity Dynamics: The General Case



Parameters: $A = 0.0667$, $\alpha = 0.33$, $\bar{\rho} = 0.0267$, $\theta = 2$, $\rho_0 = 0.1189$ and $(\eta = 0.35, B = 1)$ for solid lines and $(\eta = 0.33, B = 1.042)$ for dashed lines.

the higher propensity consume $c'(k)$. Here, the neoclassical element in production additionally implies that interest rates are (slightly) lower at the steady-state of stagnation of the less patient economy. Again, the model predicts that the more patient country takes over (here in the 17th century) and experiences industrialization, i.e. a visible take off towards balanced growth earlier (by about three generations). Because of the dampening neoclassical mechanism of falling interest rates the difference in performance of the two countries is somewhat less pronounced than for the basic model.⁸

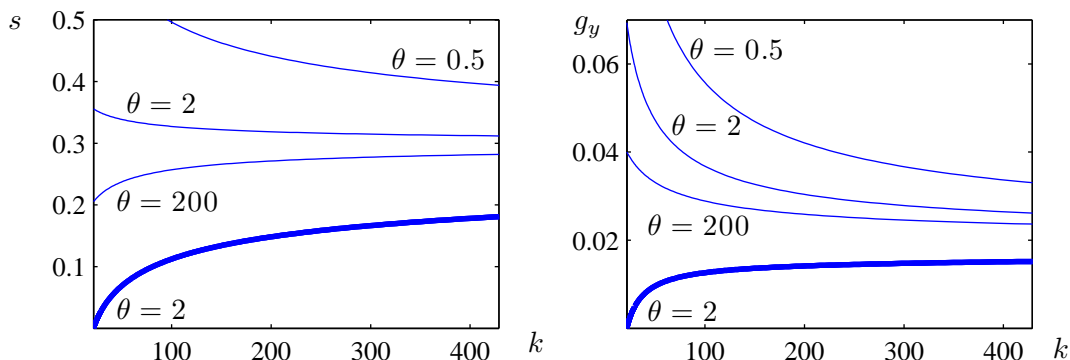
The numerical solution documents that the model is capable to produce the historical fact of simultaneously falling interest rates and rising growth rates. This interesting result is far from self-evident since falling interest rates per se lead to falling growth rates. More patience (decreasing time-preference), however, is capable to dominate the negative effect of falling capital productivity on growth.

To see the necessity of endogenous time preference for these results it is instructive to compare with the standard Jones-Manuelli model. The panel on the left hand side of Figure 5 shows the policy function $s(k)$ for the benchmark calibration and alternative specifications of the standard Jones-Manuelli model. The policy function relates the savings rate to the state of the economy (subsumed in state variable k). The bold line shows the policy function as implied by endogenous time preference. Since time preference is constant in the standard case, the only parameter that can be manipulated in order to get the shape of the policy function right is θ , i.e. the degree of relative risk aversion (the inverse of the intertemporal elasticity of substitution)

One sees, that if the patience channel is closed, there is too little curvature of the policy function. Poor people are predicted to save too much. For the benchmark setup of $\theta = 2$ (and lower values) the standard model even produces adjustment from the “wrong side”, i.e. falling savings rates during economic development. As shown by Barro and Sala-i-Martin (2004) this shortcoming can be prevented by imposing (unrealistically) high values for θ , i.e. (unrealistically) low values for the intertemporal elasticity of substitution. But, as the example shows, even if $\theta = 200$ (or higher) there is still too little curvature of the savings function such that the model, in general, predicts too fast adjustment from stagnation to balanced growth as compared with the historical evidence.

⁸The initial interest rate differential is not causal for the later take off. This has been verified by adjusting B such that both economy start out with the same initial interest rate (not shown in the Figure). Performance differences are driven by difference in patience.

FIGURE 5. Savings and Growth: Comparison with Standard Model



Parameters: $A = 0.0667$, $B = 1$, $\alpha = 0.33$, $\bar{\rho} = 0.0267$, $\theta = 2$, $\rho_0 = 0.12$, and $\eta = 0.35$. k is measured relative to k^* . Bold line: wealth-dependent preference. Thin lines: standard Jones-Manuelli model, alternative θ 's and otherwise same parameters.

The panel on the right hand side of Figure 2 shows the associated income growth rates g_y . When the patience channel is closed, diminishing capital productivity is the only impact on growth and the model counter-factually predicts that richer economies grow at lower rates. For low values of θ , the standard model gets the positive correlation between savings and growth right but this is by counterfactually predicting that both rates are *falling* during economic development. For high values of θ the standard model counterfactually predicts a negative correlation between savings and growth. Endogenous, wealth-dependent patience is capable to revert these results and let savings rates rise in accordance with economic growth.

7. THE POVERTY TRAP

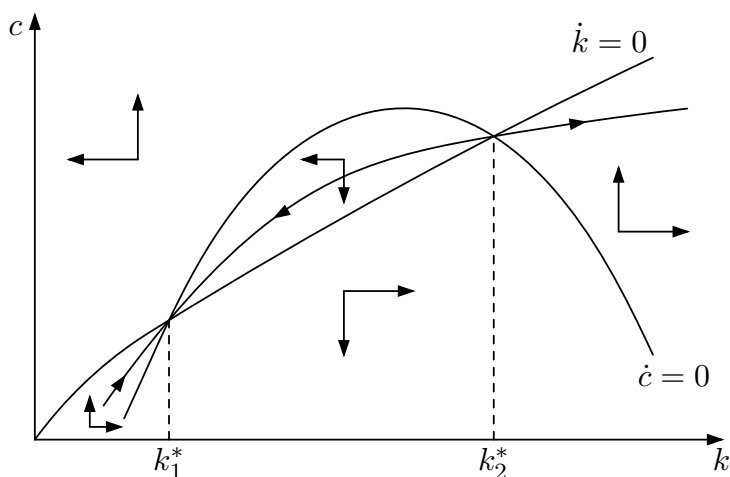
In principle, equation (13) may have two positive roots and the possibility of a locally stable poverty trap arises. In order to investigate this case in more detail in the $k - c$ space, we derive from (11) the $\dot{k} = 0$ isocline as $c = Ak + Bk^\alpha$. The isocline is positively sloped everywhere and approaches the slope A as k goes to infinity. Above the isocline we observe $\dot{k} < 0$ and vice versa below.

From (10) we get the $\dot{c} = 0$ -isocline

$$c = \frac{1 - \theta}{\theta} \left[(A + \alpha Bk^{\alpha-1} - \rho) \frac{\rho}{\rho'} - Ak - Bk^\alpha \right]. \quad (14)$$

As k goes to infinity and $\rho'(k)$ goes to zero the first term in brackets vanishes and c approaches $+\infty$ if $\theta > 1$ and $-\infty$ if $\theta < 1$.

FIGURE 6. Phase Diagram



A necessary, not sufficient condition for (13) to have two positive roots is $\theta < 1$. This can be seen by noting that the roots solve $0 = \rho^2 - a\rho + b$ with $b \equiv \eta\bar{\rho}(A + Bz)/(1 - \theta)$ and that two positive roots necessarily require $b > 0$. If $\theta < 1$, then coming from low k the $\dot{c} = 0$ -curve intersects the $\dot{k} = 0$ -curve first from below and then from above as shown in Figure 6. Finally, inspect (10) to verify that $\dot{c} > 0$ above the $\dot{c} = 0$ -curve and $\dot{c} < 0$ below and conclude from the resulting arrows of motion that the first equilibrium at k_1^* is a saddlepoint and the second equilibrium at k_2^* is unstable.

If an economy starts with capital stock below k_2^* a movement along any trajectory other than the stable manifold can again be excluded for violating in finite time either the first order conditions or the transversality condition. The unstable equilibrium at k_2^* separates convergence towards the poverty trap from convergence towards balanced growth. In the Appendix it is shown that as long as there is potential for growth a saddlepoint-stable development trap cannot occur in the standard Jones-Manuelli model augmented by subsistence consumption. The current model thus seems to be the minimum setup that allows for both convergence towards subsistence and convergence towards balanced growth, depending on initial conditions.

Intuitively, if an economy situated at k_1^* experiences a positive shock of capital and the interest rate falls, consumption decreases and exerts a negative level effect on patience if $\theta < 1$. This, however, is not sufficient to generate stagnation since the level effect has to be strong enough in order to overcompensate the positive growth effect on patience as explained in connection with the Ramsey rule (4).

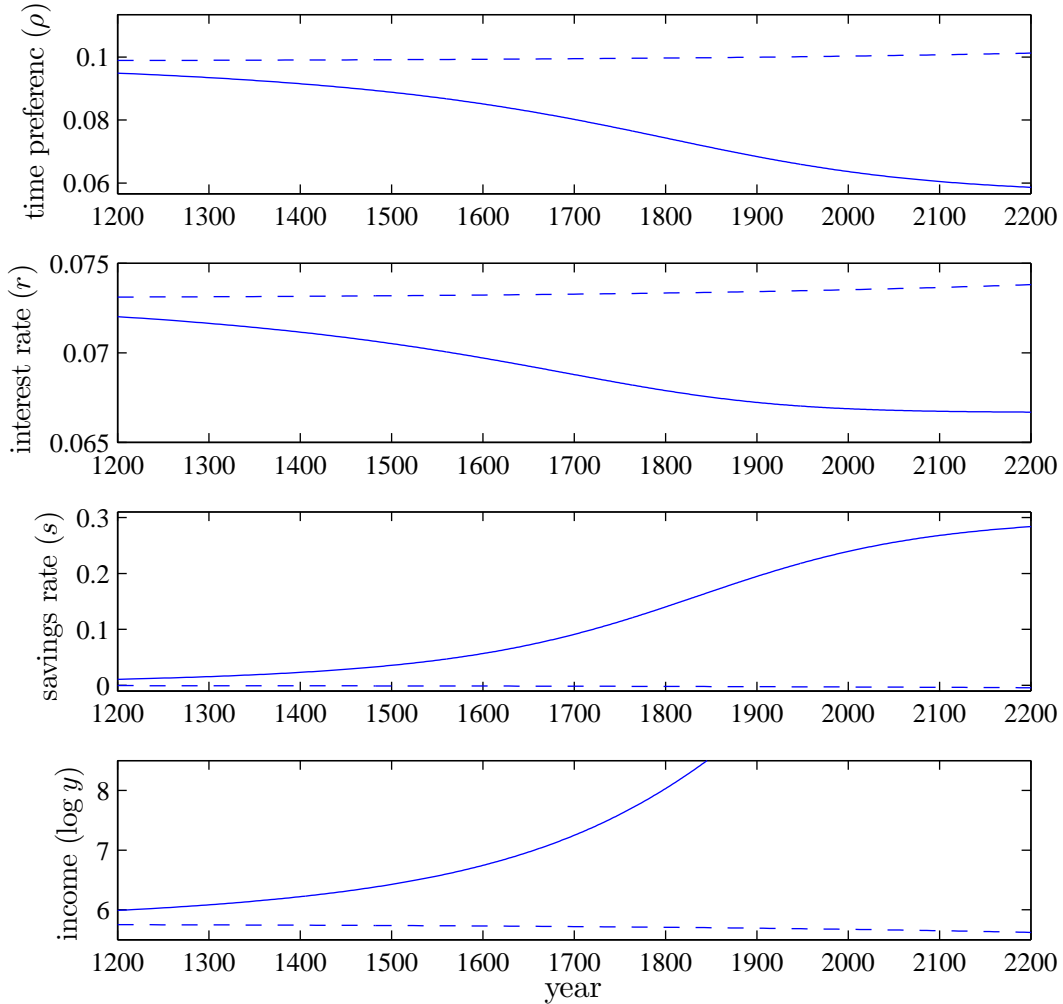
Figure 7 shows dynamic behavior of two identical economies, one starting slightly above k_2^* , the other slightly below. In order to create a locally stable poverty trap θ has been set to 0.5. The balanced growth path is kept such that $s = 0.3$ and $g_y = 0.02$. For that purpose the long-run time preference rate has been adjusted to 0.0567, a high yet not completely incomprehensible value. The low value of θ generates the “right” curvature of the \dot{c} isocline but is not sufficient to produce two intersections. For that purpose ρ_0 has been increased to 0.33, a change that does not affect balanced growth. All other parameters are kept from the last section.

The time paths generated for the successful developing economy are somewhat less satisfying than for the earlier case of $\theta = 2$. Adjustment is “too linear”, i.e. there is too little curvature during the time of the historical industrial revolution. Also, income for Model-England is overestimated in the 19th century. On the other hand, the produced time paths are still *comparatively* good approximation of actual behavior given that standard models of economic growth get adjustment dynamics completely wrong when θ is below unity (for savings and growth in the Standard Jones Manuelli setup this has been demonstrated in Figure 5). The power of endogenous time preferences is obviously sufficient to override the strong tendency for savings rates and growth rates to fall jointly with interest rates, a result which is usually generated when θ is below unity.

Qualitatively the paths for the developing economy differ from those obtained earlier for $\theta > 1$. The time preference rate lies now above the interest rate for most parts of the development process (until the 20th century). For an assessment recall that any standard model would not be capable of producing positive growth if the time preference rate exceeds the interest rate. Intuitively, successful development is possible here, because individuals take into account that time preference rates will be lower in the future such that their future selves (their offspring) will appreciate future income more, a fact that triggers already savings today. The fact that high time preferences rates are compatible with positive growth may help to reconcile the growth literature with the frequently high estimates of ρ in empirical studies and experiments conducted in rich and growing economies. For example, in a recent article Andersen et al. (2000) estimate from field experiments in Denmark that $\rho = 0.10$ (and $\theta = 0.74$).

It is telling that ρ_0 has to be adjusted to a high value in order to get the possibility of a poverty trap. This means that in addition to the negative level effect of consumption on patience (stemming from θ below unity) time preference has to be assumed to be particularly

FIGURE 7. Convergence Clubs



Solid lines: economy starting above k_2^* . Dashed lines: economy starting below k_2^* .
 Parameters for both cases: $A = 0.0667$, $B = 1$, $\alpha = 0.33$, $\bar{\rho} = 0.0267$, $\theta = 0.5$,
 $\rho_0 = 0.33$, and $\eta = 0.35$.

high at low levels of wealth. In the introduction it has been argued that there are potential determinants of time preference other than individual wealth. In particular geographic location may have an influence either directly through longevity or indirectly through institutions and the risk or expropriation. This lets us expect that for *given* wealth the rate of time preference is higher at low geographic latitudes (the tropics). Naturally we would expect this effect to show up in ρ_0 rather than in $\bar{\rho}$, i.e. we would expect the influence of geography on time preference to vanish when people become infinitely rich. If one is willing to accept this argument, the current model provides a rationale for a local poverty trap at low geographic latitudes.

8. DISCUSSION

The current paper has proposed a theory of endogenous patience that explains the long-run evolution of savings and growth in the Western world reasonably well. Actually the approximation of the actual transition is perhaps too good since there is little growth left to be explained for other important drivers of economic development. A reduction of η , for example, would slow down the adjustment speed and open room for factors like structural change, demographic transition, and TFP growth to improve the adjustment dynamics. A reasonable assessment of the results is thus to view endogenous patience as a complementing channel for the explanation of long-run economic development, a channel that has been largely overlooked so far.

Some of the results presented here can alternatively be generated by introducing habit formation or subsistence consumption into the linear growth model. It seems thus worthwhile to compare. A common characteristic of all three approaches is that they produce reasonable adjustment dynamics in the sense that savings and growth are jointly rising during the process of development.

The modeling of habit formation is technically more involved than the present approach since it requires a second state variable, the habit stock. When augmented with endogenous habit formation already the linear Ak model transforms into a three dimensional dynamic system (Carroll et al., 2000). The higher formal complexity possibly explains why some interesting features obtained here like the savings rate –interest rate dynamics, the poverty trap, or overshooting savings rates, have not (yet) been explained within the habit formation model. Although ex post adjustment dynamics look similar, they are conceptually different under habit formation. Adaptive behavior implies that the model generates serial correlation of current savings and consumption decisions with those in the past. With contrast, endogenous patience generates serial correlation of current and *future* savings and consumption decisions. If an economy traveling along its balanced growth path is hit by a shock we would thus expect current consumption to be correlated with past consumption under habit formation but not under endogenous time-preference.⁹

If the strength of wealth-dependent savings is determined by geographic location (either through mortality or expropriation risk) then an explanation of temporarily diverging time

⁹Moreover, in order to generate realistic adjustment dynamics the habit formation model relies on $\theta > 0$. The present model generates plausible adjustment dynamics irrespective of the size of θ .

preference rates and savings behavior must not rely on culture. In particular, country-specific savings behavior would be compatible with the observation that there is no fundamental difference of savings behavior between natives and immigrants (Carroll et al., 1994). With contrast if savings are driven by culture (consumption habits) we would expect savings behavior to differ for immigrants, at least for a while and in particular if habits adjust sluggishly.

The introduction of subsistence consumption in economic models comes in two conceptually different variants. The first variant tries to understand how human energy consumption affects economic choices (for example, fertility or effort at work) and how these choices feed back on macroeconomic behavior and the available diet (Dalgaard and Strulik, 2007). This is, of course, a totally different research agenda than pursued in the present paper. The second variant, which appears more often in economic modeling, introduces a constant (\bar{c}) into the utility function and calls this constant (possibly misleadingly) subsistence consumption. An exploration of subsistence needs, however, is not the purpose of the modeling. The purpose is to get adjustment dynamics right, i.e. to generate savings and growth rates that are jointly increasing with economic development.

Subsistence consumption in form of Stone-Geary utility, $u(c) = (c - \bar{c})^{1-\theta}/(1-\theta)$, implies that the intertemporal elasticity of substitution rises from zero where $c = \bar{c}$ to $1/\theta$ for infinite c . This produces trajectories similar to the ones for wealth-dependent time preference and the question occurs whether the ad hoc modeling of subsistence is acceptable as a shortcut to endogenous patience. The answer is probably yes on the level of (introductory) macroeconomics textbooks because the “shortcut”-model is formally much less involved. On the level of a serious research question the answer is certainly no. This is instructively demonstrated by Kraay and Raddatz’ (2007) attempts to calibrate the neoclassical growth model with “subsistence consumption” and in particular to find a proper \bar{c} . They conclude: “In summary, the model with subsistence consumption could be used to rationalize the persistence of low levels of consumption and capital observed in low income African countries by assuming that they are close to subsistence level, but this requires either to assume that there is significant variation in the levels of subsistence consumption across African countries or that half of these countries should exhibit high savings and growth rates.”

With respect to the Ak growth model with subsistence consumption it can be shown that adjustment dynamics of the savings rate and the rate of economic growth are independent from

the specification of \bar{c} . In fact, adjustment dynamics of these variables are completely determined by the specification of the balanced growth path (Strulik, 2010). This implies that either all countries adjust alike toward the same balanced growth path or that countries that lag behind will never catch up. The model cannot explain *temporarily* divergent behavior. With contrast, the current model can explain temporarily different behavior, in particular why originally leading countries are overtaken before and during industrial revolution and why they later catch up again. Additionally, it can explain a patience-driven, locally stable poverty trap, independent from subsistence needs, which might be relevant for some of the African countries discussed by Kray and Raddatz.

Finally the concept of endogenous time preference is comprehensive. It directly affects all intertemporal decisions alike whereas the subsistence approach affects other choice variables only through urgent consumption needs. For example, there is no “subsistence education” or “subsistence fertility”. An extension of the present model towards the analysis of other intertemporal decisions is an interesting subject for future research.

Appendix

Two-Period Finite-Life Approximation. Consider an economy where people live for two periods, childhood and adulthood. Suppose for simplicity a family consists of one parent and one child and that rearing children entails no cost and no (empathically felt) benefits for the parent. Yet parents care about utility of the offspring when offsprings themselves become parents. It is easy to imagine that nature has fostered this trait in humans. An adult of generation t maximizes

$$U_t = \frac{c_t^{1-\theta}}{1-\theta} + \frac{1}{1+\rho(k_t)}U_{t+1}.$$

With contrast to the standard model it is here assumed that wealthier parents discount utility of their offspring by less and are thus inclined to leave larger bequests k_{t+1} . Given a linear production technology, bequests evolves according to

$$k_{t+1} = Ak_t - c_t - k_t.$$

By iteratively substituting U_{t+i} , $i = 1, \dots, \infty$ in the utility function, the objective function can be written as

$$\max \sum_{i=0}^{\infty} \frac{1}{[1+\rho(k_{t+i})]^i} \frac{c_{t+i}^{1-\theta}}{1-\theta}.$$

Solving this with respect to the above equation of motion is the discrete time equivalent to problem (1)-(2) in the main text.

The Standard Model Cannot Explain A Development Trap.

PROPOSITION 4. *If there is potential for long-run growth then there cannot simultaneously exist a local (saddlepoint-) stable poverty trap in the Jones Manuelli model with constant time preference and subsistence consumption, i.e. when instantaneous utility is of the Stone-Geary type, $u(c) = (c - \bar{c})^{1-\theta}$ where \bar{c} denotes the subsistence level.*

Proof. With instantaneous utility of the Stone-Geary type the Ramsey rule in general reads $\dot{c} = (f'(k) - \rho)(c - \bar{c})/\theta$ and with production of the Jones Manuelli type it becomes

$$\dot{c} = (A + \alpha Bk^{\alpha-1} - \rho) \left(\frac{c - \bar{c}}{\theta} \right).$$

The equation of motion for capital remains to be (11). Generally, equilibria of stagnation could be where $c = \bar{c}$ or where $A - \rho - \alpha Bk^{\alpha-1} = 0$. However, under the requirement that there is potential for long-run, the second option for an equilibrium of stagnation ceases to exist. For positive growth in the long run we must have $A - \rho > 0$ which is incompatible with $A - \rho - \alpha Bk^{\alpha-1} = 0$ at some positive k .

The elements of the Jacobian evaluated at the equilibrium $c = \bar{c}$, $k = A^\alpha + Bk^{\alpha-1}$ are $J_{11} = (A + \alpha Bk^{\alpha-1} - \rho)/\theta$, $J_{12} = \alpha(\alpha - 1)Bk^{\alpha-2}(c - \bar{c})/\theta = 0$, $J_{21} = -1$, and $J_{22} = A + \alpha Bk^{\alpha-1}$. The eigenvalues are $a \equiv A + \alpha Bk^{\alpha-1} > 0$, and $a(a - \rho)\theta > 0$ for $A - \rho > 0$, i.e. when there is potential for long-run. The equilibrium of stagnation is unstable. \square

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