The Formation of Urban Centers under R&D and Spillover Externalities

Jan Kranich*

Leibniz University of Hannover

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Abstract

This paper proposes a model of urban agglomeration in conjunction with imperfect competition and endogenous product R&D of firms. The quality of differentiated manufacturing goods is a result of R&D services provided by research firms. Sectoral interactions are subject to spatially dependent transaction costs and (knowledge) spillover externalities. The paper analyzes the existence of fundamental city patterns with respect to R&D intensity and the degree of localization in knowledge production. The model features three equilibrium formations: a monocentric, a mixed, and a perfectly integrated pattern, whereas the R&D intensity always increases towards the city center. However, product quality and the corresponding R&D expenditures of firms are not necessarily increasing with the city size; a result, which also renders decisive implications of local innovation policy.

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JEL classifications: R1, R14

*Jan Kranich, Chair of Microeconomics, D-30 167 Hannover, Germany, email: kranich@mik.uni-hannover.de

1 Introduction

Along present definitions (see OECD (1999), Porter (2000)), industrial clusters, especially high-tech clusters, share a number of commonalities: 1) Knowledge production, application, and the production of new or improved goods and services require an intensive interaction between firms and people involved in the innovation process. 2) In many cases, the core of high-tech clusters consists of universities and other research institutions surrounded by a larger number of R&D firms, e.g., research labs, engineering consultants, technology platform providers etc.. 3) In particular knowledge-intensive technologies imply positive externalities across firms and industries located close to the knowledge and innovation source. 4) Finally, depending upon the technological stage of research, the location of R&D is due to the trade off between sectoral specialization and diversification.

As Feldman (1993) points out, a crucial input for innovation of product and processes is specific technical and scientific knowledge. These knowledge inputs have a cumulative character and become relatively immobile and place specific (see, e.g. Grossman and Helpman (1989), Lundvall (1988)). As summarized by Dosi (1988), a set of *stylized* facts with respect to innovation and location sheds some light on the mechanisms and determinants for the emergence of high-tech centers. First of all, innovation is an uncertain and complex process. Information exchange between knowledge producers and applicants over long distances turns out to be difficult unless impossible in case of new and highly knowledge intensive technologies. In this context, innovations involving tacit knowledge, which must be learned and practised and cannot be easily codified, occurs to be one of the major determinants responsible for keeping R&D and manufacturing close together.[1] Another crucial attribute of high technologies are customer and product specific specialties in the adaption of inventions and innovations, which are decisive in the pre-phase of standardization and mass production and require intensive communication between R&D and manufacturing. While pure transport costs have declined during the past decades, the costs for developing and transacting highly complex and skill-intensive products have significantly increased.[2]

[1] See, e.g.,Nelson and Winter (1982).

[2] See, e.g., Glaeser (1998) for a discussion.

The strong localization of R&D stems also from the presence of universities (see Nelson (1988)). On one hand, university provide public goods, e.g., in form of fundamental research, network access etc. On the other hand, they also act as a source of highly qualified and specialized labor As another aspect of agglomeration, also matching and pooling effects play an important role. The higher is the number of agents in one region the higher is also the probability for an optimum fit between economic agents. This does not only hold for the (skilled) labor market, but also for the knowledge diffusion between R&D and manufacturing due to lower search costs.[3]

As mentioned above, another determinant having an impact on firm clustering are positive localization externalities, also known as spillover effects. Spillovers are classified by Glaeser et al. (1992) with respect to their dynamic or static nature. The category of static spillovers contains external economies, e.g., pooling effects as discussed above, or urbanization economies due to higher local demand as suggested by Henderson (1986). Dynamic spillovers are information externalities, also referred to as knowledge spillovers. In this regard, two types of knowledge spillovers are fairly discussed in the literature. One type, which is defined by Glaeser et al. (1992) as **M**arshall-**A**rrow-**R**omer externalities, arises within the same industry. The second type introduced by Jacobs (1969) assumes that positive information externalities occur across firms in different industries as a result of a larger variety of potential innovation sources. Both types of knowledge externalities are not mutually exclusive. In fact, empirical studies find evidences for both: While Glaeser et al. (1992) as well as Feldman and Audretsch (1999) provide support only for Jacobs-externalities, the study by Henderson et al. (1995) reveals that MAR-externalities are relevant in traditional sectors and Jacobs-externalities in high-tech industries only. In contrast, Combes (2000) concludes based on a data set including French firms that Jacobs-externalities work only in the service sector. For Germany, Suedekum and Blien (2005) find that MAR-spillovers are prevalent in service related industries, in contrast to Jacobs-externalities in the manufacturing industries. However, these studies consider employment and city growth; in critical response Cingano and Schivardi (2004) and Henderson (2003) use productivity growth as dependent variable and

[3]See, e.g., Helsley and Strange (1990), Krugman (1991), Berliant et al. (2006).

find significant evidence für MAR-externalities contrary to Jacobs-spillovers. Finally, with respect to spillovers and space, Jaffe (1989), Jaffe et al. (1993), Feldman (1994), Audretsch and Feldman (1996) show by means of patent data that knowledge externalities tend to be strongly localized within the region, where the innovation was initially created.

As highlighted in the much noticed article by Duranton and Puga (2001), firms in more innovative and agglomerative activities show a greater tendency to relocate from diversified to specialized regions giving rise to urban specialization. The authors show by means of their urban-system model that under uncertainty in (process) R&D a coexistence of specialized and diversified cities is possible. In this constellation, firms locate in diversified cities in order to benefit from Jacobs-spillovers until they have found their ideal production process (nursery state). Thereafter firms relocate to specialized cities in order to gain specialization advantages in mass-production. Nonetheless it can be observed that also in specialized cities a certain extent of research is undertaken, which requires a further differentiation of R&D. In contrast to the diversified cities, where fundamental and early-stage research is the key issue of the innovation process, firms in the specialized regions utilize quality improving R&D for vertical differentiation in the subsequent stages of their life cycle.

In this context, product R&D within specialized cities is the main concern of this paper. The modeling objectives follow four leading questions: *1) Which equilibrium city structures are sustainable and under which conditions? 2) How does the spatial distribution of R&D expenditures and product quality correspond to the city structure? 3) How do labor market effects, differing technological stages, and innovation policy affect the long-run equilibrium? 4) How are city size and R&D intensity interrelated?*

Starting from the seminal work of Fujita (1988), we model within a monopolistic-competitive framework a research sector providing quality improving R&D services to a manufacturing sector, which does not only horizontally but also vertically differentiate its products sold on the exogenous world market.[4] In order to catch the

[4] In fact, we have a model of the *pure agglomeration* type as classified by Iammarino and McCann (2006).

key characteristics of high-tech clusters as discussed above, we implement MAR-spillovers as well as a spatial transaction cost representing the communication during the innovation process. In this regard, research and manufacturing firms are in the trade-off between i) saving spatial interaction costs by locating close to the opponent vertically linked sector; and ii) gaining productivity advantages due to intra-industry spillovers by locating close to firms of its own kind. Spillover externalities are modeled in the same manner as suggested by Berliant et al. (2002), whereas the labor input of firms is dependent upon the relative position to the mean location of all firms within the same industry as well as the total distribution of this industry. After deriving the bid-rent functions, we analyze the short run equilibrium states based upon fixed firm numbers in the R&D and manufacturing sectors. The model features three possible equilibrium states: a monocentric, a mixed, and a perfectly integrated pattern. As a central result, R&D expenditures of manufacturing firms are highest in the city center (if the manufacturing sector is located in the central business district) and quadratically decrease towards the city frontiers. In the long run, where firm numbers are variable, the parameter range of steady states becomes more restrictive. Nonetheless, we face two equilibria for each pattern in the long run: one stable and one unstable equilibrium. As a central implication, the lower are the costs of spatial interaction the higher are the firm numbers in both sectors and the larger the city. Finally, the R&D intensity quantified by the product quality is positively correlated to the city size if the manufacturing sector is located only within the central business district (CBD). However, the relationship between city size and R&D intensity can also be negative in the perfect-integration pattern as well as for equilibrium formations, in which the R&D sector is agglomerated in the CBD.

The paper is structured as follows: In the next section we discuss empirical results on innovation, urbanization and spillover externalities also by means of an own appraisal using German patent data. Section 3 introduces the assumptions and the structure of the theoretical model. In Section 4 and 5 we discuss its short and long run equilibria, respectively. Finally, Section 6 summarizes the results and draws conclusions for regional and innovation policy.

2 Empirical Findings

The relationship between innovation and urbanization has been intensely discussed in several empirical studies. As one of the early contributions, Higgs (1971) found evidence that patent activity positively responds to urbanization. In a more recent study Jaffe et al. (1993) used patent traces and found that citations of patents are likely to come from the same metropolitan area. In this context, also Acs et al. (2002) show that patents are concentrated in metropolitan areas.

As discussed above, spillover externalities are suspected to be responsible for the agglomeration of innovating activities. In this regard, Feldman and Audretsch (1999) find a positive correlation between urban employment and invention. Furthermore, Carlino et al. (2001, 2007) also used American data to show that the rate of patenting is positively correlated to the employment density of metropolitan areas.

In this paper we take a brief look on German patent data in the same manner as Bettencourt et al. (2007) and find also similar results. The calculations included data from the regional data base of the German Federal Statistical Office combined with patent statistics by the German Patent and Trade Mark Office. The data set contains information about firm numbers, population, employment, business patents as well as average land prices of more than hundred urban districts ("kreisfreie Städte") as summarized by the descriptive statistics shown in Table 1 in the Appendix. Based on these data, the Figures 1, 2, and 3 plot the patent intensity (patents per manufacturing firm) with respect to i) employment density (average employment per km^2), ii) city size (population), and iii) the average land prices of sold estates.

[Insert Figures 1-3 about here.]

As apparent, the patents per firm increase with the urbanization given by the employment density on one hand and the city size on the other hand. Table 2 in the Appendix shows the correlation coefficients and the t-values of a simple OLS-regression. The results reveal a significant positive correlation between city size and patent intensity, which supports the hypothesis of spillover effects discussed above. The finding can be summarized as follows: the larger is the city and, thus, the larger are average land prices the higher are R&D activities of firms due to stronger spillover externalities leading to the observed patent applications.

The results presented here confirm the empirical findings of the other studies mentioned above.[5] Despite these empirical indications we still do not know much about the internal structures of high-tech clusters, and how spillovers, production linkages and policy efforts affect R&D productivity within the city. Apparently, the empirical results support a positive correlation between R&D intensity and city size. Nonetheless, also a negative relationship might be conceivable given the role of spillover effects. For an illustration, if the city grows in space, the overall firm distribution becomes larger, which under certain conditions reduces inter-sectoral spillovers and, thus, the R&D performance of firms. Finally, these considerations motivate the theoretical part of this paper.

3 The Model

Space is assumed to be a continuous, infinite, and featureless plain $X = [-\infty, +\infty]$, wherein the city occupies a certain land area $[-a, a] \in X$. For sake of analytical simplicity, the city center is located in the origin of X. In consequence, we obtain the identical structural in the positive and negative part of the continuum. The land is owned by absentee landlords and bought/rented by firms whose shareholders are also absent. The city is characterized to be a specialized city according to Duranton and Puga (2001), in which specialized goods are produced by a manufacturing sector (henceforth the $\mathbb{S}$-sector). The specialized varieties are cross-differentiated and exported to the exogenous world market. As an illustration, products do not only differ in terms of horizontal attributes, like color, taste, and design, they also differ in terms of a vertical dimension, which can be interpreted as the quality of goods. The quality is a result of R&D expenditures of $\mathbb{S}$-firms, which are paid to a research sector (from now on $\mathbb{R}$-sector). The $\mathbb{R}$-sector in turn provides quality improving R&D services to the local manufacturing industry only. As a result of differentiation as well as increasing returns, both sectors are assumed to be monopolistic-competitive

[5]In view of the theoretical concern of this paper, we restrict ourselves to these sketchy results, which can surely be enhanced using more advanced econometric techniques.

in tradition of Dixit and Stiglitz (1977), whereas one firm produces only one variety and locates only at one location (no multi-plant operations). Because of analytical tractability it is additionally assumed that firms occupy only a fixed lot size at $x, y \in X$.

World market demand

The export market (corresponding variables are denoted by the subscript E) demands the whole continuum N_S of $\mathbb{S}$-goods as an intermediate input and sells its output q_E at a constant price $p_E \equiv 1$. The world production function is given by:

$$(1) \quad q_E = \int_0^{N_s} v\left[q_s(i), u_S(i)\right] di \ \ \forall \ \ i \in N_s,$$

whereas v denotes the share in output of a single specialized variety i:

$$(2) \quad v = \begin{cases} \frac{q_s}{\alpha}\left[1 + log\left(\beta u_s\right)\right] - \frac{q_S}{\alpha} log\left(\frac{q_S}{\alpha}\right) \ \ \forall \ \ q_S < \alpha\beta \\ \beta \ \ \forall \ \ q_S \geq \alpha\beta. \end{cases}$$

The parameters α and β represent preference for diversity; the higher are these values the lower is the impact of a single variety on total E-production. Equation (2) is entropy-typed, monotonously increasing in quantity q_S, and constant for $x \geq \alpha\beta$.[6] The quality of specialized varieties is denoted by u_S, which positively effects the output share of a single specialized input.

From equations (1) and (2) the world profit function can be expressed as:

$$(3) \quad \pi_E = \int_X \{v\left[q_S(x)\, u_s(x)\right] - p_s(x)\, q_s(x)\} n_s(x)\, dx.$$

Since each firm locates only at one location $x, y \in X$, the integral over N_S in equation (1) can be rearranged as a spatial integral over X in equation (3). In this context, $n_S(x)$ denotes the density of $\mathbb{S}$-firms at location x. Finally, $p_S(x)$ represents the price of a specialized variety produced at x.

[6] See Fujita (1988) for a discussion.

Maximizing profits leads to the optimum world demand for a specialized variety:

$$(4) \quad q_S^*(x) = \alpha\beta u_S(x)\, exp\left[-\alpha p_s(x)\right].$$

Thus, the optimum input share is:

$$(5) \quad \upsilon(q^*) = \beta u_S(x)\, exp\left[-\alpha p_S(x)\right]\left[1 + \alpha p_S(x)\right].$$

Specialized Sector

The profit function of monopolistic-competitive $\mathbb{S}$-firms is given by:

$$(6) \quad \pi_S(x) = \left[p_S(x) - c\right] q_S(x) - C(u) - R(x) - w l_S\left[\phi_S(x)\right].$$

In this regard, the first term on the right hand side in equation (6) reflects operating profits depending upon the constant marginal production costs, c. The second term gives the costs of quality – the R&D expenditures of a $\mathbb{S}$-firm. The third term represents the land rent paid at location x. Finally, the last term includes the labor costs, whereas w is the exogenous wage rate for unskilled labor, and l_S the labor demand depending upon spillover effects, which are discussed later on in this paper. Profit maximization leads to standard monopolistic mark-up pricing. Since marginal production costs are constant, the optimum price is independent from x and, thus, the same wherever $\mathbb{S}$-firms locate in X. After normalizing $c \equiv (\alpha - 1)/\alpha$, the price for the specialized good is simply equalized to $p_S = 1$. In consequence, the demand for a $\mathbb{S}$-firm variety in equation (4) can be simplified to:

$$(7) \quad q_s(x) = \alpha\delta u_S(x) \quad , \quad \delta \equiv \frac{\beta}{exp[\alpha]}.$$

Turning to the firm decision on the degree of vertical differentiation, the quality of specialized goods depends upon the input of the whole continuum N_R of horizontally differentiated R&D services available in the city:

$$(8) \quad u_s(x) = \int_0^{N_R} \upsilon\left[q_R(i)\right] di = \int_{y \in X} \upsilon\left[q_R(y)\right] n_R(y)\, dy.$$

The share υ in total quality generation is again entropy-typed and given by:

$$(9) \quad \upsilon\left[q_R(y)\right] = \begin{cases} \frac{q_R}{\kappa}\left[1 + log(\lambda)\right] - \frac{q_R}{\kappa} log\left(\frac{q_R}{\kappa}\right) & \forall \ q_R < \kappa\lambda \\ \lambda \ \forall \ \ q_R \geq \kappa\lambda. \end{cases}$$

Here, κ and λ are technological preference parameters again.

The knowledge created by $\mathbb{R}$-firms is totally private and specific to $\mathbb{S}$-firms. Hence, there are no public good effects of knowledge production. Knowledge exchange between manufacturing and research sectors causes a spatial transaction cost $t|x-y|$, which depends upon the distance between $\mathbb{R}$-firms at y and $\mathbb{S}$-firms at x. In this context, the relevance of transfer costs might be point of discussion in the face of the dramatic decline of pure transport costs during the last decades. Nonetheless, the transaction action costs for non-standard, non-routine, and knowledge-intensive products and services have increased as discussed in the Introduction. This phenomenon is apparently supported by the increasing importance of face-to-face contacts and strong interaction between innovation producers and applicants, especially in the case of products featuring a high uncertainty and complexity in the R&D process.

Optimizing of equation (6) leads to the demand of a $\mathbb{S}$-firm at x for the R&D service of a $\mathbb{R}$-firm at y:

$$(10) \quad q_R(x,y) = \kappa\lambda exp\left[-\frac{\kappa}{\lambda}\left[p_R(y) + t|x-y|\right]\right],$$

whereas p_R is the price of the corresponding variety. Thus, from equation (8), the optimum quality depends upon the R&D costs and the spatial distance to the continuum of $\mathbb{R}$-firms:

$$(11) \quad u_S^*(x) = \lambda \int_{y\in X} \left\{ \frac{1 + \frac{\kappa}{\delta}\left[p_R(y) + t|x-y|\right]}{exp\left[\frac{\kappa}{\delta}\left[p_R(y) + t|x-y|\right]\right]} n_R(y) \right\} dy.$$

As a result of functional assumptions, the quality of a specialized variety produced at x is increasing with decreasing research prices and the overall distance to the mass of $\mathbb{R}$-firms. Equation (11) also allows to distinguish between different technological levels. In this context, a high-tech industry is commonly characterized by a high level of R&D expenditures. Based upon the quality equation, which determines R&D expenditures, a high-tech industry in the present context may be characterized by a high R&D differentiation, which goes along with low values of κ. In addition, high spatial transaction costs between research and manufacturing firms may also be used as an indicator for the technological level. As discussed in the introduction of this paper, high-tech industries tend to feature high transaction costs due to higher complexity of products and services.

Research sector

The monopolistic-competitive $\mathbb{R}$-sector faces an aggregate demand of all manufacturing firms for the services of a particular $\mathbb{R}$-firm:

$$(12) \quad Q_R(x,y) = \int_X q_R(x,y)\, n_S(x)\, dx.$$

The profit function of a $\mathbb{R}$-firm at y is given by:

$$(13) \quad \pi_R(y) = [p_R(y) - c_R]\, Q_R(x,y) - r l_R[\phi_R(y)] - R(y).$$

Again, it is assumed that marginal production costs (of knowledge) are constant; the firm also pays a land rent, R, depending upon the location. The second term on the right hand side represents the costs of the skilled labor input, whose wage rate is exogenously given by r.

Maximizing equation (13) with use of equation (12), we obtain mark-up pricing again:

$$(14) \quad p_R = c_R + \frac{\delta}{\kappa}.$$

Thus, the price of R&D services is also constant and independent from firm location, which simplifies algebra to a high extent again. Re-substituting equations (14), (12) into the profit function (13) provides:

$$(15) \quad \pi_R(y) = \eta \int_X exp\left[-\frac{\kappa}{\delta} t|x-y|\right] n_S(x)\, dx - r l_R[\phi_R(y)] - R(y),$$

whereas η can be interpreted as a parameter for monopoly power, which is defined as: $\eta \equiv (\lambda\delta)/exp\left[1+\frac{\kappa}{\delta}c_R\right]$.
Similarly, the $\mathbb{S}$-firm profit function can be expressed as:

$$(16)\quad \pi_S(x) = \eta \int_{y\in X} exp\left[-\frac{\kappa}{\delta}t|x-y|\right] n_R(x)\, dy - wl_S\left[\phi_S(x)\right] - R(x)\,.$$

Spillover and labor demand

Spillover externalities are implemented following the concept of Berliant et al. (2002), who use an overall dispersion index. As the authors conclude such a measurement needs to meet three requirements: i) it should be an absolute value; ii) it should be decomposable into subgroups; and iii) it should be symmetric to a mean location. They find that the absolute deviation might be a suitable measure:

$$(17)\quad \sigma = \frac{2}{N}\int_{y\in X} n(y)\,|y-\mu| dy,$$

whereas the mean location is:

$$(18)\quad \mu = \int_{y\in X} yn(y)\, dy \Rightarrow 0.$$

Equation (17) simply implies that the more dispersed an industry is located with respect to a mean location the higher is the corresponding dispersion index. If firms cluster within a certain interval $[-b, b]$, the dispersion index takes the value b. If σ is equal to zero, the whole sector agglomerates only at one location x.
In the present model we simply assume that the extent of spatial spreading has an impact on the labor demand. This is motivated by the observation that firms locating close to each other are in position to reduce labor costs by research cooperations, joint fundamental research or simply a better matching of (skilled) labor. In this regard, the labor requirement functions used in equations (15) and (16) are defined as:

$$(19)\quad l\left[\phi(y)\right] = \bar{l} + (y-\mu)^2 + \epsilon\sigma^2 \quad , \quad \bar{l} \equiv 1 \quad , \quad \mu = 0.$$

Equation (19) implies that the more distant the firm is to the mean location and the more its industry is dispersed in space the higher is the corresponding labor demand on top a minimum input $\bar{l}$. The parameter ϵ denotes a sector-specific penalty on overall firm dispersion.

Sectoral bid-rent functions

The central concept of urban economics can be illustrated by means of sectoral profit functions as given by equations (15) and (16). The profit function provide information about the maximum willingness to pay for a certain lot size at location x and y, respectively. Rearranging these functions by solving for the land rent, R, gives the commonly known bid-rent functions, here denoted by Ω_R and Ω_S:

$$(20) \quad \Omega_R\left(y\right) = \eta \int_X exp\left[-\frac{\kappa}{\delta}t|x-y|\right] n_S\left(x\right)dx - rl_R\left[\phi_R\left(y\right)\right] - \pi_R\left(y\right)$$

$$(21) \quad \Omega_S\left(x\right) = \eta \int_{y \in X} exp\left[-\frac{\kappa}{\delta}t|x-y|\right] n_R\left(x\right)dy - wl_S\left[\phi_S\left(x\right)\right] - \pi_S\left(x\right).$$

Land market equilibrium

The firm decision on location determines the land market equilibrium, which in turn affects firm profits due to the spatial costs of interacting with firm of the vertically linked industry as well as with firm of their own industry. For illustration, a simple example is considered: the manufacturing industry is located within a central city business district surrounded by the research sector within the outskirts of the CBD. A R&D firm located close to the city center faces a better access to its downstream industry, but suffers from the distance to the firms of its own industry – a result of lower spillover effects. In contrast, the closer the R&D firm is located to its own sector, the higher is the local competition for land, workers (in a general equilibrium), and access to manufacturing firms.

However, a land market equilibrium needs to meet four conditions:

$$\begin{array}{rl}
i) & R^*(x) = max\left\{\Omega_S\left(x, \Pi_S^*, n_R^*\right), \Omega_R\left(x, \Pi_R^*, n_S^*\right)\right\} \\
ii) & \Omega_S\left(x, \Pi_S^*, n_R^*\right) = R^*(x) \quad if \quad n_S^*(x) > 0 \\
 & \Omega_R\left(x, \Pi_R^*, n_S^*\right) = R^*(x) \quad if \quad n_R^*(x) > 0 \\
iii) & n_S^*(x) + n_R^*(x) \leq 1 \\
 & n_S^*(x) + n_R^*(x) = 1 \quad if \quad R^*(x) > R_A \\
iv) & \int_x n_R^*(x)\, dx = N_R \\
 & \int_x n_S^*(x)\, dx = N_S.
\end{array}$$

The first condition simply states that the equilibrium land rents for a lot at location x is given by the maximum bid-rent value of firms in both sectors. If this holds for either the $\mathbb{S}$-sector or the $\mathbb{R}$-sector, the corresponding firm density at this locations is positive (condition ii)). The third condition is on one hand a physical constraint: the firm density at one location cannot be higher than a certain level, here 1. On the other hand, the firm density is only positive if the bid-rent value is higher than an outside option given by the exogenous reservation land price, R_A. Condition iv) reflects market clearing, whereas the mass of firms must correspond to their land use. From this condition follows:

$$(22) \quad a = \frac{N_R + N_S}{2}.$$

In order to solve the model, some further assumptions need to be made. First, we assume a linear interaction term: $exp\left[-\frac{\kappa}{\delta}t|x-y|\right] = 1 - \tau|x-y|$. This implies that for non-negativity: $1 < 2\tau a$. Second, since the extent of land usage cannot be greater than the city itself: $N_R + N_S \leq 2a$. Thus, in combination with the non-negativity condition gives:

$$(23) \quad N_R + N_S \leq 1/\tau.$$

Hence, the sectoral bid-rent functions become:

$$(24) \quad \Omega_R\left(y, \pi_R^*, n_S^*\right) = \eta \int_X \left(1 - \tau|x-y|\right) n_S\left(y\right) dy - rl_R\left(y\right) - \pi_R\left(y\right)$$

$$(25) \quad \Omega_S\left(y, \pi_S^*, n_R^*\right) = \eta \int_X \left(1 - \tau|x-y|\right) n_R\left(y\right) dy - wl_S\left(y\right) - \pi_S\left(y\right).$$

From equations (24) and (25) follows: the bid-rent functions of $\mathbb{S}$- and $\mathbb{R}$-firms decrease with $|x|$ and are concave on a $\mathbb{S}$-district as well as on a $\mathbb{R}$-district.

The further approach is to determine a certain land use pattern and to derive the corresponding bid-rent schedules. If the land market equilibrium conditions hold, the city structure is sustainable. In total, three equilibrium formations exist: A) a monocentric city, in which one industry occupies a central business district; B) a mixed pattern, where firms of both industries share a common district; and C) as a knife-edge case of pattern B: perfect integration over the total city area.

4 Short Run Equilibrium

Monocentric pattern

Let us assume that the city structure follows type A: a central R&D district is surrounded by two manufacturing districts (henceforth, this pattern is denoted as the SRS-equilibrium). Figure 4 shows an illustration:

[Insert Figure 4 about here.]

Here, the city fringes are indicated by $-a$ and a; $-b$ and b denote inner-city district frontiers, whereas $b < a$. This city formation implies that $\sigma_R = b$, and $\sigma_S = a + b$. The corresponding bid-rent functions are:

$$(26) \quad \Omega_R\left(x\right) = \eta \int_{X-[-b,b]} \left(1 - \tau|x-y|\right) n_S\left(y\right) dy - rl_R\left(x\right) - \pi_R\left(x\right)$$

$$(27) \quad \Omega_S\left(x\right) = \eta \int_{-b}^{b} \left(1 - \tau|x-y|\right) n_R\left(y\right) dy - wl_S\left(x\right) - \pi_S\left(x\right).$$

In this context, Figure 5 shows the corresponding bid-rent schedule for a numerical example. As apparent, the bid-rent function of the $\mathbb{R}$-sector in the CBD $[-b, b]$ is higher than of the $\mathbb{S}$-sector determining the R&D district, while in the outskirts bid-rents of $\mathbb{S}$-firms are higher than of $\mathbb{R}$-firms giving rise to specialized manufacturing districts.

[Insert Figure 5 about here.]

The city structure is sustainable if and only if:

$$\begin{aligned}
&i) && \Omega_R(x) \geq \Omega_S(x) \Rightarrow n_R(x) = 1, n_S(x) = 0 \;\; \forall \;\; x \in [-b, b] \\
&ii) && \Omega_S(x) \geq \Omega_R(x) \geq R_A \\
& && \Rightarrow n_R(x) = 0, n_S(x) = 1 \;\; \forall \;\; x \in [-a, -b] \cup [b, a] \\
&iii) && \Omega_R(x = b) = \Omega_S(x = b) \\
&iv) && \Omega_S(x = a) = R_A
\end{aligned}$$

Taking conditions iii) and iv) into account, we are in position to determine equilibrium firm profits:

$$\pi_S^* = 2\eta b(1 - \tau a) - w\left[1 + a^2 + \epsilon_S \sigma_S^2\right] - R_A \tag{28}$$

$$\begin{aligned}
\pi_R^* &= \eta\left[2(a-b) + \tau\left(b^2 - a^2\right) - 2b(1 - \tau b)\right] - r\left(1 + b^2 + \epsilon_R \sigma_R^2\right) \\
&\quad + w\left[1 + b^2 + \epsilon_S \sigma_S^2\right] + \pi_S^*.
\end{aligned} \tag{29}$$

Based on these results it can be concluded:

Proposition 1. *The monocentric SRS-formation is sustainable if and only if:* $r - w \geq \eta\tau$. *The monocentric RSR-formation is sustainable if and only if:* $w - r \geq \eta\tau$.

Proof. The inequality in Proposition 1 simply follows from solving condition i). Condition ii) can be proved as follows. The bid-rent functions for $x \geq b$ are:

$$\Omega_R(x) = \eta\left[2(a-b) - \tau\left(x^2 + a^2\right) + 2\tau b x\right] - r\left(1 + x^2 + \epsilon_R \sigma_R^2\right) - \pi_R^*$$

$$\Omega_S(x) = 2\eta b(1 - \tau x) - w\left(1 + x^2 + \epsilon_S \sigma_S^2\right) - \pi_S^*.$$

After substituting equations (28) and (29), condition ii) leads to:

$$Z \equiv \frac{b}{x}\left(\eta\tau + w - r\right) + \frac{x}{b}\left(\eta\tau - w + r\right) - 4\eta\tau \geq 0.$$

The term on the right-hand side features two roots at: $x = b$ and $x = b\left(\frac{3\eta\tau+w-r}{\eta\tau-w+r}\right)$. Since the sign of the first derivative at $x = b$ is positive, the function Z is positive for $x > b$, which implies that the bid-rent of $\mathbb{S}$-firms is higher than of $\mathbb{R}$-firms in $[-a, -b] \cup [b, a]$. The second root is negative because $r - w$ in the denominator is greater $\eta\tau$ as required in Proposition 1. Thus, the SRS-formation is sustainable for $r - w \geq \eta\tau$.

The stability of the RSR-formation can be proven in the same way by simply exchanging indices R and S, as well as r and w in the profit functions. □

In terms of R&D and product quality we can conclude:

Proposition 2. *Given the SRS-formation, the quality of manufacturing products (and the corresponding R&D expenditures) quadratically increases towards the central $\mathbb{R}$-district. Given the RSR-formation, the quality of manufacturing products takes a maximum value in the city center and quadratically decreases towards the outer $\mathbb{R}$-districts.*

Proof. Given the assumptions made above, the optimum quality in equation (11) can be expressed as:

$$\text{(30)} \quad u_S\left(x\right) = \frac{\eta}{\delta}\int_{y\in X}\left[2 + \frac{\kappa}{\delta}\left(1 + \tau|x-y|\right)\right]\left(1 - \tau|x-y|\right) n_R\left(y\right) dy.$$

Under the SRS-formation equation (30) becomes:

$$\text{(31)} \quad u_S\left(x\right) = \begin{cases} 2b\frac{\eta}{\delta}\left\{2\left(1-\tau x\right) + \frac{\kappa}{\delta}\left[1 - \tau^2\left(x^2 + \frac{b^2}{3}\right)\right]\right\} & \forall \;\; x \geq b \\ 2b\frac{\eta}{\delta}\left\{2\left(1+\tau x\right) + \frac{\kappa}{\delta}\left[1 - \tau^2\left(x^2 + \frac{b^2}{3}\right)\right]\right\} & \forall \;\; x \leq -b. \end{cases}$$

As apparent, the quality for $x \geq b$ is characterized by a parable, which has a maximum for $x < 0$. Similarly, the quality for $x \leq -b$ has a maximum in $x > 0$. Figure 6 shows an illustration for the quality schedule of the SRS-formation.

[Insert Figure 6 about here.]

In case of the RSR-formation, the quality of manufactures produced in $[-b, b]$ can be derived from equation (11) again.

$$u_S(x) = 2b\frac{\eta}{\delta}\left\{(a-b)\left[2+\frac{\kappa}{\delta}\left(1-\tau^2x^2\right)\right]+\tau\left(b^2-a^2\right)+\frac{\kappa\tau^2}{3\delta}\left(b^3-a^3\right)\right\}.$$

The quality schedule follows a parable with a maximum at $x = 0$. $\square$

Mixed pattern

Turning to the formation B, Figure 7 shows an illustration of the mixed pattern equilibrium, in which firms of both industries share a common central district.

[Insert Figure 7 about here.]

In consequence, the CBD frontier is given by: $b = N_R/(2\rho)$, whereas ρ denotes the share of $\mathbb{R}$-firms and $1-\rho$ the share of $\mathbb{S}$-firms at location x. The bid-rent functions are now:

$$\begin{aligned}\Omega_R(x) = {} & \eta(1-\rho)\int_{-b}^{b}(1-\tau|x-y|)\,dy + \eta\int_{X-[-b,b]}(1-\tau|x-y|)\,n_S(y)\,dy \\ & - rl_R(x) - \pi_R(x)\end{aligned} \tag{32}$$

$$\Omega_S(x) = \eta\rho\int_{-b}^{b}(1-\tau|x-y|)\,dy - wl_S(x) - \pi_S(x). \tag{33}$$

In this context, Figure 8 illustrates a bid-rent schedule for the mixed pattern equilibrium.

[Insert Figure 8 about here.]

As apparent, in the integrated CBD both bid-rent functions are identical, while in the outer $\mathbb{S}$-districts the bid-rents of manufacturing firms are higher than of R&D firms. As a result of the concentration of the $\mathbb{R}$-sector in the CBD, the overall dispersion index is $\sigma_R = b$. The dispersion index of the $\mathbb{S}$-industry can now be derived as: $\sigma_S = \frac{a^2-\rho b^2}{a-\rho b}$.

In contrast to the SRS-formation, the first equilibrium condition for the mixed pattern is:

$$i) \qquad \Omega_R(x) = \Omega_S(x) \Rightarrow n_R(x) = \rho, n_S(x) = 1-\rho \ \ \forall \ \ x \in [-b, b].$$

Similarly as in case of pattern A, conditions iii) and iv) allow to determine equilibrium firm profits:

$$\text{(34)} \quad \pi_S^* = 2\rho\eta b(1-\tau a) - w\left[1 + a^2 + \epsilon_S \sigma_S^2\right] - R_A$$

$$\text{(35)} \quad \begin{aligned} \pi_R^* &= \eta\left[2(a-b) + \tau\left(b^2 - a^2\right) - 2b(1-2\rho)(1-\tau b)\right] - r\left(1 + b^2 + \epsilon_R \sigma_R^2\right) \\ &+ w\left[1 + b^2 + \epsilon_S \sigma_S^2\right] + \pi_S^*. \end{aligned}$$

From these settings it can be concluded:

Proposition 3. *The mixed pattern formation is sustainable if and only if: i)* $\rho = 0.5$*; ii)* $r = w$*; and iii)* $N_S > N_R$ *for the* $S/RS/S$*-formation and* $N_S < N_R$ *for the* $R/RS/R$*-formation. Under the same conditions but identical firm numbers,* $N_S = N_R$*, the common district covers the whole city area.*

Proof. In the first step, equality of the bid-rent functions (32) and (33) requires that the first terms equalize, which holds for: $\rho = 0.5$. In the next step, the second term in equation (35) can be expressed as: $\eta\left[2(a-b) + \tau\left(b^2 - a^2\right)\right]$. Thus, bid-rent equalization requires:

$$\eta\left[2(a-b) + \tau\left(b^2 - a^2\right)\right] - r\left(1 + x^2 + \epsilon_R \sigma_R^2\right) - \pi_R^* = -w\left(1 + x^2 + \epsilon_S \sigma_S^2\right) - \pi_S^*.$$

Substituting equilibrium profits (34) and (35) into this expression leads to identical factor prices: $r = w$. In the fourth step, it can easily be proved that condition iv) always holds given the requirements of Proposition 3. The last step ensures that $a > b$. Substituting $a = \frac{N_R+N_S}{2}$ and $b = \frac{N_R}{2\rho}$ for the $S/RS/S$-formation ($b = \frac{N_S}{2\rho}$ for the $R/RS/R$-formation) provides condition iii) in Proposition 3. For the knife-edge case of identical firm numbers, the CBD frontier b is shifted onto the city fringe a, which finally makes the integrated CBD occupy the total city interval. □

In terms of quality and R&D expenditures follows:

Proposition 4. *Given the mixed $S/RS/S$-formation, the quality of manufacturing products (and the corresponding R&D expenditures) takes a maximum value at $x = 0$ and quadratically decreases to the city fringe. Given the $R/RS/R$-formation, the quality of manufacturing products takes a maximum value in the city center and quadratically decreases towards the outer $\mathbb{R}$-districts.*

Proof. Similarly as in the case of the monocentric pattern, the specified quality integral given by equation (11) can be solved with respect to the spatial city structure. The quality becomes for the $S/RS/S$-formation:

$$u_S(x) = \begin{cases} \frac{\eta b}{3\delta}\left\{6\left[1+\tau x\right] + \frac{\kappa}{\delta}\left[3-\tau^2\left(b^2+3x^2\right)\right]\right\} & \forall \ \ x \leq -b \\ \frac{\eta}{3\delta^2}\left\{3b\left[2\delta+\kappa\right] - 3\delta\tau\left[b^2+x^2\right] - b\kappa\tau^2\left(b^2+3x^2\right)\right\} & \forall \ \ -b \leq x \leq b \\ \frac{\eta b}{3\delta}\left\{6\left[1-\tau x\right] + \frac{\kappa}{\delta}\left[3-\tau^2\left(b^2+3x^2\right)\right]\right\} & \forall \ \ x \geq b. \end{cases}$$

In terms of the $R/RS/R$-formation, the quality schedule can be derived as:

$$u_S(x) = \frac{\eta}{3\delta^2}\left[3b\left(2\delta+\kappa\right) - 3\delta\tau\left(b^2+x^2\right) - b\kappa\tau^2\left(b^2+3x^2\right)\right] \quad \forall \ \ -b \leq x \leq b.$$

As apparent, both schedules are quadratic and symmetric to the origin again. □

Figure 9 shows an illustration of the $S/RS/S$-formation by means of a numerical example again. Since the manufacturing industry is located across the total city area, the corresponding quality functions are continuous. In case of the $R/RS/R$-formation, the quality is a parable over the CBD.

[Insert Figure 9 about here.]

Comparative statics

For illustrating model mechanisms, the comparative statics of firm profits are to be summarized at this stage.

- $\mathbb{R}$-firm profits decrease in the skilled wage rate r, $\mathbb{S}$-firm profits decrease in the unskilled wage rate w.

- $\mathbb{S}$-firm profits decrease with the mass of $\mathbb{S}$-firms, while the direction of the backward linkage, which denotes the dependency on the mass of $\mathbb{R}$-firms, is ambiguous in sign. $\mathbb{S}$-firm profits decrease for a low monopolistic scope (small values of η), strong spillover effects (high values of w and ϵ_S), and high costs of vertical interaction, τ. As apparent, the standard backward linkage mechanism of monopolistic competition with vertical linkages (downstream profits positively depend upon the mass of upstream firms, as a result of equation (8) and (1), respectively) can be interfered by spillover effects within the manufacturing sector implying stronger competition on the land market.

- $\mathbb{R}$-firm profits decrease with the mass of $\mathbb{R}$-firms. The forward linkage, which is the dependency of upstream profits on the size of the downstream market given by the mass of $\mathbb{S}$-firms, is also dependent upon the monopoly power, the costs of vertical interaction as well as the degree of spillover effects in the $\mathbb{S}$-sector. In this context, the unskilled wage rate, w, controls via spillovers the pressure of manufacturing firms to locate close to each other, which reinforces the land market competition as discussed above.

5 Long Run Equilibrium

Free market entry and exits

In the long run the mass of firms in both sectors is endogenous. Thus, the equilibrium firm profits become zero. Starting with the the simplest case, the perfect integration equilibrium denoted by pattern C, the profit functions must be identical. At the city fringe they can be expressed as:

$$(36) \quad \pi_R = \eta a \left(1 - \tau a\right) - r \left(1 + a^2 + \epsilon_R a^2\right) - R_A$$

$$(37) \quad \pi_S = \eta a \left(1 - \tau a\right) - w \left(1 + a^2 + \epsilon_S a^2\right) - R_A.$$

As apparent, equality of equilibrium profits requires: $r = w$, as also derived in Proposition 3, as well as identical dispersion penalties ϵ_R and ϵ_S. Finally solving equations (36) and (37) for the firm number yields:

$$(38) \quad N_{S,R} = \frac{\eta \pm \sqrt{\eta^2 - 4\left(R_A + w\right)\left(w + \eta\tau + w\epsilon\right)}}{2\left(w + \eta\tau + w\epsilon\right)}.$$

Nonnegativity of the square root in equation (38) provides a further sufficient equilibrium condition: $\eta^2 \geq 4\left(R_A + w\right)\left(w + \eta\tau + w\epsilon\right)$. If the root in equation (38) is smaller than η, we obtain two possible values for the equilibrium firm numbers in both sectors, otherwise only one value.

In case of the monocentric pattern A and the mixed pattern B, the algebra becomes more complex so that further analysis relies on numerical calibrations. Nonetheless, it can be shown that given the conditions of Propositions 1 and 3 (and $\epsilon_S < \epsilon_R$ for the $S/RS/S$-formation), the same equilibrium structure as in pattern C appears. In this context, Figure 10 illustrates for the monocentric SRS-formation the long-run firm numbers in both sectors with respect to the spatial transaction cost ($\eta = 10$, $\epsilon_R = 0.8$, $\epsilon_S = 0.1$, $R_A = 1$, $r = 8$, $w = 2$).

[Insert Figure 10 about here.]

As apparent, pattern A exists for this specific numerical example only for $\tau \leq 0.14$. For decreasing transaction costs both equilibrium firm numbers diverge. This bifurcation occurs also for patterns B and C. In general, manufacturing firms in the long run face either a relative weak or a strong competition corresponding with the mass of firms in their industry. Furthermore, the lower is the firm number the higher is the market share of each firm and, thus, the larger is the financial capacity for R&D investments. As a result, the long-run equilibria are also characterized by a divergence of R&D investments and the quality of manufacturing products.

Stability of equilibria

Assuming a simple out-of-equilibrium adjustment dynamics:

$$\dot{N} = f(\pi)\ ,\ f(0) = 0\ ,\ \partial f / \partial \pi > 0,$$

firms enter the market until their profits become zero. In this context, Figure 11 shows by means of the same parameter constellation as in Figure 10 the zero-profit isoclines of both industries with respect to the sectoral firm numbers.

[Insert Figure 11 about here.]

As apparent, the curves are concave with respect to the firm number of the opponent sector.[7] As discussed above, three factors affect the shape of the zero-profit isoclines: i) the forward and backward linkages (the higher the firm number in one sector the higher the profits in the other sector); ii) the land market competition (the higher the firm numbers the lower the firm profits due to higher land rents); and iii) spillover effects (the higher the firm number the higher potential agglomeration externalities). For a critical level of transaction costs, both curves determine at their intersections the steady states illustrated in Figure 7. For decreasing transaction costs the isoclines of $\mathbb{R}$-profits move upward, while the isocline of $\mathbb{S}$-profits move to the right-hand side. In consequence, the equilibria diverge. Applying the adjustment dynamics, for a $\mathbb{R}$-firm number above the corresponding zero-profit isoclines, profits are negative and $\mathbb{R}$-firms leave the market, until the profits are zero again. The opposite holds if the $\mathbb{R}$-firm number is below the zero-profit isocline. In case of the manufacturing industry, a $\mathbb{S}$-firm number on the right(left)-hand side of the corresponding isocline implies negative (positive) $\mathbb{S}$-firm profits, which leads to market exits (entries) out of (in) the specialized sector. It is straightforward to see that the upper equilibrium is (locally) stable, while the lower equilibrium turns out to be unstable as also illustrated in Figure 7 by the dashed lower equilibrium path.

Labor market effects

So far, supplies and wages of skilled and unskilled labor have been treated to be exogenous. Under the assumption of perfectly competitive labor markets, an increase in the local research workforce, e.g., as a result of migration from other cities, leads to a decrease in the corresponding wage rate r.[8] As discussed in the course of

[7] In fact, both curves feature also a maximum, which is neglected for sake of analytical simplicity.

[8] For simplicity, we assume that residential districts are outside the city and may be considered to correspond with the reservation land rent R_A. For a more detailed analysis including commuting see Fujita and Thisse (2002), Ch. 7.5.1, which promises similar results for the model discussed in this paper.

the comparative statics in the previous section, an increase in r implies an increase in $\mathbb{R}$-firm profits, which leads via market entries to an increase in the number of $\mathbb{R}$-firms. The next effect depends on the strength of spillovers, monopolistic power and transaction costs. For low values of η, high values of τ, and strong spillover effects, $\mathbb{S}$-firm profits decrease, which goes along with an decreasing $\mathbb{S}$-firm number. For high values of η, weak spillovers, and low transaction costs τ, the $\mathbb{S}$-firm number increases due to increasing $\mathbb{S}$-firm profits. Similarly, the same effects are at work for a change in the unskilled wage rate, w.

However, variations of the wage rate may have a destabilizing impact on the city pattern. Considering the monocentric SRS-formation, for instance, if the change in r exceeds the value of $\eta\tau$ (see Proposition 1), the equilibrium structure becomes unstable. Similarly, a slight change of r in case of the mixed pattern B or C immediately leads to a breakdown of the corresponding city pattern.

City size and R&D intensity

Another aspect of the model discussed in this paper are potential statements on the relationship between city size and the extent of R&D activities of firms. It is obvious that the number of firms, which determines the size of the city, has an impact on the research efforts through different channels. On one hand, the firm number controls the degree of competition within the same industry; on the other hand, it affects the strength of vertical linkages to the other industry. This implies that due to a higher firm number in the manufacturing industry a larger city may imply lower R&D expenditures of each firm and a lower corresponding quality of products. In contrast, an increasing firm number in the $\mathbb{R}$-sector makes R&D costs decrease, which leads to quality improvements according to equations (10) and (11).

Taking the quality as an indicator for the R&D intensity of firms into account, we consider two measures: i) the maximum quality that can be achieved; and ii) the average quality across all firms within the city. From these we can derive the following results for i):

Proposition 5. *1) If the manufacturing sector is solely located within the CBD, which includes the monocentric RSR- and the mixed $R/RS/R$-formation, the maximum quality increases with an increasing city size determined by the city frontier* *a. 2) The maximum quality in the equilibrium SRS- and $S/RS/S$-formations is independent from the city size. 3) In case of the perfect-integration pattern C, the maximum quality increases until a critical city size, from which one it decreases again.*

Proof. The highest quality produced within the city occurs at $x = 0$ for the monocentric RSR-formation, for patterns B and C as well as at $x = b$ for the monocentric SRS-formation. Simply differentiating the corresponding valuations of equation (11) with respect to a reveals either zero for the SRS- and the $S/SR/S$ formation, or $a < 1/\tau$ in case of the RSR- and the $R/SR/R$ formations, which always holds due to equation (23). Similarly, we obtain a positive correlation between maximum quality and city size for pattern C if: $a < \sqrt{\frac{2\delta+\kappa}{\kappa\tau}}$. □

And in terms of the average quality in ii) we obtain:

Proposition 6. *1) If the manufacturing sector is solely located within the CBD, the average quality increases with the city size. 2) The average quality in the equilibrium SRS- and $S/RS/S$-formations monotonously decreases with the city size. 3) 2) In case of the perfect-integration pattern C, the average quality increases until a critical city size, from which on it decreases again.*

Proof. The average quality of products manufactured in the city is given by:

$$(39) \quad AM\left(u_S\right) = \frac{1}{NS} \int_{-a}^{a} u_S\left(x\right) dx.$$

Exemplarily considering pattern C, equation (39) can be expressed as:

$$(40) \quad AM\left(u_S^C\right) = \frac{2\eta a}{3\delta^2} \left[6\delta + \kappa\left(3 - 2a^2\tau^2\right)\right].$$

Differentiating equation (40) with respect to the city size, a, reveals a non-monotonous behavior, whereas the change in sign of the first derivative occurs at:

$$(41) \quad \frac{\partial AM\left(u_S^C\right)}{\partial a} \gtrless 0 \quad \leftrightarrow \quad a^* \lessgtr \frac{1}{\tau}\sqrt{\frac{2\delta+\kappa}{2\kappa}}.$$

The corresponding threshold for the RSR- and $R/RS/R$-formations can be determined in the same manner:

$$(42)\quad \frac{\partial AM\left(u_S\right)}{\partial a} \gtrless 0 \quad \leftrightarrow \quad a^* \lessgtr \sqrt{\left(\frac{\delta+\kappa}{\kappa\tau}\right)^2 - \frac{b^2}{3}} - \frac{\delta}{\kappa\tau} < \frac{1}{\tau}.$$

As apparent, as long as the terms on the right hand side of equation (41) is smaller than the city frontier a, the city size and quality are positively correlated. This holds as long as: $2\delta < \kappa$. The threshold, at which the first derivative of equation (40) becomes zero, increases with decreasing spatial transaction costs, τ, decreasing preference parameter (for R&D services), κ, and with an increasing preference parameter (world market), δ.
As it can be easily be reproduced, the term on the right hand side of equation (42) is always lower than $1/\tau$, which is according to equation (23) always lower than the upper bound of the city size, $2a \leq 1/\tau$. Thus, the average quality is always increasing with the city size in the case of the RSR- and $R/RS/R$-formations. □

6 Concluding Remarks

Considering at first the results derived in the previous section, we are in position to draw the following conclusions: In terms of the perfect-integration pattern C, which involves a critical threshold of the city size, a^*, the average quality is increasing for low and medium technologies, which may be characterized by low values for the transactions costs, τ and high values for the preference parameter, κ. Based on these findings, the model predicts also a positive correlation between R&D intensity and city size for the monocentric RSR- and the mixed $R/RS/R$-formations. This implies: if the manufacturing sector is only located within the CBD and (parts of) the research sector in the outskirts, which probably holds for the majority of real cities, the maximum quality as well as the average quality increases with the city size.
In regard to the empirical findings, also land-prices are positively correlated with the R&D intensity via the city size as discussed in Section 2. As it is easy to reproduce, the bid-rent functions are positively dependent upon the city size given by a. This

means that the larger the city (and the higher the quality) the higher are also the corresponding bid-rent schedules.

At this point, the unavoidable question arises, why we do not empirically observe a negative correlation between city size and R&D intensity, which according to the modeling results should occur in case of the SRS- and $S/RS/S$-formations? It might stand to reason that the corresponding city formations generally do not appear very often in the real world, which makes it difficult to find significant empirical evidence. Propositions 1 and 3 may provide some indications: In case of the SRS-formation the research wage rate r has to be by the value $\eta\tau$ higher than the wage rate of manufacturing workers, w, which strengthens the impact of spillover effects in the $\mathbb{R}$-sector. In terms of the mixed pattern, the wage rates need to be equalized, but the manufacturing firm number must be higher than the number of research firms, which makes it profitable for $\mathbb{S}$-firms to locate around and amid the scarce research capacities. In view of the spatial distribution, if the manufacturing industry is such far-scattered in the city as it is in these both formations, spillovers are weak and become even weaker when the city grows, which finally makes the R&D intensity decline. In combination with the non-monotonous relationship in case of the perfect integration pattern, these aspects rather argue for high-tech clusters, which compared to other cluster categories appear only rarely in reality. However, these findings suggest a further empirical investigation.

Turning to policy implications, it can recently be observed that local policymakers utilize R&D promoting policy instruments in order to generate additional production and employment by offering R&D providers incentives to enter the local market. Such policy instruments are often tax reductions, public research infrastructure, research grants, land provision in form of technology parks etc..

However, in this paper we simply assume the impact of a lump-sum subsidy for $\mathbb{R}$-firms. Having a look on Figure 8 again, the subsidy shifts the zero-profit isocline of $\mathbb{R}$ upward. At a given level of transaction costs, this leads to an increase in the firm numbers of the upper stable equilibrium (and to a downshift of the unstable lower equilibrium). All in all, a subsidy leads to a rightward shift of the equilibrium bifurcation in Figure 7.

Nonetheless the political objective of attracting new R&D firms into the city is achieved, the subsidy causes a number of side-effects. As more firm enter the land market, the city fringe a as well as the central district b increase, which also gives rise to the land rents for firms in both industries. A further issue is the financing of the subsidy, which was neglected in the previous considerations. Taking social welfare into discount, the income of local private households provides the required tax base because firm profits become zero in the long run. In this regard, households face an increase in utility due to an increase in their working income; on the other hand the net income shrinks by the tax, which reduces the welfare again.[9] As another side-effect, the inflow of new firms and the corresponding increase in the demand for skilled and unskilled workers also increases their wage rates. This effect has either a dampening or reinforcing impact on the firm numbers as discussed in the course of the previous subsection.

Finally, in consideration of the correlation between R&D intensity and city size, attracting new research firms implies only in case of the RSR- and $R/RS/R$-formations an increase in the average and maximum qualities via a larger city size. However, as discussed above, these city formations occur rather in case of low and medium technologies. In fact, regional innovation policy often aims at stimulating growth of high-tech locations. Since these tend to form SRS and $S/RS/S$-configurations, larger cities weakens spillovers and, thus, the R&D productivity and average quality, while the maximum quality is not affected. In summary, this trade-off between city size and R&D performance restricts the outcome of political efforts.

[9]See Fujita (1988) for a further discussion of welfare issues, which also holds for this model. For discussion of welfare with respect to cross-differentiation and monopolistic competition, see Kranich (2009).

7 Figures and Tables

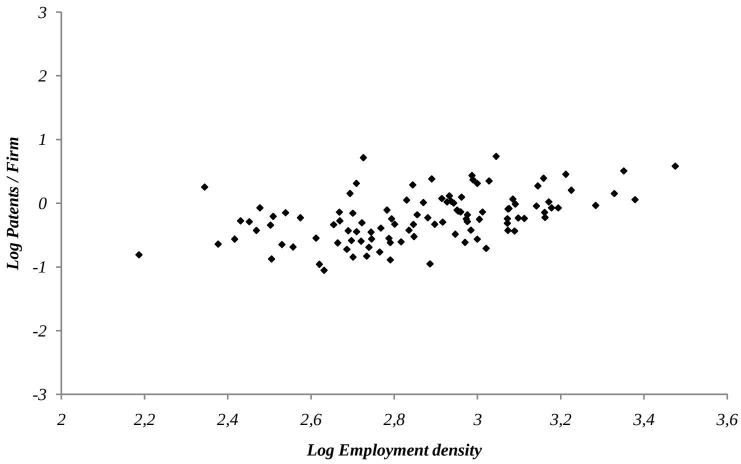

Figure 1: *Patent intensity and employment*

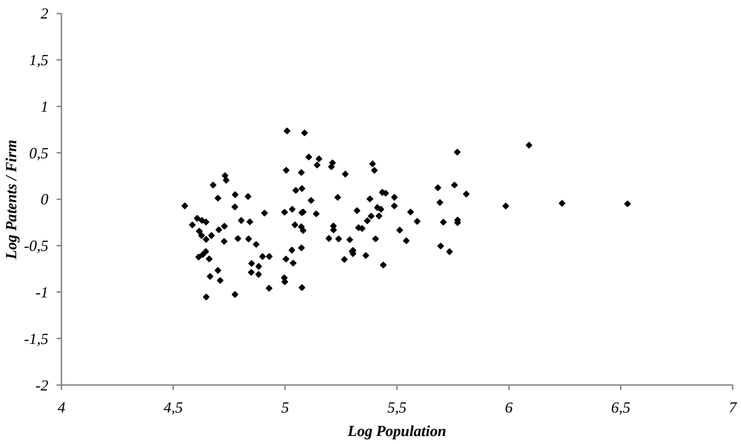

Figure 2: *Patent intensity and city size*

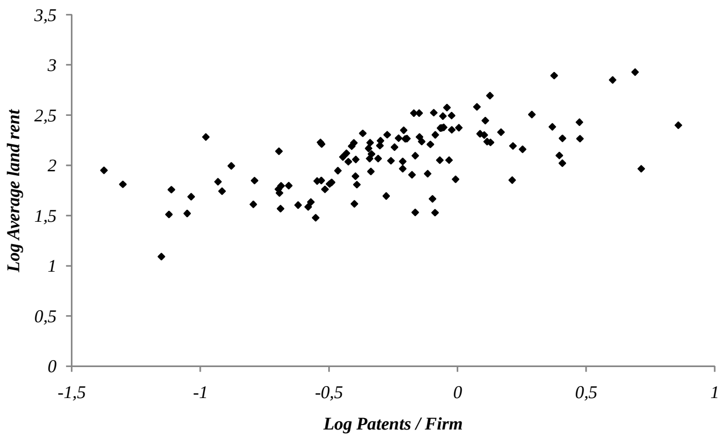

Figure 3: *Average land price and patent intensity*

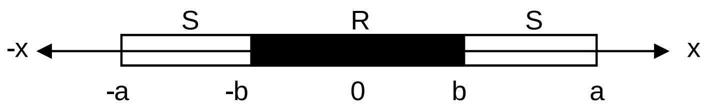

Figure 4: *Pattern A: SRS-formation*

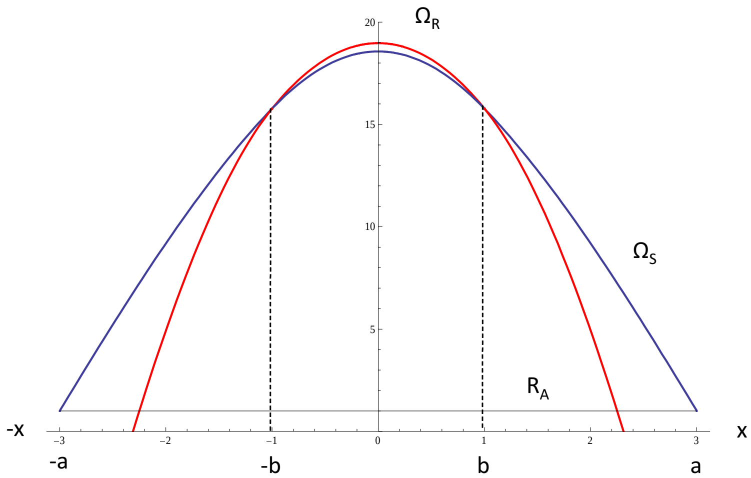

Figure 5: *Pattern A: bid-rent schedules*

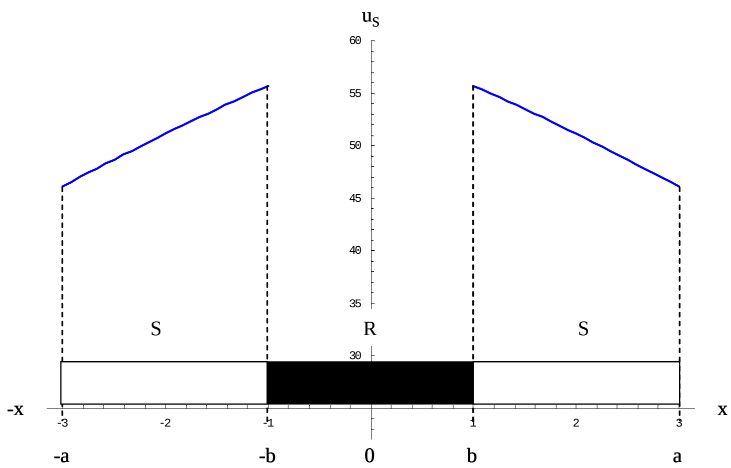

Figure 6: *Pattern A: quality schedules*

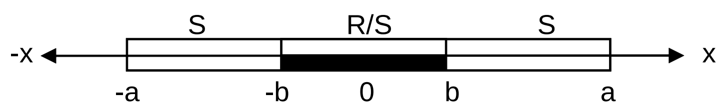

Figure 7: *Pattern B: S/RS/S-formation*

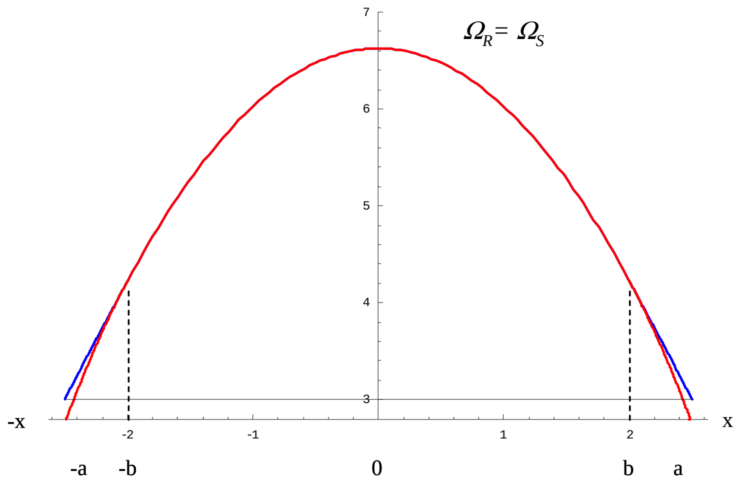

Figure 8: *Pattern B: bid-rent schedules*

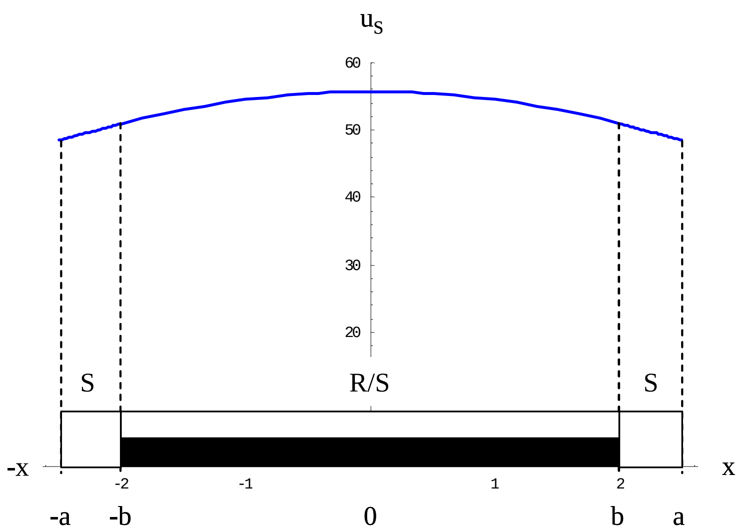

Figure 9: *Pattern B: quality schedules*

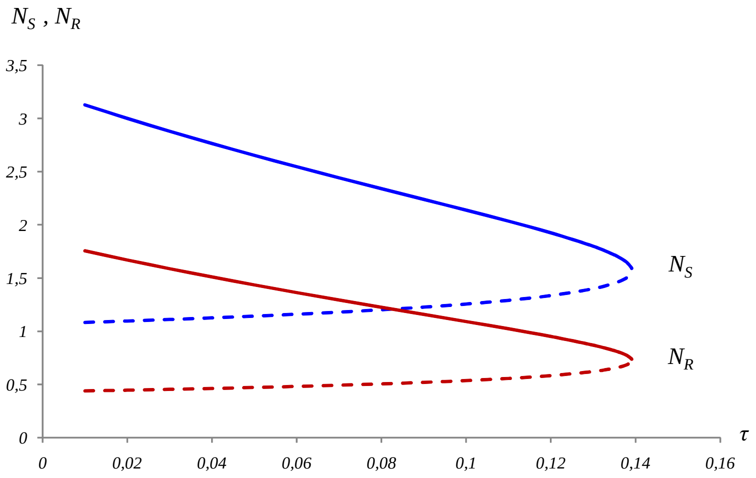

Figure 10: *Long run firm numbers of pattern A*

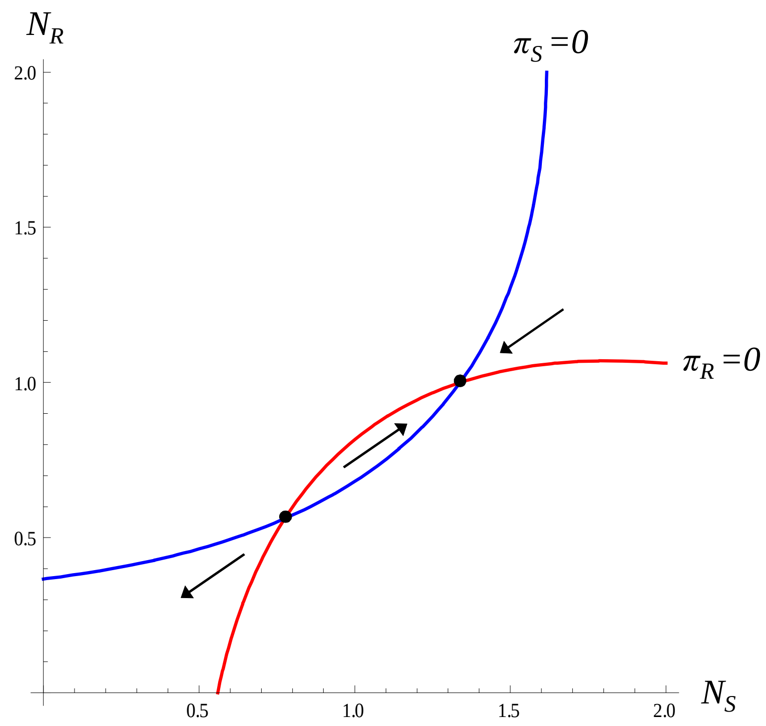

Figure 11: *Long run equilibria*

Table 1: *Descriptive statistics for Figures 1–3*

	Obs.	Mean	Std. Dev.	Min.	Max.
City area (km^2)	115	144.74	120.71	35.71	891.85
Population*	115	229,121	380,639.07	35,627.67	3,388,405
Firms*	115	93.49	105.89	13.83	836.67
Employment	108	139,994.44	211,787.44	19,050	1,550,383.33
Patents (Business)*	115	91.75	171.50	1.93	1358.82
Land price (€)**	106	162.27	141.01	12.37	849.17

* Average values for 2000-2005.
** Average values for 2005.

Sources: own calculations, based on data from the Federal Statistical Office Germany and the German Patent and Trade Mark Office.

Table 2: *Regression statistics*

	Figure 1	Figure 2	Figure 3
Correl. coeff.	0.4840	0.3055	0.6370
t-statistics	5.6678	3.4105	8.4277

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