Quantifying Optimal Growth Policy*

Volker Grossmann,† Thomas M. Steger,‡ and Timo Trimborn§

March 8, 2010

Abstract

This paper develops a comprehensive endogenous growth framework to determine the optimal mix of growth policies. The analysis is novel in that we capture important elements of the tax-transfer system and fully take into account transitional dynamics in our numerical analysis. Currently, for calculating corporate taxable income US firms are allowed to deduct approximately all of their capital and R&D costs from sales revenue. Our analysis suggests that the status quo policy leads to severe underinvestment in both R&D and physical capital. We find that firms should be allowed to deduct between 2-2.5 times their R&D costs and about 1.5-1.7 times their capital costs from sales revenue. Implementing the optimal policy mix is likely to entail huge welfare gains.

Key words: Economic growth; Endogenous technical change; Optimal growth policy; Tax-transfer system; Transitional dynamics.

JEL classification: H20, O30, O40.

* *Acknowledgements:* We are grateful to Holger Strulik for illuminating discussions.

† University of Fribourg; CESifo, Munich; Institute for the Study of Labor (IZA), Bonn. Address: University of Fribourg, Department of Economics, Bd. de Pérolles 90, 1700 Fribourg, Switzerland. E-mail: volker.grossmann@unifr.ch.

‡ University of Leipzig; CESifo, Munich. Address: Institute for Theoretical Economics, Grimmaische Strasse 12, 04109 Leipzig, Germany, Email: steger@wifa.uni-leipzig.de.

§ University of Hanover. Address: Institute for Macroeconomics, Königsworther Platz 1, 30167 Hannover, Germany, Email: trimborn@vwl.uni-hannover.de.

1 Introduction

There is common sense among economists that business enterprises in advanced economies conduct too little R&D. This conviction can be substantiated by noting that the social rate of return to business enterprise R&D is far above the private rate of return. The empirical productivity literature has identified social rates of return to R&D between 70 percent and more than 100 percent (e.g., Scherer, 1982; Griliches and Lichtenberg, 1984). Jones and Williams (1998) argue that, due to methodical shortcomings, these estimates should indeed be viewed as lower bounds. Hall (1996) reports that estimates of the private rate of return to R&D cluster around 10 percent to 15 percent. It is also widely believed that this R&D underinvestment bias is likely to cause a substantial loss in economic efficiency and social welfare. Moreover, there is now strong evidence showing that fiscal incentives are effective in increasing the economy-wide R&D intensity (e.g., Bloom, Griffith and van Reenen, 2002). Now, if one wants to turn all this research effort and the associated findings into something of practical value, one final question must be answered: What is the level of fiscal intervention which is required to remove the R&D underinvestment gap?

To answer this question, it is necessary to take both the general equilibrium dimension and the intertemporal dimension associated with R&D into account. Hence, endogenous growth theory provides a natural analytical framework for studies that aim at advising policy makers about the design of welfare-maximizing growth policy. However, any such analysis faces severe difficulties. The first important problem is to meet a balance between maintaining analytical tractability and avoiding that the model is too stylized to base policy recommendations upon it. It is true that any specific policy advise (like the calculation of the optimal R&D subsidy rate) requires numerical evaluations at some stage of the analysis. Nevertheless, we want to limit ourselves to models where the steady state can be derived analytically. For instance, the steady state seems to be a natural anchor in the context of the US economy to match endogenous variables to observables when calibrating the model. At the same time, it seems indicated to account for all three major engines of economic growth: private investment in physical capital, human capital, and R&D. Moreover, the model should capture the important elements of the tax-transfer system in order to account for existing tax distortions when designing growth policy. Taxes on labor income, capital income, capital gains and corporate income may be levied for other (e.g., redistributive) purposes than stimulating economic growth. For instance, an education subsidy is not only required to offset human capital externalities but should also alleviate negative education investment incentives which arise if labor income is taxed. In addition, the way income is taxed affects the calibration of the model when we take the steady state under the current tax-transfer system as an anchor. Failing to account for income taxation thus potentially gives rise to misleading growth policy implications.

A second problem concerns the numerical challenges associated with the analysis of transitional dynamics in complex growth models, arising in response to policy shocks, which are characterized by highly dimensional differential equation systems. It is well-known that, in growth models with decreasing marginal productivity of capital, it may take a long time (possibly more than hundred years) after some shock until per capita income adjusts to anywhere near the new steady state. It is thus salient to compute the policy mix which maximizes the intertemporal welfare gain from policy reform and not just focus on maximization of steady state welfare.

This paper develops a comprehensive endogenous growth framework to evaluate reforms of US growth policy and to derive the optimal policy mix. We allow for investment in physical capital, human capital and R&D as engines of economic growth. The analysis is novel in that we capture important elements (i.e., existing distortions) of the tax-transfer system and take into account transitional dynamics in our numerical analysis. We are also careful to calibrate the model by making use of steady state values under the status quo policy mix of a variety of endogenous variables which can be observed in the data. We thereby substantially limit the degree of freedom in the numerical analysis.

Technically, the major challenge is to calculate the entire transition path in response to policy shocks. The underlying R&D-based growth model represents a non-linear, highly dimensional, saddle-point stable, differential-algebraic system. Moreover, for plausible calibrations, the stable eigenvalues differ substantially in magnitude; hence, the dynamic system belongs to the class of stiff differential equations. Simulating such a dynamic model is all but trivial. The growth literature has used the techniques of linearization, time elimination, or backward integration. Linearization delivers bad approximations if the deviation from the steady state is large, time elimination does not work if there are non-monotonic adjustments, and backward integration fails in case of stiff differential equations. For these reasons we employ a recent procedure, called relaxation algorithm (Trimborn, Koch and Steger, 2008), which can deal with the conceptual difficulties mentioned above.

Our analysis suggests that the current R&D subsidization in the US leads to dramatic underinvestment in R&D. We find that innovating firms should be allowed to deduct from sales revenue more than twice their R&D costs for calculating corporate income, rather than just 1.1 times their R&D costs under the current policy. Interestingly, raising the R&D subsidy rate from its current level may entail no or just a small trade-off between long-run gains and short-run losses with respect to per capita consumption. There may even be an immediate increase in the level of per capita consumption. Such a possibility of an "intertemporal free lunch" arises because individuals anticipate the substantial future productivity gains and therefore may reduce savings immediately after the reform.

Moreover, we find that the US stimulus for investment in physical capital, and therefore the investment rate, is suboptimally low. The investment rate is biased downwards due to price setting power of firms. The underinvestment problem is less severe than for R&D, but is still substantial. Currently, for calculating corporate taxable income firms are able to deduct basically all of their capital costs from sales revenue. Our analysis suggests that they should be able to deduct somewhat more than 1.5 of their costs. Investment in human capital should also be subsidized, roughly to the extent labor income is taxed.

A policy reform targeted to all three growth engines simultaneously entails huge welfare gains. According to our preferred calibration, an appropriate policy reform could achieve an (intertemporal) welfare gain which is equivalent to a permanent increase in per capita consumption by about 86 percent. Unlike the optimal policy mix, the potential welfare gains are, however, quite sensitive to the underlying calibration.

There are only a few endogenous growth models which encompass all three growth engines. Funke and Strulik (2000) employ a three engines of growth model to show that economic development can be represented as a sequence of growth stages. Each stage is characterized by one of the growth engines being the dominant growth driver. Jones (2002) sets up a Solovian-type model (i.e., allocation variables are fixed) with physical capital accumulation, education, and R&D. Employing a growth accounting exercise, he shows that the average growth rate of US output per working hour (1953-1993) has a large transitional dynamics component. This is due to rising educational attainment over time and an increasing share of labor devoted to R&D. Papageorgiou and Perez-Sebastian (2006) set up a non-scale, R&D-based growth model with endogenous human capital à la Bils and Klenow (2000). The model is employed to replicate several empirical regularities of economic development in Japan and South Korea during the Post-WWII period. These contributions thus focus on different questions than our study, which is concerned with the optimality of the resource allocation in the economy and policy implications associated with it.

Our paper is most closely related to the literature which studies the R&D underinvestment problem in a steady state. Our analysis suggests that R&D underinvestment is more severe than previously found. Our main point of reference is the innovative study by Jones and Williams (2000). Like we do, they build on a horizontal innovation model without strong scale effects à la Jones (1995). Other contributions in this direction are Steger (2005) and Strulik (2007) who find an even smaller degree of underinvestment in R&D than Jones and Williams (2000). Similar to our steady state analysis, however, Steger (2005) finds that the market economy quite heavily underinvests in physical capital accumulation.

There are several differences of our analysis to these contributions. First, we also allow for human capital accumulation. Second, we use a different calibration strategy which does not require to calibrate the economy's labor share, being notoriously difficult to pin down empirically (see, e.g., Krueger, 1999). Third, and importantly, we account for existing distortions from income taxation. We demonstrate that doing so is the main reason why our (steady state) analysis suggests a more dramatic R&D underinvestment problem in the US economy than Jones and Williams (2000). Fourth, our analysis explicitly suggests how to correct underinvestment in the various growth channels, by introducing policy instruments targeted to the market failures captured in the model and calculate the optimal growth policy mix. Fifth, we are able to solve numerically also for the transition path. This allows us to evaluate policy reforms and compute welfare-maximizing values of policy instruments not only from a steady state perspective but also from a dynamic one.

The plan of the paper is as follows. Section 2 describes the underlying model. Section 3 derives the steady state solution for the market economy and for the social planning optimum. The calibration strategy is outlined in Section 4, while the optimal long run policy mix is presented in Section 5. The dynamic evaluation of policy reforms is discussed in Section 6. The main conclusions are summarized in Section 7. Technical details have been relegated to an appendix.

2 The Model

Consider the following continuous-time model with three engines of economic growth: horizontal innovations, physical capital accumulation and human capital formation. There is a homogenous final output good with price normalized to unity. Final output is produced under perfect competition according to

$$Y = \left(\int_0^A (x_i)^{\frac{\beta-1}{\beta}} di \right)^{\frac{\alpha\beta}{\beta-1}} (H^Y)^{1-\alpha}, \tag{1}$$

$0 < \alpha < 1$, $\beta > 1$, where H^Y is human capital (efficiency units of labor) in the manufacturing sector, A is the mass ("number") of intermediate goods and x_i denotes the quantity of intermediate good i. (Time index t is omitted whenever this does not lead to confusion.) The number of varieties, A, expands through horizontal innovations, protected with (potentially) infinite patent length. As usual, A is interpreted as the economy's stock of knowledge. $A_0 > 0$ is given. The labor market is perfect.

In each sector i there is one firm − the innovator or the buyer of a blueprint for an intermediate good − which has access to a one-to-one technology: one unit of foregone consumption (capital) can be transformed into one unit of output. Capital depreciates at rate $\delta_K \geq 0$. Capital supply in the initial period, K_0, is given. The capital market is perfect.

Moreover, in each sector i there is a competitive fringe which can produce a perfect substitute for good i (without violating patent rights) but is less productive in manufacturing the good: one unit of output requires κ units of capital; $1 < \kappa \leq \frac{\beta}{\beta-1}$.[1]

There is free entry into the R&D sector. Suppose that in each point of time, $(1+\psi)\dot{A}$ patents are generated. As in Jones and Williams (2000), $\psi\dot{A}$ of these patents replace existing patents, such that there will be "business stealing". Thus, in each point of time, the probability of an existing innovator to be replaced is equal to the fraction of firms driven out of business, $\psi\dot{A}/A$; the expected effective patent life is therefore limited to the inverse of this probability. Ideas for new intermediate goods are generated according to

$$(1+\psi)\dot{A} = \tilde{\nu} A^{\phi} H^A, \text{ with } \tilde{\nu} \equiv \nu \left(H^A\right)^{-\theta}, \tag{2}$$

where H^A is the human capital level in the R&D sector, $\nu > 0$, $\phi < 1$, $0 \leq \theta < 1$, $\psi \geq 0$. $\tilde{\nu}$ is taken as given in the decision of R&D firms; that is, similar to Jones and Williams (2000), R&D firms perceive a constant-returns to scale R&D technology, although the social return to higher R&D input is decreasing whenever $\theta > 0$. The wedge between private and social return may arise because firms do not take into account that rivals

[1] See Aghion and Howitt (2005), among others, for similar way of capturing a competitive fringe.

may work at the same idea such that from a social point of view some of the R&D input is duplicated ("duplication externality"). Parameter θ captures the extent of this externality. If $\phi > 0$, there is the standard "standing on shoulders" effect, whereas the case $\phi < 0$ implies by contrast that R&D productivity declines with the number of preceding innovations (possibly because the most obvious innovations are detected first; see Jones, 1995, for a discussion).

There is an infinitely-living, representative dynasty with initial per capita wealth, $a_0 > 0$. Household size, N, grows with constant exponential rate, $n \geq 0$. N_0 is given and normalized to unity. Preferences are represented by the standard utility function

$$U = \int_0^\infty \frac{(c_t)^{1-\sigma} - 1}{1-\sigma} e^{-(\rho-n)t} dt, \tag{3}$$

$\rho > n$, $\sigma > 0$, where c is consumption per capita. Households take factor prices as given.

The process of skill accumulation depends on the amount of human capital input per capita in the education sector, h^H. Moreover, it is characterized by human capital transmission within the representative household.[2] We also assume that human capital depreciates over time at rate $\delta_H > 0$. Formally, suppose that the human capital level per capita, h, evolves according to

$$\dot{h} = \xi \left(h^H\right)^\gamma h^\eta - \delta_H h, \tag{4}$$

$\gamma, \eta, \xi > 0$, $\gamma + \eta < 1$. $\gamma < 1$ captures decreasing returns to teaching input. Parameter η is associated with human capital transmission within the household over time. $\gamma + \eta < 1$ (thus, $\eta < 1$) implies that, on a balanced growth path, h assumes a stationary long-run value.[3]

[2] There is overwhelming evidence for the hypothesis that the education of parents affects the human capital level of children, even when controlling for family income. For recent studies, also providing an overview of the previous literature, see Plug and Vijverberg (2003) as well as Black, Devereux and Salvanes (2005).

[3] We abstract from human capital externalities in education of the kind formulated by Lucas (1988). In fact, there seems to be little evidence in favor of such externalities (see, e.g., Acemoglu and Angrist,

The government possesses a variety of policy instruments which potentially affect the three engines of growth in the model. At the household level it may subsidize education at rate s_H per unit of educational input. At the firm level, we assume that there is corporate income taxation. The corporate tax rate is identical across sectors and denoted by τ_c. Intermediate good firms may deduct at least part of their capital costs (for instance, via depreciation allowances or an investment tax credit), at rate $1 + s_d$. If $s_d = 0$, capital costs are fully deductible; if $s_d < (>)0$, they are less than (more than) fully deductible. Similarly, the R&D sector may deduct $1 + s_R$ of their R&D spending from sales revenue. Households are taxed in various ways. There is a tax on wage income at rate τ_w, a tax on income from asset holdings at rate τ_r, and a capital gains tax paid on increases in share prices. To be able to calibrate all the tax instruments at realistic levels, we also allow for redistribution via a lump sum transfer to households.[4] The government balances the budget in each point of time.

Let w and r denote the wage rate per unit of human capital and the interest rate, respectively. Moreover, denote by T the transfer per capita, which equals the sum of tax revenue minus subsidies, both divided by N. Financial wealth per individual, a, accumulates according to

$$\dot{a} = [(1 - \tau_r)r - n]\, a + (1 - \tau_w)wh - (1 - s_H)wh^H - c + T. \tag{5}$$

It turns out that, for the transversality conditions of both the household optimization problem and the social planner problem to hold and the value of the utility stream, U, to be finite, we have to restrict the parameter space such that

$$\rho - n + (\sigma - 1)g > 0 \text{ with } g \equiv \frac{\alpha(1-\theta)n}{(1-\alpha)(\beta-1)(1-\phi)}. \tag{A1}$$

As will become apparent, g is the economy's long run growth rate both in decentralized equilibrium and in social planning optimum. We maintain assumption A1 throughout.

2000).

[4]Note that there may well be heterogeneity of individuals, despite the assumption that there exists a (positive) representative consumer (consistent with the homothetic utility function (3)). See Mas-Colell, Whinston and Green (1995) for a discussion.

3 Steady State Analysis

The long run, decentralized equilibrium is compared to the social planning optimum. We also conduct a comparative-static analysis of the impact of changes of tax parameters on the equilibrium allocation of human capital.

3.1 Market Equilibrium

We start with intermediate goods producers. Denote by $R \equiv r + \delta_K$ the user cost of capital for an intermediate good firm (before taxation). As one unit of capital is required for one unit of output and firms are eligible to deduct capital costs at subsidy rate s_d from pre-tax profits to obtain the corporate tax base, producer i has profits

$$\begin{aligned} \pi_i &= p_i x_i - Rx_i - \tau_c \left[p_i x_i - (1+s_d)Rx_i\right] && (6) \\ &= (1-\tau_c)\left[p_i - (1-s_K)R\right] x_i, && (7) \end{aligned}$$

where we defined $s_K \equiv \frac{\tau_c s_d}{1-\tau_c}$ for the latter equation.

According to (1), the demand function for intermediate good i reads

$$x_i = \frac{\alpha Y (p_i)^{-\beta}}{P^{1-\beta}}, \tag{8}$$

where p_i is the price of good i and

$$P \equiv \left(\int_0^A (p_i)^{1-\beta} di \right)^{\frac{1}{1-\beta}} \tag{9}$$

is a price index. Profit maximization implies that the optimal price of each firm i is given by

$$p_i = p = \kappa(1-s_K)R. \tag{10}$$

To see this, note that a firm which owns a blueprint would choose a mark-up factor which is equal to $\frac{\beta}{\beta-1}$ if it were not facing a competitive fringe.[5] Moreover, the competitive fringe would make losses at a price lower than $\kappa(1-s_K)R$. Thus, as $\kappa \leq \frac{\beta}{\beta-1}$, each firm i sets the maximal price allowing it to remain monopolist. According to (8) and (9), resulting output is given by

$$x_i = x = \frac{\alpha Y}{A\kappa(1-s_K)R}. \tag{11}$$

Substituting (11) into (1) and solving for Y implies

$$y \equiv \frac{Y}{N} = A^{\frac{\alpha}{(1-\alpha)(\beta-1)}} \left(\frac{\alpha}{\kappa(1-s_K)R}\right)^{\frac{\alpha}{1-\alpha}} h^Y \tag{12}$$

for per capita income, where $h^Y \equiv H^Y/N$. Thus, the total amount of physical capital, $K = \int_0^A x_i di = Ax$, divided by population size, is given by

$$k \equiv \frac{K}{N} = A^{\frac{\alpha}{(1-\alpha)(\beta-1)}} \left(\frac{\alpha}{\kappa(1-s_K)R}\right)^{\frac{1}{1-\alpha}} h^Y. \tag{13}$$

Expressions (12) and (13) suggest that, if the interest rate r is stationary in the long run, the capital stock per capita and per capita income grow at the same rate along a balanced growth path.

Let P^A denote the present discounted (after-tax) value of the profit stream generated by an innovation. Thus, P^A is the price an intermediate good producer pays to the R&D sector for a new blueprint as well as the stock market evaluation of an intermediate good firm. In equilibrium, arbitrage possibilities in the capital market are absent. Thus, the dividends paid out by an intermediate good firm (being identical for all i due to symmetry, i.e., $\pi_i = \pi$), π/P^A, plus the growth rate of P^A after capital gains are taxed, $(1-\tau_g)\dot{P}^A/P^A$, must be equal to the sum of the after-tax interest rate, $(1-\tau_r)r$, and the probability that an existing innovator is driven out of business,

[5] As each firm is small, and thus takes aggregates Y and P as given, the perceived price elasticity of demand is $-\beta$.

$\psi\dot{A}/A$.[6] The no arbitrage condition for the capital market therefore reads

$$(1-\tau_g)\frac{\dot{P}^A}{P^A}+\frac{\pi}{P^A}=(1-\tau_r)r+\frac{\psi\dot{A}}{A}. \tag{14}$$

In the R&D sector, where firms are eligible to deduct R&D costs at subsidy rate s_R from pre-tax profits to obtain the corporate tax base, a representative firm maximizes

$$\begin{aligned}\Pi &= P^A(1+\psi)\dot{A}-wH^A-\tau_c\left[P^A(1+\psi)\dot{A}-(1+s_R)wH^A\right] &(15)\\ &= (1-\tau_c)\left[P^A\tilde{\nu}A^{\phi}H^A-(1-s_A)wH^A\right], &(16)\end{aligned}$$

taking A and $\tilde{\nu}$ as given, where we defined $s_A\equiv\frac{s_R\tau_c}{1-\tau_c}$ and used (2) for the latter equation. Rates s_A and s_K are referred to as behaviorally relevant subsidies of R&D costs and capital costs, respectively.

The household's problem is to solve

$$\max_{\{c_t,h_t^H\}}\int_0^{\infty}\frac{(c_t)^{1-\sigma}-1}{1-\sigma}e^{-(\rho-n)t}dt \quad \text{s.t. (4), (5),} \tag{17}$$

$$h_t \geq 0,\ \lim_{t\to\infty}a_t\exp\left(-\int_0^t[(1-\tau_r)r_s-n]\,ds\right)\geq 0. \tag{18}$$

The household chooses the optimal consumption path, where savings are supplied to the financial market, and the optimal (path of) education investment.

As will become apparent, in steady state, the per capita human capital level is stationary. According to (4), $\dot{h}/h=0$ implies that

$$h=\left(\frac{\xi}{\delta_H}\right)^{\frac{1}{1-\gamma-\eta}}\left(\mathfrak{h}^H\right)^{\frac{\gamma}{1-\gamma-\eta}}, \tag{19}$$

in the long run, where $\mathfrak{h}^H\equiv h^H/h$ is the fraction of human capital devoted to the education sector.

[6] Note that the after-tax income from asset holding of a household is $(1-\tau_r)rK/N+(1-\tau_g)\dot{P}^AA/N+\pi A/N-P^A\psi\dot{A}/N$. Under (14) and since $Na=K+P_AA$, this equals $(1-\tau_r)ra$, as reflected in the budget constraint (5) of a household.

In the proof of the first proposition we derive the full dynamical system (employed in the numerical analysis of Section 6), given initial conditions A_0, h_0, N_0 and K_0, as well as the steady state equilibrium. The following holds in a steady state:

Proposition 1. (Long run market equilibrium) *There exists a unique balanced growth equilibrium, which is characterized as follows.*

(i) The number of ideas grows with rate

$$\frac{\dot{A}}{A} = \frac{(1-\theta)n}{1-\phi} \equiv g_A. \tag{20}$$

(ii) Equity wealth per capita ($q \equiv P^A A/N$), the wage rate (w), income per capita (y), consumption per capita (c), financial wealth per capita (a), and the physical capital stock per capita (k) grow with rate

$$g = \frac{\alpha g_A}{(1-\alpha)(\beta-1)}. \tag{21}$$

(iii) The human capital level per capita (h) is stationary and we have

$$\frac{h^H}{h} = \frac{1-\tau_w}{1-s_H}\frac{\gamma\delta_H}{\rho-n+g(\sigma-1)+\delta_H(1-\eta)} \equiv \mathfrak{h}^{H*}, \tag{22}$$

and, defining $h^A \equiv H^A/N$,

$$\frac{h^A}{h} = \frac{1-\mathfrak{h}^{H*}}{\frac{1-s_A}{1-\tau_c}\Lambda(\tau_g)+1} \equiv \mathfrak{h}^{A*} \textit{ with} \tag{23}$$

$$\Lambda(\tau_g) \equiv \frac{\sigma g+\rho+\psi g_A-(n+g-g_A)(1-\tau_g)}{(1-1/\kappa)(\beta-1)(1+\psi)g}. \tag{24}$$

(iv) The savings and investment rate, $sav \equiv 1-c/y$, is given by

$$sav = \frac{\alpha(n+g+\delta_K)}{(1-s_K)\kappa\left(\frac{\sigma g+\rho}{1-\tau_r}+\delta_K\right)} \equiv sav^*. \tag{25}$$

Proof. See Appendix. ■

Like in Jones (1995), the growth rate of per capita income along a balanced growth path is independent of economic policy (in contrast to the level of income). This is an attractive feature for our numerical analysis. It allows us to attribute growth effects of policy shocks (starting from balanced growth equilibrium) entirely to the transitional dynamics. Proposition 1 also implies that life-time utility (3) is finite if and only if assumption (A1) holds.

Moreover, Proposition 1 shows that the three policy instruments which are targeted to the three engines of growth in the model affect only the respective engine where it is designed for in the long run. For instance, subsidizing physical capital does neither affect the level nor the allocation of human capital in long run equilibrium. But an increase in the behaviorally relevant capital cost subsidy, s_K, raises the long run savings rate and investment share, sav^*. Similarly, an increase in the education subsidy rate, s_H, raises the long run fraction of human capital devoted to education, $\mathfrak{h}^{H*}$, and therefore also raises the long run level of human capital per capita, according to (19). A change in s_H has no effect, however, on the long run fraction of human capital devoted to R&D, $\mathfrak{h}^{A*}$, or on the investment rate, sav^*. Analogously, an increase in the behaviorally relevant R&D subsidy, s_A, stimulates R&D activity of firms (i.e., $\mathfrak{h}^{A*}$ increases) but does not affect incentives to invest in education or physical capital in long run equilibrium.

Furthermore, we find that taxing wages gives a disincentive to invest in education, i.e., an increase in τ_w lowers $\mathfrak{h}^{H*}$. Similarly, an increase in the corporate tax rate (entering arbitrage condition (14) via instantaneous profits of intermediate good firms, π) gives a disincentive to invest in R&D; consequently, $\mathfrak{h}^{A*}$ is decreasing in τ_c, all other things equal. Moreover, an increase in the rate at which capital gains are taxed (τ_g) lowers R&D incentives, leading to a decline in $\mathfrak{h}^{A*}$, if $n + g > g_A$ (which turns out to hold with our calibration outlined in Section 4). Finally, the long run savings rate, sav^*, is decreasing in the capital income tax rate, τ_r.

3.2 Social Planning Optimum

A social planner chooses a symmetric capital allocation across intermediate firms, i.e., $x_i = K/A$ for all i. Noting the output technology (1), per capita output ($y = Y/N$) may be expressed as:

$$y = A^{\frac{\alpha}{\beta-1}} k^{\alpha} (h^Y)^{1-\alpha}. \tag{26}$$

Thus, the capital stock per capita ($k = K/N$) evolves according to

$$\dot{k} = A^{\frac{\alpha}{\beta-1}} k^{\alpha} (h^Y)^{1-\alpha} - (\delta_K + n)k - c. \tag{27}$$

Also note that the social planner takes R&D externalities into account. Using (2), he observes the knowledge accumulation condition

$$\dot{A} = \frac{\nu}{1+\psi} A^{\phi} (N h^A)^{1-\theta}. \tag{28}$$

The social planner's problem thus is to solve

$$\max U \ \text{ s.t. } (4), (27), (28), h^A = h - h^Y - h^H, \tag{29}$$

and non-negativity constraints, where c, h^A, h^H, h^Y are control variables and h, k, A are state variables.

Proposition 2. (Long run social optimum) *There exists an interior, unique long-run solution of the social planner problem (29) which is characterized as follows:*

(i) As in decentralized long run equilibrium, the growth rate of A is given by g_A (see (20)) and the growth rates of c, k, y are given by g (see (21)).

(ii) The fraction of human capital devoted to education and R&D are given by

$$\frac{h^H}{h} = \frac{\gamma \delta_H}{\rho - n + g(\sigma - 1) + \delta_H(1-\eta)} \equiv \mathfrak{h}^H_{opt}, \tag{30}$$

$$\frac{h^A}{h} = \frac{1 - \mathfrak{h}^H_{opt}}{\Gamma + 1} \equiv \mathfrak{h}^A_{opt} \ \textit{with} \ \Gamma \equiv \frac{\rho + g\sigma - \theta n - g}{(1-\theta)g}. \tag{31}$$

(ii) The savings and investment rate reads as follows

$$sav = \frac{\alpha(n+g+\delta_K)}{\sigma g+\rho+\delta_K} \equiv sav_{opt}. \tag{32}$$

Proof. See Appendix. ■

As for the decentralized equilibrium, the productivity of R&D and education, parameterized by ν and ξ, respectively, do neither affect the allocation and level of human capital nor the investment rate in the long run social optimum. Unlike in steady state market equilibrium, also parameter ψ, which captures the strength of the business stealing effect, and the mark-up factor κ do not affect the optimal resource allocation. These results parallel those of Jones and Williams (2000).

Like in their model, there are four R&D externalities. The duplication externality ($\theta > 0$) promotes overinvestment in R&D, whereas a standing on shoulders effect ($\phi > 0$) promotes underinvestment. (In the case where $\phi < 0$, there is a force towards overinvestment.) The business stealing effect ($\psi > 0$) gives rise to two counteracting effects on the human capital allocation in the market economy (relative to the unaffected social optimum). On the one hand, existing intermediate good firms are at risk of being replaced by future innovators. An increase in ψ thus lowers the value of patents (P^A) by raising the effective discount rate (right-hand side of (14)) and therefore depresses the long run equilibrium fraction of human capital devoted to R&D, h^A/h. On the other hand, an innovator obtains a rent from an innovation even when he does not contribute to the knowledge stock of the economy, A. To achieve the same increase in A, more R&D labor is required if ψ increases, which tends to raise the equilibrium value of h^A/h. If and only if the latter effect dominates, the fraction of human capital devoted to R&D in decentralized equilibrium increases in ψ. In this case, $\psi > 0$ promotes overinvestment. According to (23), $\mathfrak{h}^{A*}$ is increasing in ψ, for instance, if the capital gains tax rate (τ_g) is zero or small. Finally, innovators may not be able to appropriate the full economic surplus from raising the knowledge stock of the economy. To see this, note from (7), (10) and (11) that instantaneous profit of an intermediate goods firm i

reads $\pi_i = \alpha(1-\tau_c)\frac{\kappa-1}{\kappa}\frac{Y}{A}$, whereas $\frac{\partial Y}{\partial A} = \frac{\alpha}{\beta-1}\frac{Y}{A}$ holds, according to (26). If and only if $(1-\tau_c)(1-\frac{1}{\kappa}) < \frac{1}{\beta-1}$, there is a "surplus appropriability problem" which promotes underinvestment. (If $(1-\tau_c)(1-\frac{1}{\kappa}) > \frac{1}{\beta-1}$, there is a force towards overinvestment.) Thus, depending on parameter values, there may be over- or underinvestment in R&D. This leaves a critical role for the calibration strategy to obtain useful numerical results on the optimal resource allocation and policy mix.

Comparing (22) and (30), we find that in the case where the tax rate on wage income equals the effective education subsidy rate ($\tau_w = s_H$), both the long run fraction of human capital devoted to education and, according to (19), the long run level of human capital are socially optimal. That is, the distortion stemming from wage taxation can be exactly offset by an education subsidy. Generally, we find that $\mathfrak{h}^{H*} < (=,>)\mathfrak{h}^H_{opt}$ if $s_H < (=,>)\tau_w$. Finally, in absence of a capital cost subsidy ($s_d = s_K = 0$), the savings rate will be too low whenever $\tau_r \geq 0$, i.e., $sav^* < sav_{opt}$, according to (25) and (32).

We next characterize the optimal policy mix in the long run.

Proposition 3. (Optimal long run policy mix) *There exists a policy mix* $(s_H^{opt}, s_A^{opt}, s_K^{opt})$ *which for any feasible values of tax parameters* $(\tau_w, \tau_c, \tau_g, \tau_r)$ *implements the long-run social planning optimum. It is characterized as follows:*

$$
\begin{aligned}
s_H &= \tau_w, &(33)\\
s_A &= 1 - \frac{(1-\tau_c)\Gamma}{\Lambda(\tau_g)} \equiv s_A^{opt}, \text{ i.e. } s_R = \frac{1-\tau_c}{\tau_c} s_A^{opt} \equiv s_R^{opt}, &(34)\\
s_K &= 1 - \frac{\sigma g + \rho + \delta_K}{\kappa\left(\frac{\sigma g + \rho}{1-\tau_r} + \delta_K\right)} \equiv s_K^{opt}, \text{ i.e. } s_d = \frac{1-\tau_c}{\tau_c} s_K^{opt} \equiv s_d^{opt}. &(35)
\end{aligned}
$$

Proof. Set $\mathfrak{h}^{H*} = \mathfrak{h}^H_{opt}$, $\mathfrak{h}^{A*} = \mathfrak{h}^A_{opt}$ and $sav^* = sav_{opt}$ to derive (33), (34) and (35), respectively, by using the expressions in Proposition 1 and 2. ∎

How subsidies on R&D and capital costs depend on tax parameters follows from the tax distortions discussed after Proposition 1. Moreover, note that a higher mark up factor κ drives a bigger wedge between the equilibrium investment rate and the socially optimal investment rate, provided that capital income is not subsidized ($\tau_r \geq 0$). Thus, an increase in price setting power calls for a higher subsidy on capital costs.

Interestingly, in the proposed model the first-best allocation and the first-best level of human capital can be restored, despite numerous distortions from goods market imperfection, externalities and income taxation, with a very limited number of tax/subsidy instruments (one targeted to each engine of growth). This suggests, for instance, that an appropriate growth policy can mitigate distortions from redistributive policies along with correcting market failures. That is, any redistribution (which in the present context is reflected by the lump-sum transfer) can be achieved without sacrificing efficiency. In this sense there is no equity-efficiency trade-off.

4 Calibration

A calibration strategy is proposed which attempts to match observable endogenous variables for the US. We assume that the observed values correspond to the steady state in the model under the status quo policy.

4.1 Policy Parameters

In the US, the statutory tax rate on dividend income and corporate income coincide. We thus set $\tau_r = \tau_c = 0.395$, as published by the OECD tax database (federal and sub-central government taxes combined). Using the same source, the labor income tax, τ_w, is set equal to the total tax wedge (wage income tax rate including all social security contributions and from all levels of governments combined) which applies to average wage income. It is given by $\tau_w = 0.3$. The behaviorally relevant R&D subsidy rate, s_A, is (for the year 2007) taken from OECD (2007a, p.73), $s_A = 0.066$,[7] in turn implying $s_R = 0.1$.

Devereux, Griffith and Klemm (2002, p. 459) report for the US a rate of depreciation allowances for capital investments of almost 80 percent. This would suggest that s_d

[7] The OECD reports a R&D subsidy rate $RDTS = 1 - Bindex$, where the so-called B-index is given by $Bindex = \frac{1-\Xi}{1-\tau_c}$, with τ_c being the statutory corporate income tax rate and Ξ the net present discounted value of depreciation allowances, tax credits and special allowances on R&D assets. In the context of our model, $\Xi = \tau_c(1 + s_R)$. Thus, $RDTS = \frac{\tau_c s_R}{1-\tau_c} = s_A$.

is somewhat above -0.2 and thus $s_K < 0$. However, as the authors point out, the definition of corporate income tax base is very complex and there are other possibilities than depreciation allowances to deduct capital costs, which they cannot provide data on. We take into account further allowances by assuming that, initially, $s_d = s_K = 0$ (i.e. full deduction of capital costs).

As a result of the 'Jobs and Growth Tax Relief Reconciliation Act' of 2003, long-term capital gains are taxed at 15 percent if income is above some threshold. Otherwise, until 2008 it was 5 percent and until 2010 it is 0 percent. Before 2003 it was 20 percent. We calibrate τ_g to 12 percent.

Finally, we need to calibrate the education subsidy rate (s_H), which is most difficult. For instance, we observe the fraction of public education expenditure in total expenditure. In the year 2004, the average was 68.4 percent in the US (OECD, 2007b, Tab. B3.1, p. 219); among the public spending, 20.7 percent was on student loans, scholarships and other household grants (rather than direct public spending on institutions). To complicate things further, a substantial fraction of total household spending on education is unobservable, like private teachers at home, time costs of parents etc. (neither counted as education expenditure in databases nor subsidized). It is thus difficult to come up with a well-founded estimate. We assume that the education subsidy is set such that the long run equilibrium value $\mathfrak{h}^{H*}$ is socially optimal (as the long run level of human capital), given the distortion introduced by wage taxation, $s_H = \tau_w (= 0.3)$. That is, we focus on distortions of R&D investment and physical capital investment in our numerical analysis.

4.2 Other Parameters

Other parameters are calibrated as follows. First, n is set to the average population growth rate for the period 1990-2004. Taking data from the Penn World Tables (PWT) 6.2 (Heston, Summers and Baten, 2006), we find $n = 0.01$. For the same period and again from PWT 6.2, the average growth rate of per capita income is 2.1 percent. We calibrate g to match this growth rate (thereby averaging out business cycle phenomena).

We use measures for the investment rate (sav) and the capital-output ratio to calibrate the depreciation rate of physical capital, δ_K, as follows. The investment rate is given by $sav = (\dot{K} + \delta_K K)/Y = (\dot{K}/K + \delta_K)k/y$. Using $\dot{K}/K = n + g$ and solving for δ_K yields

$$\delta_K = \frac{sav}{k/y} - n - g. \tag{36}$$

Averaging over the period 1990-2004, sav is equal to about 21 percent, according to PWT 6.2. For the capital-output ratio, we take averages over the period 2002-2007 calculated from data of the US Bureau of Economic Analysis. The capital stock is taken to be total fixed assets (private and public structures, equipment and software). At current prices, this gives us $k/y = 3$. From (36), the evidence then suggests that δ_K is about 4 percent, which is a standard value in the literature. In the literature, the depreciation rate of human capital is typically set slightly lower than δ_K. We choose $\delta_H = 0.03$. This is in the range of the estimated value in Heckman (1976), who finds that δ_H is between 0.7 and 4.7 percent. For the steady state analysis in Section 5, we do not need to know δ_H, as will become apparent.

Moreover, we match the steady state interest rate to 7 percent, which coincides with the real long-run stock market return estimated by Mehra and Prescott (1985).[8] In our framework, the standard Keynes-Ramsey rule,

$$\frac{\dot{c}}{c} = \frac{(1-\tau_r)r - \rho}{\sigma}, \tag{37}$$

holds (see the proof of Proposition 1). In steady state, $\dot{c}/c = g$, according to Proposition 1. Thus, preference parameters (σ, ρ) fulfill:[9]

$$\sigma g + \rho = (1-\tau_r)r. \tag{38}$$

For $g = 0.021$, $r = 0.07$, $\tau_r = 0.395$ and a typical value for the time preference rate of

[8] Jones and Williams (2000) argue that this rate of return is more appropriate for calibration of growth models than the risk-free rate of government bonds.

[9] Rewriting assumption (A1) by using (38) implies that $(1-\tau_r)r > n+g$, i.e., the after-tax interest rate must exceed the long-run growth rate of aggregate income.

$\rho = 0.02$, we find $\sigma = 1.08$. For the steady state analysis in Section 5, we do not need to set the values for ρ and σ separately but only their combination on the left-hand side of (38), which equals the (after-tax) long run interest rate, $(1 - \tau_r)r$ $(= 0.042)$. When the transitional dynamics are fully taken into account in Section 6, we set $\rho = 0.02$ and $\sigma = 1.08$.

Production technology parameters α and β are potentially critical since they determine the elasticity of output with respect to the state of knowledge, A. To see this, use $x_i = K/A$ for all i and $H^Y = Nh^Y$ in (1) to find

$$Y = BK^{\alpha}N^{1-\alpha} \text{ with } B \equiv A^{\frac{\alpha}{\beta-1}} \left(h^Y\right)^{1-\alpha}. \tag{39}$$

We employ a relationship between α and β which can be recovered from estimates of the output elasticity with respect to the R&D capital stock. Using (39), this elasticity is equal to $\frac{\partial Y}{\partial A}\frac{A}{Y} = \frac{\alpha}{\beta-1} \equiv \varphi$. Thus,

$$\beta = 1 + \frac{\alpha}{\varphi}. \tag{40}$$

We can write $\log B = \Upsilon + \varphi \log A$, where $\Upsilon \equiv (1-\alpha)\log h^Y$, according to (39). Regressing $\log B$ (by using that the total factor productivity is given by $B = YK^{-\alpha}N^{\alpha-1}$) on a measure of knowledge capital ($\log A$), Coe and Helpman (1995) obtain $\varphi = 0.23$, which is the value we use.

The steady state fraction of intermediate good firms driven out of the market each instant is ψg_A. Its inverse is equal to the effective patent life, EPL. Thus, we have

$$\psi = \frac{1}{EPL}\frac{1}{g_A}, \tag{41}$$

where

$$g_A = \frac{(1-\alpha)(\beta-1)g}{\alpha}, \tag{42}$$

according to (21). We follow Jones and Williams (2000) in assuming an effective patent life of 10 years ($EPL = 10$).

Moreover, using (12) and (13), we find the following relationship between α and mark-up factor κ:

$$\kappa = \frac{\alpha}{(1-s_K)(r+\delta_K)\frac{k}{y}}. \tag{43}$$

A key parameter is α, which according to (40)-(43) determines β, ψ, g_A and κ, for given calibrated values of φ, EPL, g, s_K, r, δ_K, k/y. In the literature, the value of α is typically motivated by using the labor share in total income. However, due to the existence of R&D workers and teachers in the model, α is related to the fraction of income which accrues to production workers only (rather than to the entire labor share): we have $wh^Y/y = 1-\alpha$. Moreover, as pointed out by Krueger (1999), among others, there is little consensus on how to measure the total labor share as fraction of GDP. In our context, the labor share is wh/y. When two thirds of business proprietor's income is added to labor income, Krueger (1999) shows that the US labor share fluctuates over time between 75 and 80 percent. Otherwise the labor share would be significantly lower.[10] For the purposes of our model, however, income from all kinds of human capital should be taken into account; thus, wh/y may even exceed 80 percent. Due to the uncertainty about the labor share, we propose a different route than typically taken in the literature. Our calibration strategy is to determine the human capital income share endogenously, together with the salient parameter α. This is done as follows. Defining $\omega \equiv wh/y$, $\omega^H \equiv wh^H/y$ and $\omega^A \equiv wh^A/y$, we obtain from $h^Y + h^A + h^H = h$ and $wh^Y/y = 1-\alpha$ that

$$\omega = 1 + \omega^H + \omega^A - \alpha. \tag{44}$$

By definition, we have $h^A/h = \omega^A/\omega$ and $h^H/h = \omega^H/\omega$. Substituting both $\mathfrak{h}^{A*} = \omega^A/\omega$ and $\mathfrak{h}^{H*} = \omega^H/\omega$ into expression (23) for the long run equilibrium fraction of human capital devoted to R&D, and then using (44), we find

$$\frac{1-s_A}{1-\tau_c}\Lambda(\tau_g)\omega^A = 1-\alpha. \tag{45}$$

Given ω^A and taking into account relationships (38), (40), (41), (42) and (43) to find

[10] For instance, the OECD reports a labor share around 65 percent for the US.

$\Lambda(\tau_g)$ as defined in (24), α is implied by (45). Note that ω^A is the R&D intensity in the economy. For the period 1990-2006 we find that the average R&D costs of business enterprises (BERD) as fraction of GDP is 1.9 percent (OECD, 2008a). When we use gross R&D investment intensity (GERD), the figure would be higher (about 2.6 percent). As most but not all R&D costs are labor costs, this suggests to calibrate $\omega^A = 0.02$. However, one may argue that not all R&D activity in the sense of the model is captured by typical R&D intensity measures. According to OECD (2008b, Tab. 1.1), total investment in intangible assets in the US as a fraction of GDP was almost 12 percent for the period 1998-2000. However, 5 percent of GDP was spent to develop intangible assets like brand equity, firm-specific human capital and the organizational firm structure, which are not R&D activities in the sense of our model. We therefore consider $\omega^A = 0.07$ as an alternative scenario to the case of $\omega^A = 0.02$ in our numerical analysis.

Note that we do not need to know the fraction of human capital used in education, ω^H, to calibrate α. Moreover, all parameters which are needed can be led back to observables. With $n = 0.01$, $g = 0.021$, $r = 0.07$, $k/y = 3$, $sav = 0.213$ (thus, $\delta_K = 0.04$), $EPL = 10$, $\tau_r = \tau_c = 0.395$, $\tau_w = 0.3$, $\tau_g = 0.12$, we find for the case where the R&D intensity is $\omega^A = 0.02$ that $\alpha = 0.36$. In turn, this value of α implies $\beta = 2.58$, $\psi = 1.74$, $g_A = 0.06$ and $\kappa = 1.1$. If the R&D intensity is set to $\omega^A = 0.07$, we obtain $\alpha = 0.44$, $\beta = 2.93$, $\psi = 2$, $g_A = 0.05$ and $\kappa = 1.35$. Interestingly, both values for mark-up factor κ are in the range (between 1.05 and 1.4) which has been estimated by Norrbin (1993).

To calculate the long run equilibrium allocation of human capital, characterized by $\mathfrak{h}^{A*} = \omega^A/\omega$ and $\mathfrak{h}^{H*} = \omega^H/\omega$, we next need to find the human capital income share, ω, by calibrating ω^H and using (44). To calibrate ω^H, we add expenditure from public and private sources over all education levels. This gives us an average value of 7 percent for the time period 1990-2003 (OECD, 2007b, Tab. B2.1, p. 205).[11] As not all education

[11] Although there is no publicly provided education in our model, it is more appropriate to take such expenditure into account, in addition to private education spending. An underlying assumption which justifies that choice is that credit constraints are negligible for advanced economies, such that publicly provided education and private education are perfect substitutes. In fact, recent studies find

expenditure is on salary of teaching personnel, we use $\omega^H = 0.05$. For $\omega^A = 0.02$, we then find $\omega = 0.71$ and therefore $\mathfrak{h}^{H*} = \mathfrak{h}^H_{opt} = \frac{5}{70.8} = 0.071$ and $\mathfrak{h}^{A*} = \frac{2}{70.8} = 0.028$.[12] For $\omega^A = 0.07$, we obtain $\omega = 0.68$, $\mathfrak{h}^{H*} = \mathfrak{h}^H_{opt} = 0.074$ and $\mathfrak{h}^{A*} = 0.104$.

Parameters ρ, σ, γ, η, ϕ, δ_H, ν, ξ do not have to be known for the steady state analysis in Section 5. Scale parameters ν and ξ in the technology of accumulating knowledge and human capital, respectively, do not enter the long run values for the allocation variables derived in Proposition 1 (decentralized equilibrium) and Proposition 2 (social optimum). They also do not affect the allocation variables of interest in the transitional dynamics and can thus be set arbitrarily.[13]

In contrast, the duplication externality parameter θ plays an important role. Given g, n, α, β and θ, we obtain ϕ from (20) and (21):

$$\phi = 1 - \frac{\alpha n(1-\theta)}{(1-\alpha)(\beta-1)g}. \tag{46}$$

Finally, also parameters γ and η are not independent from each other when assuming that the economy initially is in steady state. According to (22), given $\rho+\sigma g = (1-\tau_r)r$, g, n, δ_H, $s_H = \tau_w$ and $h^H/h = \mathfrak{h}^{H*}$, we obtain the relationship

$$\gamma = \frac{(1-\tau_r)r - n - g + \delta_H(1-\eta)}{\delta_H}\mathfrak{h}^{H*}. \tag{47}$$

In Section 6 we consider $\eta \in \{0.15, 0.3\}$ and obtain γ by using (47). It turns out that results are basically insensitive to variations in η (and γ).

no evidence for the relevance of educational borrowing constraints in the US (see, e.g., Cameron and Taber, 2004, and the references therein).

[12] We shall note that h^A/h does not necessarily correspond to the fraction of workers in R&D and thus cannot be readily observed even under the assumption that the economy is in steady state. Although there is a representative agent, there may well be heterogeneity, such that not all individuals possess the same level of human capital. Thus, an implied h^A/h which exceeds the fraction of R&D workers (equal to about 1 percent) is consistent with the fact that the average R&D worker has a higher level of human capital than the average worker in the labor force.

[13] We can show numerically that ν and ξ do not affect the Eigenvalues of the dynamical system.

5 Optimal Long Run Policy Mix

The endogenous growth literature has discussed whether R&D activity and physical capital investment are too high or too low from a social point of view. The question has to be examined numerically. Before analyzing the transition path in the aftermath of policy reforms, we compare the long run social optimum to the decentralized steady state equilibrium, under existing US tax policy. This also allows us to compare the results with the previous literature, which exclusively focussed on a long run analysis (e.g., Jones and Williams, 2000; Steger, 2005; Strulik, 2007). Importantly, we also derive the optimal subsidy rates targeted to R&D and capital costs, by employing Proposition 3. Regarding human capital, recall that $s_H = \tau_w$ is optimal.

5.1 R&D Investment

We start with R&D investment, for different values of the degree of duplication externality θ.

θ	$\mathfrak{h}_{opt}^{A}$ $(in\%)$	$\mathfrak{h}^{A*}$ $(in\%)$	$\mathfrak{h}_{opt}^{A}/\mathfrak{h}^{A*}$	s_A^{opt}	s_R^{opt}
0	45.4	2.8	16.1	0.97	1.48
0.25	41.7	2.8	14.8	0.96	1.48
0.5	35.9	2.8	12.7	0.95	1.46
0.75	25.3	2.8	8.9	0.92	1.41
0.9	13.4	2.8	4.7	0.83	1.27
0.95	7.5	2.8	2.7	0.67	1.02
0.99	1.7	2.8	0.6	-0.61	-0.93

(a) Parameters matched to R&D intensity of 2 percent.

θ	$\mathfrak{h}^A_{opt}$ (in %)	$\mathfrak{h}^{A*}$ (in %)	$\mathfrak{h}^A_{opt}/\mathfrak{h}^{A*}$	s_A^{opt}	s_R^{opt}
0	45.3	10.4	4.4	0.88	1.34
0.25	41.6	10.4	4.0	0.86	1.31
0.5	35.8	10.4	3.5	0.81	1.25
0.75	25.2	10.4	2.4	0.69	1.05
0.9	13.3	10.4	1.3	0.30	0.46
0.95	7.5	10.4	0.7	-0.34	-0.52
0.99	1.7	10.4	0.2	-5.45	-8.35

(b) Parameters matched to R&D intensity of 7 percent.

Table 1: Long run fraction of human capital in R&D (decentralized and social optimum) and optimal R&D policy.

Note: We make use of Proposition 1-3 with the following calibration: $n = 0.01$, $g = 0.021$, $r = 0.07$, $k/y = 3$, $\delta_K = 0.04$, $EPL = 10$, $\omega^H = 0.05$, $\tau_r = \tau_c = 0.395$, $\tau_w = 0.3$, $\tau_g = 0.12$. Panel (a): $\omega^A = 0.02$, i.e., $\alpha = 0.36$, $\beta = 2.58$, $\psi = 1.74$, $g_A = 0.06$, $\kappa = 1.1$. Panel (b): $\omega^A = 0.07$, i.e., $\alpha = 0.44$, $\beta = 2.93$, $\psi = 2.00$, $g_A = 0.05$, $\kappa = 1.35$.

According to panel (a) of Tab. 1, which is based on an R&D intensity of 2 percent in long run market equilibrium, there is dramatic underinvestment in R&D in the case where the duplication externality is not very high. We find that for $\theta \leq 0.9$, the long run socially optimal human capital fraction is in the wide range of about $5 - 16$ times higher than in market equilibrium. What we would like to know, however, is how to improve the allocation of labor and to what extent which kind of tax policy should be used. Interestingly, the necessary R&D policy to restore the social optimum does not so much depend on θ, if $\theta \leq 0.9$. Our results suggest that the R&D sector should be able to deduct from pre-tax profits to obtain the corporate income tax base about $1.27 - 1.48$ the amount invested in R&D. Thus, as pre-tax profits in the sense of the model are already net of R&D costs, this suggests that firms should be allowed to deduct up to 2.5 times their R&D costs from corporate income to obtain the tax base. The current R&D subsidy policy in the US thus seems insufficient. Only if θ is very high, about 0.98 or higher, there is overinvestment in R&D such that current R&D subsidies should be cut. Such a large degree of the duplication externality does not seem to be realistic, however.

Panel (b) shows that when we assume an R&D intensity of 7 percent, the R&D underinvestment problem is less dramatic, but still substantial. For $\theta \leq 0.9$ there should be $1.3 - 4.4$ times higher human capital investment in R&D. Interestingly and importantly for a robust policy implication, the optimal R&D subsidy is not that different to the previous case. For $\theta \leq 0.75$, firms should be able to deduct $1.05 - 1.34$ times the amount of R&D costs from pre-tax profits.

In sum, it seems safe to conclude that US firms should be allowed to deduct up not less than twice their R&D costs from sales revenue for calculating corporate income.

5.2 Physical Capital Investment

For an R&D intensity of 2 percent (parameters like for Tab. 2 (a)), we find that the US economy underinvests in physical capital. Employing Proposition 2, the optimal long run investment rate, sav_{opt}, is equal to 31.3 percent, whereas in market equilibrium the investment as a fraction of GDP, sav^*, is 21.3 percent (used for the calibration of capital depreciation rate δ_K in (36)). According to Proposition 3, this means that US firms should be allowed to deduct about one and a half of their capital costs from sales revenue (i.e. $s_d^{opt} = 0.49$) rather than being allowed to deduct their capital costs by 100 percent (i.e. $s_d = 0$) for calculating corporate income. For an R&D intensity of 7 percent (parameters like for Tab. 2 (b)), the gap between the decentralized and the socially optimal investment rate is even larger ($sav^* = 0.213$, $sav_{opt} = 0.38$, $s_d^{opt} = 0.68$).

5.3 Comparison to the Literature

Previous analyses suggest that the R&D underinvestment problem is considerably less dramatic than implied by our study. There are two main differences between our analysis and the literature. First, we explicitly capture tax/subsidy policy and calibrate the economy accordingly. Second, our calibration strategy does not use some empirical measure of the labor share (or human capital income share) to calibrate the output elasticity of labor/human capital, $1-\alpha$. Our baseline calibration rather uses evidence on the R&D intensity to calibrate α for the long run, in turn determining the human capital income share, ω, endogenously.

We will now demonstrate, exemplarily, that if we followed the strategy of the important and prominent contribution of Jones and Williams (2000), we would obtain results which are similar to theirs. First, one can show how abstracting from the tax system leads to a downward bias of the extent of R&D underinvestment. According to (31) in Proposition 2, the optimal fraction of human capital in R&D, $\mathfrak{h}^A_{opt}$, is decreasing in both preference parameters, ρ and σ. That is, if individuals are less patient, the social planner devotes less resources to R&D. According to (38), ρ and σ are positively related – by the Keynes-Ramsey rule – to the after-tax interest rate, $(1-\tau_r)r$. Setting the tax rate on capital income, τ_r, to zero rather than to its actual value means that individuals are assumed to be less patient. This brings the social optimally R&D resources closer to the market equilibrium. To see the effect numerically, suppose again $n = 0.01$, $g = 0.021$, $r = 0.07$, $k/y = 3$, $\delta_K = 0.04$, $EPL = 10$, $\omega^H = 0.05$ and recalibrate the model by assuming that there are no taxes and subsidies. For an R&D intensity $\omega^A = 0.02$, we then obtain $\alpha = 0.36$ and, accordingly, $\omega = 1.07 - \alpha = 0.71$ for the labor share (recall $\omega^H = 0.05$). Thus, the equilibrium fraction of human capital in R&D, $\mathfrak{h}^{A*}$, is again about 2.8 percent ($= \frac{2}{71}$); moreover, $\mathfrak{h}^{H*} = \mathfrak{h}^H_{opt} = 0.07$ ($= \frac{5}{71}$). However, the optimal R&D effort, $\mathfrak{h}^A_{opt}$, is now given by 28 percent for $\theta = 0$, by 18 percent for $\theta = 0.5$, and by 10.3 percent for $\theta = 0.75$. Thus, the relative gap to the market equilibrium shrinks considerably compared to the case with taxes and subsidies shown in Tab. 1 (a); for instance, if $\theta = 0.75$, $\mathfrak{h}^A_{opt}/\mathfrak{h}^{A*}$ is now equal to 3.7 instead of 8.9.

If, in addition to abstracting from taxes and subsidies, we assume $\omega^A = 0.07$ instead of $\omega^A = 0.02$ (calibration as for Tab 1 (b)), then α becomes 0.42 and the implied labor share, $\omega = 1.12 - \alpha$, is 70 percent, i.e., almost equal to the labor share in the case where $\omega^A = 0.02$. Consequently, we obtain very similar values for $\mathfrak{h}^H_{opt}$ and therefore for

$\mathfrak{h}_{opt}^{A}$ as in the case where the R&D subsidy is 2 percent. However, now $\mathfrak{h}^{A*} = \frac{7}{70}$, i.e., 10 percent of human capital is allocated to R&D in market equilibrium. This means that $\mathfrak{h}_{opt}^{A}/\mathfrak{h}^{A*}$ is equal to 2.8 for $\theta = 0$, to 1.8 for $\theta = 0.5$, and to 1.03 for $\theta = 0.75$; that is, for $\theta = 0.75$ the long run equilibrium R&D intensity is about socially optimal. Interestingly, these figures almost match the results of Jones and Williams (2000) who also assume an interest rate of 7 percent and an effective patent life of 10 years in their baseline calibration. In fact, they set the output elasticity of labor such that the R&D intensity is about 7 percent and abstract from taxes or subsidies − the case just examined. As a result, for the same extent of the duplication externality which corresponds to $\theta = 0$, $\theta = 0.5$ and $\theta = 0.75$, they obtain an R&D investment in social optimum relative to the equilibrium investment of 2.2, 1.7 and unity, respectively. This demonstrates that the different results of our study, shown in Tab. 2, stem from the public finance side in the model, which is supposed to capture some key elements of the US tax-transfer system.[14]

Regarding investment in physical capital, Steger (2005) finds a similar extent of underinvestment problem like we do. He employs a general, semi-endogenous R&D-based growth model to investigate the allocative bias in the R&D share and the saving rate along the balanced growth path. The main finding is that the market economy slightly underinvests in R&D but heavily underinvests in physical capital accumulation. For his baseline calibration, the optimal investment rate along a balanced growth path should be about 15 percentage points higher than the steady state equilibrium rate. This figure is largely robust to parameter variations. Our analysis suggests a gap of 10-17 percentage points.

6 Dynamic Policy Evaluation

The analysis in Section 5 parallels the previous literature in that transitional dynamics were ignored. We therefore have, so far, abstracted from the question whether there

[14] Allowing for human capital as the third engine of growth (not considered in Jones and Williams, 2000), by contrast, does not affect the results considerably.

are intertemporal trade-offs involved with bringing the long run equilibrium allocation of labor closer to the long run social optimum. As will become apparent, there may be very slow adjustment to the new steady state in response to policy shocks. It is thus not evident whether policies which maximize steady state welfare should be implemented.

We evaluate the consequences of growth-related policy measures on intertemporal welfare and on the evolution of central macroeconomic variables, starting from an initial balanced growth path. The resulting change in intertemporal welfare is measured by the consumption-equivalent change in intertemporal welfare, Θ (see appendix for details). The transitional dynamics is simulated by applying the relaxation algorithm (Trimborn et al., 2008).[15]

6.1 Change in Single Policy Instruments

We start by investigating the consequences of small changes in the policy instruments s_R and s_d by 10 percentage points, starting from observed values (Tab. 1). The focus is at first on the dynamic consequences of consumption per capita and intertemporal welfare. Panel (a) of Fig. 1 displays the time path of consumption per capita in response to an increase in the R&D subsidy rate s_R from 0.1 to 0.2. Assuming $\omega^A = 0.07$ the level of (scale-adjusted) consumption per capita increases in the long run by about 12.9 percent.[16] The associated change in intertemporal welfare, Θ, is about 3.8 percent. Compared to the reference scenario of no policy change, as represented by the horizontal line, initial consumption drops. That is, the usual intertemporal consumption trade-off (gains in the longer run at the expense of short-term losses) can be observed. For $\omega^A = 0.02$ the long run increase in consumption per capita is about 15 percent and the associated change in intertemporal welfare, Θ, amounts to 4.2 percent. It is remarkable that there is an "intertemporal free lunch" in this case. Initial consumption does not drop but jumps up instead; however, the upward jump is small, i.e., about 0.17 percent.

[15] Details of the numerical evaluations presented in this section are discussed in supplementary material available on request.

[16] In other words, consumption per capita along the (new) balanced growth path is by about 12.9 percent higher compared to consumption per capita along the initial balanced growth path (reference scenario of no policy shock).

The large gain in intertemporal welfare, as well as the "intertemporal free lunch", is of course due to the substantial R&D underinvestment in the market economy, as suggested by Tab. 1. An increase in s_R reduces this allocation bias; the economic intuition behind the intertemporal free lunch is sketched below. Finally, it should be observed that the speed of convergence appears fairly small, the half life amounts to about 160 years (assuming $\omega^A = 0.07$).

Panel (b) of Fig. 1 shows the consequences of an increase in s_d from 0 to 0.1. Assuming $\omega^A = 0.07$ per capita consumption increases in the long run by about 3.6 percent and the associated change in welfare, Θ, amounts to 2.5 percent. If one assumes $\omega^A = 0.02$, the long run increase in per capita consumption is about 1.9 percent and welfare rises by about 1.2 percent. The positive welfare effect is due to the fact that decentralized saving rate is too low compared to the social optimum. This effect is stronger for $\omega^A = 0.07$ compared to $\omega^A = 0.02$ since the saving bias is more pronounced in this case (see Section 5.1.2). The half life is about 21 years in this case (assuming $\omega^A = 0.07$). The speed of convergence obviously depends on the specific shock under study, i.e. it is quite low in the case of an expansionary R&D-policy and fairly high for an expansionary investment policy.

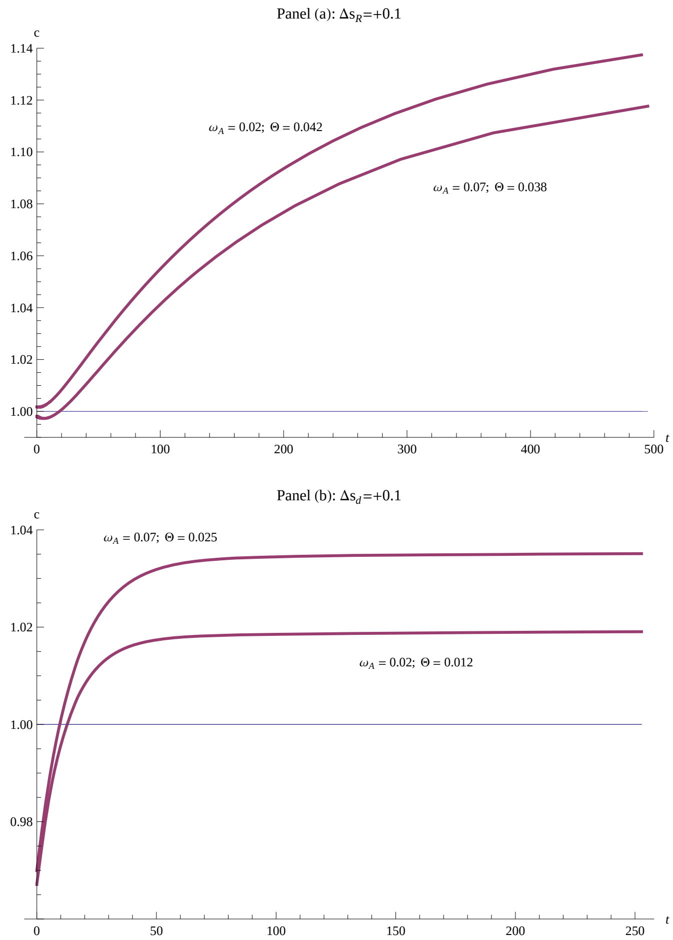

Figure 1: Per capita consumption (scale adjusted) in response to policy shocks. Note: $\rho = 0.02$, $\sigma = 1.08$, $\delta_H = 0.03$, $\theta = 0.5$ and $\eta = 0.15$; corresponding to θ and η, we have $\phi = 0.91$, $\gamma = 0.087$ for $\omega^A = 0.02$ and $\phi = 0.9$, $\gamma = 0.091$ for $\omega^A = 0.07$. All other parameter values are like in Tab. 1.

Fig. 2 describes how the market economy responds to an expansionary R&D-policy ($\Delta s_R = +0.1$) by focusing on the time paths of h^A/h, h^H/h, and sav. In addition, the time path of (scale-adjusted) per capita income y is displayed. The immediate response to the policy shock under study comprises an increase in the share of human capital devoted to R&D (h^A/h) as well as a drop in the saving rate (sav). The share of human capital devoted to education (h^H/h) is increased initially; the quantitative effect is, however, small. Assuming $\omega^A = 0.07$ ($\omega^A = 0.02$) the long run increase in the level of (scale-adjusted) per capita income amounts to about 15 percent (12.9 percent).

Fig. 2, panel (a), helps understanding the economic reasons behind the intertemporal free lunch mentioned above (which occurs for $\omega^A = 0.02$). The immediate reallocations in human capital imply a drop in h^Y (i.e., y falls initially). However, the expansionary R&D policy under study attenuates the substantial R&D underinvestment in the market economy. Rational, forward-looking agents understand that there is an associated wealth effect. They therefore reduce the saving rate (i.e., increase the rate of consumption). Since the implied proportional increase in the rate of consumption (wealth effect) exceeds the proportional decrease in per capita income (due to the reallocation effect), per capita consumption jumps up (see Fig. 1, panel (a)). It is also remarkable to notice that h^A/h jumps up and then converges quite rapidly to its final steady state value. This implies that the results on the steady state level of basic allocation variables (see Proposition 1) are of high relevance. The associated build up in the stock of knowledge A (not displayed) as well as the stock of capital per capita k (not displayed) require a substantial amount of time and therefore the rise in per capita income y proceeds comparably slowly.

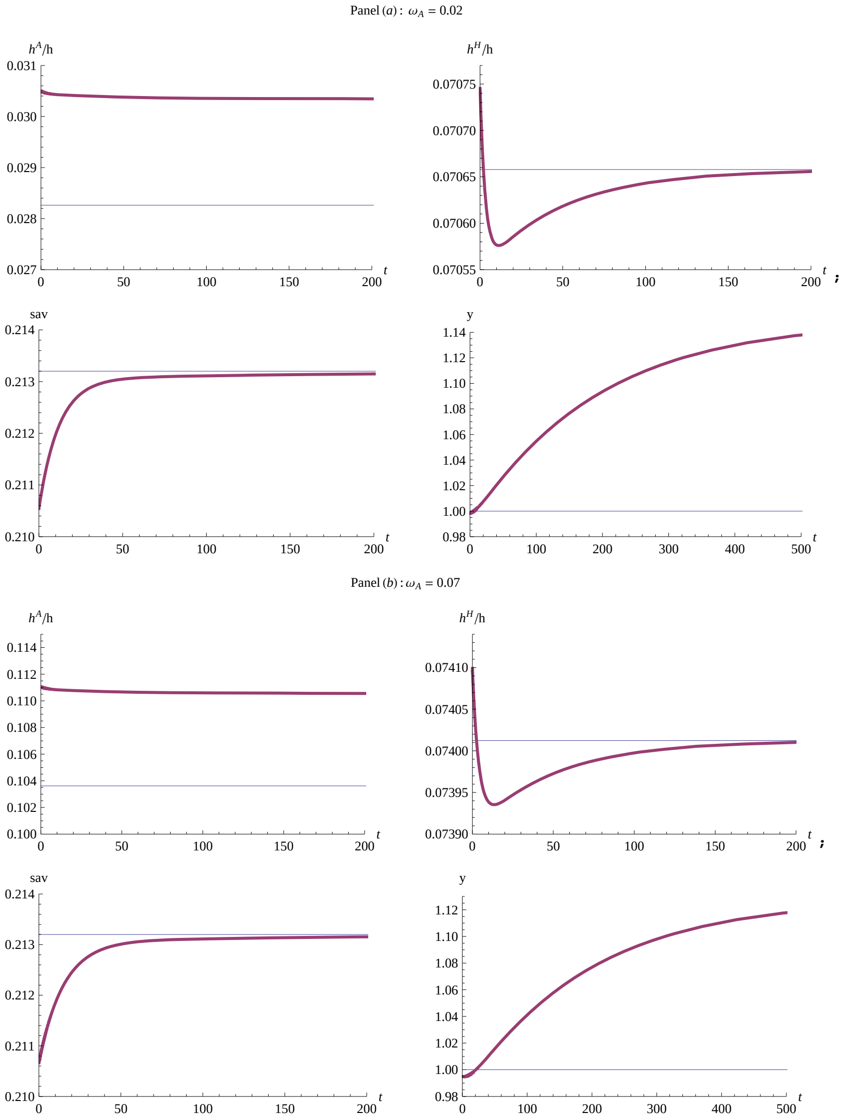

Figure 2: Impulse responses resulting from expansionary R&D-policy ($\Delta s_R = +0.1$). Parameter values as in Fig. 1.

Fig. 3 describes how the market economy responds to an expansionary investment policy ($\Delta s_d = +0.1$). The saving rate jumps up in this case and then converges from above towards its final steady state level, which is higher than the initial steady state value. The share of human capital devoted to R&D (h^A/h) as well as the share of human capital devoted to education (h^H/h) drop and then converge back to their original steady state levels. The long run increase in the level of income per capita y in response to $\Delta s_d = 0.1$ is much smaller. Since both h^A and h^H drop initially, h^Y increases, implying that y jumps up immediately, too. The saving rate increases substantially (associated with an increase in the rate of return to capital, not displayed) such that per capita consumption falls (see Fig. 1, panel (b)). For $\omega^A = 0.02$ ($\omega^A = 0.07$) per capita income y increases in the long run by about 3.7 percent (5.2 percent) compared to the reference scenario of no policy change.

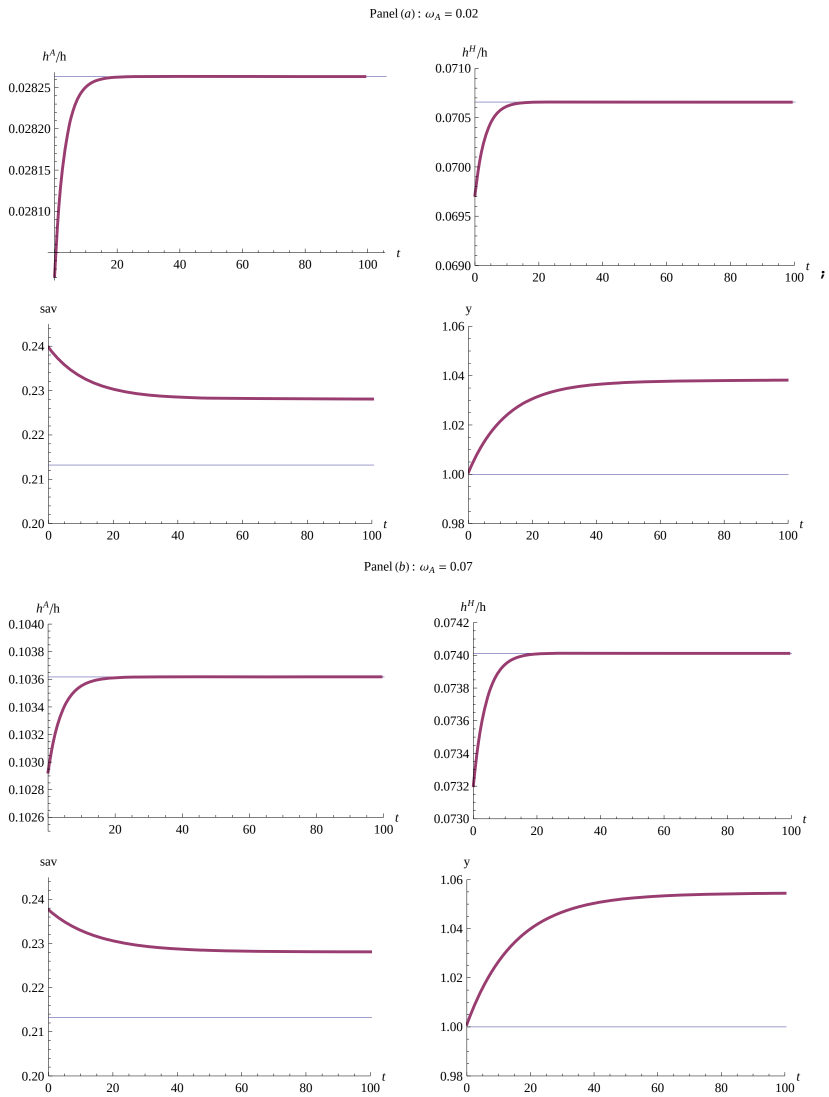

Figure 3: Impulse responses resulting from expansionary investment policy ($\Delta s_d = +0.1$). Parameter values as in Fig. 1 and 2.

6.2 Optimal Policy Mix

In Section 5, we have computed optimal long-run policies by investigating the effect of policy changes on steady state values of consumption. This means that we neglected transitional dynamics implied by a parameter change, but instead assumed that the economy reaches its steady state immediately. We now compute the dynamically optimal policies, i.e. the optimal policy mix by taking into account transitional dynamics. Since the speed of convergence is fairly low, dynamically optimal policies may considerably deviate from optimal steady state policies.

For tractability reasons we restrict the attention to the case where subsidy rates are time-invariant.[17] That is, we start from an initial steady state under the status quo policy and calculate the time path of consumption in response to a one-time change in the subsidy rates. The policy mix which maximizes the welfare gain is denoted by $(\bar{s}_R^{opt}, \bar{s}_d^{opt}, \bar{s}_H^{opt})$.

θ	η	ϕ	γ	s_R^{opt}	$\bar{s}_R^{opt}$	s_d^{opt}	$\bar{s}_R^{opt}$	s_H^{opt}	$\bar{s}_H^{opt}$	Θ
0.5	0.15	0.91	0.09	1.46	1.49	0.49	0.54	0.3	0.27	4.16
0.5	0.3	0.91	0.08	1.46	1.49	0.49	0.54	0.3	0.31	4.16
0.75	0.15	0.95	0.09	1.41	1.44	0.49	0.52	0.3	0.30	0.98
0.75	0.3	0.95	0.08	1.41	1.44	0.49	0.52	0.3	0.32	0.98

(a) Parameters matched to R&D intensity of 2 percent.[18]

[17] The optimal rates may change over time during the transition path. See Grossmann, Steger and Trimborn (2010) for a first analysis of optimal dynamic subsidies in a semi-endogenous growth model.

[18] For $\theta = 0.25$ and $\omega^A = 0.02$ the algorithm does not converge. The initial policy is too far away from the optimum to find a numerical solution.

θ	η	ϕ	γ	s_R^{opt}	$\bar{s}_R^{opt}$	s_d^{opt}	$\bar{s}_R^{opt}$	s_H^{opt}	$\bar{s}_H^{opt}$	Θ
0.25	0.15	0.84	0.09	1.31	1.37	0.68	0.71	0.3	0.30	1.62
0.25	0.3	0.84	0.08	1.31	1.37	0.68	0.71	0.3	0.28	1.62
0.5	0.15	0.90	0.09	1.25	1.31	0.68	0.71	0.3	0.31	0.86
0.5	0.3	0.90	0.08	1.25	1.31	0.68	0.71	0.3	0.30	0.86
0.75	0.15	0.95	0.09	1.05	1.10	0.68	0.70	0.3	0.33	0.32
0.75	0.3	0.95	0.08	1.05	1.10	0.68	0.70	0.3	0.27	0.32

(b) Parameters matched to R&D intensity of 7 percent.

Table 2: Optimal growth policy mix and welfare gain, Θ.

Note: $\rho = 0.02$, $\sigma = 1.08$, $\delta_H = 0.03$. Other parameters as in Tab. 1.

Results are presented in Tab. 2. We find that the dynamically optimal subsidy rates, when restricted to be time-invariant, are not much different from those suggested by the steady state analysis ($s_R^{opt}, s_d^{opt}, s_H^{opt}$). Both $\bar{s}_R^{opt}$ and $\bar{s}_d^{opt}$ are slightly higher than optimal long run values s_R^{opt} and s_d^{opt}, respectively. Deviation of $\bar{s}_H^{opt}$ from the optimal long run education subsidy ($s_H^{opt} = 0.3$) is overall negligible and does not seem to follow a pattern. Also note that the results do not critically depend on η (and thus not on γ). Like in the steady state analysis, the only important parameter we could not satisfactorily calibrate is the extent of the duplication externality θ. Fortunately, the optimal policy mix does not critically depend on θ for intermediate values of this parameter. Thus, we can safely conclude that the underinvestment problem is severe for R&D and substantial for physical capital. The policy implications outlined in Section 5 roughly apply.

The potential welfare gains when implementing the optimal growth policy mix are remarkable. For instance, for $\theta = 0.5$, the intertemporal welfare gain is equivalent to a permanent increase in the annual consumption level per capita, Θ, of about 86 percent if we start out with an R&D intensity of $\omega^A = 0.07$; it even equals 416 percent for the case $\omega^A = 0.02$. Unlike the optimal policy mix, the welfare gain from implementing an appropriate policy reform critically depends on both θ and ω^A. As discussed in Section

4, it may make more sense to view R&D activity in a broader way as measured by the officially reported R&D intensity. Therefore, we prefer the case $\omega^A = 0.07$ to the case $\omega^A = 0.02$. This suggests that for an intermediate value of $\theta \approx 0.5$, the welfare gain from an appropriate policy reform is roughly equivalent to a permanent doubling of per capita consumption.

7 Conclusion

This paper has employed a comprehensive endogenous growth model to examine the impact of changes in incentives to invest in the three major engines of economic growth and to derive the optimal growth policy mix. The main innovation of our study was to capture important elements of the tax-transfer system and to account for transitional dynamics induced by policy shocks. We identified a dramatic underinvestment problem with respect to R&D activity and a substantial one with respect to physical capital accumulation under the status quo US growth policy mix and the present distortions from income taxation. Our analysis suggests that there are huge welfare losses from insufficiently supporting investment in R&D and physical capital, which are far too large to be ignored and call for a significant policy reform.

One may argue that the results are model-specific and hence not robust with respect to the underlying endogenous growth model. Admittedly, this objection is (potentially) correct. However, this characteristic is unavoidable and applies to any numerical analysis which aims at coming up with specific recommendations. Further studies in this direction, which appropriately account for the tax/ transfer system as well as for transitional dynamics, could indeed contribute to yield a more complete picture on the appropriate growth policies.

Appendix

Proof of Proposition 1: The current-value Hamiltonian which corresponds to the household optimization problem (17), (18) is given by

$$\begin{aligned}\mathcal{H} \;=\; & \frac{c^{1-\sigma}-1}{1-\sigma}+\mu\left[\xi\left(h^{H}\right)^{\gamma}h^{\eta}-\delta_{H}h\right]+ \\ & \lambda\left(\left[(1-\tau_{r})r-n\right]a+(1-\tau_{w})wh-(1-s_{H})wh^{H}-c+T\right), \qquad (48)\end{aligned}$$

where λ and μ are multipliers (co-state variables) associated with constraints (4) and (5), respectively. Necessary optimality conditions are $\partial\mathcal{H}/\partial c=\partial\mathcal{H}/\partial h^{H}=0$ (control variables), $\dot{\mu}=(\rho-n)\mu-\partial\mathcal{H}/\partial h$, $\dot{\lambda}=(\rho-n)\lambda-\partial\mathcal{H}/\partial a$ (state variables), and the corresponding transversality conditions. Thus,

$$\lambda=c^{-\sigma} \tag{49}$$

$$\mu\xi\gamma(h^{H})^{\gamma-1}h^{\eta}=\lambda(1-s_{H})w, \tag{50}$$

$$\frac{\dot{\mu}}{\mu}=\rho-n-\xi\left(h^{H}\right)^{\gamma}\eta h^{\eta-1}+\delta_{H}-\frac{\lambda}{\mu}w(1-\tau_{w}), \tag{51}$$

$$\frac{\dot{\lambda}}{\lambda}=\rho-(1-\tau_{r})r, \tag{52}$$

$$\lim_{t\to\infty}\mu_{t}e^{-(\rho-n)t}h_{t}=0, \tag{53}$$

$$\lim_{t\to\infty}\lambda_{t}e^{-(\rho-n)t}a_{t}=0. \tag{54}$$

Differentiating (49) with respect to time and using (52), we obtain the Euler equation

$$\frac{\dot{c}}{c}=\frac{(1-\tau_{r})r-\rho}{\sigma}. \tag{55}$$

Now, define $\tilde{z}\equiv zA^{-\frac{\alpha}{(1-\alpha)(\beta-1)}}$ for $z\in\{w,c,a,T\}$; we will show that the adjusted values ($\tilde{z}$) of these variables are stationary in the long run. From (55),

$$\frac{\dot{\tilde{c}}}{\tilde{c}}=\frac{(1-\tau_{r})r-\rho}{\sigma}-\frac{\alpha}{(1-\alpha)(\beta-1)}\frac{\dot{A}}{A}. \tag{56}$$

Differentiating (50) with respect to time and making use of (4), (50), (51) and (52) we obtain:

$$\frac{\dot{h}^H}{h^H} = \frac{1}{1-\gamma}\left[(1-\tau_r)r - n + (1-\eta)\delta_H - \frac{1-\tau_w}{1-s_H}\frac{\xi\gamma h^\eta}{(h^H)^{1-\gamma}} - \frac{\dot{w}}{w}\right]. \tag{57}$$

Moreover, with $H^A = Nh^A$, (2), (4), (5) can be written as

$$\frac{\dot{A}}{A} = \frac{\nu}{1+\psi}A^{\phi-1}(Nh^A)^{1-\theta}, \tag{58}$$

$$\frac{\dot{h}}{h} = \xi(h^H)^\gamma h^{\eta-1} - \delta_H, \tag{59}$$

$$\frac{\dot{\tilde{a}}}{\tilde{a}} = (1-\tau_r)r - n + (1-\tau_w)\frac{\tilde{w}h}{\tilde{a}} - (1-s_H)\frac{wh^H}{\tilde{a}} - \frac{\tilde{c}}{\tilde{a}} - \frac{\alpha}{(1-\alpha)(\beta-1)}\frac{\dot{A}}{A} + \frac{\tilde{T}}{\tilde{a}}. \tag{60}$$

Next, substitute (10) and (11) into (7) and use both (12) and $R = r + \delta_K$ to obtain the following expression for the profit of each intermediate goods producer i:

$$\pi_i = \pi = A^{\frac{\alpha}{(1-\alpha)(\beta-1)}-1}(1-\tau_c)(\kappa-1)\left(\frac{\alpha}{\kappa}\right)^{\frac{1}{1-\alpha}}\left[(1-s_K)(r+\delta_K)\right]^{-\frac{\alpha}{1-\alpha}}H^Y. \tag{61}$$

Now define $\tilde{q} \equiv P^A A^{1-\frac{\alpha}{(1-\alpha)(\beta-1)}}/N$ and differentiate $\tilde{q}$ with respect to time; then use the resulting expression as well as (61) to rewrite (14) as

$$\begin{aligned}\frac{\dot{\tilde{q}}}{\tilde{q}} &= \left(1 - \frac{\alpha}{(1-\alpha)(\beta-1)}\right)\frac{\dot{A}}{A} - n + \frac{1}{1-\tau_g}\times \\ &\left((1-\tau_r)r + \psi\frac{\dot{A}}{A} - \frac{(1-\tau_c)(\kappa-1)\left(\frac{\alpha}{\kappa}\right)^{\frac{1}{1-\alpha}}\left[(1-s_K)(r+\delta_K)\right]^{-\frac{\alpha}{1-\alpha}}h^Y}{\tilde{q}}\right)\end{aligned} \tag{62}$$

The capital market clearing condition reads $Na = K + P^A A$; it implies, by using (13) and $R = r + \delta_K$ (as well as the definitions of $\tilde{a}$ and $\tilde{q}$), that

$$\tilde{a} = \left(\frac{\alpha}{\kappa(1-s_K)(r+\delta_K)}\right)^{\frac{1}{1-\alpha}}h^Y + \tilde{q}. \tag{63}$$

The wage rate equals the marginal product of human capital in the final goods sector, i.e., $w = (1-\alpha)Y/H^Y$. Using (12) we obtain

$$\tilde{w} = (1-\alpha)\left(\frac{\alpha}{\kappa(1-s_K)(r+\delta_K)}\right)^{\frac{\alpha}{1-\alpha}}. \tag{64}$$

Moreover, in equilibrium, $\Pi = 0$ holds. This leads to

$$w = \frac{P^A \tilde{\nu} A^\phi}{1-s_A}, \tag{65}$$

according to (16). Combining (65) with (2) and using both $\tilde{q} = P^A A^{1-\frac{\alpha}{(1-\alpha)(\beta-1)}}/N$ and $\tilde{w} = A^{-\frac{\alpha}{(1-\alpha)(\beta-1)}} w$, we can write

$$h^A = \frac{\tilde{q}(1+\psi)\frac{\dot{A}}{A}}{(1-s_A)\,\tilde{w}}. \tag{66}$$

We next derive steady state values. In steady state, the growth rate of A must be equal to zero. Differentiating the right-hand side of (58) with respect to time and setting the resulting term to zero leads to $\dot{A}/A = g_A$ as given by (20), provided that $\dot{h}^A = 0$. In the following we show that $\dot{h}^A = 0$ indeed holds if $\dot{A}/A = g_A$; we therefore set $\dot{A}/A = g_A$ to derive the following (candidates of) steady state values. Setting $\dot{\tilde{c}} = 0$ in (56) and using $g = \frac{\alpha g_A}{(1-\alpha)(\beta-1)}$, we find

$$r = \frac{\sigma g + \rho}{1-\tau_r}. \tag{67}$$

Note that substituting (67) into (64) also gives us a stationary value for $\tilde{w}$ in terms of exogenous parameters only. According to (59) and $\dot{h} = 0$, we obtain

$$h = \left(\frac{\xi}{\delta_H}\right)^{\frac{1}{1-\eta}} (h^H)^{\frac{\gamma}{1-\eta}}. \tag{68}$$

Setting $\dot{h}^H = 0$ in (57) (which holds in steady state, as will become apparent) and employing both $\dot{w}/w = g$ and (67) implies[19]

$$h^H = \left(\frac{1-\tau_w}{1-s_H} \frac{\gamma}{(\sigma-1)g+\rho-n+(1-\eta)\delta_H} \right)^{\frac{1-\eta}{1-\gamma-\eta}} \left(\frac{\xi}{(\delta_H)^\eta} \right)^{\frac{1}{1-\gamma-\eta}}. \tag{69}$$

Combining (68) and (69) gives us expression (22) for the equilibrium fraction of human capital devoted to education.

Using $\dot{A}/A = g_A$ in (66), we furthermore obtain

$$h^A = \frac{(1+\psi)g_A\tilde{q}}{(1-s_A)\,\tilde{w}} \tag{70}$$

To find the steady state values for h^Y and $\tilde{q}$, first substitute (70) into labor market clearing condition $h^Y = h - h^H - h^A$, which gives us

$$h^Y = h - h^H - \frac{(1+\psi)g_A\tilde{q}}{(1-s_A)\,\tilde{w}}. \tag{71}$$

Also set $\dot{\tilde{q}} = 0$ in (62) and use $\dot{A}/A = g_A$ to find $\tilde{q} = \Omega h^Y$ with

$$\Omega \equiv \frac{(1-\tau_c)(\kappa-1)\left(\frac{\alpha}{\kappa}\right)^{\frac{1}{1-\alpha}}}{\left[(1-\tau_r)r+\psi g_A-(n+g-g_A)(1-\tau_g)\right]\left[(1-s_K)(r+\delta_K)\right]^{\frac{\alpha}{1-\alpha}}}. \tag{72}$$

Substituting $\tilde{q} = \Omega h^Y$ into (71) and solving for h^Y yields

$$h^Y = \frac{h-h^H}{1+\frac{(1+\psi)g_A\Omega}{(1-s_A)\tilde{w}}} \tag{73}$$

and thus

$$\tilde{q} = \frac{\Omega(h-h^H)}{1+\frac{g_A\Omega}{(1-s_A)\tilde{w}}}. \tag{74}$$

Substituting (74) into (70) yields

$$h^A = \frac{h-h^H}{\frac{(1-s_A)\tilde{w}}{\Omega(1+\psi)g_A}+1}. \tag{75}$$

[19] That the wage rate grows with rate g in steady state follows from $w = \tilde{w}A^{-\frac{\alpha}{(1-\alpha)(\beta-1)}}$ and $\dot{\tilde{w}} = 0$.

Dividing by both sides of (75) by h, substituting into it both expressions (64) for $\tilde{w}$ and (72) for Ω as well as using $(1-\tau_r)r = \sigma g + \rho$ from (67) gives us expression (23) for the steady state fraction of human capital devoted to R&D.

Equations (68), (73), (74) and (75) give us explicit expressions for h, h^Y, $\tilde{q}$ and h^A, respectively, noting that h^H is explicitly given by (69) and $\tilde{w}$ by (64), using (67) for the latter. Setting next $\dot{\tilde{a}} = 0$ in (60) and using (67), $\dot{A}/A = g_A$ as well as $g = \frac{\alpha g_A}{(1-\alpha)(\beta-1)}$ yields

$$\tilde{c} = [(\sigma-1)g + \rho - n]\,\tilde{a} + (1-\tau_w)\tilde{w}h - (1-s_H)\tilde{w}h^H + \tilde{T}. \tag{76}$$

We also need to show that the adjusted lump-sum transfer per capita, $\tilde{T}$, is stationary in the long run when r, h, h^A, h^Y, h^H, $\tilde{w}$, $\tilde{c}$, $\tilde{a}$, $\tilde{q}$ are stationary. Under a balanced government budget it must hold that the sum of education subsidy payments ($s_H wNh^H$) and lump-sum transfer payments (TN) is equal to the sum of revenue from labor income taxation ($\tau_w wNh$), taxation of capital income from asset holding ($\tau_r rK$), taxation of capital gains ($\tau_g \dot{P}^A A$), and corporate income taxation of intermediate good firms after depreciation allowances ($\int_0^A \tau_c(p_i x_i - Rx_i - s_d Rx_i)di$), and of R&D firms after R&D subsidy ($\tau_c\left(P^A\dot{A} - wH^A - s_R wH^A\right)$). Hence, using $p_i = \kappa(1-s_K)R$ for all i, $K = \int_0^A x_i di$, $R = r + \delta_K$ as well as expressions (14) and (61), we have

$$\begin{aligned}\tilde{T} &= \tau_w\tilde{w}h + \tau_r r\tilde{k} + \tau_c\left[\kappa(1-s_K) - (1+s_d)\right](r+\delta_K)\tilde{k} + \frac{\tau_g}{1-\tau_g}\times \\ &\left(\left[(1-\tau_r)r + \psi\frac{\dot{A}}{A}\right]\tilde{q} - (1-\tau_c)(\kappa-1)\left(\frac{\alpha}{\kappa}\right)^{\frac{1}{1-\alpha}}\left[(1-s_K)(r+\delta_K)\right]^{-\frac{\alpha}{1-\alpha}} h^Y\right) \\ &\tau_c\left[\tilde{q}(1+\psi)\frac{\dot{A}}{A} - (1+s_R)\tilde{w}h^A\right] - s_H\tilde{w}h^H,\end{aligned} \tag{77}$$

where $\tilde{k} \equiv A^{-\frac{\alpha}{(1-\alpha)(\beta-1)}}k$. According to (13), $\tilde{k}$ is stationary in the long run if h^Y is; thus, provided that $\dot{A}/A = g_A$ as claimed, $\tilde{T}$ is stationary. We also see that, in steady state both per capita capital stock k and, according to (12), per capita income grow with rate g as given by (21).

The investment share is given by $sav = (\dot{K} + \delta_K K)/Y = (\dot{K}/K + \delta_K)k/y$. Using

$\dot{K}/K = n + g$ together with expressions (13) and (12) for k and y, respectively, we obtain

$$sav = \frac{\alpha(n+g+\delta_K)}{\kappa(1-s_K)R}. \tag{78}$$

Using $R = r + \delta_K$ and expression (67) for r confirms (25).

Finally, it remains to be shown that the transversality conditions (53) and (54) hold under assumption (A1). Differentiating (50) with respect to time and using $\dot{h} = \dot{h}^H = 0$ as well as $\dot{w}/w = g$ implies that, along a balanced growth path, $\dot{\mu}/\mu = \dot{\lambda}/\lambda + g$. From (52) and (67) we find $\dot{\lambda}/\lambda = -\sigma g$ and thus $\dot{\mu}/\mu = (1-\sigma)g$. As h becomes stationary, (53) holds if $\lim_{t\to\infty} e^{[(1-\sigma)g+n-\rho]t} = 0$, or $\rho > (1-\sigma)g+n$. Using the expression for g in (21) shows that the latter condition is equivalent to (A1). Similarly, using $\dot{\lambda}/\lambda = -\sigma g$ and the fact that a grows with rate g in the long run, we find that also (54) holds if $\rho > (1-\sigma)g + n$. This concludes the proof. ■

Proof of Proposition 2: The current-value Hamiltonian which corresponds to the social planning problem (29) is given by

$$\begin{aligned}
\mathcal{H} &= \frac{c^{1-\sigma}-1}{1-\sigma} + \lambda_k(\underbrace{A^{\frac{\alpha}{\beta-1}}k^{\alpha}(h^Y)^{1-\alpha}}_{=y} - (\delta_K+n)k - c) + \\
&\quad \lambda_h\left[\xi\left(h^H\right)^{\gamma}h^{\eta} - \delta_H h\right] + \lambda_A \overbrace{\bar{\nu}A^{\phi}N^{1-\theta}\underbrace{\left(h - h^Y - h^H\right)}_{=h^A}{}^{1-\theta}}^{=\dot{A}},
\end{aligned} \tag{79}$$

$\bar{\nu} \equiv \frac{\nu}{1+\psi}$, where λ_k, λ_h and λ_A are co-state variables associated with constraints (27), (4) and (28), respectively. Necessary optimality conditions are $\partial\mathcal{H}/\partial c = \partial\mathcal{H}/\partial h^H = \partial\mathcal{H}/\partial h^Y = 0$ (control variables), $\dot{\lambda}_z = (\rho - n)\lambda_z - \partial\mathcal{H}/\partial z$ for $z \in \{k, h, A\}$ (state variables), and the corresponding transversality conditions. Thus,

$$\lambda_k = c^{-\sigma} \tag{80}$$

$$\lambda_h \xi\gamma(h^H)^{\gamma-1}h^{\eta} = \lambda_A(1-\theta)\bar{\nu}A^{\phi}N^{1-\theta}(h^A)^{-\theta}, \tag{81}$$

$$\lambda_k(1-\alpha)A^{\frac{\alpha}{\beta-1}}k^{\alpha}(h^Y)^{-\alpha} = \lambda_A(1-\theta)\underbrace{\bar{\nu}A^{\phi}N^{1-\theta}(h^A)^{-\theta}}_{=\dot{A}/h^A}, \tag{82}$$

$$\frac{\dot{\lambda}_k}{\lambda_k} = \rho - \frac{\alpha y}{k} + \delta_K, \tag{83}$$

$$\frac{\dot{\lambda}_h}{\lambda_h} = \rho - n - \xi\left(h^H\right)^{\gamma}\eta h^{\eta-1} + \delta_H - \frac{\lambda_A}{\lambda_h}(1-\theta)\bar{\nu}A^{\phi}N^{1-\theta}(h^A)^{-\theta}, \tag{84}$$

$$\frac{\dot{\lambda}_A}{\lambda_A} = \rho - n - \frac{\lambda_k}{\lambda_A}\frac{\alpha}{\beta-1}A^{\frac{\alpha}{\beta-1}-1}k^{\alpha}(h^Y)^{1-\alpha} - \phi\frac{\dot{A}}{A} \tag{85}$$

$$\lim_{t\to\infty}\lambda_{z,t}e^{-(\rho-n)t}z_t = 0,\ \ z \in \{k, h, A\}. \tag{86}$$

($\lambda_{z,t}$ denotes the co-state variable associated with state variable z at time t.)

We exclusively focus on the long run. In steady state, with h^A being stationary, A must grow with rate g_A. Moreover, y, k, and c must grow at the same rate g, if h^Y is stationary. Differentiating (80) with respect to time, we obtain

$$\frac{\dot{\lambda}_k}{\lambda_k} = -\sigma\frac{\dot{c}}{c} = -\sigma g, \tag{87}$$

where we used $\dot{c}/c = g$ for the latter equation. Combining (87) with (83) implies a capital output ratio

$$\frac{k}{y} = \frac{\alpha}{\rho + \delta_K + \sigma g}. \tag{88}$$

Next, differentiate (81) with respect to time to find that in steady state, under a stationary allocation of human capital,

$$\frac{\dot{\lambda}_h}{\lambda_h} = \frac{\dot{\lambda}_A}{\lambda_A} + g_A \tag{89}$$

holds, where we used $\dot{A}/A = g_A$, $\dot{N}/N = n$ and the fact that $(1-\theta)n = (1-\phi)g_A$, according to (20). Making use of the same properties, differentiating (82) with respect to time leads to

$$\frac{\dot{\lambda}_k}{\lambda_k} + \left(\frac{\alpha}{\beta-1} - 1\right)g_A + \alpha g = \frac{\dot{\lambda}_A}{\lambda_A}. \tag{90}$$

Using (87) and the definition of g in (21), we can rewrite (90) to

$$\frac{\dot{\lambda}_A}{\lambda_A} = (1-\sigma)g - g_A \tag{91}$$

and thus, according to (89),

$$\frac{\dot{\lambda}_h}{\lambda_h} = (1-\sigma)g. \tag{92}$$

Moreover, substituting the right-hand side of (81) into (84) as well as using both (92) and the fact that $\xi \left(h^H\right)^\gamma h^\eta = \delta_H h$ when $\dot{h} = 0$, eventually confirms the expression for h^H/h in (30).

Next, rewrite (82) to

$$\frac{\lambda_k}{\lambda_A} = \frac{(1-\theta)\frac{\dot{A}}{A}\frac{h^Y}{h^A}}{(1-\alpha)A^{\frac{\alpha}{\beta-1}-1}k^\alpha(h^Y)^{1-\alpha}}. \tag{93}$$

Substituting (93) into (85) and using $\dot{A}/A = g_A$ together with the definition of g in (21) leads to

$$\frac{\dot{\lambda}_A}{\lambda_A} = \rho - n - (1-\theta)g\frac{h^Y}{h^A} - \phi g_A. \tag{94}$$

Combining (94) with (91) and using the fact that $(1-\phi)g_A = (1-\theta)n$ leads to

$$\frac{h^Y}{h^A} = \frac{\rho - \theta n + g(\sigma-1)}{(1-\theta)g} = \Gamma. \tag{95}$$

Using $h^Y = h - h^A - h^H$ then confirms the expression for h^A/h in (31).

To confirm the socially optimal savings and investment rate ($sav = 1-c/y$) as well, note from (27) that

$$sav = \left(\frac{\dot{k}}{k} + \delta_K + n\right)\frac{k}{y}. \tag{96}$$

Using $\dot{k}/k = g$ and expression (88) for k/y confirms (32).

Finally, it is easy to see from (87), (92) and (91) that, under assumption (A1), transversality conditions (86) hold for k, h and A, respectively (using $\dot{k}/k = g$, $\dot{h} = 0$ and $\dot{A}/A = g_A$). This concludes the proof. ■

References

[1] Acemoglu, Daron and Joshua Angrist (2000). How Large Are Human-Capital Externalities? Evidence from Compulsory Schooling Laws, *NBER Macroeconomics Annual* 15, 9-59.

[2] Aghion, Philippe and Peter Howitt (2005). Growth with Quality-improving Innovations: An Integrated Framework, in: P. Aghion and S. Durlauf (eds.), *Handbook of Economic Growth*, North-Holland, Amsterdam.

[3] Bils, Mark and Peter J. Klenow (2000). Does Schooling Cause Growth?, *American Economic Review* 90, 1160-1183.

[4] Black, Sandra E., Paul Devereux and Kjell G. Salvanes (2005). Why the Apple Doesn't Fall Far: Understanding Intergenerational Transmission of Human Capital, *American Economic Review* 95, 437-449.

[5] Bloom, Nick and Rachel Griffith and John Van Reenen (2002). Do R&D tax credits work? Evidence from a panel of countries 1979-1997, *Journal of Public Economics*, 85(1), 1-31.

[6] Cameron, Stephen V. and Christopher Taber (2004). Estimation of Educational Borrowing Constraints Using Returns to Schooling, *Journal of Political Economy* 112, 132–182.

[7] Coe, David T. and Elhanan Helpman (1995). International R&D Spillovers, *European Economic Review* 39, 859-887.

[8] Devereux, Michael P., Rachel Griffith and Alexander Klemm (2002). Corporate Income Tax Reforms and International Tax Competition, *Economic Policy* 17, 449-495.

[9] Funke, M. and H. Strulik (2000). On Endogenous Growth with Physical Capital, Human Capital and Product Variety, *European Economic Review* 44, 491-515.

[10] Griliches, Zvi and Frank R. Lichtenberg (1984). Interindustry Technology Flows and Productivity Growth: A Reexamination, *Review of Economics and Statistics* 66, 324-9.

[11] Grossmann, Volker, Thomas M. Steger and Timo Trimborn (2010). Dynamically Optimal R&D Subsidization, University of Fribourg (mimeo).

[12] Hall, Bob (1996). The Private and Social Returns to Research and Development, in: Smith, B. and C. Barfield (Eds.) Technology, R&D, and the Ecoonomy, Brookings Institution and American Enterprise Institute: Washington DC.

[13] Heckman, James (1976). A Life-Cycle Model of Earnings, Learning and Consumption, *Journal of Political Economy* 84, S11-S44.

[14] Heston, Alan, Robert Summers and Bettina Aten (2006). Penn World Table Version 6.2, Center for International Comparisons of Production, Income and Prices at the University of Pennsylvania.

[15] House, Christopher L. and Matthew D. Shapiro (2008). Temporary Investment Tax Incentives: Theory with Evidence from Bonus Depreciation, *American Economic Review* 98, 737-768.

[16] Jones, Charles I. (1995). R&D-based Models of Economic Growth, *Journal of Political Economy* 103, 759–784.

[17] Jones, Charles I. (2002). Sources of U.S. Economic Growth in a World of Ideas, *American Economic Review* 92, 220-239.

[18] Jones, Charles I. (2005). Growth and Ideas, in: P. Aghion and S. Durlauf (eds.), *Handbook of Economic Growth*, North-Holland, Amsterdam.

[19] Jones, Charles I. and John C. Williams (1998). Measuring The Social Return To R&D, *The Quarterly Journal of Economics*, 113(4), 1119-1135.

[20] Jones, Charles I. and John C. Williams (2000), Too Much of a Good Thing? The Economics of Investment in R&D, *Journal of Economic Growth* 5, 65-85.

[21] Krueger, Alan B. (1999). Measuring Labor's Share, *American Economic Review Papers & Proceedings* 89, 45-51.

[22] Lucas, Robert E. (1988). On the Mechanics of Economic Development, *Journal of Monetary Economics* 22, 3-42.

[23] Mas-Colell, Andreu, Michael D. Whinston and Jerry R. Green (1995). Microeconomic Theory, New York.

[24] Mehra, Rajnish and Edward C. Prescott (1985). The Equity Premium: A Puzzle, Journal of Monetary Economics 15, 145-61.

[25] Norrbin, Stefan C. (1993). The Relationship Between Price and Marginal Cost in U.S. Industry: A Contradiction, *Journal of Political Economy* 101, 1149-1164.

[26] OECD (2007a). Science Technology and Industry Scoreboard 2007, Paris.

[27] OECD (2007b). Education at a Glance 2007, Paris.

[28] OECD (2008a). Main Science and Technology Indicators 2008-1, www.sourceoecd.org.

[29] OECD (2008b). Science, Technology and Industry: Outlook 2008, Paris.

[30] Papageorgiou, Chris and Fidel Perez-Sebastian (2006). Dynamics in a Non-Scale R&D Growth Model with Human Capital: Explaining the Japanese and South Korean Development Experiences, *Journal of Economic Dynamics and Control* 30, 901-930.

[31] Plug, Erik and Wim Vijverberg (2003). Schooling, Family Background, and Adoption: Is It Nature or Is It Nurture?, *Journal of Political Economy* 111, 611-641.

[32] Romer, Paul M. (1990). Endogenous Technological Change, *Journal of Political Economy* 98, 71-101.

[33] Scherer, Frederic (1982). Inter-industry Technology Flows and Productivity Growth, *Review of Economics and Statistics* 64, 627-634.

[34] Steger, Thomas M. (2005). Welfare Implications of Non-Scale R&D-based Growth Models, *Scandinavian Journal of Economics* 107, 737-757.

[35] Strulik, Holger (2007). Too Much of a Good Thing? The Quantitative Economics of R&D-driven Growth Revisited, *Scandinavian Journal of Economics* 109, 369-386.

[36] Trimborn, Timo, Karl-Josef Koch and Thomas M. Steger (2008). Multi-Dimensional Transitional Dynamics: A Simple Numerical Procedure, *Macroeconomic Dynamics* 12, 301–319.

Supplementary Material (not intended for publication)

This supplement provides details of numerical evaluations in Section 6.

Consumption-equivalent change in intertemporal welfare - derivation of Θ: First, adjust per capita consumption to $\tilde{c} \equiv cA^{-\frac{\alpha}{(1-\alpha)(\beta-1)}}$, which is stationary in the long run (Proposition 1). Moreover, denote the change in life-time utility by ΔU and the (hypothetical) change in adjusted steady per capita consumption by $\Delta\tilde{c}$. Initially, there is an adjusted steady consumption stream $\tilde{c}_0$, as we start from an initial balanced growth path. Then we have

$$\Delta U = \int_0^\infty \frac{\left((\tilde{c}_0 + \Delta\tilde{c})e^{gt}\right)^{1-\sigma} - 1}{1-\sigma} e^{-(\rho-n)t} dt - \int_0^\infty \frac{\left(\tilde{c}_0 e^{gt}\right)^{1-\sigma} - 1}{1-\sigma} e^{-(\rho-n)t} dt \tag{97}$$

which we can solve to find

$$\Theta \equiv \frac{\Delta\tilde{c}}{\tilde{c}_0} = \frac{\left(\tilde{c}_0^{1-\sigma} + \Delta U(\sigma-1)(g(1-\sigma)+n-\rho)\right)^{\frac{1}{1-\sigma}}}{\tilde{c}_0} - 1. \tag{98}$$

We numerically find $\tilde{c}_0$ under the status quo policy and obtain the change in welfare ΔU which results from a policy reform. In turn, we get $\Theta = \Delta\tilde{c}/\tilde{c}_0$ from (98).

Consistency checks and the goods market clearing condition: To rule out errors from typos in the program code or due to software limitations, we took caution as follows. First, we implemented the model in two different software packages, Mathematica and Matlab, by two different authors separately. Second, we checked numerically whether the goods market clears (which must hold true due to Walras' law). We now show that the goods market clearing condition $Y = C + I$, where $I \equiv \dot{K} + \delta_K K$ is total investment and $C \equiv Nc$ is total consumption spending, is fulfilled. That is, $\dot{K} = Y - C - \delta_K K$. This condition is not used in the program codes and therefore can be used as a test of correct implementation.

The following relationships will be used to show that $Y = C + I$.

- The no-arbitrage condition (14) implies

$$(1 - \tau_r) r P_A - \dot{P}^A = \pi - \tau_g \dot{P}^A - \psi P^A \frac{\dot{A}}{A}. \tag{99}$$

- Tax revenue in intermediate goods sector: From profit function of an intermediate good producer i,

$$\pi_i = \pi = p_i x_i - R x_i - \tau_c (p_i x_i - R x_i - s_d R x_i),$$

we obtain

$$\begin{aligned} & \int_0^A \tau_c (p_i x_i - R x_i - s_d R x_i) di \\ = & \int_0^A (p_i x_i - R x_i - \pi_i)\, di \\ = & \left[\kappa(1 - s_K) - 1\right] RK - \pi A. \end{aligned} \tag{100}$$

To derive the latter equation we used the facts that $p_i = \kappa(1 - s_K) R$ for all i and $K = \int_0^A x_i di$.

- Tax revenue from R&D firms: From the expression for profits of the representative

R&D firm,

$$\Pi = P^A(1+\psi)\dot{A} - wH^A - \tau_c\left(P^A(1+\psi)\dot{A} - wH^A - s_R wH^A\right),$$

and the fact that $\Pi = 0$ in equilibrium, we have

$$\tau_c\left(P^A(1+\psi)\dot{A} - wH^A - s_R wH^A\right) = P^A(1+\psi)\dot{A} - wH^A. \tag{101}$$

- From the asset accumulation equation (5) of households:

$$N\dot{a} = \left[(1-\tau_r)r - n\right]Na + (1-\tau_w)wNh - (1-s_H)Nwh^H - Nc + NT. \tag{102}$$

Now we proceed in several steps:

- Step 1: Given the linear-homogenous production function (1) and due to perfect competition we know that profits are zero in the final goods sector. Thus,

$$Y = \int_0^A p_i x_i di + wH^Y.$$

Using $p_i = \kappa(1-s_K)R$, $K = \int_0^A x_i di$ and $H^Y = N(h - h^A - h^H)$, we obtain

$$Y = \kappa(1-s_K)RK + wN(h - h^A - h^H)$$

and thus

$$Y - wNh - RK + wNh^H = \left[\kappa(1-s_K) - 1\right]RK - wH^A. \tag{103}$$

- Step 2: Under a balanced government budget, the aggregate lump-sum transfer to households is

$$
\begin{aligned}
NT &= \tau_w wNh + \tau_r rK + \tau_g \dot{P}^A A - s_H Nwh^H + \\
&\quad \int_0^A \tau_c (p_i x_i - Rx_i - s_d Rx_i) di + \\
&\quad \tau_c \left(P^A (1+\psi)\dot{A} - wH^A - s_R wH^A \right) \\
&= \tau_w wNh + \tau_r rK + \tau_g \dot{P}^A A - s_H Nwh^H + \\
&\quad [\kappa(1-s_K) - 1] RK - \pi A + P^A(1+\psi)\dot{A} - wH^A, \qquad (104)
\end{aligned}
$$

where we used (100) and (101) for the latter equation. Using (103), we can rewrite (104) as

$$
\begin{aligned}
NT &= \tau_w wNh + \tau_r rK + \tau_g \dot{P}^A A - s_H Nwh^H + \\
&\quad Y - wNh - RK + wNh^H - \pi A + P^A(1+\psi)\dot{A},
\end{aligned}
$$

or, equivalently,

$$
\begin{aligned}
Y &= (1-\tau_w) wNh + RK + \pi A - P^A(1+\psi)\dot{A} + \\
&\quad NT - \tau_r rK - \tau_g \dot{P}^A A - (1-s_H) Nwh^H. \qquad (105)
\end{aligned}
$$

This gives us an expression for GDP from the distribution side.

- Step 3: Total assets of households are given by $Na = K + P^A A$. Differentiating with respect to time yields:

$$\dot{N}a + N\dot{a} = \dot{K} + \dot{P}^A A + P^A \dot{A}.$$

Using $\dot{N} = nN$ we can write

$$N\dot{a} = \dot{K} + \dot{P}^A A + P^A \dot{A} - nNa. \qquad (106)$$

Combining (106) with asset accumulation equation (102) we find

$$\begin{aligned} & \dot{K} + \dot{P}^A A + P^A \dot{A} - nNa \\ = & \left[(1-\tau_r)r - n\right] Na + (1-\tau_w)wNh - (1-s_H)Nwh^H - Nc + NT. \end{aligned}$$

Using $Na = K + P^A A$ and rearranging terms we obtain

$$\begin{aligned} \dot{K} = & (1-\tau_r)rK + (1-\tau_w)wNh - (1-s_H)Nwh^H - Nc + NT + \\ & \left[(1-\tau_r)rP^A - \dot{P}^A\right] A - P^A \dot{A}. \end{aligned}$$

Using (99) and $r = R - \delta_K$ this leads to

$$\begin{aligned} \dot{K} = & RK - \delta_K K - \tau_r rK + (1-\tau_w)wNh - (1-s_H)Nwh^H + \\ & \pi A - \tau_g \dot{P}^A A - P^A(1+\psi)\dot{A} - Nc + NT. \end{aligned}$$

Finally, using expression (105) for Y confirms the goods market clearing condition

$$\dot{K} = Y - Nc - \delta_K K,$$

or,

$$Y = C + I,$$

where $I \equiv \dot{K} + \delta_K K$ is aggregate gross investment and $C \equiv Nc$ is total consumption. *Q.E.D.*

Dynamical system: The following equations are employed for numerical simulations in Section 6.1.

$$\frac{\dot{c}}{c} = \frac{(1-\tau_r)r - \rho}{\sigma}. \tag{107}$$

$$\mu\xi\gamma(h^H)^{\gamma-1}h^\eta = c^{-\sigma}(1-s_H)w, \tag{108}$$

$$\frac{\dot{\mu}}{\mu} = \rho - n - \xi \left(h^H\right)^\gamma \eta h^{\eta-1} + \delta_H - \frac{c^{-\sigma}}{\mu} w(1-\tau_w), \tag{109}$$

$$\begin{aligned}\frac{\dot{P}^A}{P^A} \quad = \quad & \frac{1}{1-\tau_g}\left((1-\tau_r)r + \frac{\psi\nu}{1+\psi}A^{\phi-1}(Nh^A)^{1-\theta} - \right. \\ & \left. \frac{(1-\tau_c)(\kappa-1)\left(\frac{\alpha}{\kappa}\right)^{\frac{1}{1-\alpha}}\left[(1-s_K)(r+\delta_K)\right]^{-\frac{\alpha}{1-\alpha}} Nh^Y}{P^A A^{1-\frac{\alpha}{(1-\alpha)(\beta-1)}}}\right), \end{aligned} \tag{110}$$

$$\frac{\dot{A}}{A} = \frac{\nu}{1+\psi}A^{\phi-1}(Nh^A)^{1-\theta}, \tag{111}$$

$$\frac{\dot{h}}{h} = \xi(h^H)^\gamma h^{\eta-1} - \delta_H, \tag{112}$$

$$\frac{\dot{N}}{N} = n, \tag{113}$$

$$\dot{a} = \left[(1-\tau_r)r - n\right]a + (1-\tau_w)wh - (1-s_H)wh^H - c + T, \tag{114}$$

$$\begin{aligned} T \quad = \quad & \tau_w wh + \tau_r rk + \tau_c\left[\kappa(1-s_K) - (1+s_d)\right](r+\delta_K)k + \\ & \frac{\tau_g}{1-\tau_g}\left((1-\tau_r)rP^A\frac{A}{N} + \frac{P^A}{N}\frac{\psi\nu}{1+\psi}A^\phi(Nh^A)^{1-\theta} - \right. \\ & \left. A^{\frac{\alpha}{(1-\alpha)(\beta-1)}}(1-\tau_c)(\kappa-1)\left(\frac{\alpha}{\kappa}\right)^{\frac{1}{1-\alpha}}\left[(1-s_K)(r+\delta_K)\right]^{-\frac{\alpha}{1-\alpha}} h^Y\right) + \\ & \tau_c\left[P^A\nu A^\phi N^{-\theta}\left(h^A\right)^{1-\theta} - (1+s_R)wh^A\right] - s_H wh^H, \end{aligned} \tag{115}$$

$$k = A^{\frac{\alpha}{(1-\alpha)(\beta-1)}}\left(\frac{\alpha}{\kappa\left[(1-s_K)(r+\delta_K)\right]}\right)^{\frac{1}{1-\alpha}} h^Y, \tag{116}$$

$$a = k + \frac{P^A A}{N}, \tag{117}$$

$$w = A^{\frac{\alpha}{(1-\alpha)(\beta-1)}}(1-\alpha)\left(\frac{\alpha}{\kappa(1-s_K)(r+\delta_K)}\right)^{\frac{\alpha}{1-\alpha}}. \tag{118}$$

$$h^A = \frac{1}{N}\left(\frac{P^A\nu A^\phi}{(1-s_A)\,w}\right)^{\frac{1}{\theta}}. \tag{119}$$

$$h^A + h^H + h^Y = h. \tag{120}$$

Initial conditions are: a_0, K_0, A_0, N_0, h_0.

We obtain the following (initial) steady state values:

$$r = \frac{\sigma g + \rho}{1 - \tau_r}, \tag{121}$$

$$w = A^{\frac{\alpha}{(1-\alpha)(\beta-1)}} (1-\alpha) \left(\frac{\alpha}{\kappa(1-s_K)(r+\delta_K)} \right)^{\frac{\alpha}{1-\alpha}}, \tag{122}$$

$$h^H = \left(\frac{1-\tau_w}{1-s_H} \frac{\gamma}{(\sigma-1)g + \rho - n + (1-\eta)\delta_H} \right)^{\frac{1-\eta}{1-\gamma-\eta}} \left(\frac{\xi}{(\delta_H)^{\eta}} \right)^{\frac{1}{1-\gamma-\eta}}, \tag{123}$$

$$h = \left(\frac{\xi}{\delta_H} \right)^{\frac{1}{1-\eta}} (h^H)^{\frac{\gamma}{1-\eta}}, \tag{124}$$

$$\Omega = \frac{(1-\tau_c)(\kappa-1)\left(\frac{\alpha}{\kappa}\right)^{\frac{1}{1-\alpha}}}{\left[(1-\tau_r)r + \psi g_A - (n+g-g_A)(1-\tau_g)\right]\left[(1-s_K)(r+\delta_K)\right]^{\frac{\alpha}{1-\alpha}}}. \tag{125}$$

$$h^A = \frac{(1+\psi) g_A \Omega (h - h^H)}{(1-s_A)\, w A^{-\frac{\alpha}{(1-\alpha)(\beta-1)}} + (1+\psi) g_A \Omega}, \tag{126}$$

$$h^Y = \frac{h - h^H}{1 + A^{\frac{\alpha}{(1-\alpha)(\beta-1)}} \frac{(1+\psi) g_A \Omega}{(1-s_A) w}}, \tag{127}$$

$$P^A = \frac{N A^{\frac{\alpha}{(1-\alpha)(\beta-1)}-1} \Omega (h - h^H)}{1 + A^{\frac{\alpha}{(1-\alpha)(\beta-1)}} \frac{(1+\psi) g_A \Omega}{(1-s_A) w}}, \tag{128}$$

$$c = \left[(\sigma-1)g + \rho - n\right] a + (1-\tau_w) w h - (1-s_H) w h^H + T, \tag{129}$$

$$\begin{aligned} T = {} & \tau_w w h + \tau_r r k + \tau_c \left[\kappa(1-s_K) - (1+s_d)\right](r+\delta_K)k + \\ & \frac{\tau_g}{1-\tau_g} \Bigg(\left[(1-\tau_r)r + \psi g_A\right] P^A \frac{A}{N} - \qquad (130) \\ & A^{\frac{\alpha}{(1-\alpha)(\beta-1)}} (1-\tau_c)(\kappa-1) \left(\frac{\alpha}{\kappa}\right)^{\frac{1}{1-\alpha}} \left[(1-s_K)(r+\delta_K)\right]^{-\frac{\alpha}{1-\alpha}} h^Y \Bigg) + \\ & \tau_c \left[P^A (1+\psi) g_A \frac{A}{N} - (1+s_R) w h^A \right] - s_H w h^H, \qquad (131) \end{aligned}$$

$$k = A^{\frac{\alpha}{(1-\alpha)(\beta-1)}} \left(\frac{\alpha}{\kappa\left[(1-s_K)(r+\delta_K)\right]} \right)^{\frac{1}{1-\alpha}} h^Y, \tag{132}$$

$$a = k + \frac{P^A A}{N}. \tag{133}$$

(Also recall that $g_A = \frac{(1-\theta)n}{1-\phi}$ and $g = \frac{\alpha g_A}{(1-\alpha)(\beta-1)}$.)

Numerical procedure for the optimal dynamic policy mix: As a reference point of our analysis in Section 6.2, we compute the transition path from the initial steady state to the steady state, implied by the parameters $(s_R^{opt}, s_d^{opt}, s_H^{opt})$, which maximize steady state consumption. Obviously, this transition path does not yield the maximum welfare for time-invariant policies. Then, a search algorithm is applied to find a set of parameters, which induces a transition path yielding a higher level of welfare. This transition path starts at the initial steady state and terminates at the steady state implied by the set of test parameters. We then vary one parameter at a time and calculate the implied transition as well as the associated welfare. If this welfare is higher, the reference set of policy parameters is updated accordingly. The procedure is terminated when no further significant improvements in welfare can be achieved.

The parameters are varied stochastically. A deterministic search grid would either lead to a high computational demand (in case of a "fine grid"), or a low accuracy (in case of a "rough grid"). By varying the policy parameters stochastically, the algorithm uses small and large steps to improve welfare.