Dynamically Optimal R&D Subsidization

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Abstract

Previous research on optimal R&D subsidies has focussed on the long run. This paper characterizes the optimal time path of R&D subsidization in a semi-endogenous growth model, by exploiting a recently developed numerical method. Starting from the steady state under current R&D subsidization in the US, the R&D subsidy should significantly jump upwards and then slightly decrease over time. There is a negligible loss in welfare, however, from immediately setting the R&D subsidy to its optimal long run level, compared to the case where the dynamically optimal policy is implemented.

Key words: R&D subsidy; Transitional dynamics; Semi-endogenous growth; Welfare.

JEL classification: H20; O30; O40.

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1 Introduction

A large body of empirical evidence suggests that the social return to R&D exceeds the private return by a wide margin (e.g., Scherer, 1982; Grilichis and Lichtenberg, 1984).[1] By focussing on the long run, a similar finding is typically derived from calibrated endogenous growth models. In fact, positive externalities from R&D seem to substantially outweigh negative externalities (e.g., Romer, 2000; Jones and Williams, 2000; Steger, 2005; Grossmann, Steger and Trimborn, 2010). For instance, Jones and Williams (2000) argue on basis of a semi-endogenous growth model in the spirit of Jones (1995), that the optimal long run R&D effort may be about twice the decentralized R&D effort.

One widely discussed policy implication from the apparent R&D underinvestment problem is to provide R&D subsidies to innovating industries. According to empirical evidence, such tax incentives indeed seem to be successful in stimulating R&D investments (e.g., Bloom, Griffith and van Reenen, 2002). The question is thus not so much whether or not R&D subsidies should be provided to innovating industries. One rather needs to know to what extent R&D should be subsidized and how R&D subsidization should change over time as the economy develops.

The existing literature has examined optimal R&D subsidization by either focussing on static models or exclusively on the steady state in dynamic models. However, as it is well known, in R&D-based models of economic growth the speed of convergence is typically low.[2] Thus, any attempt to provide a careful policy recommendation requires to investigate the entire time path of the first-best R&D subsidy along the transition to the steady state. This is true even for advanced economies, which may have come close to their decentralized steady state. The reason is that current R&D subsidy rates may be far away from their long run social optimum. In this case, implementing the optimal long run policy would induce transitional dynamics with potentially long

[1] See also Jones and Williams (1998) for a discussion of the empirical literature in the light of endogenous growth theory.

[2] See, for instance, Steger (2003) who examines the speed of convergence in the endogenous growth model of Segerstrom (1998). Grossmann, Steger and Trimborn (2010) find a similar pattern for an extended Jones (1995) model.

lasting adjustment to the new steady state. Hence, it is a priori not clear whether looking at the long run optimal R&D subsidization is indeed meaningful when it comes to providing careful policy recommendations.

This paper attempts to characterize the dynamically optimal, i.e. first-best, R&D subsidy on the basis of a calibrated R&D-based growth model with accumulation of both knowledge and physical capital. We show how the optimal time path of the R&D subsidy and important allocation variables depend on the initial conditions of the economy. For instance, we take the steady state under the status quo R&D subsidization of a calibrated US economy as anchor (i.e., we assume that the US is in long-run equilibrium) to investigate the dynamics induced by a shift towards the dynamically optimal R&D subsidization.

We also calculate the welfare loss, in terms of permanent consumption-equivalent changes, which results from implementing the long run optimal (time-invariant) policy rather than the dynamically optimal one (i.e. the time-varying, first-best policy). This exercise may be of high relevance for real-life policy. Policy makers may be constrained to set policy instruments at time-invariant levels for the sake of simplicity. We would like to have an idea about the magnitude of the loss from such a political constraint. Moreover, policy makers do not know at which point along the transition path the economy is located.

We deliberately choose to analyze the optimal dynamic R&D tax in a standard growth model, i.e., a standard version of the semi-endogenous growth model by Jones (1995). The reason is that, for a first analysis of a dynamically optimal R&D policy, we consider it an advantage to be able to analytically derive the steady state (decentralized equilibrium and social planner solution) and to draw on a deep understanding of its properties (e.g., Jones, 2005). Nevertheless, to examine the evolution of the variables of interest along the transition to the steady state requires the solution of highly dimensional, non-linear differential equation systems. We are able to do so by applying a novel numerical procedure which was suggested recently by Koch, Steger and Trimborn (2008).

Our results indicate that the optimal R&D subsidy is time-dependent and adjusts monotonically towards the steady state. Whether it should decrease or increase over time depends on the gap in the knowledge stock and the capital stock to their optimal steady state levels. For instance, if the knowledge stock is far away from the optimal steady state relative to the capital stock, the optimal R&D subsidy should initially start above its long run level and decrease over time when the economy approaches the steady state. For the US, our analysis suggests that the R&D subsidy should jump up significantly from its current level and then slightly decrease over time. The striking result from our analysis is that the welfare loss from immediately setting the R&D subsidy to its optimal long run level (i.e. the constrained optimum) is negligible compared to the case where the dynamically optimal policy path is implemented. In other words, the error of neglecting the transitional dynamics when designing the optimal R&D subsidy is very small, despite the fact that the speed of convergence to the steady state is fairly low.[3]

Dynamic optimal tax problems have been extensively studied in neoclassical growth models with fixed government expenditure (e.g., Judd, 1985; Chamley, 1986). For instance, it is well known that the optimal Ramsey-tax on capital income is highly time-variant in a closed-economy neoclassical growth framework with endogenous labor supply. Early in the transition, capital should be taxed at high rates due to the lump-sum tax character of taxing the existing capital stock. However, in an infinite horizon context the optimal capital tax becomes zero when the economy approaches the steady state (i.e., required tax revenue is exclusively financed by labor income taxation).[4] In an endogenous growth model there is no analogy with respect to R&D subsidies.[5] Rather than studying a second-best Ramsey-tax problem, we seek for dynamically optimal Pigouvian subsidies which eliminate inefficient investment incentives under

[3] Our analysis suggests that after a policy reform in the US which implements the optimal R&D subsidization it takes more than 200 years to close half of the gap of per capita consumption to the new steady state.

[4] The result is potentially modified in overlapping-generation models (e.g., Conesa, Kitao and Krueger, 2009).

[5] Moreover, the transitional dynamics are more complex when, in addition to physical capital accumulation, R&D-based innovations are a second engine of growth.

laisser-faire R&D incentives are distorted, for instance, by intertemporal knowledge spillovers. Inefficiency of capital accumulation arises from market power of capital good producers. We show that the first-best allocation can be restored with two linear policy instruments in our framework.

Our paper is most closely related to studies which numerically derive the long run optimal R&D subsidy in an endogenous growth framework. For instance, Sener (2008) studies an endogenous growth model without scale effects where the steady state growth rate depends on the R&D subsidy. Calibration exercises indicate that the optimal steady state R&D subsidy should range between 5 and 25 percent. However, the analysis abstracts from transitional dynamics. Grossmann, Steger and Trimborn (2010) extend the semi-endogenous growth framework of Jones and Williams (2000) to capture distortions from the tax-transfer system. Their results suggest that in the long run firms should be allowed to deduct at least twice the R&D costs from sales revenue to calculate corporate income. This policy recommendation still holds when the whole transition path is taken into account, provided that the subsidy rates are constrained to be time-invariant. In this paper, we relax the latter restriction and analyze the socially optimal transitional dynamics. To the best of our knowledge, the optimal time path of the R&D subsidy and the welfare loss arising from neglecting the dynamic nature of the optimal R&D subsidy has not previously been analyzed.

The paper is organized as follows. Section 2 presents the endogenous growth model with linear subsidies on capital costs and R&D costs, derives the dynamic system in decentralized equilibrium and analytically characterizes the dynamically optimal capital and R&D subsidization. Section 3 presents and discusses the calibration of the model. Based on the calibration strategy, section 4 numerically analyzes the socially optimal evolution of important allocation variables and the R&D subsidy when varying initial conditions. Most importantly, it also compares the optimal dynamics with the one resulting from implementing the time-invariant, optimal long run R&D subsidy from the start. The last section concludes.

2 The Model

Consider the following continuous-time model with semi-endogenous economic growth, based on Jones (1995). There is a homogenous final output good with price normalized to unity. Final output is produced under perfect competition according to

$$Y = (L^Y)^{1-\alpha} \int_0^A (x_i)^\alpha di, \tag{1}$$

$0 < \alpha < 1$, where L^Y is labor input in the manufacturing sector, A is the mass ("number") of intermediate goods and x_i denotes the quantity of intermediate good i. (Time index t is omitted whenever this does not lead to confusion.) The number of varieties, A, expands through horizontal innovations, protected with patent rights of infinite length. As usual, A is interpreted as the economy's stock of knowledge. $A_0 > 0$ is given. The labor market is perfect.

In each sector i there is one firm − the innovator or the buyer of a blueprint for an intermediate good − which can produce good x_i with a one-to-one technology: one unit of foregone consumption (capital) can be transformed into one unit of output. Capital depreciates at rate $\delta \geq 0$. Capital supply in the initial period, K_0, is given. The capital market is perfect.

Moreover, in each sector i there is a competitive fringe which can produce a perfect substitute for good i (without violating patent rights) but is less productive in manufacturing the good: one unit of output requires κ units of capital; $1 < \kappa \leq 1/\alpha$.[6]

There is free entry into the R&D sector. Ideas for new intermediate goods are generated according to

$$\dot{A} = \tilde{\nu} A^\phi L^A, \text{ with } \tilde{\nu} \equiv \nu \left(L^A\right)^{-\theta}, \tag{2}$$

where L^A is the labor input in the R&D sector, $\nu > 0$, $\phi < 1$, $0 \leq \theta < 1$. $\tilde{\nu}$ is taken as given in the decision of the representative R&D firm; that is, similar to Jones

[6]See Aghion and Howitt (2005), among others, for similar way of capturing a competitive fringe.

and Williams (2000), R&D firms perceive a constant returns to scale R&D technology, although the social return to higher R&D input is decreasing when $\theta > 0$. The wedge between the private and social return may arise because firms do not take into account that rivals may work on the same idea such that there is redundant R&D in market equilibrium. θ measures the strength of such "duplication externality". If $\phi \neq 0$, there is a second R&D externality; $\phi > 0$ captures a standard "standing on shoulders" effect, whereas the case $\phi < 0$ reflects by contrast that R&D productivity declines with the number of preceding innovations (possibly because the most obvious innovations are detected first; see Jones, 1995, for a discussion).[7]

There is an infinitely-living, representative dynasty with initial per capita wealth, $a_0 > 0$. Household size, N, grows with constant exponential rate, $n \geq 0$. N_0 is given and normalized to unity. Preferences are represented by the standard utility function

$$U = \int_0^\infty \frac{(c_t)^{1-\sigma} - 1}{1 - \sigma} e^{-(\rho - n)t} dt, \tag{3}$$

$\sigma > 0$, where c is consumption per capita. Households take factor prices as given.

The government may subsidize both R&D costs (R&D sector) and capital costs (intermediate goods sector), financed by lump-sum taxes (T) on households. The subsidy rates are independent of total costs at one point in time, but possibly time-variant. They are denoted by s_A (R&D) and s_K (capital), respectively.

Let w and r denote the wage rate and the interest rate, respectively. Financial wealth per individual, a, accumulates according to

$$\dot{a} = (r - n)a + w - c - T, \tag{4}$$

with a_0 being given.

It turns out that, for the transversality conditions to hold and the value of the

[7] In his seminal contribution on endogenous technical change, Romer (1990) assumes $\phi = 1$. This assumption has been modified by Jones (1995) as $\phi = 1$ implies that the economy's growth rate depends on the aggregate human capital level ("strong scale effect"), a prediction which seems to be largely inconsistent with the data. We therefore follow Jones (1995).

utility stream, U, to be finite, we have to restrict the parameter space such that

$$\rho - n + (\sigma - 1)g > 0 \text{ with } g \equiv \frac{(1-\theta)n}{1-\phi}. \tag{A1}$$

As will become apparent, g is the economy's long run growth rate in decentralized equilibrium as well as in social planning optimum. We maintain assumption A1 throughout.

2.1 Market Equilibrium

We first derive the decentralized equilibrium and show how the steady state allocation of labor and the steady state savings rate (equal to the investment rate) depends on policy parameters.

We start with intermediate goods producers. Note that $r + \delta$ is the user cost per unit of capital for an intermediate good firm. As one unit of capital is required for one unit of output, if the government subsidizes capital costs at rate s_K, producer i has profits

$$\pi_i = [p_i - (1 - s_K)(r + \delta)]\, x_i, \tag{5}$$

where p_i is the price of good i. According to (1), the inverse demand function for intermediate good i reads $p_i = \alpha(L^Y/x_i)^{1-\alpha}$.

Profit maximization implies that the optimal price of each firm i is given by

$$p_i = p = \kappa(1 - s_K)(r + \delta). \tag{6}$$

To see this, note that a firm which owns a blueprint would choose a mark-up factor which is equal to $1/\alpha \geq \kappa$ if it were not facing a competitive fringe. Moreover, the competitive fringe would make losses at a price lower than $\kappa(1-s_K)(r+\delta)$. Thus, each firm i sets the maximal price allowing it to remain monopolist. We can substitute (6) into the inverse demand function and solve for x_i to obtain output

$$x_i = x = \left(\frac{\alpha}{\kappa(1 - s_K)(r + \delta)}\right)^{\frac{1}{1-\alpha}} L^Y. \tag{7}$$

Substituting (7) into (1) gives

$$Y = A\left(\frac{\alpha}{\kappa(1-s_K)(r+\delta)}\right)^{\frac{\alpha}{1-\alpha}} L^Y \tag{8}$$

Moreover, as the total amount of physical capital is $K \equiv \int_0^A x_i di = Ax$, the capital-output ratio is given by

$$\frac{K}{Y} = \frac{\alpha}{\kappa(1-s_K)(r+\delta)}. \tag{9}$$

Thus, if the interest rate r is stationary in the long run, the total capital stock and aggregate income grow at the same rate along a balanced growth path.

Let us denote the present discounted value of the profit stream generated by an innovation by P^A (being equal to the price an intermediate good producer pays to the R&D sector for a new blueprint and to the stock market evaluation of a firm). In equilibrium, there are no arbitrage possibilities in the capital market. Noting that all intermediate goods producers have the same profit due to the symmetry in their sector, i.e., $\pi_i = \pi$ for all i, this implies the standard capital market equilibrium condition

$$\frac{\dot{P}^A}{P^A} + \frac{\pi}{P^A} = r. \tag{10}$$

Let us define $\tau \equiv 1 - s_A$. In the R&D sector, under R&D subsidy rate s_A, a representative firm maximizes

$$\Pi = P^A \underbrace{\tilde{\nu} A^\phi L^A}_{=\dot{A}} - \tau w L^A, \tag{11}$$

taking A, $\tilde{\nu}$ and prices as given. That is, in equilibrium, $\Pi = 0$.

The household's problem is to solve

$$\max_{\{c_t\}} \int_0^\infty \frac{(c_t)^{1-\sigma} - 1}{1-\sigma} e^{-(\rho-n)t} dt \text{ s.t. } (4), \ \lim_{t\to\infty} a_t e^{-\int_0^t (r_s - n) ds} \geq 0. \tag{12}$$

The household chooses the optimal consumption path, where savings are supplied to the financial market.

Definition. A market equilibrium in this economy consists of time paths for the quantities $\{L_t^A, L_t^Y, c_t, \{x_{it}\}_{i=0}^A, a_t, Y_t, K_t, A_t\}_{t=0}^{\infty}$ and prices $\{P_t^A, \{p_{it}\}_{i=0}^A, w_t, r_r\}_{t=0}^{\infty}$ such that

1. final goods producers, intermediate goods producers and R&D firms maximize profits,

2. households maximize intertemporal welfare,

3. the capital resource constraint $\int_0^A x_{it} di = K_t$ holds,

4. the capital market equilibrium condition, equ. (10), holds,

5. the labor market, the intermediate goods market, and the financial market clear.[8]

We define per capita measures $l^A \equiv L^A/N$, $l^Y \equiv L^Y/N$, $k \equiv K/N$, $y \equiv Y/N$ and $p^A \equiv P^A/N$. Clearing of the financial market requires $aN = K + P^A A$. Moreover, in labor market equilibrium, $l^A + l^Y = 1$. Along a balanced growth path, all variables grow at a constant (possibly zero) rate. We descale those variables which turn out to grow with rate $g = \frac{(1-\theta)n}{1-\phi}$ in steady state and define $\tilde{A} = A/N^{\frac{1-\theta}{1-\phi}}$, $\tilde{k} = k/N^{\frac{1-\theta}{1-\phi}}$ and $\tilde{c} = c/N^{\frac{1-\theta}{1-\phi}}$. Proposition 1 (below) presents the full dynamical system which governs the evolution of the market equilibrium and its steady state.

Proposition 1. (Dynamic system for market equilibrium)

(i) Given the time path of τ, the evolution of $\tilde{A}$, p^A, $\tilde{k}$, $\tilde{c}$, r, $l^A = 1 - l^Y$ is governed

[8] According to Walras' law, the final goods market then clears as well.

by the following dynamic system (together with appropriate boundary conditions)

$$\frac{\dot{\tilde{A}}}{\tilde{A}} = \nu \tilde{A}^{\phi-1}\left(l^A\right)^{1-\theta} - g, \tag{13}$$

$$\frac{\dot{p}^A}{p^A} = r - n - \frac{(\kappa-1)(\alpha/\kappa)^{\frac{1}{1-\alpha}}(1-l^A)}{[(1-s_K)(r+\delta)]^{\frac{\alpha}{1-\alpha}} p^A}, \tag{14}$$

$$\frac{\dot{\tilde{k}}}{\tilde{k}} = \left(\frac{\tilde{A}(1-l^A)}{\tilde{k}}\right)^{1-\alpha} - \frac{\tilde{c}}{\tilde{k}} - \delta - n - g, \tag{15}$$

$$\frac{\dot{\tilde{c}}}{\tilde{c}} = \frac{r-\rho}{\sigma} - g, \tag{16}$$

$$r + \delta = \frac{\alpha}{\kappa(1-s_K)}\left(\frac{\tilde{A}(1-l^A)}{\tilde{k}}\right)^{1-\alpha}, \tag{17}$$

$$p^A \nu \tilde{A}^{\phi-1}(l^A)^{-\theta} = \tau(1-\alpha)\left(\frac{\tilde{k}}{\tilde{A}(1-l^A)}\right)^{\alpha}. \tag{18}$$

(ii) In the long run, there exists a unique balanced growth equilibrium, where

$$r = \sigma g + \rho \equiv r^*, \tag{19}$$

$$l^A = \frac{1}{\frac{\tau(1/\alpha-1)(\sigma g+\rho-n)}{(1-1/\kappa)g} + 1} \equiv l^{A*}, \tag{20}$$

$$\tilde{A} = \left(\frac{\nu\left(l^{A*}\right)^{1-\theta}}{g}\right)^{\frac{1}{1-\phi}} \equiv \tilde{A}^*, \tag{21}$$

$$p^A = \frac{(\kappa-1)(\alpha/\kappa)^{\frac{1}{1-\alpha}}(1-l^{A*})}{[(r^*+\delta)(1-s_K)]^{\frac{\alpha}{1-\alpha}}(r^*-n)} \equiv p^{A*}, \tag{22}$$

$$\tilde{k} = \tilde{A}^*(1-l^{A*})\left(\frac{\alpha}{\kappa(1-s_K)(r^*+\delta)}\right)^{\frac{1}{1-\alpha}} \equiv \tilde{k}^*, \tag{23}$$

$$\tilde{c} = (\tilde{k}^*)^{\alpha}(\tilde{A}^*)^{1-\alpha}(1-l^{A*})^{1-\alpha} - (\delta+n+g)\tilde{k}^* \equiv \tilde{c}^*. \tag{24}$$

In the long run, k, y, c and A grow at rate $g = \frac{(1-\theta)n}{1-\phi}$. The savings and investment rate, $sav \equiv 1 - \frac{c}{y}$, is given by

$$sav = \frac{\alpha(n+g+\delta)}{\kappa(1-s_K)(\sigma g+\rho+\delta)} \equiv sav^*. \tag{25}$$

Proof. See Appendix. ■

Like in Jones (1995), the growth rate of per capita income along a balanced growth path is independent of economic policy (in contrast to the level of income). Proposition 1 also implies that life-time utility (3) is finite if and only if assumption (A1) holds.

Moreover, Proposition 1 shows that subsidizing physical capital does not affect the allocation of labor in long run equilibrium, but an increase in s_K raises the long run savings and investment rate, sav^*. Similarly, an increase in the R&D subsidy rate s_A (i.e., a decline in τ) stimulates R&D activity of firms (i.e., l^{A*} increases); it does not, however, affect the long run equilibrium rate of investment in physical capital, sav^*.

2.2 Social Planning Optimum

A social planner chooses a symmetric capital allocation across intermediate firms, i.e., $x_i = K/A$ for all i. Using this in production function (1) yields per capita output ($y = Y/N$):

$$y = k^{\alpha}(Al^{Y})^{1-\alpha}. \tag{26}$$

Thus, using the goods market clearing condition $\dot{K} = Y - Nc - \delta K$, the capital stock per capita ($k = K/N$) evolves according to

$$\dot{k} = k^{\alpha}(Al^{Y})^{1-\alpha} - (\delta + n)k - c. \tag{27}$$

Also note that the social planner takes R&D externalities into account such that the relevant knowledge accumulation technology is

$$\dot{A} = \nu A^{\phi}(Nl^{A})^{1-\theta}. \tag{28}$$

The social planner's problem thus is to solve

$$\max U \text{ s.t. } (27), (28), l^{A} = 1 - l^{Y}, \tag{29}$$

and non-negativity constraints, where c, l^A, l^Y are control variables and k, A are state variables.

Comparing the social planning optimum to the decentralized equilibrium, we may ask if the two policy instruments, a subsidy to R&D and capital costs, can restore the first best optimum, in view of the following market failures. First, due to monopolistic competition as compared to marginal cost pricing, intermediate goods supply and therefore the demand for capital is inefficiently low. Thus, savings (and thus capital investment) may be too low, calling for a capital cost subsidy. Moreover, there are three sources of inefficient R&D incentives. The duplication externality ($\theta > 0$) promotes overinvestment in R&D, whereas a standing on shoulders effect ($\phi > 0$) promotes underinvestment. (In the case where $\phi < 0$, there is a force towards overinvestment.) Finally, innovators can only appropriate part of the economic surplus from raising the knowledge stock of the economy. To see this, first note that $x_i = x = \frac{K}{A} = \frac{K}{Y}\frac{Y}{A}$. Substituting this into (5) and using (6) and (9) implies that instantaneous profit of an intermediate goods firm reads $\pi = \alpha(1 - \frac{1}{\kappa})\frac{Y}{A}$. Moreover, according to (8), we have $\frac{\partial Y}{\partial A} = \frac{Y}{A}$. Since $\alpha(1 - \frac{1}{\kappa}) < 1$, the per-period profit π for an innovator is lower than the contribution of an additional blueprint to output, $\frac{\partial Y}{\partial A}$. In other words, there is a "surplus appropriability problem" which promotes underinvestment.

The next proposition shows that appropriately setting $\tau = 1 - s_A$ and s_K in the market economy can indeed implement the first best optimum. Moreover, it turns out that the optimal R&D subsidy is time-variant rather than being equal to the time-invariant long run optimum, whereas the optimal capital subsidy does not change over time. We will closely examine the implications of this insight below.

Proposition 2. (Social optimum) *The first-best optimal evolution of the economy can be implemented by setting a time-invariant capital cost subsidy*

$$s_K = 1 - \frac{1}{\kappa} \equiv s_K^{opt} \tag{30}$$

together with a time-variant R&D subsidy $s_A = 1 - \tau$, where the optimal τ evolves *according to*

$$\frac{\dot{\tau}}{\tau} = \left[\left(1 - \theta - \frac{1}{\tau} \frac{1 - 1/\kappa}{1/\alpha - 1} \right) \left(\frac{1}{l^A} - 1 \right) + \phi \right] \nu \tilde{A}^{\phi - 1} \left(l^A \right)^{1-\theta} \tag{31}$$

with terminal condition (long-run optimal R&D subsidy)

$$\tau = \frac{1 - 1/\kappa}{1/\alpha - 1} \frac{(\sigma - 1)g + \rho - \theta n}{(1-\theta)(\sigma g + \rho - n)} \equiv \tau^{opt}. \tag{32}$$

Proof. See Appendix. ■

A higher mark up factor κ drives a bigger wedge between the equilibrium investment rate and the socially optimal investment rate, calling for a higher subsidy on capital costs. At the same time, if κ rises, the surplus appropriability problem, which promotes sub-optimally low investment in R&D, becomes less severe, such that the optimal long run R&D subsidy, $1 - \tau^{opt}$, decreases. This suggests that stronger patent protection should be accompanied by lower R&D subsidies. As long run growth is policy-independent and the dynamically optimal Pigouvian subsidies implement the first best allocation, Proposition 2 implies that the long-run growth rate in social optimum coincides with the one arising in decentralized equilibrium.

The important questions we examine below is whether the optimal R&D subsidy should increase or decrease over time, given that the capital cost subsidy is set optimally. Moreover, as previous studies of optimal R&D subsidies have exclusively focussed on the long run, it is interesting to compare the evolution of important variables under the first-best (i.e. time-varying) R&D subsidy and the optimal steady state (i.e. the time-invariant) R&D subsidy. As both requires numerical analysis, we need to calibrate the model first.

3 Calibration

We calibrate the model for the US economy. The strategy is to match steady state values of important variables. First, g is set to the average US GDP per capita growth rate for the period 1990-2004. Taking data from the Penn World Tables (PWT) 6.2 (Heston, Summers and Baten, 2006), we find that approximately $g = 0.02$.[9] Moreover, for the same period and again from PWT 6.2, the average population growth rate is approximately $n = 0.01$.

We use measures for the investment rate and the capital-output ratio to calibrate the depreciation rate of physical capital as follows. The investment share is given by $sav = (\dot{K} + \delta_K K)/Y = (\dot{K}/K + \delta)K/Y$. Using $\dot{K}/K = n + g$ and solving for δ yields

$$\delta = \frac{sav}{K/Y} - n - g. \tag{33}$$

Averaging over the period 1990-2004, sav is equal to about 21 percent, according to PWT 6.2. For the capital-output ratio, K/Y, we take averages over the period 2002-2007 calculated from data of the US Bureau of Economic Analysis. The capital stock is measured by total fixed assets (private and public structures, equipment and software). When measuring K and Y in current prices, this gives us $K/Y = 3$. From (33), the evidence thus suggests $\delta = 0.04$, a standard value in the literature.

Moreover, the steady state interest rate is set to seven percent, which coincides with the real long-run stock market return estimated by Mehra and Prescott (1985).[10] According to (19), for $g = 0.02$, $r = 0.07$, and a typical value for the time preference rate of $\rho = 0.02$, one gets $\sigma = 2.5$.

We also assume that the capital cost subsidy rate, s_K, is at the optimal level, for two reasons: First, this allows us to focus on the consequences of deviating from the optimal path of the R&D subsidy rate. Second, one may argue that s_K actually is at the optimal level (s_K^{opt}) at present in the US. In line with estimates for the average mark up factor in the economy (e.g. Norrbin, 1993), setting $\kappa = 4/3$ implies $s_K^{opt} = 1 - 1/\kappa = 0.25$. Now, given a rate of depreciation allowances for capital investments, s_d, and a corporate income tax rate, τ_c, the behaviorally relevant capital cost subsidy is $s_K = \frac{\tau_c s_d}{1-\tau_c}$ (e.g., Grossmann, Steger and Trimborn, 2010). According to Devereux, Griffith and Klemm

[9] Averaging over some years takes out business cycle phenomena.

[10] Jones and Williams (2000) argue that this rate of return is more appropriate for calibration of growth models than the risk-free rate of government bonds.

(2002), in the US, we approximately have $s_d = 0.75$. For large corporations, the federal US statutory corporate income tax rate is 35 percent (and about 39 percent including sub-governments).[11] For small corporations, it is 15 (with sub-governments, about 20) percent. We may thus base our calibration on $\tau_c = 0.25$, which together with $s_d = 0.75$ indeed implies $s_K = 0.25$.

Next, using (9), we find that $\kappa(1 - s_K) = 1$, $r = 0.07$, $\delta = 0.04$ and $K/Y = 3$ implies $\alpha = 0.33$, which is a typical value of the output elasticity of capital used in the growth literature.[12]

Parameter ν is irrelevant for the steady state allocation of labor in both market equilibrium and social optimum. Moreover, for a given growth rate of per capita income (g) and population size (n), we can use the relationship between the standing on shoulders parameter (ϕ) and the duplication externality parameter (θ) which is implied by $g = \frac{(1-\theta)n}{1-\phi}$. Hence, from $g = 0.02$ and $n = 0.01$ we get

$$\phi = 0.5(1 + \theta). \tag{34}$$

This leaves us with one degree of freedom. We focus our discussion on the case where $\theta = 0.5$ (medium degree of duplication externality), which implies that $\phi = 0.75$. According to (32), this implies an optimal long run R&D subsidy rate, $s_A^{opt} \equiv 1 - \tau^{opt}$, of 81.5 percent.[13] In a model with corporate income taxation, assuming a corporate tax rate $\tau_c = 0.25$, this would mean that innovating firms should be allowed to deduct $1 + \frac{(1-\tau_c)s_A}{\tau_c} = 3.4$ times their R&D costs from sales revenue to compute the corporate income tax base (Grossmann, Steger and Trimborn, 2010). Our main results and conclusions are unchanged when we use other values for θ, as long as θ is not too high

[11] See the OECD tax database.

[12] We consider it to be an advantage of our calibration strategy that we do not have to assume a value for α, but infer it from observables. Although α is one minus the labor share of income in neoclassical models, we cannot use this standard argument in our context where labor is not only used in final goods production. Moreover, as discussed by Krueger (1999), measurement of the labor share is difficult and inevitably depends on strong assumptions.

[13] This may seem high at the first glance. However, it is in line with previous calibration exercises. For instance, Grossmann, Steger and Trimborn (2010) show that a R&D subsidy of that kind of magnitude is required to solve the R&D underinvestment problem which is identified in the seminal paper by Jones and Williams (2000).

(obviously, for $\theta \to 1$ no R&D should be conducted). For instance, for $\theta = 0.25$ we find that 3.6 times R&D costs and for $\theta = 0.75$ about three times the R&D costs should be deductible. The current US R&D subsidy rate is given by $s_A = 0.066$ (OECD, 2009),[14] which means that in the US only 1.2 times the R&D costs are deductible. With $s_A = 0.066$, we find that the share of labor devoted to R&D, l^{A*}, is about 4.2 percent for our calibration, according to (20). This corresponds to a steady state R&D intensity, wl^A/Y, of 2.9 percent.

4 Quantitative Analysis

In this section, we make use of Proposition 1 and 2 to examine, based on the calibration described above, the optimal time path of the R&D subsidy rate (s_A) and main allocation variables and compare the results to the case where the optimal steady state R&D policy $\tau = \tau^{opt}$ is implemented from the start. We also discuss policy implications from our analysis for the US economy.

4.1 The Role of Initial Conditions

Panel (a) of Fig. 1 shows how the optimal level of $\tau = 1 - s_A$ evolves over time when the adjusted capital stock per capita ($\tilde{k}$) and/or the adjusted stock of knowledge ($\tilde{A}$) start below their optimal long run levels. For instance, suppose the initial knowledge stock is at 50 percent of its optimal long run level ($\tilde{A}_0 = 0.5\tilde{A}^*$, where the long run level $\tilde{A}^*$ is evaluated at $\tau = \tau^{opt}$) and the capital stock is initially optimal ($\tilde{k}_0 = \tilde{k}^*$, where $\tilde{k}^*$ is evaluated at $\tau = \tau^{opt}$), which is *Scenario (i)* in Fig. 1. Then τ should start below the long run optimum, τ^{opt}, and increase over time. That is, the R&D subsidy rate, s_A, should be high initially in view of the high initial gap in the knowledge stock

[14] OECD (2009) reports a R&D subsidy rate $RDTS = 1 - Bindex$, where the so-called B-index is given by $Bindex = \frac{1-\Xi}{1-\tau_c}$, with τ_c being the statutory corporate income tax rate and Ξ the net present discounted value of depreciation allowances, tax credits and special allowances on R&D assets. We have $\Xi = \tau_c(1 + s_R)$, where s_R is the subsidy rate at which R&D costs which can be deducted from pre-tax profits. Thus, $RDTS = \frac{\tau_c s_R}{1-\tau_c}$. Grossmann et al. (2010) show that $RDTS$ is equal to the behaviorally relevant R&D subsidy rate, s_A.

(“knowledge gap”) and decrease over time when $\tilde{A}$ comes closer to the steady state.[15] Quantitatively, however, the variation in the dynamically optimal s_A over time is small.

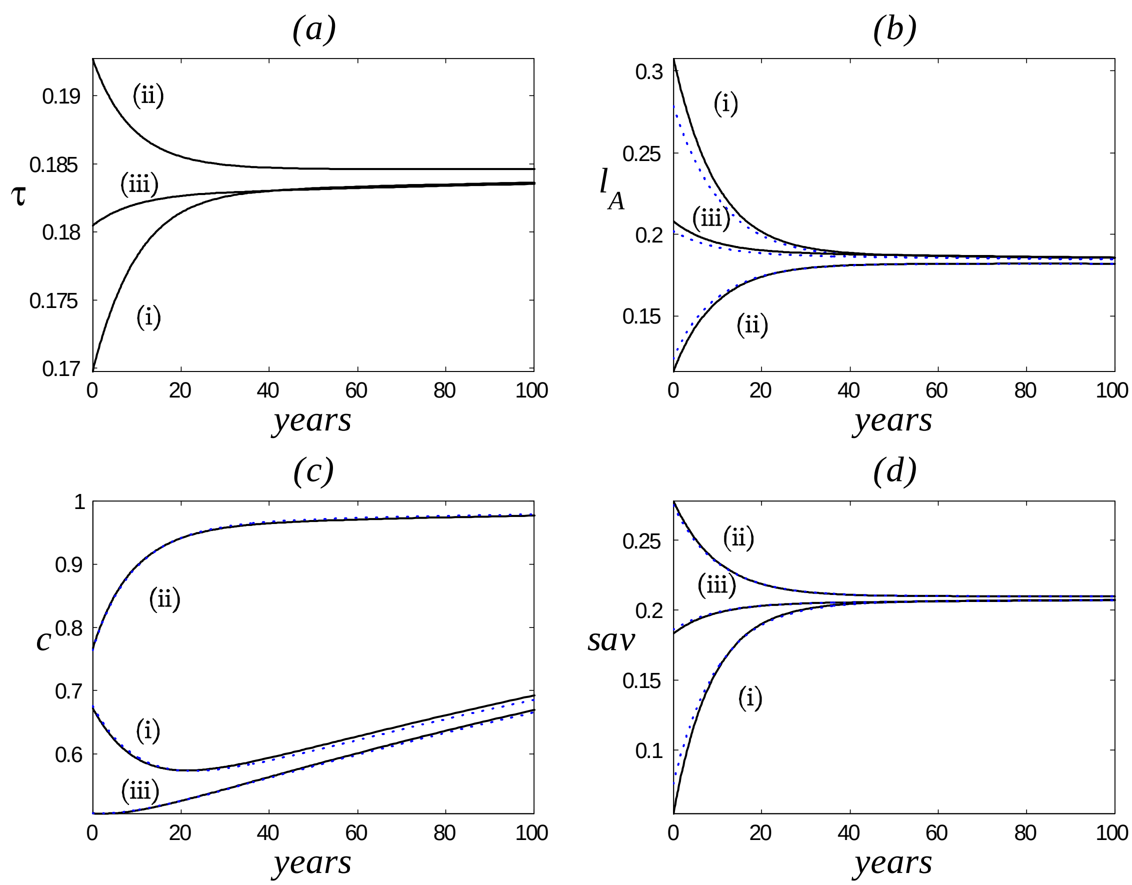

Figure 1: Transitional dynamics (solid lines: first-best solution, dashed lines: constrained optimal policy). Scenario (i): $\tilde{A}_0 = 0.5\tilde{A}^*$ and $\tilde{k}_0 = \tilde{k}^*$; Scenario (ii): $\tilde{A}_0 = \tilde{A}^*$ and $\tilde{k}_0 = 0.5\tilde{k}^*$; Scenario (iii): $\tilde{A}_0 = 0.5\tilde{A}^*$ and $\tilde{k}_0 = 0.5\tilde{k}^*$. The time path for c is normalized such that the final steady state equals unity.

A qualitatively similar evolution is induced for the fraction of R&D labor, l^A, as displayed by the solid line in panel (b) for *Scenario (i)*. In this scenario the social planner reallocates labor in favor of R&D to close the initial knowledge gap, as seen in panel (b). This implies a drop in final output production and, holding the saving rate constant, would also imply a low level of consumption. To achieve a comparably smooth consumption path, however, the social planner reduces the savings rate, sav,

[15] A gap expresses the proportional difference between the initial value of the state variable under consideration and its socially optimal steady state value.

in parallel to high R&D investment. Subsequently, *sav* rises quite quickly over time. Panel (c) shows the evolution of per capita consumption, c, relative to its optimal long run level. One recognizes that consumption decreases initially and then starts to increase.

In panels (b)-(d) of Fig. 1, we also compare the time paths of the allocation variables (l^A and sav) and consumption under optimal R&D subsidization (solid lines) with the ones under the steady state R&D subsidy (dashed lines). A time-invariant R&D subsidy rate which is set at its long run level may be referred to as "constrained optimal policy".[16] The constraint captures the potential difficulty to write tax laws which specify how policy rates change over time as well as the difficulty to know at which point along the transition path the economy is located. We see rather small differences in the respective time paths. It should also be observed that the difference between the first-best solution (solid line) and the solution under the constrained optimal steady state policy (dashed line) appears small.[17] This raises the question about the welfare gain which results from implementing the first-best rather than the constrained optimal steady state policy. This welfare gain is expressed in terms of the permanent percentage gain in consumption (see the appendix for details). It turns out that this consumption equivalent welfare gain is negligible. For *Scenario (i)*, the permanent increase in consumption is merely about 1.1 *per mill*. This result is in line with our finding that, although an unconstrained social planner would like to reduce the R&D subsidy rate as the knowledge gap narrows, quantitatively the initial dynamically optimal R&D subsidy rate is close to the optimal long run value, s_A^{opt}. Intuitively, the result is an implication of the standard consumption-smoothing motive which prevents the social planner to close the knowledge gap too fast at the cost of lower consumption early on.

[16] There is an alternative notion of a constrained optimum, where the welfare gain from a policy reform is maximized (taking into account the transition phase and assuming the economy is initially in steady state under the status quo policy). As shown in Grossmann, Steger and Trimborn (2010), a R&D policy which is optimal in this sense is very close to the optimal long run policy.

[17] Moreover, a detailed inspection shows that a dynamically optimal R&D policy implies to give up slightly more consumption in the beginning than under the constrained optimal policy. As the knowledge gap is closed faster, the consumption level in the first-best optimum overtakes the one under the constrained optimal R&D subsidy at a later stage of development.

Fig. 1 also contains the scenarios where only the capital stock is at 50 percent of its optimal long-run level (*Scenario (ii)*) and the scenario where both the capital and the knowledge stock are at 50 percent of their long run levels (*Scenario (iii)*). In *Scenario (ii)*, the dynamically optimal R&D subsidy rate and the fraction of labor devoted to R&D both increase over time, whereas the savings rate decreases over time. Hence, these variables qualitatively follow the opposite paths than in *Scenario (i)*. As a result, the consumption level relative to the long run optimal value is monotonically increasing over time. In *Scenario (iii)*, neither the allocation variables (l^A and sav) nor the first-best R&D subsidy change much over time and are close to their steady state values right from the start. This is unsurprising given the contrary evolution of these variables over time in *Scenario (i)* and *(ii)*. In *Scenario (iii)* the gaps in both stock variables (knowledge and capital) have to be closed simultaneously. Fig. 1 thus suggests that when the gap (i.e. the proportional difference between the initial value of the state variable and its socially optimal steady state) in knowledge is large relative to the gap in the capital stock, then in the beginning one should, relative to the steady state optimum, save little and invest much in R&D via a high R&D subsidy, whereas the opposite holds when the initial conditions constitute a relatively large gap in the capital stock.

Interestingly, the welfare gain from choosing the dynamically optimal R&D subsidy instead of the constrained optimal steady state policy is again very small in *Scenario (ii)* and *(iii)*. The potential gain in the per capita consumption level is 0.3 and 0.1 *per mill*, respectively, and thus even smaller than in *Scenario (i)* where there was a knowledge gap only.

4.2 The US Economy

We now consider the optimal R&D policy in the US. Obviously, as shown in section 3, the long run optimal subsidy rate $s_A^{opt} = 0.815$ (for $\theta = 0.5$) exceeds the current one, $s_A = 0.066$, dramatically. Consequently, we find that the current knowledge stock and capital stock are merely at 5.4 percent and 6.3 percent of the long run optimal levels, respectively.

Starting from these initial conditions, a policy reform which implements the dynamically optimal R&D subsidy is characterized as follows, shown in Fig. 2 (solid lines). The R&D subsidy rate, $s_A = 1 - \tau$, should initially jump upwards significantly (to about 83.6 percent) and then slightly decrease over time to s_A^{opt} (panel (a)). Again, the optimal change in s_A over time is small, despite the fact that we start far away from the long run optimum. Also the fraction of labor devoted to R&D (initially at about 4.2 percent under current R&D subsidization in the US), l^A, should jump upwards dramatically and then decrease considerably over time towards 18.2 percent (panel (b)). Thus, in steady state, the US should devote about four times as much labor to R&D than at present. The importance of knowledge accumulation for the growth process in the model is so high, that the initial savings rate should be slightly negative in the beginning under the optimal resource allocation (panel (d)). Overall, a steep increase in per capita consumption is induced (panel (c)). Notably, there is a long transition towards the new steady state with a half life of more than 200 years.

With respect to the welfare gain when the dynamically optimal policy rather than the constrained steady state optimal R&D subsidy is implemented, the insights of the previous subsection are confirmed also for the US. Despite the dramatic R&D underinvestment in the US, which is suggested by the model, the welfare difference is equivalent to a permanent change in per capita consumption of merely 0.4 *per mill*. Notice that the time paths for per capita consumption and the savings rate are strikingly similar in the first-best and the constrained-optimum case. This observation is in line with the negligible welfare gain reported above. The fraction of R&D labor under the first-best policy should be somewhat higher than in the constrained optimum, especially in the beginning, but the difference in the two paths is rather small.

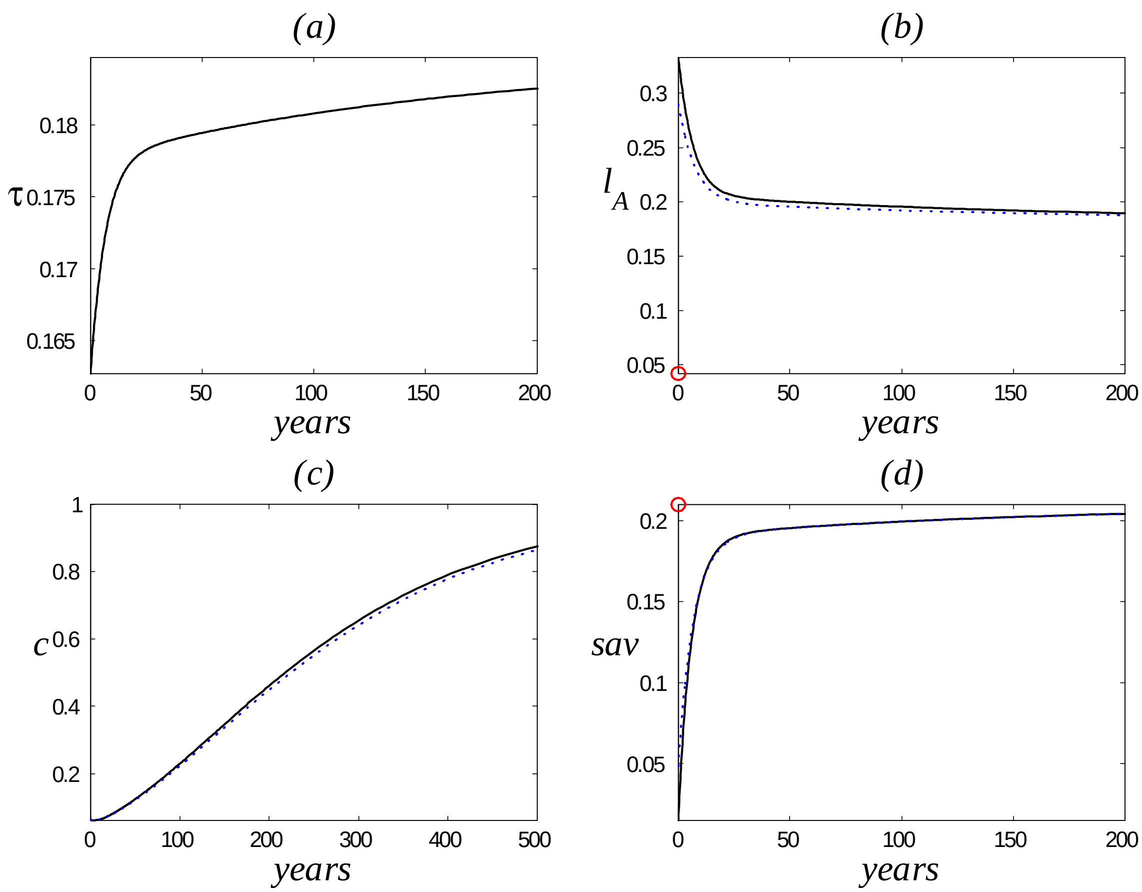

Figure 2: Transitional dynamics for the US economy (solid lines: first-best solution, dashed lines: constrained optimal policy). The time path for c is normalized such that the final steady state equals unity. In panel (b) and (d) the circles indicate the initial steady state values under the status quo policies in the US.

5 Conclusion

We characterized the time path of first-best (i.e. time-varying) R&D subsidization in a semi-endogenous growth model and compared the allocative impact to the one arising from implementing the optimal steady state (i.e. time-invariant) policy from the start. We find that the differences in the time paths of per capita consumption and the allocation variables which result from the comparison between the first-best and the constrained optimal steady state policy are rather small. Our results suggest that the optimal R&D subsidization should change little over time, even when the economy starts far away from its socially optimal steady state. As a result, the welfare loss from a possible political constraint to use time-invariant policies is negligible. This insight is striking, and at a first glance surprising, given the slow speed of convergence to the steady state in R&D-based endogenous growth models. The economic reasoning for this observation lies in the standard consumption-smoothing motive which prevents the social planner to sacrifice a high level of consumption in the early transition phase.

Our analysis also suggests that the US economy is currently far away from its socially optimal steady state. Starting from the steady state under current R&D subsidization, the R&D subsidy should significantly jump upwards and then slightly decrease over time. Again, the welfare loss from implementing the long run optimal policy from the start compared to the dynamically optimal one is almost zero.

Our results may have important implications for further research on optimal policy design. In line with previous research which employs semi-endogenous growth models, we identify a huge R&D underinvestment gap. This suggests to dramatically raise R&D subsidies. Such an analysis is of major importance in view of the huge potential welfare gains from a reform of growth policy. Future research should investigate optimal growth policy in alternative models of endogenous technical change in order to obtain a robust picture on the optimal policy programme. Moreover, although our paper suggests that policy makers may focus on policy rates which are optimal in the long run, it is necessary to substantiate also this conclusion in alternative frameworks.

Appendix

Proof of Proposition 1: The current-value Hamiltonian which corresponds to the household optimization problem (12) is given by

$$\mathcal{H} = \frac{c^{1-\sigma} - 1}{1-\sigma} + \lambda \left[(r-n)a + w - c - T \right], \tag{35}$$

where λ is the co-state-variable associated with constraint (4). Necessary optimality conditions are $\partial\mathcal{H}/\partial c = 0$, $\dot{\lambda} = (\rho - n)\lambda - \partial\mathcal{H}/\partial a$, and the corresponding transversality condition. Thus,

$$\lambda = c^{-\sigma}, \text{ i.e., } \frac{\dot{\lambda}}{\lambda} = -\sigma \frac{\dot{c}}{c}, \tag{36}$$

$$\frac{\dot{\lambda}}{\lambda} = \rho - r, \tag{37}$$

$$\lim_{t \to \infty} \lambda_t e^{-(\rho - n)t} a_t = 0. \tag{38}$$

Combining (36) with (37), we obtain the standard Euler equation

$$\frac{\dot{c}}{c} = \frac{r - \rho}{\sigma}. \tag{39}$$

Using $\dot{N}/N = n$ together with definitions $\tilde{c} = c/N^{\frac{1-\theta}{1-\phi}}$ and $g = \frac{(1-\theta)n}{1-\phi}$ confirms (16). In a similar fashion, (13) can be derived from (2).

Moreover, it can be shown (available upon request) that the capital stock per capita ($k = K/N$) evolves according to

$$\dot{k} = y - (\delta + n)k - c. \tag{40}$$

Using $x_i = K/A$ for all i in production function (1) gives us for per capita output ($y = Y/N$) the expression

$$y = k^{\alpha}(Al^Y)^{1-\alpha}, \tag{41}$$

where $l^Y = L^Y/N$ has been used. Combining (40) and (41) as well as using $l^Y = 1 - l^A$, $\dot{N}/N = n$, $\tilde{k} = k/N^{\frac{1-\theta}{1-\phi}}$ and $g = \frac{(1-\theta)n}{1-\phi}$ confirms (15). From (9), we find in addition that

$$r = \frac{\alpha}{\kappa(1 - s_K)} \frac{y}{k} - \delta. \tag{42}$$

Substituting (41) into (42) and using that $A/k = \tilde{A}/\tilde{k}$ then confirms (17).

Next, substitute (6) and (7) into (5) to obtain the following expression for the profit of each intermediate goods producer i:

$$\pi_i = \pi = (\kappa - 1) \left(\frac{\alpha}{\kappa}\right)^{\frac{1}{1-\alpha}} \left[(1 - s_K)(r + \delta_K)\right]^{-\frac{\alpha}{1-\alpha}} L^Y. \tag{43}$$

Now recall definition $p^A = P^A/N$ as well as $L^Y/N = 1 - l^A$ to rewrite (10) such as to confirm (14).

Since final goods producers take the wage rate as given, we have $w = (1-\alpha)Y/L^Y$. Thus,

$$w = (1-\alpha)A^{1-\alpha}\left(\frac{k}{l^Y}\right)^{\alpha}, \tag{44}$$

according to (41) and the fact that $Y/L^Y = y/l^Y$. Moreover, due to free entry in the R&D sector, in equilibrium, $\Pi = 0$ holds, i.e., $p^A \nu A^{\phi} N^{1-\theta}(l^A)^{-\theta} = \tau w$, according to (11). Inserting (44) implies

$$(1-\alpha)A^{1-\alpha}\left(\frac{k}{l^Y}\right)^{\alpha} = \frac{p^A \nu A^{\phi} N^{1-\theta}(l^A)^{-\theta}}{\tau}. \tag{45}$$

Using the definitions of $\tilde{k}$ and $\tilde{A}$ (thus, $A^{\phi-1}N^{1-\theta} = \tilde{A}^{\phi-1}$), we then obtain

$$(1-\alpha)\left(\frac{\tilde{k}}{\tilde{A}l^Y}\right)^{\alpha} = \frac{p^A \nu \tilde{A}^{\phi-1}(l^A)^{-\theta}}{\tau}. \tag{46}$$

Substituting $l^Y = 1 - l^A$ into (46) confirms (18). This concludes the proof of part (i).

To prove part (ii), first, set $\dot{\tilde{c}} = 0$ (implying that c grows at rate g in steady state) in (16) to confirm (19). Next, set $\dot{p}^A = 0$ in (14) to find

$$p^A = \frac{(\kappa - 1)\,(\alpha/\kappa)^{\frac{1}{1-\alpha}}\,(1 - l^A)}{(r-n)[(1-s_K)(r+\delta)]^{\frac{\alpha}{1-\alpha}}}. \tag{47}$$

Moreover, substituting (8) into $w = (1-\alpha)Y/L^Y$ implies

$$\frac{w}{A} = (1-\alpha)\left(\frac{\alpha}{\kappa(1-s_K)(r+\delta)}\right)^{\frac{\alpha}{1-\alpha}}. \tag{48}$$

It will become apparent, that in steady state knowledge stock A grows with rate g. From condition $\Pi = 0$, i.e., $p^A\dot{A} = \tau w l^A$, we find that $\dot{A}/A = g$ implies $p^A g = l^A \tau w/A$. Substituting both (47) and (48) into this expression and using (19) leads to (20). According to (13), $\dot{\tilde{A}} = 0$ implies (21). (22) immediately follows from (47). (23) follows from rearranging (17). To confirm (24), set $\dot{\tilde{k}} = 0$ in (15).

The savings rate (and investment share) is given by

$$sav = (\dot{K} + \delta K)/Y = (\dot{K}/K + \delta)K/Y. \tag{49}$$

As $\dot{\tilde{k}} = 0$, we have $\dot{K}/K = n + g$ in steady state. Substituting this and (9) into (49) and using (19) confirms (25).

Finally, using $\dot{\lambda}/\lambda = -\sigma g$ from (36) and the fact that $\dot{c}/c = g$, we find that if a grows with rate g in the long run, the transversality condition (38) holds under assumption (A1). Rewriting (4) to

$$\frac{\dot{a}}{a} = r - n + \frac{w}{a} - \frac{c}{a} - \frac{T}{a} \tag{50}$$

reveals that $\dot{a}/a = g$ indeed holds in steady state, if both the lump-sum tax per household (T) and wage rate w grow at rate g (recall that c grows at rate g and r is time-invariant in steady state). That w grows at the same rate as A (namely at rate g) in steady state can be seen from (48), since r is time-invariant in the long run. Moreover, we have

$$T = s_K(r + \delta)k + s_A w l^A, \tag{51}$$

which also confirms that, in steady state, $\dot{T}/T = g$. This concludes the proof. ■

Proof of Proposition 2: The current-value Hamiltonian which corresponds to the social planning problem (29) is given by

$$\mathcal{H} = \frac{c^{1-\sigma} - 1}{1 - \sigma} + \lambda_k(\overbrace{\underbrace{k^\alpha (A l^Y)^{1-\alpha}}_{=y} - (\delta + n)k - c}^{\dot{k}}) + \lambda_A \overbrace{\nu A^\phi N^{1-\theta} (\underbrace{1 - l^Y}_{=l^A})^{1-\theta}}^{=\dot{A}}, \tag{52}$$

where λ_k and λ_A are co-state variables associated with constraints (27) and (28), respectively. Necessary optimality conditions are $\partial\mathcal{H}/\partial c = \partial\mathcal{H}/\partial l^Y = 0$ (control variables), $\dot{\lambda}_z = (\rho - n)\lambda_z - \partial\mathcal{H}/\partial z$ for $z \in \{k, A\}$ (state variables), and the corresponding transversality conditions. Thus,

$$\lambda_k = c^{-\sigma}, \text{ i.e., } \frac{\dot{\lambda}_k}{\lambda_k} = -\sigma\frac{\dot{c}}{c}, \tag{53}$$

$$(1-\alpha)A^{1-\alpha}\left(\frac{k}{l^Y}\right)^{\alpha} = \frac{\lambda_A}{\lambda_k}(1-\theta)\underbrace{\nu A^{\phi}N^{1-\theta}(l^A)^{-\theta}}_{=\dot{A}/l^A}, \tag{54}$$

$$\frac{\dot{\lambda}_k}{\lambda_k} = \rho - \alpha\left(\frac{Al^Y}{k}\right)^{1-\alpha} + \delta, \tag{55}$$

$$\frac{\dot{\lambda}_A}{\lambda_A} = \rho - n - \frac{\lambda_k}{\lambda_A}(1-\alpha)\underbrace{\left(\frac{k}{A}\right)^{\alpha}(l^Y)^{1-\alpha}}_{=y/A} - \phi\frac{\dot{A}}{A} \tag{56}$$

$$\lim_{t\to\infty}\lambda_{z,t}e^{-(\rho-n)t}z_t = 0,\ \ z\in\{k,A\}. \tag{57}$$

($\lambda_{z,t}$ denotes the co-state variable associated with state variable z at time t.)

To find the optimal capital cost subsidy, first note that from (36) and (53) that we must have $\lambda = \lambda_k$ in social optimum; thus, according to (37) and (55),

$$r + \delta = \alpha\left(\frac{Al^Y}{k}\right)^{1-\alpha}. \tag{58}$$

Comparing (58) with (17), by using $l^Y = 1 - l^A$ and the definitions of $\tilde{c}$, $\tilde{k}$, $\tilde{A}$, we find that $\kappa(1-s_K) = 1$ must hold in social optimum at all times, which is equivalent to (30).

Next, note from (45) and (54) that a R&D subsidy which implements the social optimum must fulfill

$$p^A = (1-\theta)\tau\frac{\lambda_A}{\lambda_k}, \text{ i.e.,} \tag{59}$$

$$\frac{\dot{\tau}}{\tau} = \frac{\dot{p}^A}{p^A} - \left(\frac{\dot{\lambda}_A}{\lambda_A} - \frac{\dot{\lambda}_k}{\lambda_k}\right). \tag{60}$$

Moreover, substituting optimality conditions $1 - s_K = 1/\kappa$ and (59) into (14), we find

$$\frac{\dot{p}^A}{p^A} = r - n - \frac{(1 - 1/\kappa)\alpha(1 - l^A)}{(Al^Y/k)^\alpha \tau(1 - \theta)} \frac{\lambda_k}{\lambda_A}. \tag{61}$$

Rewriting (54) to

$$\frac{\lambda_k}{\lambda_A} = \frac{(1 - \theta)\left(Al^Y/k\right)^\alpha}{(1 - \alpha)l^A} \frac{\dot{A}}{A} \tag{62}$$

and substituting into (61) leads to

$$\frac{\dot{p}^A}{p^A} = r - n - \frac{(1 - 1/\kappa)(1 - l^A)}{\tau(1/\alpha - 1)l^A} \frac{\dot{A}}{A}. \tag{63}$$

Moreover, combining (55) and (56) by subtracting both sides of the equations from each other and substituting (62), we have

$$\frac{\dot{\lambda}_A}{\lambda_A} - \frac{\dot{\lambda}_k}{\lambda_k} = \alpha \left(\frac{Al^Y}{k}\right)^{1-\alpha} - \delta - n - (1 - \theta)\frac{l^Y}{l^A} \frac{\dot{A}}{A} - \phi\frac{\dot{A}}{A}. \tag{64}$$

Substituting (63) and (64) into (60) and making use of (58) and $l^Y = 1 - l^A$ then leads to

$$\frac{\dot{\tau}}{\tau} = \left[\left(1 - \theta - \frac{1}{\tau}\frac{1 - 1/\kappa}{1/\alpha - 1}\right) \frac{1 - l^A}{l^A} + \phi\right] \frac{\dot{A}}{A}. \tag{65}$$

From (28) and the definition of $\tilde{A}$ we find

$$\frac{\dot{A}}{A} = \nu \tilde{A}^{\phi-1} \left(l^A\right)^{1-\theta}. \tag{66}$$

Substituting (66) into (65) confirms (31).

It remains to confirm terminal condition (32), i.e., the optimal long run R&D policy. We seek for a steady state where A, k and c all grow at rate g and $\dot{l}^Y = \dot{l}^A = \dot{\tau} = 0$. Setting $\dot{\tau} = 0$ in (31) implies that, in long run social optimum,

$$\left(1 - \theta - \frac{1}{\tau}\frac{1 - 1/\kappa}{1/\alpha - 1}\right) \left(\frac{1}{l^A} - 1\right) + \phi = 0. \tag{67}$$

To infer (32) from (67) we need to find the steady state value for the fraction of R&D

labor, l^A. From (53), $\dot{\lambda}_k/\lambda_k = -\sigma g$; combining with (55) implies

$$\alpha \left(\frac{Al^Y}{k}\right)^{1-\alpha} - \delta = \sigma g + \rho. \tag{68}$$

From (62) and $\dot{A}/A = g$, together with the property that Al^Y/k are l^A are constant in the long run, we find $\dot{\lambda}_k/\lambda_k = \dot{\lambda}_A/\lambda_A$. Using $\dot{\lambda}_k/\lambda_k = \dot{\lambda}_A/\lambda_A$, $\dot{A}/A = g$, (68) and $l^Y = 1 - l^A$ in (64) we can solve for l^A. Doing so and using $(1-\phi)g = (1-\theta)\,n$ from the definition of g in (A1) implies

$$l^A = \frac{1}{\frac{(\sigma-1)g+\rho-\theta n}{(1-\theta)g} + 1}. \tag{69}$$

Substituting (69) into (67) and using $(1-\phi)g = (1-\theta)\,n$ confirms (32).

From (53) and $\dot{c}/c = g$, we have $\dot{\lambda}_k/\lambda_k = -\sigma g$. Using that $\dot{\lambda}_k/\lambda_k = \dot{\lambda}_A/\lambda_A = -\sigma g$ for $t \to \infty$ and $\dot{k}/k = \dot{A}/A = g$, transversality condition (57) is fulfilled under assumption (A1) for both state variables, k and A.

So far we have shown that the policy mix in Proposition 2 is necessary for a first-best optimum. To show that it is also sufficient, we need to prove that under this policy mix, the market equilibrium is the same as the social planning optimum.

From (27) we obtain

$$\frac{\dot{k}}{k} = \left(\frac{Al^Y}{k}\right)^{1-\alpha} - \delta - n - \frac{\tilde{c}}{\tilde{k}}, \tag{70}$$

which coincides with (15) by using $l^Y = 1 - l^A$ and the definitions of $\tilde{c}$, $\tilde{k}$, $\tilde{A}$. Similarly, (66) coincides with (13) and combining (53) with (55) leads to (16) when using (58). Finally, combining (59) with (62) and using (66) leads to (18). This concludes the proof. ■

Measuring Welfare Differences: To quantify the welfare difference between implementing the dynamically optimal policy mix and the optimal steady state policy mix, we express it as the percentage change of a hypothetical steady state per capita consumption stream when implementing the dynamic optimal policy rather than the steady state optimal policy. Formally, recall that in the long run adjusted per capita consumption, $\tilde{c} = c/N^{\frac{1-\theta}{1-\phi}}$, is stationary and per capita consumption c grows with rate $g = \frac{(1-\theta)n}{1-\phi}$. Thus, in the long run, $c = \tilde{c}^* e^{gt}$. Moreover, denote the steady state value of $\tilde{c}$ when using the (suboptimal) steady state optimal policy by $\tilde{c}^*_{sub}$, the change in life-time utility when switching to the dynamically optimal policy by ΔU, and the associated hypothetical increase in adjusted steady per capita consumption by $\Delta\tilde{c}$. Then we have:

$$\Delta U = \int_0^\infty \frac{((\tilde{c}^*_{sub} + \Delta\tilde{c})e^{gt})^{1-\sigma} - 1}{1-\sigma} e^{-(\rho-n)t} dt - \int_0^\infty \frac{(\tilde{c}^*_{sub} e^{gt})^{1-\sigma} - 1}{1-\sigma} e^{-(\rho-n)t} dt \qquad (71)$$

which we can solve to find

$$\frac{\Delta\tilde{c}}{\tilde{c}^*_{sub}} = \frac{(\tilde{c}^{*1-\sigma}_{sub} + \Delta U(\sigma-1)(g(1-\sigma)+n-\rho))^{\frac{1}{1-\sigma}}}{\tilde{c}^*_{sub}} - 1, \qquad (72)$$

which is the measure we use to quantify welfare differences.

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Supplementary Material (not intended for publication)

Here we show that the goods market clearing condition (40) holds in equilibrium.

- Given the linear-homogenous production function (1) and due to perfect competition we know that profits are zero in the final goods sector. Thus,

$$Y = \int_0^A p_i x_i di + wL^Y. \tag{73}$$

Using (5), i.e., $\pi_i = p_i x_i - (1-s_K)(r+\delta)x_i$ together with $K = \int_0^A x_i di$ and $\pi_i = \pi$ for all i (firms are symmetric) we obtain

$$\int_0^A p_i x_i di = \pi A + (1 - s_K)(r + \delta)K \tag{74}$$

and thus

$$\pi A = Y - (1 - s_K)(r + \delta)K - wL^Y. \tag{75}$$

- Under a balanced government budget, the aggregate lump-sum tax faced by households is

$$NT = s_K(r+\delta)K + s_A w L^A. \tag{76}$$

- According to (11) and $\Pi = 0$, we have (recall $\tau = 1 - s_A$):

$$P^A \dot{A} = (1 - s_A) w L^A. \tag{77}$$

- Total assets of households are given by $Na = K + P^A A$. Differentiating with respect to time yields:

$$\dot{N}a + N\dot{a} = \dot{K} + \dot{P}^A A + P^A \dot{A}. \tag{78}$$

  Using $\dot{N} = nN$ we can write

$$N\dot{a} = \dot{K} + \dot{P}^A A + P^A \dot{A} - nNa. \tag{79}$$

  Combining (79) with asset accumulation equation (4) we find

$$\dot{K} + \dot{P}^A A + P^A \dot{A} = rNa + wN - Nc - NT.$$

  Using $Na = K + P^A A$ and $\pi = rP^A - \dot{P}^A$ from (10) we obtain

$$\dot{K} = rK + wN - Nc + NT + \pi A - P^A \dot{A}.$$

  Substituting (75), (76), (77) and using $N = L^Y + L^A$ implies

$$\dot{K} = Y - Nc - \delta K.$$

  Using the definition of per capita measures k and y confirms (40). *Q.E.D.*