Long memory and changing persistence*

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Abstract

We study the empirical behaviour of semi-parametric log-periodogram estimation for long memory models when the true process exhibits a change in persistence. Simulation results confirm theoretical arguments which suggest that evidence for long memory is likely to be found. A recently proposed test by Sibbertsen and Kruse (2009) is shown to exhibit noticeable power to discriminate between long memory and a structural change in autoregressive parameters.

Key Words: Long memory; changing persistence; structural break; semi-parametric estimation.
JEL codes: C12, C22.

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1 Introduction

Long memory models receive considerable attention in the empirical literature on economics and finance. Their successful applications justify the large body of literature dealing with spurious detections of long memory. Diebold and Inoue (2001) among others demonstrate that evidence for long memory can be falsely ascribed to structural break models with short memory. Among these models are ones with occasional mean shifts and other non-linear models like the sign model (see Granger and Teräsvirta 1999).

In this article we consider a simple changing persistence model which has not been analyzed, at least to the best of our knowledge, in the related literature so far. This autoregressive time series model describes a switch from stationarity ($I(0)$) to non-stationarity ($I(1)$) over time, or vice versa. In addition, we study the case of stable shifts. They are defined as a structural change in the autoregressive parameters which does not constitute a change in persistence as the process is $I(0)$ throughout the entire sample. In a related article, Leybourne and Taylor (2004) provide a comprehensive study on the behaviour of some changing persistence tests under stable shifts. They consider processes with an integer degree of integration instead of fractional integration.

Our simulation results show that the estimated memory parameter is located in the region of non-stationarity, i.e. $d \in (0.5, 1)$. Theoretical explanations are provided and a bias formula is derived in the case of stable shifts. The results of this analysis are empirically relevant and important given the wide application of estimators for long memory to time series where changes in persistence are likely to be present. A leading example are inflation rates which are modeled by either (i) changing persistence (Halunga et al. 2008 and Noriega and Ramos-Francia 2009) or (ii) long memory models (see Hassler and Wolters 1995, Hsu 2005 and Lee 2005). In order to discriminate between long memory and changing persistence, or stable shifts, we suggest to use a CUSUM of squares-based test proposed by Sibbertsen and Kruse (2009). Further simulation results show that it has remarkable power to detect spuriously generated long memory due to structural changes in the autoregressive parameters.

2 Autoregressive changing persistence model

We consider a first-order autoregressive model that has a change in persistence at the breakpoint $T_B = [\tau T]$ with $\tau \in (0, 1)$:

$$y_t = \alpha_1 y_{t-1} + \varepsilon_t, \qquad \text{for } t = 1, ..., T_B \tag{1}$$

$$y_t = \alpha_2 y_{t-1} + \varepsilon_t, \qquad \text{for } t = T_B + 1, ..., T\ . \tag{2}$$

The innovation process ε_t is assumed to be stationary, short memory and linear. In this model, persistence is determined through the autoregressive parameters $|\alpha_1| \leq 1$ and $|\alpha_2| \leq 1$. As long as $\alpha_1 \neq \alpha_2$, a structural change occurs at time T_B. The special case where $|\alpha_1| = 1$ and $|\alpha_2| < 1$ hold, is called a decline in persistence because the AR model is $I(1)$ during time $t = 1, \ldots, T_B$ and $I(0)$ afterwards. Analogously, an increase in persistence takes place if $|\alpha_1| < 1$ and $|\alpha_2| = 1$ hold, i.e. the process switches from stationarity to a unit root process. A stable shift is defined as a structural change where both autoregressive parameters satisfy the stationarity condition, i.e. $|\alpha_1| < 1$ and $|\alpha_2| < 1$ hold.

3 Semi-parametric GPH-estimator and its bias

The widely applied $I(d)$ model with long memory is given by

$$(1 - L)^d y_t = \varepsilon_t \tag{3}$$

where ε_t has zero mean and is i.i.d. with variance σ^2. A popular estimator for d is the one proposed by Geweke and Porter-Hudak (1983). It is based on the spectral density of a long-memory model which is given by

$$f(\lambda) = |1 - \exp(-i\lambda)|^{-2d} f^*(\lambda), \qquad -\pi \leq \lambda \leq \pi. \tag{4}$$

Here, the first term determines the long-range behaviour of the process and the remaining spectral density $f^*(\lambda)$ determines the short-run behaviour of the process, which can be autoregressive for instance. The GPH-estimator neglects the short-run behaviour and focusses on the long-run part of the spectral density. This may introduce a serious bias in the estimation (see Hurvich et al., 1998, or Davidson and Sibbertsen, 2009, for a discussion).

More specifically, the GPH-estimator is based on the regression

$$\log(I_j) = \log c_f - 2dX_j + \log \xi_j \ , \quad j = 1, 2, \dots, m \tag{5}$$

where $I_j = \frac{1}{2\pi n} \left| \sum_{t=0}^{T-1} y_t \exp\left(\frac{i2\pi jt}{T}\right) \right|^2$ is the j-th periodogram ordinate, X_j denotes the j-th Fourier frequency and ξ_j are assumed to be i.i.d. with $-E(\log \xi_j) = 0.577216...$ which is known as the Euler constant. The GPH-estimator for d equals the $-1/2$ times the OLS estimator of the slope parameter in the log-peridogram regression (6). A common choice for the number of periodogram ordinates is the MSE-optimal rate of $m = T^{4/5}$ which is applied in the Monte Carlo study below.

We analyze the bias of the GPH-estimator when the true model is the one given in equations (1) and (2) with a stable shift and hence, short memory, i.e. $d = 0$. The model can be interpreted as a time-varying autoregressive process. Moulines et al. (2006) introduce a time-dependent local spectral density which is given by

$$f^*(\lambda) \equiv f^*(\lambda, t) = f_1^*(\lambda) 1(t \leq [\tau T]) + f_2^*(\lambda) 1(t > [\tau T]), \qquad -\pi \leq \lambda \leq \pi, \tag{6}$$

where $f_1^*(\lambda)$ denotes the spectral density of process (1) and $f_2^*(\lambda)$ this of (2); $1(A)$ is the indicator function of the set A. We assume the following condition, see Hurvich et al. (1998):

Condition 1: $m \to \infty$, $T \to \infty$, with $m/T \to 0$ and $(m \log m)/T \to 0$.

Condition 2 in Hurvich et al. (1998) is automatically fulfilled here as we deal with local autoregressive processes. By similar arguments as in Hurvich et al. (1998), we derive the bias expression for the GPH-estimator. Let us denote $a_j = X_j - \bar{X}$ and $S_{XX} = \sum_{k=1}^{m} a_k^2$. The bias of $\widehat{d}$ is given by

$$E(\widehat{d} - d) = -\frac{1}{2S_{XX}} \sum_{j=1}^{m} a_j \log f_i^*(\omega_j) - \frac{1}{2S_{XX}} \sum_{j=1}^{m} a_j E(\epsilon_j). \tag{7}$$

Now, for $1 \leq j \leq m$ there exists a ξ_j with $0 \leq \xi_j \leq \omega_j$ such that

$$\log f_i^*(\omega_j) = \log f^*(0) + \frac{\omega_j}{2} \frac{f_i^{*''}(0)}{f_i^*(0)} + \frac{\omega_j^3}{6} g(\xi_j) \tag{8}$$

with

$$g(\omega) = \frac{f_i^{*'''}(\omega)}{f_i^*(\omega)} - \frac{3 f_i^{*'}(\omega) f_i^{*''}(\omega)}{[f_i^*(\omega)]^2} + \frac{2[f_i^{*'}(\omega)]^3}{[f_i^*(\omega)]^3} \tag{9}$$

as in Hurvich et al. (1998). We obtain

$$E(\widehat{d} - d) = -\frac{2\pi}{9} \frac{f^{*''}(0)}{f^*(0)} \frac{m^2}{T^2} + o\left(\frac{m^2}{T^2}\right). \tag{10}$$

In our case it can furthermore be seen that

$$\frac{f_i^{*''}(0)}{f_i^*(0)} = \frac{-2\alpha_i}{(1-\alpha_i)^2}, \qquad i = 1, 2. \tag{11}$$

This gives our bias expression to be

$$E(\widehat{d} - d) = \frac{2\pi}{9} \left[\frac{2\alpha_1}{(1-\alpha_1)^2} 1(t \leq [\tau T]) + \frac{2\alpha_2}{(1-\alpha_2)^2} 1(t > [\tau T]) \right] \frac{m^2}{T^2} + o\left(\frac{m^2}{T^2}\right). \tag{12}$$

4 Monte Carlo study

Data is generated according to the AR model in equations (1) and (2). The sample sizes $T = 250, 500$ and 750 are usual in economics for daily, weekly, monthly and quarterly recorded data. The breakpoint is located in the first half of the sample ($\tau = 0.3$), in the middle ($\tau = 0.5$) and the second half ($\tau = 0.7$). The autoregressive parameter $\alpha_{1,2}$ takes the value 0.5, while $\alpha_{2,1} \in \Theta = \{0.9, 0.905, \ldots, 0.995, 1.0\}$.

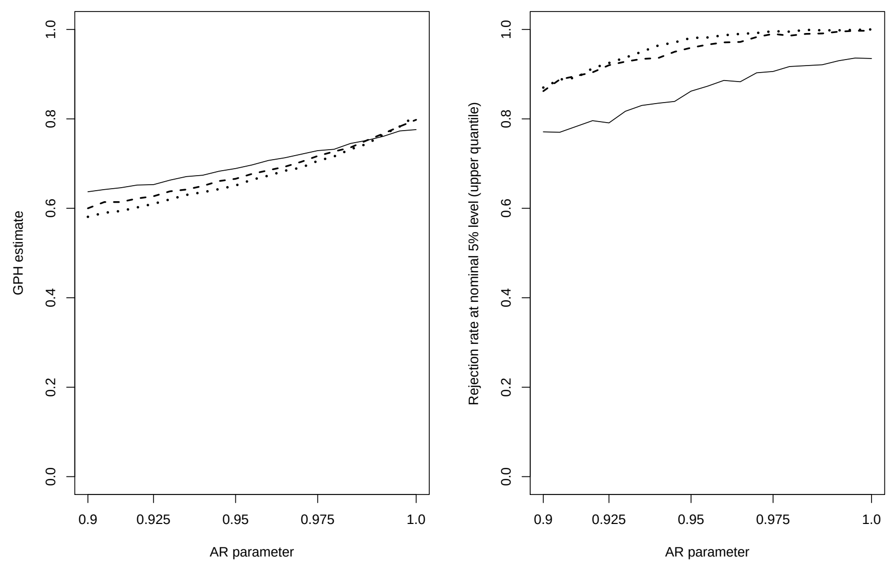

Figure 1: $\tau = 0.3$, solid line: $T = 250$, dashed line: $T = 500$, dotted line: $T = 750$

Hence, we consider a range of stable shifts and a change in persistence in the limit. The innovations ε_t are drawn from a standard normal distribution. The number of Monte Carlo repetitions is 5000 for each single experiment. We report the Monte Carlo mean of the GPH-estimator for d. As there the true value for d is zero in the case of stable shifts, the simulated bias of the GPH-estimator simply equals its Monte Carlo average. In the case of a change in persistence from $I(0)$ to $I(1)$ (or vice versa), no bias statistic can be computed. Therefore, we focus on the pure estimates of d in this case. Results for the GPH-estimator are reported in the left part of Figures 1, 2 and 3 for the breakpoints $\tau = \{0.3, 0.5, 0.7\}$, respectively. We only report the results for the case of decreasing persistence as the results for increasing persistence are symmetric and do not convey any further insights.[1]

The results suggest that spurious evidence for long memory can easily be found. Irrespective of the particular value of $\alpha_2 \in \Theta$, the Monte Carlo averages of the GPH-estimates are located in the non-stationary region $(0.5, 1)$. Thus, stable shifts and

[1]Full results are available from the authors upon request.

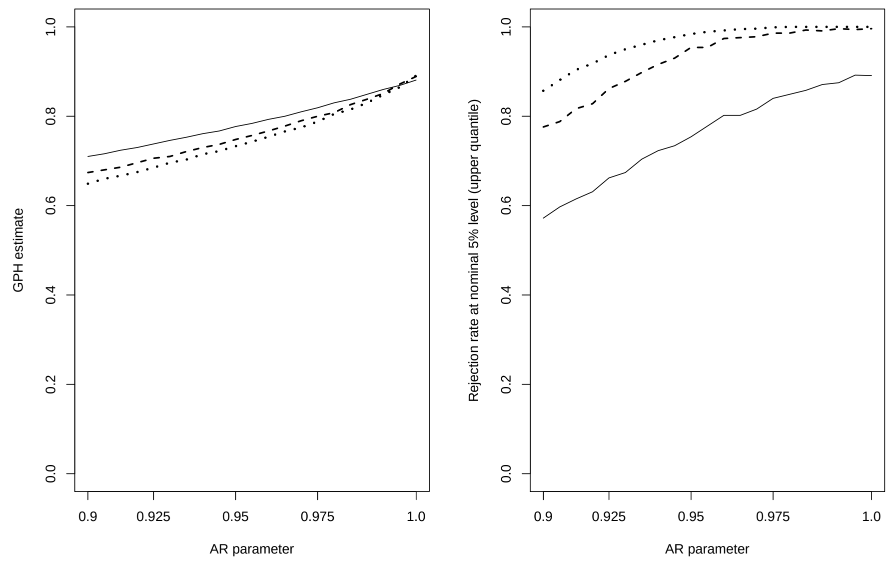

Figure 2: $\tau = 0.5$, solid line: $T = 250$, dashed line: $T = 500$, dotted line: $T = 750$

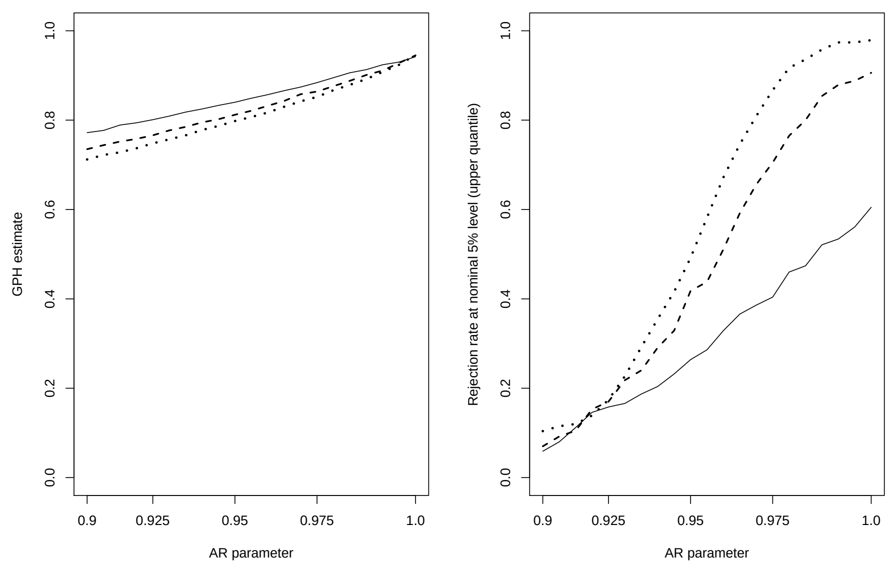

Figure 3: $\tau = 0.7$, solid line: $T = 250$, dashed line: $T = 500$, dotted line: $T = 750$

changes in persistence are easily confused with long memory. We observe three general tendencies in the results which are confirmed by our bias formula (13): (i) the later the break from $\alpha_1 \in \Theta$ to $\alpha_2 = 0.5$ occurs, the larger is the bias, (ii) a larger value of α_1 leads to a larger bias, but (iii) an increasing sample size T leads to a smaller, but still remarkable bias. Moreover, the results for the limiting case of a change in persistence appear to be very similar to the ones for the local-to-unity cases although the bias formula (13) does not apply as the spectral densities do not exist for non-stationary processes.

Given the fact that long memory may be easily confused with stable shifts and changes in persistence, it is important to discriminate between these two types of processes. To this end, we study the behaviour of a CUSUM of squares-based test suggested by Sibbertsen and Kruse (2009). This testing procedure is originally designed to test the null hypothesis of long memory against a change in the d parameter. The following simulations shed light on the tests' ability to distinguish long memory models (under H_0) and stable shifts or changes in persistence (under H_1). The test is carried out by computing the statistic

$$R = \frac{\inf_{\tau \in \Lambda} K^f(\tau)}{\inf_{\tau \in \Lambda} K^r(\tau)}, \tag{13}$$

where $K^f(\tau)$ and $K^r(\tau)$ are CUSUM of squares-based statistics. In more detail, $K^f(\tau)$ and $K^r(\tau)$ are given by

$$K^f(\tau) = \frac{1}{[\tau T]^2} \sum_{t=1}^{[\tau T]} \hat{v}_{t,\tau}^2$$

and

$$K^r(\tau) = \frac{1}{(T - [\tau T])^2} \sum_{t=1}^{T-[\tau T]} \tilde{v}_{t,\tau}^2.$$

Here, $\hat{v}_{t,\tau}$ are the residuals from the OLS regression of y_t on a constant based on the observations up to $[\tau T]$. Similarly $\tilde{v}_{t,\tau}$ is defined for the reversed time series. More details on the asymptotic distribution of R and critical values can be found in Sibbertsen and Kruse (2009). The simulation results therein show that the test is correctly sized for the sample sizes considered here. The right part of Figures 1, 2 and 3 show the empirical power of the CUSUM of squares-based test against stable shifts and changes in persistence. The test has monotone power with respect to the breakpoint: the earlier the breakpoint, the higher is the power of the test. It is also monotonically increasing with the magnitude of the AR parameter. For $T = 250$ we find the following results: For an early breakpoint ($\tau = 0.3$), the empirical power varies from 77.1% to 93.5% (for $\alpha_1 = 0.9$ to $\alpha_1 = 1.0$); if the breakpoint is located in the middle of the sample the tests' power ranges from 57.2% to 89.1%; for a late break ($\tau = 0.7$), the power varies from 5.9% to 60.5%. For larger sample sizes, the power increases, as one may expect. Especially in the case of a late break, the power increases strongly with the sample size. The simulation results suggest that the test is powerful in distinguishing long memory and stable shifts or changes in persistence and is therefore of empirical usefulness.

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