The Dark Side of Fiscal Stimulus

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Abstract. The output multiplier turns negative before a deficit spending program expires. We show the generality of this unpleasant finding for the standard real business cycle model. We then calibrate an extended model for the US and demonstrate how fiscal stimulus slows down economic recovery from recession in the medium-run. We discuss the slowdown from recovery w.r.t. alternative assumptions about the size and persistence of the initial shock (severity of the recession), the assumed power of the impact multiplier, and the scale and duration of the stimulus program. We also show that results are quantitatively very similar independent from whether a recession was caused by an efficiency wedge (input-financing frictions) or a labor wedge (labor market frictions). Capital stock and output are always below their laissez-faire level of recovery when fiscal stimulus expires.

Keywords: fiscal stimulus, government spending, output multiplier, economic recovery.
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1. Introduction

The recent economic recession has resuscitated a debate which not long ago seemed to have been settled among mainstream economists, a debate about the need for and the capability of deficit spending. The conclusion that fiscal policy is not useful as a counter-cyclical instrument suddenly seemed to be less obvious in the year 2008 and many economists have uttered second thoughts about the usefulness of fiscal stimulus (Feldstein, 2010 Auerbach et al., 2010). The U.S. government has launched a large spending program, which the economic advisors of the President predicted to be highly effective in speeding up economic recovery (Romer and Bernstein, 2009). Many other governments around the world have displayed similar fiscal activities and passed large-scale deficit spending programs. These observations have motivated the present research, which reinvestigates the consequences of deficit spending in the framework of the neoclassical business cycle model.

In this endeavor we are "standing on the shoulders of giants".[1] In contrast to most of the purely quantitative business cycle literature, we want to continue a line of papers, which tried to make a general, theoretical point about the output multiplier of government spending (Hall, 1980, Barro, 1981, Aiyagari et al. 1992, Baxter and King, 1993, Angeletos and Panousi, 2009).

The earlier literature focussed mainly on whether the output multiplier for a temporary increase of expenditure can exceed that for a permanent increase and whether and how the multiplier can exceed unity. We take up this discussion with a novel insight. Shifting the focus from the impact multiplier to the moment of termination of an expenditure program we argue that capital stock and output fall short of their laissez-faire levels before a fiscal stimulus program expires.

Although the full story is more complex (see Section 3) the basic mechanism generating the negative multiplier is straightforwardly explained. When a deficit spending program is implemented, the added supply of government bonds exceeds the added need for savings (which may actually decline). Consequently, households restructure their portfolio and save more in terms of bonds and less in terms of investment in private capital. This process continues as long as the deficit spending program is operative, implying that when the fiscal stimulus expires the aggregate capital stock is smaller than before. This in turn means that output is lower than it could be without fiscal stimulus.

[1] An incomplete list of studies of fiscal policy in the neoclassical growth model includes Judd (1985), Greenwood and Huffman (1991), Cooley and Hansen (1992), Baxter and King (1993), McGrattan (1994), Ludvigson (1996), Burnside et al. (2004), Turnovsky (2004), Mankiw and Weinzierl (2006) and McGrattan and Ohanian (2010), Traband and Uhlig (2010), and Uhlig (2010).

The mechanism is so general that we believe that it is not confined to the neoclassical framework. It should be visible in a new-Keynesian setting as well. Maybe even more so since in a new-Keynesian world increasing government spending frequently leads to higher private consumption such that less aggregate savings amplifies the loss of capital. Of course, in a model that takes capital stock as exogenous, the mechanism cannot be identified. A complex new-Keynesian model with endogenous capital stock, which reached a kind of state-of-the-art status in the New-Keynesian community is the Smets-Wouters (2007) model. Taking this model and analyzing the 4 year ARRA fiscal stimulus program, Cogan et al. (2010) find that the output multiplier turns negative after about 3 years and that GDP remains below its laissez faire path "for many years beyond 2013" (p. 285). Earlier, Baxter and King (1993) have found within a very different, purely neoclassical context, that GDP turns negative just before a 4 years expenditure program expires (p. 325). Uhlig (2010) arrives at a similar result in his study of ARRA. Thus, models with very different predictions about the impact multiplier of deficit spending seem to agree about the multiplier at the time of expiry. This is perhaps not surprising since "the long-run feature of typical New-Keynesian models is neoclassical in nature" (Uhlig, 2010).[2]

In the present paper we show – to our best knowledge for the first time – that there is a systematic mechanism behind the phenomenon that the output multiplier turns negative before expiry of fiscal stimulus. Given the generality of the result it could appear puzzling that it has not (yet) been uniquely confirmed by the empirical VAR literature. A recent study confirming our result is provided by Mountford and Uhlig (2010) who estimate the multiplier for a one-year deficit spending program, which then phases out gradually. They find an output multiplier of 0.65 on impact (after one quarter), 0.27 (and insignificantly different from zero) after 4 quarters and -0.74 after 8 quarters (Table IV).

One problem with the empirical VAR literature, which has not yet converged towards a univocal assessment of government shocks (see the discussion in Hall, 2009), is that according to the impulse response function any temporary government expenditure is assumed to phase out gradually. There is no clear-cut deadline of spending. If the time window of analysis ends before the government shock has returned sufficiently close to zero, the impulse response of GDP is still positive. The

[2]In New-Keynesian models the result survives the dilution of the population with non-Ricardian rule-of-thumb consumers who do not invest. (Cogan et al., p. 286). See also Forni et al. (2009) who investigate a phasing-out government expenditure shock in the Smets-Wouters setup when 50 percent of the population are rule-of-thumb consumers. There, GDP turns negative when spending is still active and about 90 percent of the spending plan have been realized (inferred from Fig. 3, p. 572).

negative part is off the time window shown.[3] Another problem is the high degree of uncertainty involved in VAR estimates. Consider, for example the recent study by Ramey (2011, Figure IX). The impulse response of the government shock for the period 1947-2008 touches the zero-line after 20 quarters and the impulse response of GDP becomes negative after 12 quarters. However, the confidence bands are so wide that this behavior is identified as“ return to normal”.

Given the high uncertainty entailed by VAR estimates, in particular at longer lags (i.e. around what would be the “time of expiry” of spending plans with a given end), it is particularly useful that the general mechanism turning the multiplier negative can be identified by theory. We show below that the result holds also if fiscal activism kicks in when the economy is in deep recession. A negative multiplier then means that GDP falls below its level under laissez-faire recovery before deficit spending expires. For example, if we consider two identical economies which experienced the same fall in output, theory predicts that the economy with a 4 year fiscal stimulus plan will be overtaken by the laissez-faire economy before the 4 year plan has been fully realized.

In Section 4 and 5 we calibrate an extended version of the model for the US and show how fiscal stimulus slows down recovery from recession in the medium run with respect to alternative assumptions about the size and persistence of the initial shock (severity of the recession), the assumed power of the impact multiplier, and the scale and duration of the stimulus program. We also show that results are quantitatively very similar independent from whether a recession was caused by an efficiency wedge or a labor wedge.

Our modeling of wedges is inspired by Chari et al. (2007) who show that any actual business cycle (and thus any reasonable business cycle model) can be reproduced exactly by the standard real business cycle model with four wedges and that for the two examples investigated, the Great Depression and the 1982 recession, the efficiency wedge and the labor wedge account for essentially all of the fluctuations.[4] We are thus confident that our result, which we can prove – constrained by mathematical complexity – only within the neoclassical framework, is much broader and covers all kinds of real world recessions and deficit spending policies.

[3] In Section 5 we address spending programs that are phasing out gradually and show that the general mechanism is at work there as well. In particular, if a spending program phases out sufficiently fast, results differ insignificantly from spending programs of the same present value, which are actually terminated.

[4] Ohanian (2010) applies the methodology to the recession of 2007-2009 in high income countries. He finds that most of the recession is captured by an increasing efficiency wedge. A notable exception is the US, where the bulk of the recession is captured by an increasing labor wedge. There exists a large literature applying the neoclassical business cycle model to the Great Depression. An incomplete list includes Cole and Ohanian (2002, 2004), Kehoe and Prescott (2002, 2007), and Chari et al. (2002).

2. The Model

In order to convey the intuition for our main result easily we begin with reconsidering the standard real business cycle model (King et al., 1988, King and Rebelo, 1999). Later on, in the numerical section, we add various taxes and frictions and investigate quantitative implications. In order to make inferences from phase diagram analysis the model is stated in continuous time. The economy is populated by a continuum of identical households of measure 1. Each household has preferences over private and public consumptions goods and leisure and maximizes intertemporal utility (1).

$$\max_{c,\ell} \int_0^\infty \left(\log(c) + \beta \frac{(1-\ell)^{1-\gamma} - 1}{1-\gamma} + \xi \frac{G^{1-\eta} - 1}{1-\eta} \right) \mathrm{e}^{-\rho t} \mathrm{d}t. \tag{1}$$

Here, c denotes consumption of private goods, G consumption of public goods (government purchases), and ℓ labor supply, ρ is the time preference rate, $1/\gamma$ the elasticity of intertemporal substitution for leisure, and $1/\eta$ the elasticity of intertemporal substitution for consumption of public goods. The assumption of power utility for leisure nests three popular cases (Ludvigson, 1996). For $\gamma \to \infty$, labor is inelastically supplied, for $\gamma = 1$ utility is logarithmic in leisure as frequently assumed in the real business cycle literature, and for $\gamma = 0$ utility is linear in leisure as suggested by the indivisible labor model of Hansen (1985) and Rogerson (1988). The latter constitutes an important limiting case for the subsequent analysis since it implies the most elastic labor supply (an infinite Frisch elasticity).

Households face the budget constraint (2).

$$\dot{a} = w\ell + ra - c - T, \tag{2}$$

where a denotes asset holdings, w is the wage rate, r the interest rate, and T is a lump-sum tax paid to the government. As usual, the first order conditions for problem (1)–(2) provide the Ramsey rule (3) and the optimal trade-off between consumption and leisure (4).

$$\frac{\dot{c}}{c} = r - \rho \tag{3}$$

$$w = \frac{\beta c}{(1-\ell)^{\gamma}}. \tag{4}$$

The government finances public goods expenditure G and interest payments on debt rb by taxes T and new debt $\dot{b}$. Its budget constraint is thus given by (5).

$$\dot{b} + T = G + rb. \tag{5}$$

Government bonds are held by households together with capital k implying that total assets are defined by (6).

$$a \equiv k + b. \tag{6}$$

A representative firm employs capital and labor and produces output according to a Cobb-Douglas technology $y = Ak^{\alpha}\ell^{1-\alpha}$ where A denotes aggregate factor productivity. At an equilibrium on the factor market, factor prices equal their marginal product such that $r = \alpha A k^{\alpha-1}\ell^{1-\alpha} - \delta$ and $w = (1-\alpha)Ak^{\alpha}\ell^{-\alpha}$. At an equilibrium on the goods market, output is used for private and public consumption, and gross investment $\dot{k} + \delta k$, where δ denotes the rate of depreciation. This means that aggregate capital evolves according to (7).

$$\dot{k} = y - c - \delta k - G. \tag{7}$$

Finally we re-establish the intertemporal budget constraints for government and households. Integrating the flow budget constraint (5) and applying the no-Ponzi-game condition $\lim_{t\to\infty} \mathrm{e}^{-\int_0^t r(u)du} b(t)\mathrm{d}t = 0$ we obtain (8) and integrating the flow budget constraint (2), applying the no-Ponzi-game condition $\lim_{t\to\infty} \mathrm{e}^{-\int_0^t r(u)du} k(t)\mathrm{d}t = 0$, using the capital market equilibrium (6), and inserting the government budget constraint (8) we obtain the household budget constraint (9).

$$0 = b(0) + \int_0^{\infty} (G - T) e^{-\int_0^s r(u)du} ds \tag{8}$$

$$\int_0^{\infty} c\, e^{-\int_0^s r(u)du} ds = k(0) + \int_0^{\infty} w\ell\, e^{-\int_0^s r(u)du} ds - \int_0^{\infty} G\, e^{-\int_0^s r(u)du} ds. \tag{9}$$

3. The Government Spending Multiplier: Theory

3.1. **Intuition.** In the tradition of the theoretical literature on the output multiplier of government spending we begin with a subsection that develops some preliminary intuition for our main result. With contrast to the earlier literature we shift the focus from the impact multiplier to the multiplier at the end of a deficit spending program and argue that capital stock and output fall short of their laissez-faire values when a fiscal stimulus program expires.

In order to develop the intuition it is instructive to compare a temporary expenditure program with a permanent increase of government expenditure under the normalizing assumption that both policies entail the same present value $\int_0^\infty G\, e^{-\int_0^s r(u)du} ds$. Obviously, this implies that expenditure is (much) higher for the temporary spending program during the periods in which spending is operative. Furthermore, suppose that both permanent and temporary expenditures are financed with the *same* time series of lump-sum taxes. This means for the temporary expenditure program that tax revenue during the spending phase is much lower than expenditure. The policy thus constitutes a deficit-financed fiscal stimulus.

Recall that within the neoclassical setup a permanent increase of government expenditure increases long-run output permanently through the wealth effect, i.e. the long-run multiplier is positive. Focussing on a temporary expenditure program of the same present value is thus a disciplining device. It guarantees that our result that the multiplier turns negative before the expenditure program expires is not driven by the wealth effect. If we furthermore assume for a moment that labor is supplied inelastically, we can infer from (9) that the two expenditure programs reduce private consumption in present value terms by exactly the same amount.

Generally, if labor is supplied elastically, households react on the negative wealth effect not only by consuming less but also by supplying more labor since consumption and leisure are normal goods. More labor supply leads to a higher marginal product of capital and a higher interest rate. In case of a permanent rise of government expenditure, it is a well-known result that higher interest rates spur investment such that the capital stock raises gradually. Since this in turn depresses the interest rate, the economy moves towards a new steady-state at which the interest rate returns to its initial level (and equals the time preference rate) whereas both capital and labor are permanently higher than initially. The output multiplier is positive (see e.g. Aiyagari et al., 1992, Baxter and King, 1993).

Turning towards the temporary expenditure program, imagine for a moment that households choose the same response of consumption and labor supply as under the permanent program. Facing the same present value of government expenditure, households would indeed response in exactly the same way if interest rates stay constant. Actually, however, there is a dampening effect via the interest rate at work, but its magnitude is of second order such that the approximation error seems to be justified by the insight gained. The fact that households choose the same path of consumption and labor supply under both government expenditure policies implies that savings $\dot{a}$ also follows the same path under both expenditure policies. Yet, households facing the temporary expenditure policy buy (much) more $\dot{b}$.

In order to understand why households restructure their asset holdings, recall that government expenditure is higher "per period" (per unit of time) for the temporary program because the same total expenditure in net present value terms is concentrated on a much smaller time span. Consequently, the government's demand for capital demand ($\dot{b}$) is also higher in each period for the temporary program. Formally, this can be seen by the government's budget constraint (5). On impact, taxes T and interest on debt rb are equal for both spending programs. The right hand side of the budget constraint (5) is the same for both policies. But because the government has to finance a higher level of expenditure G when the temporary policy is operative, it has to issue more new debt $\dot{b}$ (left hand side of the budget constraint). Capital market equilibrium ($\dot{a} = \dot{k} + \dot{b}$) then requires that savings in terms of government bonds are higher and savings in private capital are lower than under the permanent expenditure policy. As a result, the private capital stock declines when the fiscal stimulus program is active. For the same reason, private investment and capital stock rise again after the termination of fiscal stimulus policy. Since the temporary policy has left all "fundamentals" unchanged, capital stock and output converge from below towards their initial steady-states.

So far, capital supply was assumed to be the same for both spending policies such that the temporary expenditure program crowds out investment of $\dot{b} = G - T$; a crowding out which, by construction, does not occur under the permanent policy. In reality, fortunately, the implications of fiscal stimulus are somewhat less grim since there is a dampening effect at work. This is so because less private investment gradually reduces the capital stock, which in turn raises the return on capital and increases the incentive to save. This (second order) effect somewhat mitigates the negative effect of the temporary spending program on investment.

The result that capital stock is decreasing when a deficit spending is active and increasing after it expired is explained by the households' desire to smooth consumption over time. Trying to keep consumption smooth, they reduce private investment when the government claims additional shares of GDP and expand private investment afterwards. Because the mechanism is so general, our conclusion holds true regardless of the state of the economy. If, for example, an economy is recovering from recession, the time path of wages and interest rates differs from those at the steady state. Naturally, net present value of wage income and net present value of government expenditure deviate compared to an economy that was initially at its steady state. But since the recession affects net present value in the same way for temporary and permanent expenditure policies, we expect that the magnitude by which consumption, labor supply, and investment differs between both spending policies will be roughly the same, no matter whether the economy is initially at its steady state or in transition.

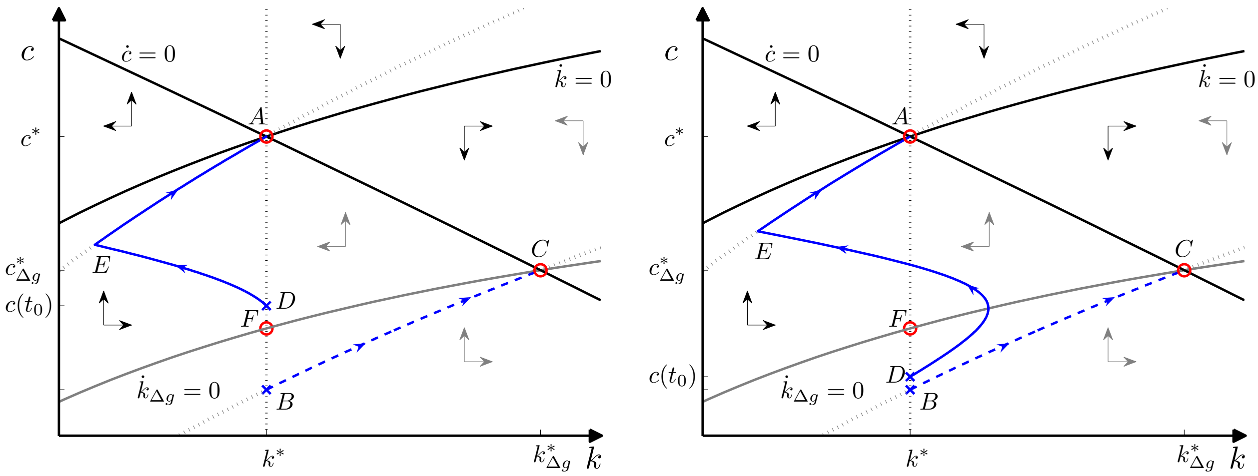
3.2. **Phase Diagram Analysis.** In this subsection we prove that the output multiplier turns negative before a temporary government spending program expires. Here we consider an economy resting at the steady-state when the government expenditure shock occurs. Later on we investigate an economy driven far its below steady-state position (by efficiency- or labor-wedges) before deficit spending is implemented. The larger part of the proof uses phase diagram analysis to show that the capital stock falls short of its steady-state value at the moment of expiry of fiscal stimulus. We begin with setting up the phase diagram for the economy, which consists of the differential equations (3) and (6), labor supply according to (4), and production $y = Ak^{\alpha}\ell^{1-\alpha}$. Impulse responses for this economy can be represented in two-dimensional $c - k$ space as displayed in the two panels of Figure 1.

The steady-state is where the $\dot{c} = 0$–isocline and the $\dot{k} = 0$–isocline intersect. Along the $\dot{c} = 0$–isocline, the capital labor ratio k^*/ℓ^* is determined by the time preference rate ρ. From (3), $\rho = \alpha A(k^*/\ell^*)^{\alpha-1} - \delta$. In contrast to the familiar phase diagram of the neoclassical growth model with exogenous labor (see e.g. Barro and Sala-i-Martin, 2004) the $\dot{c} = 0$–isocline is not vertical. It is downward sloping because consumption and leisure are normal goods and households supply more labor at lower levels of consumption (see (4)). The smaller the Frisch elasticity of labor supply, the steeper the slope of the isocline. Only in the limit, when labor is supplied inelastically, the isocline becomes a vertical line. On the other hand, if the Frisch elasticity of labor supply becomes infinite ($\gamma = 0$), the isocline becomes a horizontal line. Figure 1 shows an intermediate case. In the Appendix we derive the isoclines formally.

The $\dot{k} = 0$–isocline is obtained by setting (6) to zero and solving for c; $c = Ak^{\alpha}\ell^{1-\alpha} - \delta k - G$. Here, the analysis coincides with the familiar textbook model: the isocline is increasing when the concave part stemming from the neoclassical production function is dominating and decreasing when the linear part stemming from depreciation is dominating. There exists a unique intersection of the isoclines and the resulting arrows of motion identify the equilibrium as a saddlepoint. The unique adjustment path after a change of government behavior is given by the movement along the stable saddlepath towards the steady-state.

It is instructive to begin the analysis with an inspection of adjustment dynamics after a permanent increase of government expenditure. Suppose the economy is situated initially at steady-state A in Figure 1. More government expenditure reduces households' claims on output, $y-G$, and the $\dot{k}=0$–isocline shifts down. The $\dot{c}=0$–isocline remains unchanged. Because both consumption and leisure are normal goods, households respond to permanently lower income by consuming less and supplying more labor (the wealth effect). Higher labor supply increases the marginal product of capital and raises investment. The economy converges towards a steady state of higher aggregate capital, point C in Figure 1. Adjustment dynamics following a fiscal expansion are given by an instantaneous drop of private consumption, after which both capital and consumption increase along the stable saddlepath towards the new equilibrium. The economy jumps from A to B and then moves to C. The transition path is indicated by dashed lines in both panels of Figure 1.

Figure 1: Phase Diagram: Permanent and Temporary Rise of Government Purchases



The black vector field is associated with the initial equilibrium A and the grey vector field is associated with the equilibrium assumed under higher government expenditure C. If the fiscal expansion is permanent, the grey vector field applies always. If the fiscal expansion is temporary, the grey vector field applies only when fiscal stimulus is active. When fiscal stimulus expires, the black vector field applies again.

Turning towards a temporary increase of government spending, we begin with assuming that G increases unexpectedly at time t_0 and that the spending program is known to terminate at time $\tilde{t}$. In Section 5 we show that our main result continues to hold if the fiscal stimulus never really expires but phases out over time.[5] Because households know in advance when the stimulus program ends, they adjust labor supply and investment already from t_0 onwards. Since there are no more surprises, there is only one jump of consumption and employment, namely at the moment when the economy is unexpectedly hit by the fiscal stimulus plan. Afterwards there are – by construction – no further shocks and any further jumps are ruled out by no-arbitrage . Formally, the first order condition for a utility maximum requires that the shadow price of capital, $\lambda = 1/c$, is continuous and thus consumption is continuous.

Because there cannot be any further jumps, and because any trajectory which is not starting on the stable saddlepath will never arrive at the steady-state, the economy has to be on the saddlepath and moving towards point A when deficit spending expires (i.e. at time $\tilde{t}$) and the "pre-shock" saddlepoint dynamics apply again. The fact that the state variable capital stock cannot jump implies that initial consumption after the shock $c(t_0)$ is found on a vertical line through point k^*. In Figure 1 this point is denoted by D on the dotted line through A.

In order understand adjustment dynamics note that as long as the expenditure program is active the grey arrows of motion, associated with steady-state C, apply. When the expenditure program expires, the black arrows of motion, associated with the initial steady-state A, apply again. The impulse response dynamics of consumption can thus be figured out by following the directions indicated by the grey arrow field for the period from t_0 to $\tilde{t}$. In doing so, it is straightforward to rule out any initial consumption levels above A and below B. In these cases the consumption path would never meet the stable saddlepath at time $\tilde{t}$. Within the remaining feasible range for $c(0)$, between point A and B, we have to distinguish two cases, depending on the duration of the expenditure plan. These cases are shown in the left and right panel of Figure 1.

If the duration of the expenditure program is sufficiently short, households choose initial consumption above the $\dot{k} = 0$–isocline associated with steady-state C (i.e. within the interval AF) In this case, investment jumps down when the expenditure program starts and remains below steady-state level for the whole duration of the expenditure program. As a consequence, capital stock declines continuously until "fiscal stimulus" expires and the economy hits the stable saddlepath at a lower level of capital compared to the original steady state k^*. The resulting trajectory ADEA is shown by the solid line in the left panel of Figure 1.

[5] We could add more realism by assuming an announcement phase during which the spending program is known but not yet enacted and operative. This would increase the impact multiplier but would not affect the multiplier at time of expiry. We investigate announcement effects of fiscal policy in phase diagrams in Strulik and Trimborn (2010).

The case of a long-lasting but still temporary expenditure program is shown in the right panel of Figure 1. Here, households choose initial consumption below the $\dot{k} = 0$–isocline associated with steady-state C (i.e. within the interval FB), implying that investment increases initially. Following the grey vector field, it is evident that the impulse response of investment soon turns negative (when the trajectory crosses the $\dot{k} = 0$–isocline). Again capital stock falls below its initial steady-state while the expenditure program is still active and the economy hits the stable saddlepath in point E at a lower level of capital than k^*. The resulting trajectory ADEA for the second case is indicated by a solid line in the right panel of Figure 1.

Intuitively, investment is above steady-state level initially because a longer spending program reduces household wealth more pronouncedly and triggers more labor supply. This in turn implies a relatively high marginal product of capital. At the beginning of the spending program, households respond to this incentive by saving more. After some time, however, the crowding out effect on investment becomes dominating and investment falls again below steady-state level. The main result that capital stock falls short of its steady-state level at the time of expiry is thus independent from the duration of fiscal stimulus.

Real world fiscal stimulus, designed to shorten a recession, typically belongs to the first category (left panel). Numerical experiments with our calibration for the US from Section 4 show that expenditure programs shorter than 7.5 years lead to an immediate drop of investment. Moreover, for really long expenditure programs the initial phase of higher investment turns out to be relatively short. For example, for a 10 year expenditure program we find that investment falls below steady-state level after the first two years and stays there for the remaining eight years of "fiscal stimulus".

When the spending program expires, not only capital stock but also aggregate output falls short of its initial steady-state level to which it converges monotonically from below. In order to understand the generality of this result the following proposition, proven in the Appendix, is useful.

PROPOSITION 1. *Along the stable saddlepath, iff capital is below steady-state level, wages are below steady-state level and interest rates are above steady-state level, i.e.* $k < k^* \Leftrightarrow w < w^* \wedge r > r^*$.

Diagrammatically, this fact is reflected by the positive slope of the saddlepath. Since, empirically, aggregate output is positively correlated with wages and negatively with interest rates (see King and Rebelo, 1999), for any meaningful calibration of the real business model output is below steady-state level after expiry of fiscal stimulus.

Summarizing, capital stock and aggregate output are positively correlated. At the time when the spending program terminates, both capital stock and GDP are lower than they would have been without the spending program. Actually, GDP is already below steady-state level for some time before the spending program terminates. In order to see this, recall that the capital stock cannot jump and is thus below steady-state level already for some time before $\tilde{t}$ (moving from D to E in Figure 1). Since consumption and labor supply cannot jump (except at t_0), aggregate output moves continuously (except at t_0) and is thus in line with capital already below steady-state level before the government expenditure program terminates. In other words, the government spending multiplier is negative already for some time before deficit spending ends.

4. Fiscal Stimulus and Recovery from Recession

In this section we investigate, for the model calibrated with US data, how a temporary government expenditure program affects the speed of recovery from recession. Our objective is twofold. First, we want to substantiate the claim that the result of a negative multiplier in the medium run holds true irrespective of whether a fiscal expansion is put into effect in good times (at the steady-state as shown in the last section) or in bad times when the economy has been hit by a (deep) recession. Second, we want to provide a quantitative assessment of the slowdown from recovery originating from "fiscal stimulus".

In order to add more realism we augment the model by various taxes, an efficiency wedge and a labor wedge. We assume that a shock has driven the economy far below the steady state and that this fact has motivated the government to implement a large expenditure program. We calibrate the model such that the multiplier of a permanent expenditure program as well as the impact multiplier of a temporary expenditure program is positive and – compared to empirical studies – of average size. We then show that the government spending multiplier turns negative before fiscal stimulus expires and that the slowdown of recovery is substantial. We demonstrate the generality of this result with respect to the size and persistence of the shock (severity of the recession), the size of the impact multiplier, and the scale and duration of the stimulus program.

We also demonstrate the invariance of our main results with respect to the recession-causing shock. The modeling of shocks is inspired by Chari et al.'s (2007) "equivalence results". Chari et al. show that a large variety of business models is equivalent to a prototype model with wedges for efficiency, labor, investment, and government consumption. Moreover, they show that almost all business cycle dynamics of past US recessions can be accounted for by the efficiency wedge and the labor wedge. The efficiency wedge can (but need not) be motivated by input-financing frictions. The labor wedge can (but need not) be motivated by labor market frictions and sticky wages.

Here we assume that a shock has driven either the productivity wedge or the labor wedge out of its steady-state and that the distortion returns gradually to its steady-state position. We investigate separately recessions caused by the efficiency wedge and by the labor wedge under the normalizing assumption that the initial output gap and the government expenditure program are the same. We demonstrate that although the response of factor supply and consumption is much stronger for the labor wedge scenario, both scenarios provide very similar predictions for the deviation of GDP under fiscal activism from laissez-faire recovery. In particular, at the moment of expiry the fiscally stimulated GDP falls short of laissez-faire GDP by about the same amount for both wedges.

4.1. **Model Extension.** For the efficiency wedge we assume that total factor productivity A follows a first-order autoregressive process. Initially, at time 0 an exogenous shock has driven A down by A_0. From the level $(A^* - A_0)$ factor productivity converges towards its steady state value A^* at constant rate σ as shown in (10). Linguistically, we address $(A^* - A(t))$ as the efficiency wedge. The severity of a recession is thus increasing in the efficiency wedge. The labor wedge ω is introduced as a distortion of the marginal rate of substitution between consumption and leisure and the marginal product of labor beyond the "normal" distortion caused labor income taxes. At the steady-state $\omega = 0$. The severity of a recession is increasing in the labor wedge $\omega(t)$. From the initial value ω_0 the labor wedge returns to zero at constant rate σ as shown in (11).

$$A(t) = A^* - A_0 e^{-\sigma t} \tag{10}$$

$$\omega(t) = \omega_0 e^{-\sigma t}. \tag{11}$$

Households pay taxes on interest income, τ_a, labor income, τ_w and consumption τ_c. This implies that the household budget constraint modifies to (12) and the government budget constraint modifies to (13)

$$\dot{a} = (1 - \tau_w) w\ell + (1 - \tau_a) ra - (1 + \tau_c) c - T. \tag{12}$$

$$\dot{b} + \tau_w w\ell + \tau_a rk + \tau_c c + T = rb + G. \tag{13}$$

The households' first order conditions for maximizing (1) subject to (12) modify to (14) and (15) and aggregate capital accumulates according to (16).

$$\frac{\dot{c}}{c} = r - \rho \tag{14}$$

$$(1 - \tau_w)(1 - \omega)w = \frac{c\beta}{(1 - \ell)^\gamma} \tag{15}$$

$$\dot{k} = y - c - \delta k - gy - \omega w \ell = (1 - g - \omega)y - c - \delta y. \tag{16}$$

4.2. **Calibration.** We calibrate the model with US data in line with the existing RBC literature on fiscal policy and government expenditure. Parameter values for tax rates, government share, capital share, depreciation rate, and time preference are taken from Trabandt and Uhlig's (2009) recent calibration of the RBC model with US data. In line with standard RBC modeling (King and Rebelo, 1999) we set ρ such that the real interest equals 6 percent per year and we set β such that households in equilibrium supply a quarter of their time on the labor market. At the pre-shock steady-state the government purchases 18 percent of output, i.e. $g \equiv G/y = 0.18$. Altogether these values imply an investment rate of 20.5 percent. Table 1 summarizes the setup of parameters.

We want to model a deep recession by either an increasing efficiency wedge (the A-scenario) or an increasing labor wedge (the ω-scenario). For that purpose we adjust A_0, or ω_0 respectively, such that an output gap of 7 percent below steady-state level results. The rate of recovery, σ, is calibrated in both scenarios such that half of the original GDP gap is closed after two years by setting $\sigma = 0.4$. In Section 5 we provide sensitivity analysis with respect to the size of the initial shock and the laissez-faire speed of recovery.

Table 1. Parameter Values

τ_a	τ_w	τ_c	α	A^*	A_0	ω^*	ω_0	σ	δ	ρ	g	γ	ℓ^*
0.36	0.26	0.05	0.38	1	0.046	0	0.15	0.4	0.07	0.038	0.18	1.5	0.25

Of course, quantitative results depend crucially on the assumed power of the government spending multiplier. Unfortunately the literature has not yet converged towards a generally accepted value for the multiplier. We try to meet the involved uncertainty by defining a reasonable benchmark case and by providing sensitivity analysis. For the benchmark case we set $\gamma = 1.5$, which implies a long-run multiplier for a *permanent* increase of government purchases of 0.89. This value lies in the middle of Barro's (1981) estimate of 0.62 and Baxter and King's (1993) estimate of 1.16. Since we embarked on this project, Barro and Redlick (2011) provided new estimates of the long-run multiplier between 0.5 and 0.6 (depending on the period of investigation) and Hall (2009) provided estimates between 0.13 and 0.56. Both studies infer the multiplier from military expenditures. Hall (2009) also provides a survey of results from VAR analyses, which estimate multipliers in the range from 0.3 to 0.9 on impact. In Section 5 we provide a robustness check with respect to the size of the multiplier.

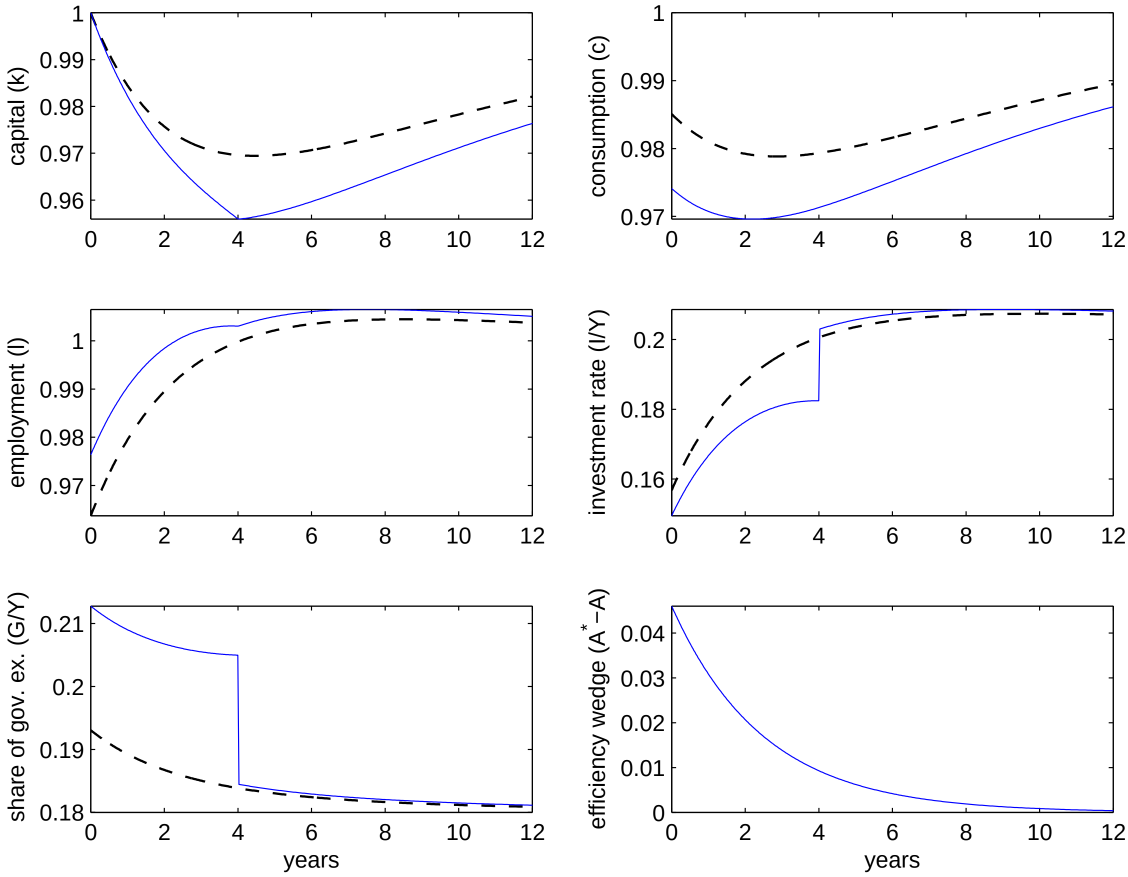
We define as "laissez-faire" the policy that keeps total purchases G constant. This assumption is in contrast to the standard RBC modeling where laissez faire means that the government maintains a constant GDP share of purchases $g = G/y$. Holding G constant instead, adds more realism to our model since it implies that the GDP share of government purchases rises after a negative shock of GDP already under laissez faire, i.e. without the *additional* expenditure triggered by the shock.[6] In the benchmark case we assume that the recession triggers a large fiscal stimulus program: government expenditure increases by 2 percent of GDP per year. The spending program becomes operative immediately after the output drop and expires after 4 years. In Section 5 we provide a sensitivity analysis with respect to the size and duration of the spending plan.

For the computation of adjustment dynamics we apply the relaxation algorithm, which is in detail described in Trimborn et al. (2008). The method does not require linearization around the steady-state or any other transformation of the original dynamic system. It provides the exact solution (up to a user-specified error) irrespective of the deviation from steady-state and is thus a reliable tool for the investigation of big shocks, i.e. of an economy far away from its steady-state.

4.3. **Benchmark Recoveries.** As motivated above, we consider in our benchmark case a collapse of GDP of 7 percentage points at time 0, after which the output gap closes with a half life of 2 years under laissez-faire. We discuss alternatively that the recession was caused by an increasing efficiency wedge and by an increasing labor wedge. We begin with discussing the A-scenario. Figure 2 shows adjustment dynamics for several variables of interest during economic recovery from a shock caused by a decline of total factor productivity. Dotted lines represent "laissez-faire" recovery and solid lines represent recovery with fiscal stimulus. The A panel at the lower right corner in Figure 2 shows the time path of productivity that produces the laissez-faire output gap.

[6] We have checked that our results remain qualitatively unchanged if "laissez faire" is defined by a constant GDP share of government purchases.

Figure 2: Recovery from Recession: A–scenario



Recovery from an output drop of 7 percent below steady-state level caused by an increasing efficiency wedge. Dotted lines: laissez faire policy, solid lines: temporary increases of government share. Capital (k), consumption (c), employment (ℓ), are measured relative to their pre-shock (steady-state) position. The investment rate (I/Y), the government expenditure share of GDP (G/Y) and the efficiency wedge ($A^* - A$) are measured in percent.

Focussing first on laissez-faire recovery, we observe that consumption and investment fall below their steady-state positions. However, consumption smoothing behavior causes – in line with findings from business cycle research – a much stronger effect of the shock on investment than on consumption. The severe drop of investment results in a prolonged decline of capital stock, which reaches a trough between year 4 and year 5 after the shock. Lower factor productivity and less investment causes employment to fall initially. The fact that the government maintains its pre-shock purchases of goods implies, together with the output fall, an increase of the government share of GDP from 0.2 to almost 0.22. Since tax revenue declines the shock entails an increasing government deficit already under laissez faire.

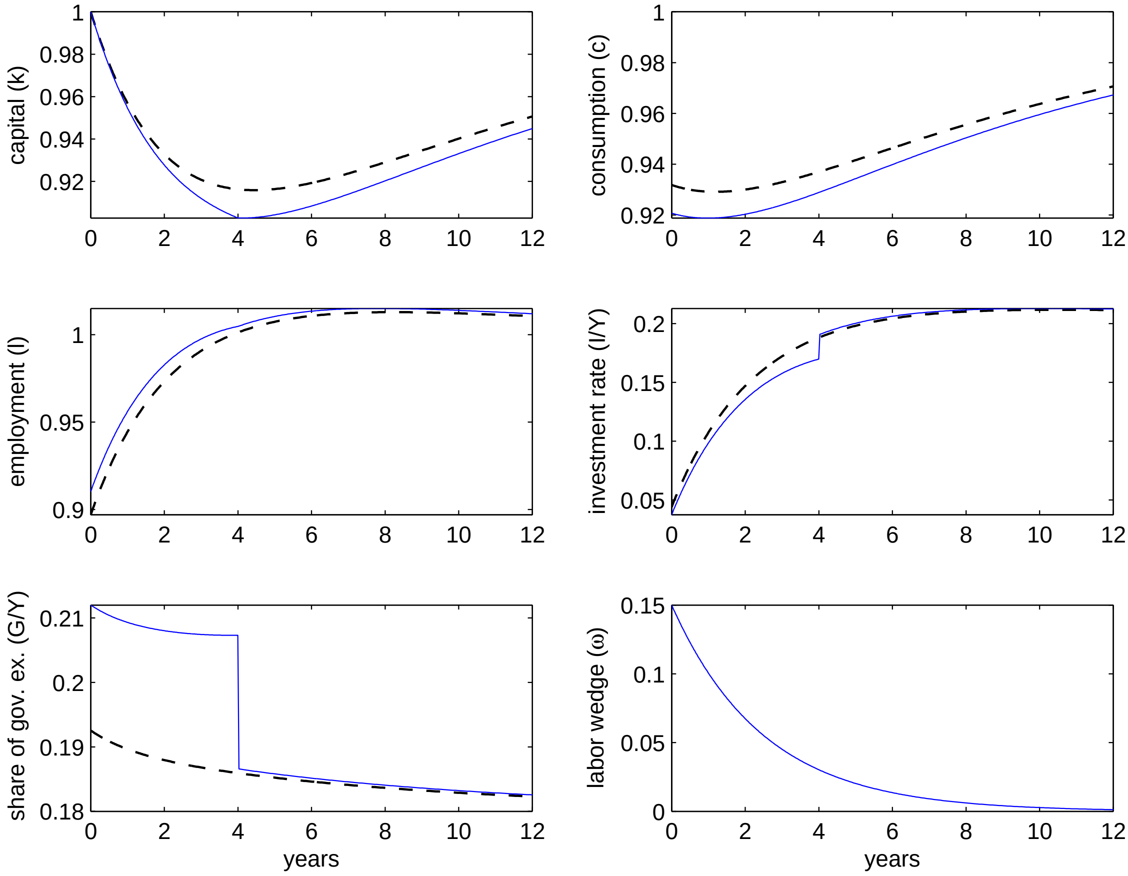
Next consider adjustment dynamics when public spending increases. In our benchmark scenario we assume that the government reacts to the shock with a deficit-financed fiscal stimulus of ΔG such that $\Delta g = 0.02$ for 4 years (solid lines). Qualitatively, impulse responses confirm our results from phase diagram analysis. Fiscal stimulus very effectively raises employment but the downside is that it also drives private investment far below the laissez-faire path. To assess the impact on investment note that if crowding-out were complete, the government expenditure would drive the investment ratio two percentage points below the laissez-faire path. Measured by the vertical distance between the dotted and solid I/Y path, we see that crowding out of the investment rate increases from about 0.7 percentage points at t_0 to 1.8 percentage points when deficit spending expires. As a consequence capital stock falls much harder than under laissez-faire.

Economic recovery from a shock caused by an increasing labor wedge is shown in Figure 3. The lower right panel shows the labor wedge needed to produce an initial output gap of 7 percent and its convergence to zero. Again, dotted lines represent laissez-faire paths and solid lines represent recovery under the benchmark government expenditure program. Compared to the A-scenario the slump of factor inputs is much heavier for the ω-scenario. This result is not surprising. It follows naturally from the fact that if A stays constant the drop of output $Ak^{\alpha}\ell^{1-\alpha}$ has to be explained solely by the drop of ℓ (and later on of k). Consequently, the government expenditure program is less successful in rising employment. But notwithstanding these large differences, fiscal stimulus crowds out about the same investment as for the A–scenario. In the ω-scenario, government spending crowds out an investment rate of almost 0.7 percentage point initially and about 1.8 percentage points at the time of expiry.

Adjustment dynamics for aggregate output result from the interaction of investment and employment in the following way. During an initial period, immediately after deficit spending becomes operative, the positive effect on GDP through employment dominates. The negative effect through falling investment is not yet effective because capital stock is a state variable, i.e. it cannot jump and adjusts slowly over time. After some time, however, the effect of dampened and delayed investment becomes dominating and not only capital stock but also GDP falls below the laissez faire path.

The deviation of GDP from laissez-faire recovery and the associated output multiplier of government expenditure $\Delta Y/\Delta G$ are shown in Figure 4. Solid lines reflect the A–scenario and dashed lines the ω–scenario. The figure visualizes for the example economy the general result from phase diagram analysis. Given the huge difference in performance of economic aggregates under the A–

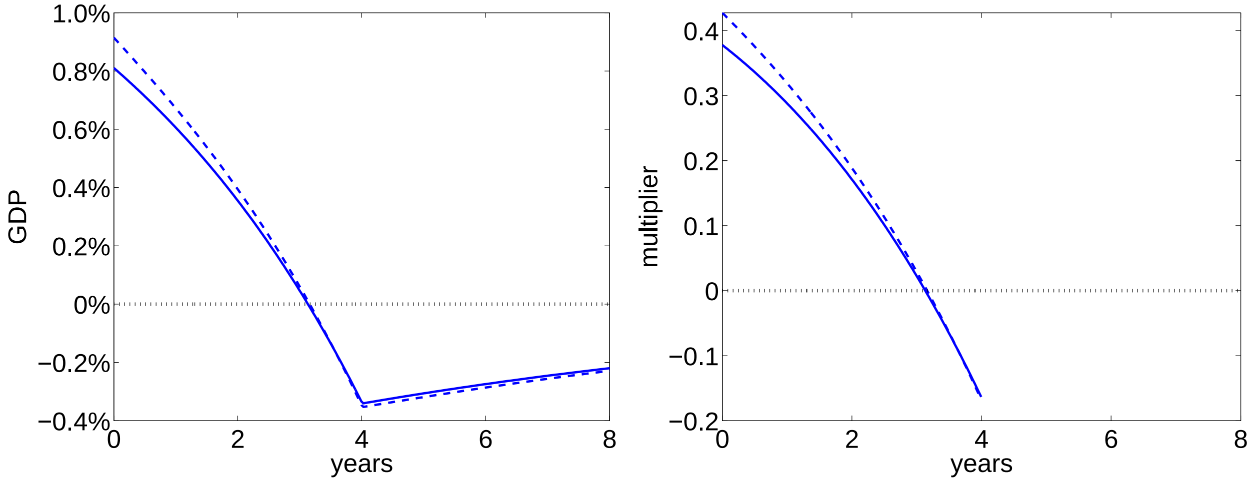
Figure 3: Recovery from Recession: ω–scenario



Recovery from an output drop of 7 percent below steady-state level caused by an increasing labor wedge. Dotted lines: laissez faire policy, solid lines: temporary increases of government share. Capital (k), consumption (c), employment (ℓ), are measured relative to their pre-shock (steady-state) position. The investment rate (I/Y), the government expenditure share of GDP (G/Y), and the labor wedge (ω) are measured in percent.

and ω–scenario, it is striking how similar the results are for the deviation of GDP from laissez-faire recovery. On impact deficit spending raises output effectively by about 0.9 percent above laissez-faire with subsequently decreasing lead. After about 3 years, at a time when the economy has closed about 60 percent of the initial gap, and when deficit spending is still running for 1 more year, the negative investment effect becomes dominating and GDP falls below its laissez-faire level. The output multiplier turns negative. The greatest distance from laissez-faire recovery is reached at the time when deficit spending is terminated, after which the "stimulated" economy converges towards the laissez-faire path from below, catching up the loss as time goes to infinity.

Figure 4: Deviation of GDP from Laissez-Faire and the Multiplier



Left panel: Deviation of GDP from laissez-faire recovery. Right panel: government spending multiplier. Solid lines: A–scenario (efficiency wedge causes recession). Dashed lines: ω–scenario (labor wedge causes recession).

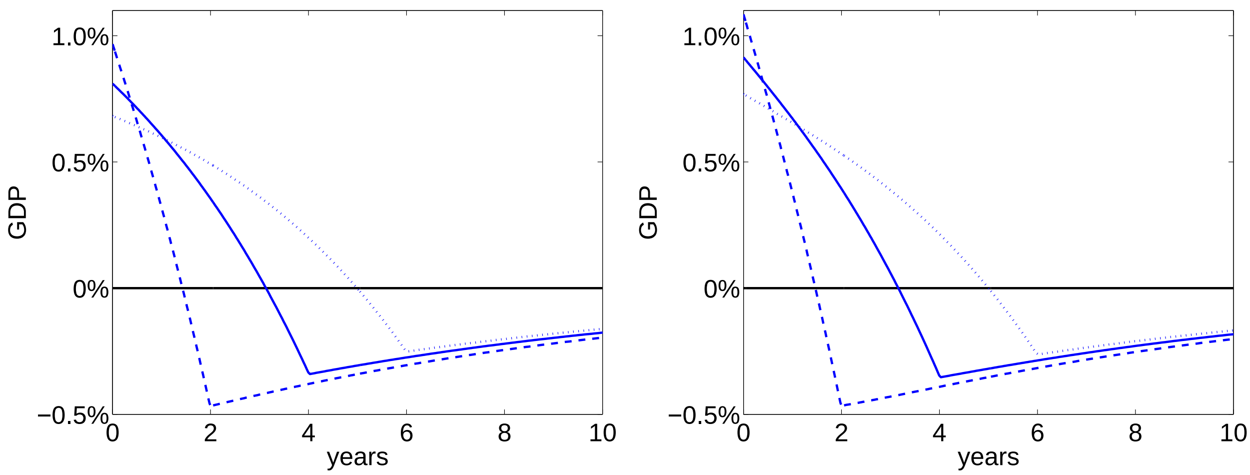
Incidently, the impact multiplier predicted by the model is between 0.38 and 0.43 and coincides approximately with Barro and Redlick's (2011) point estimate. Over time the multiplier is continuously declining and turns negative before fiscal stimulus expires. The multiplier at expiry is about -0.2. Our result can be compared to Cogan et al.'s (2010) study of the ARRA spending program within the new-Keynesian Smets-Wouters (2007) model. Within this more complex framework Cogan et al. investigate a four year deficit spending program and predict that the multiplier turns negative after about 3 years and that GDP remains below its laissez faire path "for many years beyond 2013" (p. 285). Earlier, Baxter and King (1993) have found within a very different, purely neoclassical context, that GDP turns negative just before a four-year expenditure program expires (p. 325). Here, we have shown for the first time that there is a systematic mechanism behind these results.

5. Sensitivity Analysis

We have already argued with help of the phase diagram that our main result of a negative multiplier at time of expiry of deficit spending is of general nature rather than an artefact of the particular shock and the particular government reaction assumed in the benchmark calibration. To further validate this claim and to assess the quantitative impact of alternative assumptions we now present a sensitivity analysis with respect to the duration of the deficit spending program, the size of the government spending multiplier, the scale of the spending program, and the severity of the recession and its persistence.

5.1. **Duration of Deficit Spending.** We begin with investigating the impact of the duration of the spending program. For that purpose we assume that the same total increase of government outlays is stretched over an alternative number of years. A longer spread of the stimulus thus reduces extra expenditure per year. To be specific, we take total outlays from our benchmark case (where they were spent over 4 years) and assume that they are spent alternatively over 2 years, implying $\Delta g = 0.04$ annually or over 6 years, implying $\Delta g = 0.013$.

FIGURE 5: ROBUSTNESS W.R.T. TIMING OF DEFICIT SPENDING



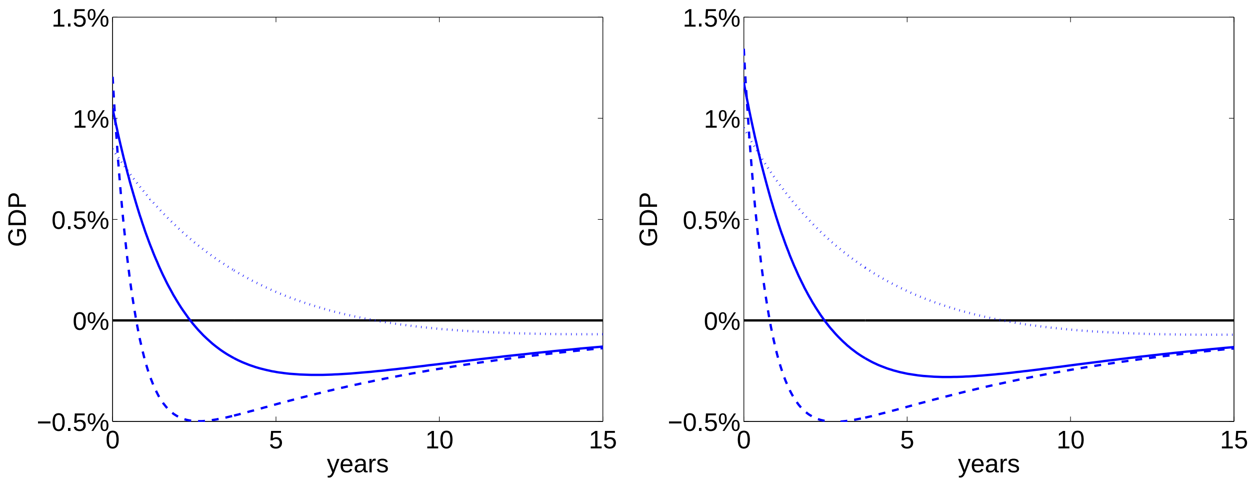
Left panel: A-scenario. Right panel: ω-scenario. Solid lines: benchmark case. dashed lines: 2 years deficit spending, dotted lines: 6 years deficit spending.

Figure 5 shows the effect of a varying length of the deficit spending period. The left hand side shows results for an increase of the efficiency wedge, and the right hand side shows results for an increase of the labor wedge. For comparison, the solid line reiterates deviation from laissez-faire GDP for our benchmark policy from Section 4. The dashed line shows the implied deviation when total extra outlays are spend over 2 years while the dotted line shows the deviation for an 6–year spending program. If deficit spending is performed over a shorter period of time, the triggered initial increase of GDP is stronger and of shorter duration, and the fall below laissez-faire is harder. The opposite is true for a prolonged spending program. Regardless of the duration of the stimulus, the deviation from laissez faire *after* expiry follows almost the same path. The result reflects the fact that the induced loss of permanent income depends mainly on total government purchases, which have been assumed to be the same in all three cases.

5.2. **Phasing Out Policies.** The standard RBC modeling of government shocks considers usually a fiscal stimulus that does not end at a certain point of time but instead phases out gradually. In order to compare with this literature we next investigate an alternative pattern of government spending, which follows a first-order autoregressive process. Initially, at time 0, the government increases spending by G_0. From then on, government spending converges towards its steady state value from above at constant rate μ, i.e. $G = G^* + G_0 e^{-\mu t}$.

We investigate three different government spending plans Medium, Fast, and Long. To make them comparable, we assume that they all sum up to the same net present value of government expenditure. According to plan Medium, government expenditure increases initially by 5% of GDP in year 0 and additional expenditure returns to zero with a half-life of six quarters (similar to our 4 year benchmark scenario in net present value terms). The Fast plan schedules an initial increase of government expenditure of 15% and reduces expenditure with a half-life of two quarters. And according to Plan Slow expenditure increases initially by 2% and declines with a half-life of 16 quarters. Results are shown in Figure 6. The left hand side shows results for an increase of the efficiency wedge, and the right hand side shows results for an increase of the labor wedge.

FIGURE 6: DEVIATION FROM LAISSEZ-FAIRE GDP: PHASING OUT FISCAL STIMULUS



Three never-ending, exponentially decaying government spending plans with the same net present value. Left panel: A-scenario. Right panel: ω-scenario. Solid line: Plan Medium: initial government expenditure of 5% of GDP declining with a half-life of 6 quarters, dashed line: Plan Fast: initial government expenditure of 2% of GDP declining with a half-life of 16 quarters, Dotted line: Plan Long: initial government expenditure of 15% of GDP declining with a half-life of 2 quarters

Apparently, each of these spending schedules causes GDP to fall below steady state at some point of time. If expenditures are concentrated on the first periods, GDP declines very fast, falls below steady state soon and goes through a deep and long-lasting slack period. If the expenditure plan is more persistent (but starts at a lower level), output gains are more sustainable and output falls below steady state at a later point of time.

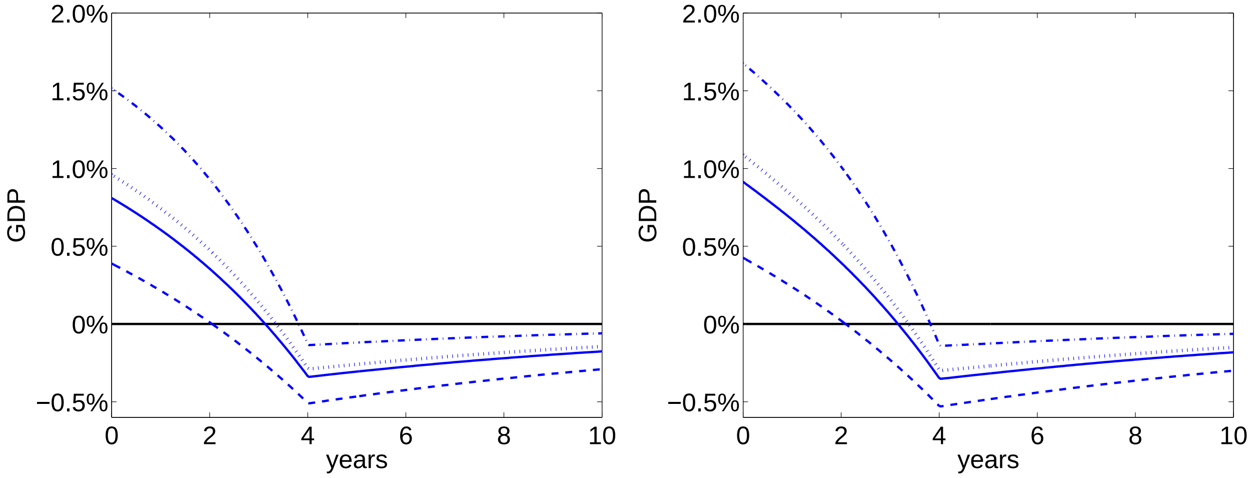
These results can be compared to Mountford and Uhlig (2010) and Uhlig's (2010) investigation of the ARRA plan. Mountford and Uhlig provide VAR estimates for the multiplier for a one-year deficit spending program, which then phases out gradually. They find an output multiplier of 0.65 on impact (after one quarter), 0.27 (and insignificantly different from zero) after 4 quarters and -0.74 after 8 quarters (Table IV). Uhlig (2010) approximates the ARRA spending plan with an AR(2) process and finds that GDP turns negative during the third year of spending.

5.3. **Power of the Multiplier.** Figure 7 show deviation from laissez-fair recovery for alternative assumptions about the power of the output multiplier of government spending. Since all multipliers are decaying and turn negative eventually, we measure the power of a multiplier by the change in output that would be provoked if the increase of government expenditure were *permanent*. As a rule, a high multiplier for permanent expenditure is associated with a high impact multiplier for temporary expenditure.

Solid lines reiterate the benchmark case. Dashed lines reflect the outcome when $\gamma = 5$. In this case the implied Frisch elasticity is 3/5, closer to the micro estimates (e.g. Pencavel, 1986). The weaker response of employment implies a reduction of the long-run multiplier from 0.89 to 0.54. The impact multiplier of the temporary program declines from 0.43 to 0.21. Dotted lines show the result for the case of $\gamma = 1$, which is frequently investigated in the RBC literature. Then, the implied elasticity of labor supply assumes a value of 3 and the long-run multiplier of government spending is 0.97 (impact multiplier 0.50). Finally, dashed-dotted lines reflect the case of $\gamma = 0$, i.e. the case when labor enters linearly in utility. This assumption gets theoretical foundation from the indivisible labor approach and has been very popular in the RBC literature as well. In this case the long-run multiplier assumes its maximum of 1.22 (impact multiplier 0.73).

Naturally, our main finding that GDP falls below its level under laissez-faire recovery before deficit spending ends (at year 4) is obtained regardless of the assumed power of the multiplier. The model produces more optimistic predictions for the short run when the multiplier is more powerful. In this case the positive effect through higher employment is stronger such that the initial impact is larger and the fall below laissez faire is smaller. The negative effect through delayed investment, however, is never overturned. On the other hand, if the implied labor supply elasticity is closer

Figure 7: Robustness w.r.t. Magnitude of the Government Multiplier



Left panel: A-scenario. Right panel: ω-scenario. Solid lines: benchmark case, dashed lines: $\gamma = 4$, dotted lines: $\gamma = 1$, dashed-dotted lines $\gamma = 0$.

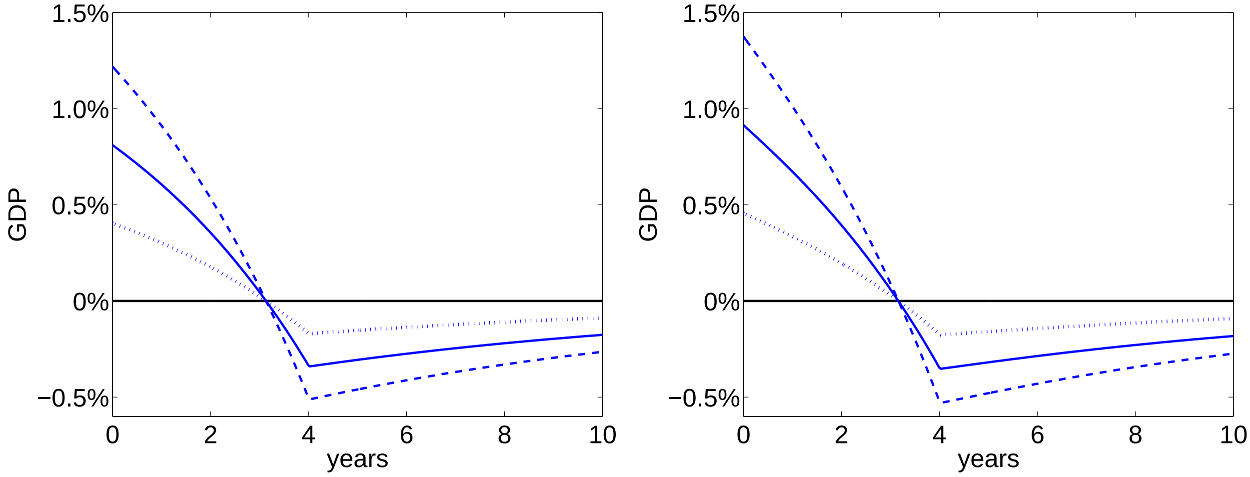
to what micro-econometricians suggest and the long-run multiplier is smaller than assumed by our benchmark scenario, the fall of GDP below laissez-faire level happens earlier and is harder.

5.4. **Scale of the Stimulus Program.** In conjunction with the surprisingly mild impact of ARRA on recovery from recession, it has been argued that the spending program was "too small". We know already from the theoretical part of the paper that our main result is independent from size. A quantitative investigation is nevertheless interesting since it reveals a trade-off: A large deficit spending program leads to a large improvement of GDP on impact and a large relapse of GDP below the laissez-faire recovery level at time of expiry.

The trade-off is demonstrated in Figure 8 where solid lines reiterate adjustment dynamics of the benchmark model. Dashed lines show adjustment dynamics when the increase of government purchases is much higher than assumed for our benchmark scenario (ΔG rises such that the GDP share Δg increases by 3 percentage points). The dotted line shows adjustment dynamics when the increase of government purchases is smaller than in our benchmark scenario (Δg increases by one percentage point). As a rule, economic recovery in the medium and long-run is the slower the larger the scale of the deficit spending program.

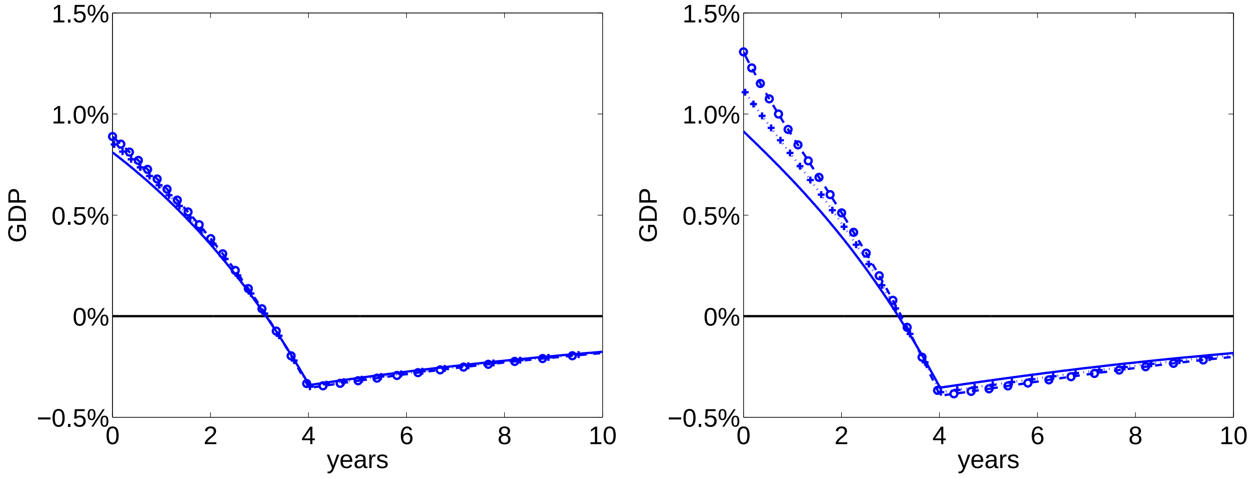
5.5. **Severity of the Recession.** In Figure 9 we investigate alternative assumptions about the severity of the recession. Solid lines reiterate the benchmark case and dashed and dotted lines show adjustment dynamics when the initial fall of output is assumed to be (much) harder, i.e. of 14 or 19 percent below steady-state level. For better visibility dashed lines are augmented with crosses and

Figure 8: Robustness w.r.t. Scale of Deficit Spending



Left panel: A-scenario. Right panel: ω-scenario. Solid lines: benchmark case, dashed lines: $\Delta g = 0.02$, dotted lines: $\Delta g = 0.08$.

Figure 9: Robustness w.r.t. Severity of the Recession

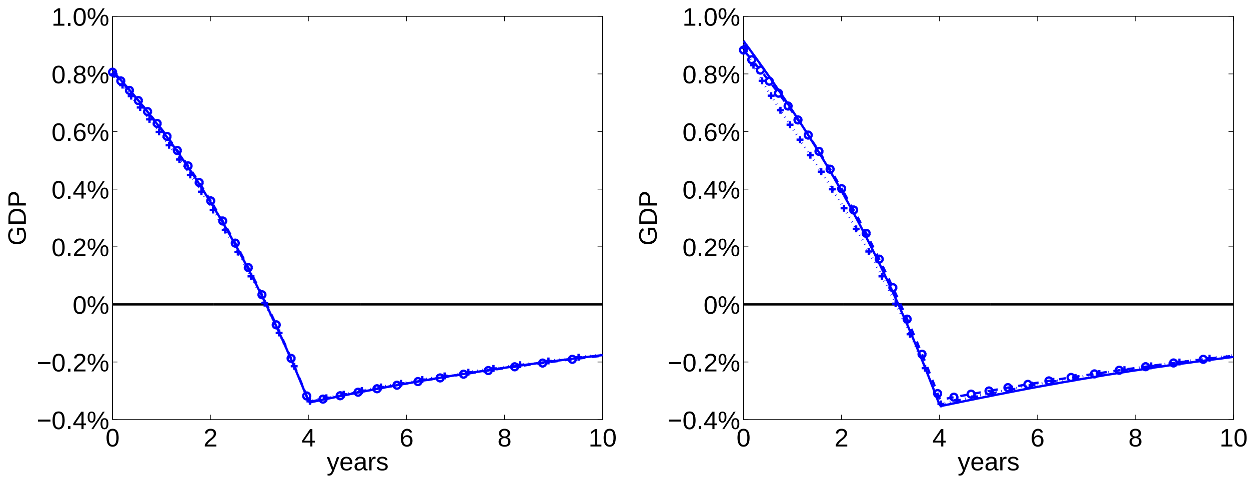


Left panel: A-scenario. Right panel: ω-scenario. Solid lines: $\Delta Y(0) = 7\%$, dashed lines (+): $\Delta Y(0) = 12\%$, dotted lines (o): $\Delta Y(0) = 18\%$.

dotted lines with circles. We see that the *relative* deviation from laissez faire is almost invariant to the assumed severity of the recession. This means that the non-linear forces in the model are not particularly strong.

5.6. **Duration of the Recession.** Finally, Figure 10 shows the implications for alternative assumptions about the duration of the recession. Solid lines show adjustment dynamics for the benchmark case of $\sigma = 0.4$, i.e. for a half-life of recovery of 2 years. Dotted lines show adjustment dynamics when recovery is assumed to be much faster with a half life of half a year ($\sigma = 1.5$,) and dashed lines show results for a much slower recovery with a half life of 12 years ($\sigma = 0.1$). The relative deviation from laissez faire is almost invariant to the assumed length of the recession. In any case the government spending multiplier turns negative before the deficit spending program expires.

Figure 10: Robustness w.r.t. Duration of the Recession (σ)



Left panel: A-scenario. Right panel: ω-scenario. Solid lines: benchmark case, dashed lines (+): $\sigma = 0.1$, dotted lines (o): $\sigma = 1.5$

6. Final Remarks

In this paper we have formally shown for the standard neoclassical business cycle model that the output multiplier of deficit spending turns negative before deficit spending expires. If deficit spending sets off in a recession this means that it slows down recovery in the medium run. Before deficit spending expires, GDP falls short of an otherwise identical laissez-faire economy.

It is perhaps appropriate to emphasize that we have never employed a welfare argument and thus have not argued against deficit spending as such. Temporarily increasing government spending could be very sensible in order to reduce individual hardships after a severe shock and may even raise aggregate welfare. We have just tried to explain why fiscal stimulus has a dark side. The price of a boost of GDP and employment on impact is paid later on, when recovery turns out to be slower than under laissez-faire. This price is not paid in a very distant future (by the next generation) but relatively soon, before deficit spending expires.

In order to mitigate or perhaps even overturn the negative result we could have given government purchases a bigger chance to boost investment by introducing elements beyond the standard neoclassical growth model (and beyond standard new-Keynesian reasoning). For example, we could have assumed that (a large part of) the additional government spending is productivity enhancing and leads through this channel to higher growth of aggregate output. But then the question arises why these expenditures are not planned to be permanent and why, in the first place, they have not been made at times before the recession when tax revenues to finance them were more plentiful.

On the other hand, we could have easily made the negative repercussions from deficit spending more severe. The most natural channel would be that deficit spending is, later on, not financed by lump sum taxes but – more realistically – by distortionary taxes on income. Such a scenario is investigated by Uhlig (2010). We have demonstrated that Uhlig's result of a multiplier turning negative after about 3 years after enactment of deficit spending accords very well with our results. We thus argue that, ultimately, the negative medium-run multiplier should not be attributed to disincentives caused by distortionary taxation. There is a deeper and more general mechanism at work: the "missing capital stock" caused by private investment below laissez-faire performance during the deficit spending phase.

Appendix

Derivation of the $\dot{c} = 0$ isocline. From $\dot{c} = 0$ the capital-labor ratio is given by

$$\frac{k}{\ell} = \left(\frac{A\alpha}{\delta + \rho}\right)^{\frac{1}{1-\alpha}} .$$

We differentiate the households' first order condition

$$c = \frac{(1-\ell)^\gamma w}{\beta} = \frac{1-\alpha}{\beta} A (1-\ell)^\gamma \left(\frac{k}{\ell}\right)^\alpha \tag{17}$$

keeping k/ℓ constant and obtain

$$\left.\frac{dc}{d\ell}\right|_{\dot{c}=0} = -\frac{\gamma c}{1-\ell} < 0 .$$

We thus calculate the slope of $\dot{c} = 0$ as

$$\frac{dc}{dk} = -\left(\frac{A\alpha}{\delta + \rho}\right)^{-\frac{1}{1-\alpha}} \frac{\gamma c}{1-\ell} < 0 .$$

For $\gamma \to \infty$ the $\dot{c} = 0$ line is vertical in the (k, c)-space, and for $\gamma = 0$ it is horizontal. For any other value it is downward sloping. Note that the $\dot{c} = 0$ isocline does not shift if government spending G increases. Consumption is rising for a capital labor ratio above steady-state level, implying that the arrows of motion point upward to the right of the isocline. Consumption is falling for a capital labor ratio below steady-state level, arrows point downward to the left of the isocline.

Derivation of the $\dot{k} = 0$ isocline. From $\dot{k} = 0$ we get

$$c = Ak^\alpha \ell^{1-\alpha} - \delta k - G \tag{18}$$

Together with equation (17) this provides (19)

$$\frac{1-\alpha}{\beta} A (1-\ell)^\gamma \left(\frac{k}{\ell}\right)^\alpha = Ak^\alpha \ell^{1-\alpha} - \delta k - G \tag{19}$$

From (19) we calculate the differential $d\ell/dk$:

$$\left.\frac{d\ell}{dk}\right|_{\dot{k}=0} = -\frac{\alpha A k^{\alpha-1}\ell^{1-\alpha} - \delta - \frac{\alpha c}{k}}{A(1-\alpha)k^\alpha \ell^{-\alpha} + \frac{\gamma c}{1-\ell} + \frac{\alpha c}{\ell k}} = -\frac{r - \frac{\alpha c}{k}}{w + \frac{\gamma c}{1-\ell} + \frac{\alpha c}{\ell k}} .$$

In combination with equation (18) we calculate the slope of the $\dot{k} = 0$ isocline as

$$\frac{dc}{dk} = A\alpha k^{\alpha-1}\ell^{1-\alpha} - \delta - Ak^{\alpha}(1-\alpha)\ell^{-\alpha}\frac{r - \frac{\alpha c}{k}}{w + \frac{\gamma c}{1-\ell} + \frac{\alpha c}{\ell k}} = r + w\frac{\frac{\alpha c}{k} - r}{w + \frac{\gamma c}{1-\ell} + \frac{\alpha c}{\ell k}}.$$

In case of inelastic labor supply the last term on the right hand side vanishes and $dc/dk = r = \partial Y/\partial K - \delta$. In this case the slope would be positive and diminishing for low k. Once the capital stock is larger than the golden rule capital stock, the slope would turn negative. The $\dot{c} = 0$ and $\dot{k} = 0$ isoclines intersects at a capital stock smaller than the golden rule capital stock where the slope of $\dot{k} = 0$ is positive.

Elastic labor supply changes the slope of the $\dot{k} = 0$ isocline. Since the economy produces with a Cobb-Douglas technology it is straightforward to calculate capital and labor income shares as αy and $(1-\alpha)y$. We take $(r+\delta)k = \alpha y$ and insert the steady state condition $y = c + \delta k + G$ and get

$$(r+\delta)k = \alpha(c + \delta k + G) \quad \Leftrightarrow \quad rk = \alpha c - (1-\alpha)\delta k + \alpha G \quad \Leftrightarrow \quad r = \alpha\frac{c}{k} - (1-\alpha)\delta + \alpha\frac{G}{k}.$$

With this expression the slope modifies to

$$\frac{dc}{dk} = r + w\frac{(1-\alpha)\delta - \alpha\frac{G}{k}}{w + \frac{\gamma c}{1-\ell} + \frac{\alpha c}{\ell k}}.$$

Hence, for $G = 0$ the slope of the isocline is steeper than for the case of inelastic labor supply. If G is sufficiently small, the intersection point of $\dot{c} = 0$ and $\dot{k} = 0$ is where $dc/dk > 0$. Capital is falling for values of consumption above the isocline (implying that arrows point to the left in this region), and rising for values of consumption below the curve (arrows point to the right).

Finally, we calculate how government spending shifts the $\dot{k} = 0$ isocline. Holding capital constant we get from $\dot{k} = 0$

$$\frac{dc}{dG} = -1 + A(1-\alpha)k^{\alpha}\ell^{-\alpha}\frac{d\ell}{dG} = -1 + w\frac{d\ell}{dG}.$$

If households would not adjust labor supply, one dollar extra government spending implies one dollar less of consumption for given capital stock. From equation (19) we get for constant k

$$\left.\frac{d\ell}{dG}\right|_{\dot{k}=0} = \frac{1}{w + \frac{\gamma c}{1-\ell} + \frac{\alpha c}{\ell k}}.$$

Together this implies

$$\frac{dc}{dG} = -1 + \frac{w}{w + \frac{\gamma c}{1-\ell} + \frac{\alpha c}{\ell k}} < 0.$$

Hence, the $\dot{k} = 0$ isocline shifts down when government spending increases.

Proof of Proposition 1. Consider an economy with capital stock below steady-state level. Phase diagram analysis shows that the economy converges towards the steady state on a saddlepath below the $\dot{c} = 0$–isocline. The proof first shows that along the saddlepath the capital labor ratio is smaller than at the steady state. From that we infer that the interest rate is higher than at the steady state, and the wage rate is smaller than at the steady state.

Recall that k/ℓ is at steady-state level along the $\dot{c} = 0$–isocline. We calculate $dc/d\ell$ for a constant capital stock. From equation (17) we obtain

$$\left.\frac{dc}{d\ell}\right|_{k=\bar{k}} = -\frac{\gamma c}{1-\ell} - \frac{c\alpha}{\ell k} < 0\,.$$

This implies that $k/\ell < (k/\ell)^*$ if the economy is located below the $\dot{c} = 0$–isocline and thus $k/\ell < (k/\ell)^*$ if the economy adjusts along the saddlepath towards the steady-state from below. Since $r = \alpha A(k/\ell)^{\alpha-1} - \delta$ and $w = (1-\alpha)A(k/\ell)^{\alpha}$, this in turn implies that $r > r^*$ and $w < w^*$ for $k < k^*$.

References

Aiyagari, S.R., L.J. Christiano, and M. Eichenbaum, 1992, The output, employment, and interest rate effects of government consumption, *Journal of Monetary Economics* 30, 73-86.

Angeletos, G.-M. and V. Panousi, 2009, Revisiting the supply side effects of government spending, *Journal of Monetary Economics* 56, 137-153.

Auerbach, A.J., W.G. Gale, and B.H. Harris, 2010, Activist fiscal policy, *Journal of Economic Perspectives* 24(4), 141-164.

Barro, R,J., 1981, Output effects of government purchases, *Journal of Political Economy* 89, 1086-1121.

Barro, R,J., and C.J. Redlick, 2011, Macroeconomic effects from government purchases and taxes, *Quarterly Journal of Economics*, forthcoming.

Barro, R.J. and X. Sala-i-Martin, 2004, *Economic Growth*, MIT Press, Cambridge, MA.

Baxter, M. and R.G. King, 1993, Fiscal policy in general equilibrium, *American Economic Review* 83, 315-334.

Burnside, C., M. Eichenbaum, and J. Fisher, 2004, Fiscal shocks and their consequences, *Journal of Economic Theory* 115, 89-117.

Chari, V.V., P.J. Kehoe, and E.R. McGrattan, 2002, Accounting for the Great Depression, *American Economic Review* 92, 22-27.

Chari, V.V., P.J. Kehoe, and E.R. McGrattan, 2007, Business cycle accounting, *Econometrica* 75, 781-836.

Cogan, J.F., T. Cwik, J.B. Taylor, V. Wieland, 2010, New Keynesian versus old Keynesian government spending multipliers, *Journal of Economic Dynamics and Control* 34, 281-295.

Cole, H.L. and L.E. Ohanian, 2002, The U.S. and U.K. Great Depressions through the lens of neoclassical growth theory, *American Economic Review* 92, 28-32.

Cole, H.L. and L.E. Ohanian, 2004, New deal policies and the persistence of the Great Depression: a general equilibrium analysis, *Journal of Political Economy* 112, 779-816.

Cooley, Th.F. and G. D. Hansen, 1992, Tax distortions in a neoclassical monetary economy, *Journal of Economic Theory* 58, 290-316.

Feldstein, M., 2010, Rethinking the role of fiscal policy, *American Economic Review: Papers and Proceedings* 99, 556-559.

Forni, L., L. Monteforte, and L. Sessa, 2009, The general equilibrium effects of fiscal policy: estimates for the Euro area, *Journal of Public Economics* 93, 559-585.

Greenwood, J. and G.W. Huffman, 1991, Tax analysis in a real business cycle model, *Journal of Monetary Economics* 27, 167-190.

Hall, R.E., 2009, By how much does GDP rise if the government buys more output? *Brookings Papers on Economic Activity* Fall 2009, 183- 228.

Hansen, G., 1985, Indivisible labor and the business cycle, *Journal of Monetary Economics* 16, 309-327.

Judd, K.L., 1985, Short-run analysis of fiscal policy in a simple perfect foresight model, *Journal of Political Economy* 95, 298-319.

Kehoe, T.J. and E.C. Prescott, 2002, Great Depressions of the 20th century, *Review of Economic Dynamics* 5, 1-18.

Kehoe, T.J. and E.C. Prescott, 2007, *Great Depressions of the Twentieth Century*, Federal Reserve Bank of Minneapolis.

King, R.G., C. Plosser, and S. Rebelo, 1988, Production, growth, and business cycles: I. The basic neoclassical model, *Journal of Monetary Economics* 21, 195-232.

King, R.G., and S. Rebelo, 1999, Resuscitating real business cycles, in: B. Taylor and M. Woodford (eds.), *Handbook of Macroeconomics* 1B, 927-1007.

Ludvigson, S., 1996, The macroeconomic effects of government debt in a stochastic growth model, *Journal of Monetary Economics* 38, 25-45.

Mankiw, N.G. and Weinzierl, M., 2006, Dynamic scoring: A back-of-the-envelope guide, *Journal of Public Economics* 90, 1415-1433.

McGrattan, E.R., 1994, The macroeconomic effects of distortionary taxation, *Journal of Monetary Economics* 33, 573-604.

McGrattan, E.R. and L.E. Ohanian, L.E., 2010, Does Neoclassical Theory Account for the Effects of Big Fiscal Shocks? Evidence from World War II, *International Economic Review* 5, 509–532.

Mountford, A. and H. Uhlig, 2008, What are the effects of fiscal policy shocks?, *Journal of Applied Econometrics* 24, 960-992.

Ohanian, L.E., 2010, The economic crisis from a neoclassical perspective, *Journal of Economic Perspectives* 24(4), 45-66.

Pencavel, J., 1986, Labor supply of men: a survey, in: O. Ashenfelter and R. Layard, eds., *Handbook of Labor Economics*, North-Holland, Amsterdam, 3-102.

Ramey, V.A., 2011, Identifying government spending shocks: it's all in the timing, *Quarterly Journal of Economics* 126, forthcoming.

Romer, C.D. and J. Bernstein, 2009, The job impact of the American recovery and reinvestment plan, Council of Economic Advisors, January 2009.

Rogerson, R., 1988, Indivisible labor, lotteries and equilibrium, *Journal of Monetary Economics* 21, 3-16.

Strulik, H. and T. Trimborn, 2010, Anticipated Tax Reforms and Temporary Tax Cuts: A General Equilibrium Analysis, *Journal of Economic Dynamics and Control*, forthcoming.

Smets, F., R. Wouters, 2007, Shocks and frictions in US business cycles: a Bayesian DSGE approach, *American Economic Review* 97, 586-606.

Trabandt, M. and H. Uhlig, 2009, How far are we from the slippery slope? The Laffer curve revisited, NBER Working Paper 15343.

Trimborn, T., K.-J. Koch, and T.M. Steger, 2008, Multi-dimensional transitional dynamics: A simple numerical procedure, Macroeconomic Dynamics, 12, 1-19.

Turnovsky, S., 2004, The transitional dynamics of fiscal policy: long-run capital accumulation and growth, *Journal of Money, Credit and Banking* 36, 883–910.

Uhlig, H., 2010, Some Fiscal Calculus, *American Economic Review: Papers and Proceedings* 100, 30-34.