The Efficiency of Unfunded Pension Schemes

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Abstract: Public pension schemes can be designed either as capital reserve systems or as unfunded (or pay-as-you-go) schemes. In the literature it has been alleged that unfunded schemes are intergenerationally efficient in Pareto's sense. Here we show that this holds only if contributions to the system are levied as lump-sum taxes. But in reality, flat-rate income taxes are normally used instead – and then, an unfunded scheme induced distortions and can completely be abolished in finite time without inflicting damage upon any generation.

JEL-Classification: H21, H55

1. Introduction

In an important article in this *Journal,* Breyer [1989] has considered the problem of the efficiency of unfunded (or pay-as-you-go: PAYG) pension schemes. He found that these schemes are intergenerationally efficient in Pareto's sense even when the rate of interest *permanently* exceeds the growth rate. In such a scenario, an infinity of generations are made worse off and only the first one is made better off by the introduction of PAYG. Nevertheless, the unfunded pension scheme turns out to be efficient for the following reason: In Breyer's model, contributions to that system are introduced as *lump-sum-taxes* and the pensions are *lump-sum-transfers.* Now, applying the second basic theorem of welfare economies, we know that any lump-sum redistribution of incomes entails an allocation which is different but also Pareto-efficient. Therefore, Breyer's result is intuitively understandable.

The present paper seeks to take into account the fact that, in reality, contributions to PAYG are *never* raised as lump-sum payments. Consider the US Social Security System, for instance. Premium payments are levied as a flat-rate tax on labor incomes whereas pension payments are virtually lump-sum – at least in the sense that they are independent of the preceding contributions. Thus, a PAYG of the American type is basically a tax-transfer-system which distributes income between generations. We will show in the sequel that such a system is always inefficient and that, moreover, it can be converted into a capital-reserve system without inflicting damage upon anyone.

2. The Basic Model

Our framework is the standard overlapping-generations model without bequests which has been widely used for analyzing the impacts of public pension schemes. We consider the case of a small open economy[1] which is characterized by exogenous sequences of wage rates w_t and interest factors $R:=1+r$. For convenience, interest factors and population are supposed to be constant. The wage rates w_t, however, may vary in a perfectly arbitrary manner; thus the growth rate may be greater or smaller than the interest rate. In every period t, a representative member of the younger generation solves

$$\begin{aligned} &\max_{c_t^1, c_{t+1}^2, l_t}! \quad U(c_t^1, c_{t+1}^2, 1-l_t) \\ (1) \qquad &\text{s.t. (i)} \quad c_t^1 + s_t = w_t l_t (1-\tau), \\ &\quad\ \ \text{(ii)} \quad c_{t+1}^2 = R s_t + p_{t+1}, \end{aligned}$$

1 The assumption of a small open economy is more favorable to a pay-as-you-go system since, with interest and wage rates given from outside, a decrease in domestic savings will not hurt later generations via depressed domestic wages. Breyer [1989] analyzes the open as well as the closed economy and finds pay-as-you-go schemes to be efficient in both cases.

where c_t^1 is consumption in the first period, c_{t+1}^2 is consumption during the retirement period, s_t denotes savings and l_t is the labor supply, the sum of labor and leisure time having been normalized to one. U(.) is assumed to be strictly monotonically increasing, strictly quasi-concave, and twice continuously differentiable. The goods shall be normal.

The variable τ is either a *premium rate* or an *income tax rate,* and p_{t+1} is a *public pension.* Thus every household must pay the amount $\tau w_t l_t$ when young and obtains a payment p_{t+1} when old. A *pay-as-you-go system* (PAYG) with a constant premium rate is simply a sequence $(\bar{\tau}, \bar{p}_t)$, where both $\bar{\tau}$ and all $\bar{p}_t$ are strictly positive and where $\bar{p}_t = \bar{\tau}\bar{w}_t\bar{l}_t$ for all t This last equation says that the pension payments which are made to the preceding generation must equal current contributions; and the contribution rate has no time index because it has been assumed constant. Note that the choice of a particular PAYG generally affects the households' labor supplies; therefore, wage income $\bar{w}_t\bar{l}_t$ must be evaluated at the resulting perfect-foresight equilibrium position which, of course, need not be uniquely determined. Henceforth, all variables which are associated with some PAYG will be signed with a bar (¯).

It is now straightforward to show that every PAYG is inefficient in the following sense: Generation t can be made better off without any other generation being made worse off. This is done using a system I call *capital reserve system cum government debt (CRCD).* Such a system will be shown to be the optimal solution to the following maximization problem of the state:

$$\text{(2)} \qquad \begin{aligned} &\max_{\tau_t, p_{t+1}, D_t}! \quad V(\tau, p_{t+1}) \\ &\text{s.t. (2.1)} \quad \tau w_t l_t = \bar{\tau}\bar{w}_t\bar{l}_t - D_t, \\ &\quad\;\;\, (2.2) \quad p_{t+1} = \bar{p}_{t+1} - RD_t, \\ &\quad\;\;\, (2.3) \quad p_{t+1} \geq 0. \end{aligned}$$

Here $V_t(\tau, p_{t+1})$ is the indirect utility function of generation t which shall be maximized by planning an optimal policy (τ, p_{t+1}, D_{t+1}) in period t. In initial equilibrium, a particular PAYG is in operation, and the amount $\bar{\tau}\bar{w}_t\bar{l}_t$ is given to the elderly. The constraint (2.1) requires that this payment may be financed either by an income tax $\tau w_t l_t$ or by government debt, D_t. The total amount $\tau w_t l_t + D_t$ however, must be equal to the initial payment $\bar{\tau}\bar{w}_t\bar{l}_t$ so that the preceding generation is not made worse off. Assume that a certain government debt $D_t > 0$ has been chosen in period t. Then, according to (2.2), generation t must repay that debt plus interest before obtaining a pension payment p_{t+1}. Put differently, the debt is not shifted into the future but is completely repaid by generation t so that the subsequent generations are not made worse off.

Finally, (2.3) demands $p_{t+1} \geq 0$, because a negative pension payment would have to be interpreted as a *lump-sum-tax.* Because it is infeasible to levy lump-sum-taxes, we exclude this possibility. The constraints (2.2) and (2.3) together show that government debt never exceeds the "implicit debt" of PAYG: RD_t will not exceed $\bar{p}_{t+1}$. Subtracting R times (2.1) from (2.2) yields the combined constraint of the above optimization problem:

$$(3) \qquad p_{t+1} - R\tau w_t l_t = \bar{p}_{t+1} - R\bar{\tau}\bar{w}_t\bar{l}_t \,.$$

This says that the choice of a particular policy (τ, p_{t+1}, D_t) may not change the generation's lifetime income. As $\bar{p}_{t+1}$ equals $\bar{\tau}$ times the wage income of the subsequent period, it is easy to see that expression (3) is positive whenever the growth rate exceeds the interest rate. The generation will then be called a *winner* of PAYG. In the more interesting case where R exceeds the growth factor $\bar{w}_{t+1}\bar{l}_{t+1}/(\bar{w}_t\bar{l}_t)$, expression (3) will become negative, and the generation will be called a *loser*.

Proposition: (i) If generation t is a *winner*, the only optimal policy $(\tau^*, p^*_{t+1}, D^*_t)$ is defined by $\tau^* = 0$, where p^*_{t+1} and D^*_t can be calculated from the constraints.

(ii) If generation t is a *loser*, the only optimal policy $(\tau^*, p^*_{t+1}, D^*_t)$ is defined by $p^*_{t+1} = 0$, where τ^* and D^*_t can be calculated from the constraints.

(iii) If generation t is neither a winner nor a loser, the only optimal policy $(\tau^*, p^*_{t+1}, D^*_t)$ is defined by $\tau^* = 0$ and $p^*_{t+1} = 0$, where D^*_t can be calculated from the constraints.

(iv) PAYG, i.e. $\bar{\tau}, \bar{p}_{t+1} > 0$ and $\bar{D}_t = 0$ is optimal in neither case.

Proof: From the household's combined budget constraint

$$(4) \qquad Rc^1_t + c^2_{t+2} = R(1-\tau)w_t l_t + p_{t+1}$$

we obtain the familiar Slutzky-equation

$$(5) \qquad \left(\frac{\partial l_t}{\partial \tau}\right)^c = \frac{\partial l_t}{\partial \tau} + Rw_t l_t \frac{\partial l_t}{\partial p_{t+1}} < 0,$$

which states that the compensated elasticity of labor supply l_t with respect to the premium rate τ is strictly negative. This follows immediately from the strict quasi-concavity of U(.). In order to solve the government's optimization problem (2) we calculate the derivative of $V_t(.)$ with respect to τ:

$$(6) \qquad \frac{dV_t}{d\tau} = \frac{\partial V_t}{\partial \tau} + \frac{\partial V_t}{\partial p_{t+1}} \frac{dp_{t+1}}{d\tau} \,.$$

Now, from (3) we obtain the derivative $dp_{t+1}/d\tau$ by applying the implicit function theorem:

$$(7) \qquad \frac{dp_{t+1}}{d\tau} = \frac{R\,w_t l_t + R\,w_t \tau\, \partial l_t/\partial\tau}{1 - R\,\tau\, w_t\, \partial l_t/\partial p_{t+1}}$$

Combining (6) with Roy's identity

$$(8) \qquad \frac{\partial V_t}{\partial \tau} = -R\,w_t l_t \frac{\partial V_t}{\partial p_{t+1}}$$

and substituting $dp_{t+1}/d\tau$ from (7) gives

$$(9) \qquad \frac{dV_t}{d\tau} = \left[-R\,w_t l_t + \frac{R\,w_t l_t + R\,w_t \tau\, \partial l_t/\partial\tau}{1 - R\,\tau\, w_t\, \partial l_t/\partial p_{t+1}}\right] \frac{\partial V_t}{\partial p_{t+1}}$$

Finally, after rearranging terms and employing the Slutzky-equation (5) we obtain

$$\text{(10)} \qquad \frac{dV_t}{d\tau} = \left(\frac{\partial l_t}{\partial \tau}\right)^c \frac{R\, w_t \tau}{1 - R\,\tau\, w_t\, \partial l_t / \partial p_{t+1}} \frac{\partial V_t}{\partial p_{t+1}}.$$

As p is a lump-sum payment, $\partial V_t/\partial p_{t+1} > 0$ and $\partial l_t/\partial p_{t+1} < 0$ (remember that leisure, $1-l_t$, was assumed to be normal). Thus, for $\tau > 0$, all terms in (10) are strictly positive except $(\partial l_t/\partial \tau)^c$ which is strictly negative. We conclude, therefore, that $dV_t/d\tau$ is strictly negative for $\tau > 0$ and strictly positive for $\tau < 0$. So $\tau^* = 0$ is the globally unique optimum. This establishes (i).

If the generation is a net loser, however, $\tau^* = 0$ implies $p^*_{t+1} < 0$, which is not possible. In this case, τ must be gradually increased until $p^*_{t+1} = 0$. The corner solution is a unique optimum because $dV_t/d\tau < 0$ for all $\tau > 0$. This proves (ii). The claims (iii) and (iv) follow at once. Q.E.D.

The intuition behind this result is very simple indeed. One has only to realize that PAYG consists of an *income tax* (the premium payment $\tau w_t l_t$) and a *lum-sum transfer* (the pension payment p_{t+1}) Replacing PAYG by government debt entails a simultaneous reduction in the income tax and the lump-sum transfer. This is obviously beneficial to the household because the income tax distorts the labor-leisure choice and hence induces an excess burden. Setting $(\partial l_t/\partial \tau)^c = 0$ in (10), i.e. assuming an exogenous labor supply, we obtain the Breyer result: Changes in τ neither increase nor reduce the household's welfare, provided that its lifetime income remains unchanged.

3. Making all Generations Better off

Subsequent to the partial analysis of the preceding section, we now consider the sequence of *all* generations, $V_t(.)$ denoting again generation t's utility level. All generations, however, are assumed to have the same direct utility function. We assume that a PAYG is already in operation and construct an alternative system which makes every generation $t \geq 1$ better off. In doing so, we deal only with situations where the interest rate exceeds the growth rate in initial equilibrium so that every generation is a *loser.* This is the case which has been considered by most authors, including Breyer. The reader will note, however, that our results also hold for the reverse case. This follows immediately from statement (i) in the above proposition. An explicit consideration of that reverse case would make the following notation rather cumbersome but would not introduce analytical difficulties. From now on, $W_t = w_t l_t$ denotes *aggregate wage income.*

Corollary 1: Assume that, with the interest rate exceeding the growth rate, some specific PAYG is already in operation: $\bar{\tau} > 0$ and $\bar{p}_t = \bar{\tau}\overline{W}_t$. Then, there exist sequences (τ^*_t and $p^*_t = (\bar{p}_1, 0, 0, \ldots)$ such that

$$\text{(11)} \qquad \tau^*_t W_t = \bar{\tau}\overline{W}_t - \bar{p}_{t+1}/R \quad \text{and}$$

$$\text{(12)} \qquad V_t(\tau^*_t, 0) > V_t(\bar{\tau}, \bar{p}_{t+1}) \quad \text{for all} \quad t \geq 1.$$

Proof: Immediate from the proposition, (ii).

The system thus constructed has already been called CRCD. From the household's view, it is a pure capital reserve system (CR) because no pension payments are made. But on the aggregate level, such a system implies a sequence of government debts which are equal to the implicit debt of PAYG. Therefore, the economy's net wealth is the same as before. Note that the presently living elderly are completely unaffected by a transition from PAYG to CRCD because they get the pension payment $\overline{p}_t$ anyway. But all subsequent generations are made better off because CRCD entails a reduction in the premium rates and thus diminishes the excess burden.

4. A Pareto-improving Conversion Policy

Among others, Breyer [1989] has posed the question whether it is possible to convert a PAYG into a *pure* capital reserve system (without government debt), provided that no generation may be made worse off. His answer has been in the negative even when every generation (except the first) is hurt by PAYG because the interest rate exceeds the growth rate. Concentrating on this case also, we now want to show that, in an economy with an endogenous labor supply, such a Pareto-improving conversion policy is indeed possible. The implicit debt of PAYG is run down in *finite time* and up to that moment no generation will be made worse off. Thereafter, all generations are better off. In the following, τ_t are the premium rates which are used for such a conversion policy. It will be seen that these rates are greater than τ_t^*, the premium rates associated with CRCD, but smaller than $\overline{\tau}$ the premium rate of PAYG. Observe that, as labor supply is endogenous, total wage income W_t will generally vary if the contribution rate changes; therefore, the W_t in (11) and (13) are not necessarily the same.

Corollary 2: Assume that, with the interest rate exceeding the growth rate, some specific PAYG is already in operation: $\overline{\tau} > 0$ and $\overline{p}_t = \overline{\tau}\,\overline{W}_t$. Then there exist sequences (τ_t), $(p_t) = (\overline{p}_1, 0, 0, ...)$ and (ε_t) ≥ ε > 0 such that

$$(13) \qquad \tau_t W_t = \overline{\tau}\,\overline{W}_t - \overline{p}_{t+1}/R + \varepsilon_t \overline{W}_t \quad \text{and}$$

$$(14) \qquad V_t(\tau_t, 0) = V_t(\overline{\tau}, \overline{p}_{t+1}) \quad \text{for all} \quad t = 1 ... T.$$

Furthermore, there exists T such that $D_t \leq 0$ for all t > T.

Proof: The existence of $\tau_t > \tau_t^*$ is readily inferred from corollary 1: If we can tax the generations in a way that $V_t(\tau_t^*, 0)$ strictly exceeds $V_t(\overline{\tau}, \overline{p}_{t+1})$ we can also tax them a bit *more* heavily so that $V_t(\tau_t, 0)$ equals $V_t(\overline{\tau}, \overline{p}_{t+1})$ As all direct utility functions are identical, the ε_t also bounded away from zero. It remains to be shown that the debt is repaid in finite time. From (2.1), we have

$$(15) \qquad D_1 = \overline{p}_1 - \tau_1 W_1 ,$$

so that, using (13) and $\overline{p}_2 = \overline{\tau}\,\overline{W}_2$

(16) $$D_1 = \bar{\tau}\,\overline{W}_2 / R - \varepsilon_1 \overline{W}_1 \,.$$

Similarly,

(17) $$D_2 = R\,D_1 - \tau_2\, W_2 \,,$$

(18) $$\Leftrightarrow \quad D_2 = \bar{\tau}\,\overline{W}_3 / R - R\,\varepsilon_1 \overline{W}_1 - \varepsilon_2 \overline{W}_2$$

and for arbitrary T,

(19) $$D_T = \bar{\tau}\,\overline{W}_{T+1} / R - \sum_{t=1}^{T} R^{T-t}\,\varepsilon_t\,\overline{W}_t$$

(20) $$\Leftrightarrow \quad D_T = \frac{\overline{W}_{T+1}}{R}\left[\bar{\tau} - \sum_{t=1}^{T-1} \varepsilon_t\, R^{T+1-t}\,\frac{\overline{W}_t}{\overline{W}_{T+1}}\right].$$

As the interest rate exceeds the growth rate, we necessarily have $R^{T+1-t}\,\overline{W}_t / \overline{W}_{T+1} > 1$. (For a steady state path, we find $\overline{W}_t / \overline{W}_{T+1} = 1/G^{T+1-t}$ and R > G, where G is the growth factor. For other paths, G is the supremum of $\overline{W}_{t+1} / \overline{W}_t$. In either case, $(R/G)^{T+1-t} > 1$.) And as $\varepsilon_t \geq \varepsilon > 0$ the terms on the right-hand side are bounded away from zero. Hence there exists finite T such that the sum on the right exceeds $\bar{\tau}$. Q.E.D.

Again, the economic idea behind this proof is relatively simple. In the preceding section we used the efficiency gains which accrue from leaving PAYG for improving all generations' welfares. In the present section, the first T generations are made as well off as in PAYG, and the efficiency gains are used instead for repaying the government debt. It is thus possible to completely abolish PAYG in finite time and to restore the economy's net wealth to the level which is associated with a pure capital reserve system. And during the transition, no one is made worse off. This option is most attractive for those generations which are living in the far future because, from period T on, the individuals will no longer be taxed. On the other hand, the presently living households – among them all current voters – will prefer CRCD since this system makes them better off. Yet, considering the polar cases τ_t and τ_t^* as defined in the corollaries, we recognize at once that there exist premium rates *in between* which make the first generations better off and at the same time allow the debt to be repaid in finite time.

Reference

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