

Overaccumulation, Public Debt, and the Importance of Land

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Abstract: In recent contributions, Weizsäcker (2014) and Summers (2014) maintain that mature economies accumulate too much capital. They suggest large and lasting public deficits as a remedy. This paper argues that overaccumulation cannot occur in an economy with land. It presents novel data of aggregate land values, analyzes the issue within a stochastic framework, and conducts an empirical test of overaccumulation.

Keywords: Dynamic efficiency; overaccumulation; land; fiscal policy; public debt

JEL-Classification: D92, E62, H63

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1. Introduction

In a recent article, Carl-Christian von Weizsäcker (2014) challenged the prevailing skeptical view of public debt. In a modernized Austrian framework, he argues that the natural real rate of interest, i.e. the rate that would emerge in the absence of public debt, has become negative in OECD economies and China. Because nominal interest rates are positive, governments face an uneasy choice between price stability and fiscal prudence: They must either raise inflation in order to make negative real and positive nominal interest rates compatible, or lift the real interest rate into the positive region via deficit spending. To substantiate his point, Weizsäcker claims that the average waiting period, the ratio of private wealth and annual consumption, has risen historically whereas the average production period, the ratio of capital and annual consumption, has remained constant. He attributes the surge in the average waiting period—to an assumed value of 12—to demographic changes such as an increased life expectancy and concludes that public debt is necessary to preserve prosperity at stable prices.

At talks before the International Monetary Fund and the American Economic Association, Lawrence Summers (2014) advanced a similar argument. He diagnoses a savings glut—due to reduced population growth, global surpluses, and other factors—which has pushed the interest rate below the growth rate. Under such circumstances, Summers believes it would be “madness not to be engaged in substantially stimulative fiscal policies”, for otherwise the economy would have to face a future of low growth and high unemployment. These gloomy prospects, of course, remind strongly of Alvin Hansen’s (1939) secular stagnation hypothesis, both with respect to the alleged crisis cause, i.e. demographic change, and to the proposed remedy, i.e. large and lasting budget deficits.

The objective of this article is to show that the Hansen-Weizsäcker-Summers stagnation hypothesis is unfounded and that the policy conclusions drawn from it are misguided: From a theoretical point of view, there are strong reasons to believe that contemporary economies do not accumulate too much capital; this assertion is supported by the empirical evidence; and hence public deficits do not represent a free lunch but mainly crowd out productive capital formation, thus aggravating sluggish growth.

Section 2 presents a brief review of the overaccumulation issue. Although the discussion mostly takes the form of examples which are easy to follow, the text also includes an elementary proof of the main result and studies the consequences for fiscal policy. Section 3 continues with material which is less well-known, showing that overaccumulation cannot occur in an economy with land. Land enters the production function as a third factor beside labor and capital, and opens a new market which absorbs any potential excess capital supply. In particular, this market keeps interest rates above the growth rates. The land theorem is seldom used in macroeconomic discussions—Hans-Werner Sinn in his exchange with Summers (2014) being a notable exception. Since this neglect may result from a doubt in empirical relevance, section 4 presents novel figures of aggregate land values. These data are impressive, if not surprising. Thereafter, two possible objections to the land theorem are discussed in section 5.

Section 6 shows that the results derived so far for deterministic economies still hold in the presence of uncertainty. Introducing uncertainty is important because it allows to make a

distinction between safe and risky interest rates. As will be shown, a sensible empirical assessment of overaccumulation must deal with risky interest rates; the safe interest rates are immaterial and misleading. Building on these findings, section 7 presents a test which reaffirms the case against overaccumulation. Section 8 concludes the paper. Technical remarks, mathematical derivations, and data descriptions are relegated to the appendix.

In order to develop the argument, three methodical premises are made which should be noted at the outset. First, a closed economy is assumed throughout. Since the world as a whole is surely closed, an open economy model which allowed excess capital to be pushed abroad circumvents rather than addresses the problem under consideration. The well-known home bias puzzle (Obstfeld and Rogoff 2001) lends additional support to the closed economy assumption. Second, the analysis uses an overlapping generations model without bequests, where people perceive government bonds as net wealth. This assumption is made because Ricardian models with an operative bequest motive (Barro 1974) or with infinitely lived households (Sinn 1981) exclude overaccumulation in the first place. And third, the term public debt is restricted to genuine liabilities such as government bonds; it does not comprise contingent liabilities like public pensions which can be changed by law.

2. Characterizing Overaccumulation

Starting with Malinvaud (1953) and Cass (1972), a comprehensive literature on overaccumulation exists. Most papers in the field aim at establishing existence of equilibrium or necessary efficiency conditions and are rather intricate, which may disguise the fact that the core argument is quite simple. To show this and to have a reference point, it seems convenient to start with a model of pure accumulation which makes no regard to market institutions. In every period t , a homogenous output $Y_t > 0$ is used for consumption $C_t \geq 0$ and capital formation $K_t \geq 0$. Under full depreciation, the capital stocks formed in period t and used in period $t+1$ evolve according to

$$(1) \quad Y_{t+1} = F_t(K_t) = C_{t+1} + K_{t+1}, \quad \text{for } t=0, 1, 2, \dots$$

The production functions F_t are time dependent, admitting other production factors or technical change. They are also concave and, except perhaps at the origin, possess strictly positive marginal productivities $dF_t(K_t)/dK_t = R_{t+1}$, referred to as *interest factors* (or gross interest rates). Subject to an initial capital stock K_0 , a feasible growth path consists of a sequence of capital stocks K_1, K_2, \dots associated with two implied sequences of output and consumption. *Overaccumulation* means the existence of another path yielding strictly more consumption in some period and no less otherwise. If no such alternative exists, a path is called *dynamically efficient*.

The idea to characterize overaccumulation is to find capital stock reductions $\varepsilon_t > 0$ such that the remaining stocks allow more consumption in the first period and no less subsequently. If one reduces K_1 by $\varepsilon_1 > 0$, consumption C_1 increases by the same amount because first-period output is exogenous. As every concave function lies below its tangents, Y_2 falls at least by $R_2 \varepsilon_1$, and with C_2 undiminished, K_2 must be reduced by $\varepsilon_2 \geq R_2 \varepsilon_1$. By the same token, K_3 must be reduced by $\varepsilon_3 \geq R_3 R_2 \varepsilon_1$. Working forward this way shows that

the capital stock reduction in any period T satisfies $\varepsilon_T \geq R_T \dots R_3 R_2 \varepsilon_1$. Defining *growth factors* $G_t = Y_t / Y_{t-1}$, output in period T equals $Y_T = Y_1 G_T \dots G_3 G_2$ along the original path. Since capital stock reductions cannot exceed output, one obtains $\varepsilon_T \leq Y_1 G_T \dots G_3 G_2$. Combining the two inequalities and rearranging yields

$$(2) \quad \varepsilon_1 \leq Y_1 \frac{G_2 G_3 \dots G_T}{R_2 R_3 \dots R_T}, \quad \text{for } T = 2, 3, \dots$$

The desired sufficient condition for dynamic efficiency follows at once. It states that the right-hand fraction in the preceding inequality, evaluated along the original path, vanishes when the time horizon T goes to infinity:

$$(3) \quad \frac{G_2 G_3 \dots G_T}{R_2 R_3 \dots R_T} \rightarrow 0.$$

Under this premise, every $\varepsilon_1 > 0$ would obviously violate (2). Thus, overaccumulation cannot occur if the compound interest factor, shown in the denominator, eventually exceeds the compound growth factor, shown in the numerator. Since the right-hand side of (2) denotes the present value of period T output, this has a neat economic interpretation: If the present value of future output vanishes in the limit, resources cannot be pulled out from behind permanently.

The proposition admits population growth, technical change of any form, and linear production functions. It does not employ Inada or boundedness assumptions and thus is pretty general. Interest and growth factors, however, are strictly positive by construction in order to exclude two less interesting situations: With a vanishing interest factor, the associated *interest rate* r_t , defined by $R_t = 1 + r_t$, amounts to minus one hundred percent—representing a perfectly unreasonable investment. With a vanishing growth factor, the associated *growth rate* g_t , defined by $G_t = 1 + g_t$, amounts to minus one hundred percent, and the economy disappears in period t . All factors and rates must be understood in real terms.

According to an equivalent condition frequently used in the literature, dynamic efficiency obtains if the reciprocal product $R_2/G_2 \dots R_T/G_T$ diverges. Here the economic interpretation reads that an initial debt which is rolled over forever would eventually exceed total output. As this is impossible, dynamic efficiency impedes Ponzi schemes.

Along a steady state path with fixed factor proportions, overaccumulation is excluded if the constant interest factor exceeds the constant growth factor or, which amounts to the same thing, if the interest rate exceeds the growth rate. This is a direct implication of (3). It should be emphasized that from a normative point of view the *sign* of the interest rate is immaterial. What counts is its relationship to the growth rate. In an economy shrinking to zero at $g = -3$ percent, a negative interest rate $r = -2$ percent raises no efficiency problem because it exceeds the growth rate. The sign of the interest rate is important only in the case of a stationary economy to which the following examples pertain.

Example 1: Employing the capital stock $K = 1$, the economy produces $Y = 8K^{1/4}$. Output equals 8, consumption equals 7, and the interest factor equals 2. With $G/R = 1/2$, the product in (3) converges to zero and the allocation is dynamically efficient.

Example 2: Under the same technology, the initial capital stock is $K=81/16>5$. Output equals 12, consumption falls slightly below 7, and the interest factor equals $16/27$, implying a negative interest rate. With $G/R=27/16$, condition (3) is violated. Indeed, reducing the capital stock to $K=1$ allows to increase consumption by $65/16$ in the respective period and by $1/16$ in all following periods, as the comparison with example 1 reveals. This presents a blatant case of overaccumulation. While more capital always increases output, it diminishes national income $Y-K$ if the interest rate happens to become negative.

Diamond (1965) was the first to demonstrate that competitive steady states may be inefficient. In his model production functions take the form $F_t(K_t) = F(N_{t+1}, K_t)$ with constant returns to scale, where N_{t+1} denotes exogenous labor input. Production is conducted by firms which, in each period t , issue bonds in order to finance investment. One period later they sell the output, pay wage rates w_{t+1} and redeem the bonds. Firms choose the inputs so as to maximize profit $F(N_{t+1}, K_t) - w_{t+1}N_{t+1} - R_{t+1}K_t$. The interest factor, which up to now has been defined as the marginal productivity of capital, will equal this productivity in equilibrium. Besides the firms there are utility maximizing households who live for two periods. When young, they consume, work, and provide for retirement by purchasing bonds of the amount S_t , their first-period budget constraint reading $C_t^1 + S_t = w_t$. When old, the individuals do not work and finance consumption with the bond redemption, so that their second-period budget constraint is $C_{t+1}^2 = R_{t+1}S_t$. A generation zero which only consumes $R_1 K_0$ in the first period completes the model. In an equilibrium, characterized by optimal behavior and market clearance, private wealth equals the capital stock, $S_t = K_t$.

Example 3: Labor is constant and normalized to one. Households maximize log-linear utility functions given by $\beta \ln C_t^1 + \ln C_{t+1}^2$, where β represents the pure rate of time preference. As shown in appendix B, the equilibrium interest factor becomes $R(\beta) = (1+\beta)/3$ and the equilibrium capital stock satisfies $K(\beta) = (6/(1+\beta))^{4/3}$. The efficient stationary state of example 1 emerges if $\beta=5$. The inefficient stationary state of example 2 emerges if $\beta=7/9$.

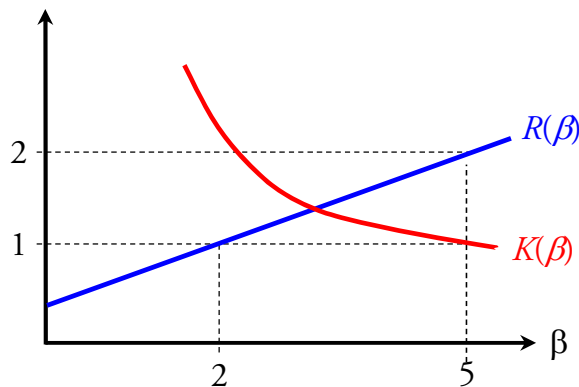


Figure 1: Stationary states without land.

To generalize the examples somewhat, figure 1 depicts stationary capital stocks and interest factors as functions of the time preference rate. When one approaches the origin from the right, the preference for present consumption becomes weaker, which will increase the capital stock and lower the interest rate. For every time preference rate $\beta < 2$, the interest factor falls below the critical value one, and the economy becomes trapped in an overac-

cumulation equilibrium with a negative interest rate. This invalidates Böhm-Bawerk's (1889) claim that "underestimation of the future" combined with "roundaboutness of production" will guarantee positive interest rates. Setting $\beta=3/2$, for instance, future goods are clearly underestimated. Nevertheless, the interest rate would amount to -17 percent because the actual rate of time preference, i.e. the expression $dC^2/dC^1 = \beta C^2/C^1$, does not depend only on the discount factor but also on the respective consumption quantities. It is easy to relate these findings to Weizsäcker's (2014) Austrian model of capital accumulation. Weizsäcker attributes the possibility of overaccumulation to a prolonged average waiting period which, in turn, results from increased life expectancy. In the present overlapping generations model—with its fixed period length—the prolonged waiting period is simulated by a decrease in the pure time preference rate, β , which induces people to increase their retirement provisions. The result is the same in both cases: Strong capital accumulation may render allocations dynamically inefficient.

This has an important implication for fiscal policy. In an efficient equilibrium, any stock of public debt raised in the first period, D_1 , must be served later on because the debt level would otherwise explode and exceed total output. But in an overaccumulation equilibrium, the government can generally improve the allocation. Capital market clearance is now defined by $S_t = K_t + D_t$, and public debt can be used to crowd out excess capital.

To accomplish a true Pareto improvement this way is, however, anything but straightforward. First of all, the efficiency test (3) refers to an infinite horizon. If the interest rate falls short of the growth rate for some years—or for some centuries—this does not logically imply overaccumulation but can only be taken as an indication. Naturally, the same holds true in the reverse case. Second, even under the very stringent assumptions made in example 2, public debt programs must be designed carefully. Raising some debt D_1 and transferring the proceeds to generation zero, for instance, would violate the Pareto principle: While generations zero and one profit from the transfer and from higher interest, respectively, generation two is made worse off via a diminished wage rate. In order to avoid this outcome, additional measures have to be taken, calling for an entire sequence of debt levels (D_1, D_2, \dots) which are feasible and avoid worsening any generation's wellbeing. Notwithstanding these caveats, overaccumulation generally presents a valid case for public debt policies.

3. Land Precludes Overaccumulation

In the preceding section, which summarized the overaccumulation issue, it turned out that aging economies may accumulate too much capital. This dismal result is overturned when one takes land as a factor of production into account. *Land* consists solely of the ground, excluding any structures situated on it and all improvements; the latter two belong to reproducible capital. The distinguishing feature of land is that in every period nature provides a constant amount $L > 0$, measured in square meters, which is not used up in the production process. Assuming free disposal, land prices q_t , measured in output units per square meter, are non-negative. The product $q_t L$ represents the land value measured in output units so that total nonfinancial wealth becomes

$$(4) \quad S_t = K_t + q_t L.$$

While private wealth coincides with the capital stock K_t in the Diamond model, it exceeds the latter in the presence of land. This has a number of striking implications. First, an approach comparing private wealth with reproducible capital only is not well taken; it overlooks that investors can buy land instead. The empirical relevance of this point will be highlighted in the next section which shows that the values of capital and land are of the same order of magnitude. For a number of countries, neglecting land misses about one half of an economy's investment opportunities.

Second, the presence of land markedly changes the effects of public debt. Rewriting (4) as $S_t = K_t + q_t L + D_t$ means that deficits will not only crowd out capital but also depress land values. With log-linear utilities, desired wealth is interest inelastic and only depends on the capital stock inherited from the past. Consequently, $K_t + q_t L$ falls exactly by the amount of public debt. Contrary to the partial equilibrium view that during crisis times, governments can create safe assets (viz. treasury bonds) out of thin air, public debt is apt to reduce land values and to extinguish a most important part of bank collateral. This impact becomes particularly pronounced in the unrealistic but instructive case of a corner solution $K=0$, where land values would plunge exactly by the amount of public debt. The burden of a rise in public expenditure could not be shifted into the future but would diminish private consumption instantaneously, which gives the land model a Ricardian spirit despite the absence of a bequest motive (on this point see also Buiter 1989).

Third, overaccumulation is impossible in a competitive economy with land, where production functions take the form $F_t(K_t) = F(N_{t+1}, K_t, L)$ with constant returns to scale. Firms now acquire capital as well as land in every period and finance the total outlay with bond issues. One period later they sell the output plus the land and redeem the bonds so that profit becomes $F(N_{t+1}, K_t, L) + q_{t+1}L - w_{t+1}N_{t+1} - R_{t+1}(K_t + q_t L)$. The marginal productivity of land $\partial F(N_{t+1}, K_t, L)/\partial L$ is abridged as ρ_{t+1} and referred to as *rent*. The time index of the rent points to the period where land is used in production; this parallels the indexation of wage rates and interest rates. Differentiating profit with respect to the land input yields

$$(5) \quad R_{t+1} = \frac{q_{t+1} + \rho_{t+1}}{q_t}.$$

As discussed below in section 5, the land income share $\rho_t L/Y_t$, i.e. the ratio of rent income and output, is assumed uniformly strictly positive. Supposing that firms are the only land-owners does not restrict the model's scope and has the advantage that no change at all must be made regarding the households: These still buy bonds when young and collect the redemption when old. The analysis starts again with a stationary economy.

Example 4: In a stationary state, firms produce according to the technology $6N^{2/3}K^{1/6}L^{1/6}$, where labor and land are normalized to one, implying that land value and land price coincide. Completing the model with the households of example 3, competitive solutions are derived in appendix C. They take the form shown in figure 2 which displays the interest factor, the land price, and the capital stock as functions of the time preference rate.

Figure 2 focusses deliberately on low time preference rates. Approaching the origin from the right, land value increases relative to capital. This mechanism keeps the interest factor

above the critical level one. Even under the extreme assumption that people attach no value whatsoever to the present, $\beta=0$, the associated interest rate remains positive. A crucial feature of an economy with land is that stationary states are dynamically efficient because the land crowds out inefficient capital formation.

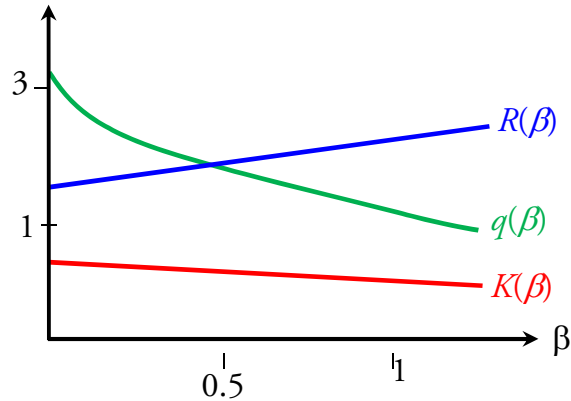


Figure 2: Stationary states with land.

This insight goes back to Turgot (1766: 85 f.). It has a simple intuition: With a constant land price, the arbitrage condition (5) becomes $1+r=(q+\rho)/q$. The left-hand side of this equation shows the interest factor as the return of a capital purchase: For each dollar expended in some period, the investor gets back $1+r$ dollars in the following period. The right-hand side represents the return of a land purchase: For each square meter purchased in some period, the investor pays q and gets back q plus the land rent in the following period. Simplifying the equation to $r=\rho/q$ demonstrates that the interest rate must be strictly positive. Solving for the price yields $q=\rho/r$ as the familiar expression of a perpetuity, which facilitates an alternative interpretation: If the interest rate approached zero, the value of land would become infinite, surpassing total output.

Transferring the previous results to a growing economy is not straightforward because the existence of land rules out fixed factor proportions, and this is arguably the main reason why land has disappeared from growth theory—mathematical convenience prevailed. In an almost forgotten paper, however, Donald A. Nichols (1970) incorporated land into a balanced growth framework, using the special assumptions of land-augmenting technical progress and a Cobb-Douglas production function.

In order to make progress, a manifest strategy is to transcend the steady state and to study arbitrary growth paths with changing factor proportions and fluctuating prices. In such a setting a competitive equilibrium consists of sequences of wage rates, interest rates, land rents, and land prices such that all markets clear while households and firms make individually optimal choices. Under this generalization the following holds true:

Land theorem: In an economy with land, every competitive equilibrium is dynamically efficient.

Rewriting the arbitrage condition (5) yields $q_t=(q_{t+1}+\rho_{t+1})/R_{t+1}$. From a forward recursion one can solve for the land price in the first period, which equals the present value of the land in some period $T\geq 2$ plus the present values of the interim land rents:

$$(6) \quad q_1 = \frac{q_T}{R_2 R_3 \dots R_T} + \sum_{t=2}^T \frac{\rho_t}{R_2 R_3 \dots R_t}.$$

With a uniformly strictly positive land income share there exists $\lambda > 0$ such that $\rho_t L / Y_t \geq \lambda$ or $\rho_t \geq Y_t \lambda / L$ for all $t \geq 2$. Using again the identity $Y_t = Y_1 G_2 G_3 \dots G_t$ introduced above, one can substitute the rent in the second summand by $\rho_t \geq Y_1 G_2 G_3 \dots G_t \lambda / L$. Doing so and dropping the non-negative first summand yields a lower estimate of the current land price:

$$(7) \quad q_1 \geq \sum_{t=2}^T \frac{Y_1 G_2 G_3 \dots G_t \lambda / L}{R_2 R_3 \dots R_t}.$$

Finally, taking the limit and rearranging gives

$$(8) \quad \frac{q_1 L}{Y_1} \geq \lambda \sum_{t=2}^{\infty} \frac{G_2 G_3 \dots G_t}{R_2 R_3 \dots R_t}.$$

As the left-hand side cannot exceed one, the infinite series remains finite and its elements, which are the terms in the efficiency condition (3), converge to zero. This completes the proof. The boundedness of the ratio $q_1 L / Y_1$ should be interpreted generically: With a period length of thirty years, for instance, land values must not exceed thirty annual outputs. The only essential point is that finite incomes keep land values finite.

Put differently, land precludes overaccumulation. This result carries over to multi-sector models with diverse land qualities, heterogeneous households and firms, changing preferences, and technical progress, see Homburg (1992). In all cases, the compound interest factor will eventually exceed the compound growth factor so that the efficiency condition (3) is preserved. As a positive by-product, the land theorem uncovers growth as a driving force of interest: The higher the growth rates are on the average, the higher will be the interest rates since otherwise the land value would exceed output. Intuitively, growth drives up land prices, and the increase in the return to land necessitates an increase in the return to capital.

Augmenting the Diamond model with land opens a new market, and as soon as people start trading in this market, overaccumulation is ruled out. This mechanism works automatically and is so powerful that the land theorem has induced some economists to view land as inimical to capital formation, e. g. Keynes (1936: 242) and to recommend partial (Crettez, Loupias, and Michel 1997) or even total (Deaton and Laroque 2001) nationalization of the land to stimulate investment. However, since private land markets play a very important allocative role at the micro-level, it seems unreasonable to transfer all decisions on land use to the political elite. Moreover, as opposed to the overaccumulation case, underaccumulation is not an inferior outcome in the sense of Pareto's criterion—and if the government wished to follow some other criterion, it could stimulate capital formation via higher taxes, saving subsidies, or budget surpluses, as recommended by Summers and Carroll (1987: 635) at a time when Summers believed interest rates were too high.

To recapitulate, inserting land into the Diamond model changes the latter's properties in a number of ways: Nonfinancial wealth exceeds the capital stock, public debt depresses the value of bank collateral, and overaccumulation becomes impossible. In light of these results, one may wonder why land remains unmentioned in macroeconomic textbooks and

plays almost no role in current research. After all, land had figured prominently in the classical triad of the factors of production studied before the 20th century. And from that time on, it was not forgotten: Samuelson (1958: 481) himself, in his famous article on fiat money, acknowledged explicitly that his analysis made sense only if land were “superabundant”. Niehans (1966) showed that in order to keep land values finite, the interest rate must exceed the growth rate of land rents, but his partial equilibrium model found little resonance. And Tobin (1980: 84) effectively smashed the entire literature on monetary overlapping generations models with the following remark: “[I]f a nonreproducible asset has been needed for intergenerational transfers of wealth, land has always been available. Quantitatively it has been a much more important store of value than money.” But none of these authors put land into a macroeconomic growth model.

A possible reason for this neglect has already been mentioned: the tension between land and fixed factor proportions in a growing economy. Perhaps more importantly, the quantitative significance of land has remained obscure until recently, making economists doubt the relevance of the above argument. The following section reveals that omitting land as a part of nonfinancial wealth yields anything but a good approximation.

4. The Empirical Significance of Land

Unlike business accounting, national accounting has been restricted to flows for long. But during the last years, balance sheets displaying stocks were added to the System of National Accounts (SNA) as the international standard, and a rising number of countries actually provide such balance sheets.

The current SNA 2008 distinguishes nonfinancial and financial assets. Balance sheets for nonfinancial assets contain produced assets, consisting mainly of fixed assets, and nonproduced assets, consisting mainly of land. Balance sheets for financial assets and liabilities report each sector’s financial net worth, including the government sector. Figure 3 shows the values of fixed assets (capital) and of land for four large countries. The private sector’s financial net worth, which coincides with the negative of government’s net worth in a closed economy, is also reported. The figures for public debt may appear unusually low because they refer to net debt whereas most publications, owing to the Maastricht criteria, document gross debt. The difference of net and gross debt is considerable especially in Japan’s case because the Japanese government runs a capital reserve system for its employees, and the public pension funds, in turn, hold a large portion of public debt. Even in a non-Ricardian framework, only net public debt can count as private wealth.

Figure 3 proves that land is of the utmost importance: Its value exceeds GDP and public debt in all cases. And except for Canada, land values come close to the value of reproducible capital. Moreover, land-output ratios have appreciated since the beginning of the 21st century except in the case of Japan, where they have declined somewhat.

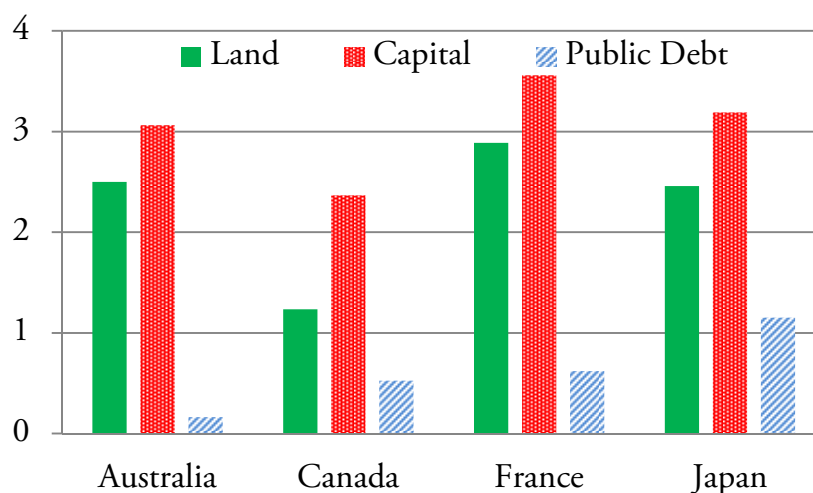


Figure 3: Capital, land, and public debt as multiples of GDP.

Inspecting France more closely, the capital-output ratio amounted to 360 percent in 2011, and the land-output ratio amounted to 290 percent. Hence, total nonfinancial wealth in relation to GDP was 650 percent whereas net public debt stood at 60 percent of GDP. Expressed in Austrian terms, and considering that France's private consumption expenditure made up 58 percent of GDP, the capital stock sufficed to finance consumption for 6 years. This is below the value of 12 years mentioned by Weizsäcker (2014: 46). But land sufficed to finance another 5 years. And as net public debt and private consumption almost coincided, the bars shown in figure 3, expressed in terms of annual private consumption, sum up to 12. The figures are in accordance with Weizsäcker's, the difference being that they take land into account, which constituted almost one half of private wealth.

What would happen if the French government repaid its debt (or had always run a balanced budget in the past)? The preceding theory suggests that the values of capital and land would increase in a way that preserves dynamic efficiency. For instance, a rise in the capital-output ratio from 360 to 390 percent and a rise in the land-output ratio from 290 to 320 percent would suffice to absorb the entire public debt, leaving private net wealth at its original value. Such an increase by an order of magnitude of ten percent does not look particularly alarming.

The main result of the preceding section, however, is more fundamental: Under the far-fetched assumptions that the French government would shortly repay its entire debt, while the present capital stock were close to its overaccumulation level, the land theorem suggests a capital-output ratio staying constant at 360 percent and a land-output ratio rising from 290 percent to 350 percent. Land values alone could absorb the entire reduction in public debt, thereby keeping interest rates above the growth rates. Under the assumption that the French economy keeps growing, negative interest rates are also precluded.

Data for the Netherlands and for South Korea are similar to those presented above, while the Czech Republic reports lower land values. Other countries do not provide harmonized land values as yet, so one has to wait until they become available.

However, the U. S. Federal Reserve Board furnishes balance sheets for nonfinancial assets covering several sectors of the economy. Compared with the SNA 2008 standard, the U.S.

figures are biased downward because they do not encompass land in possession of financial corporations and the general government. Moreover, the land values are not reported directly. Following a seminal paper by Davis and Heathcote (2007), they can be obtained when one subtracts the structures, valued at current cost, from the market value of total real estate. The long-term development of capital-output ratios and land-output ratios depicted in figure 4 has made use of this method.

During the postwar period, the capital-output ratio fluctuated around 200 percent, confirming the familiar stylized fact of its constancy. The land-output ratio amounted to 84 percent on the average, also showing no trend but with stronger fluctuations, especially during the last financial crisis: In 2006, the land-output ratio skyrocketed to 141 percent of GDP, or \$19.6 trillion in absolute terms, and then plunged to but \$8.8 trillion within four years, extinguishing over \$10 trillion of actual or potential bank collateral (on this see also Liu, Wang, and Zha 2013). While the pre-crisis gains can be attributed to irrational exuberance or to new securities pretending to be both as liquid and secure as money and as profitable as land, the truly remarkable feature is the subsequent decline in the land-output ratio to 61 percent, well below its long-term average. According to the above theory, extreme public deficits—which were used to finance bail-outs and various expenditure programs—may have induced investors to buy treasuries rather than mortgage-backed securities, thus aggravating the land price crash. This last observation does not belong to the paper's main theme but reinforces the contention that macroeconomics could be improved by paying more attention to land.

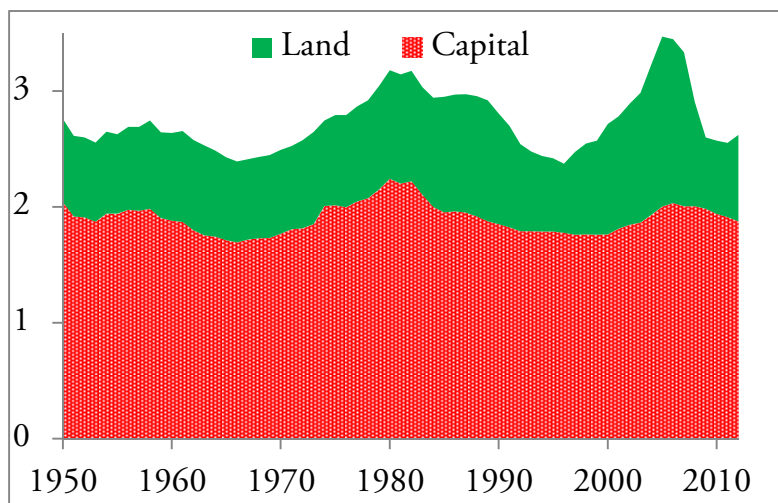


Figure 4: U.S. capital and land values as multiples of GDP.

In sum, there is overwhelming evidence supporting the macroeconomic importance of land. Land values are of the same order of magnitude as capital stocks, and there is not the slightest indication that this will change in the foreseeable future.

5. Two Possible Objections

The land theorem presented above can be challenged on two grounds which both amount to making land intrinsically worthless. First, one may assume that the land income share vanishes in finite time or in the limit. Remembering that the model's efficiency properties

do not hinge upon what happens over a bounded horizon but only upon the economy's eventual behavior, it follows that with a vanishing land income share, land simply drops out of the model. Since nothing points into this direction, however, such an allegation is similar to assuming that labor or capital would cease to remain valuable. Of course, anything can happen in the more distant future, and lacking a time constraint one could also analyze such imaginary settings. A sensible research strategy, however, relies on available evidence which does by no means indicate that one of the three factors of production will become useless henceforth. Housing services, for instance, form an important part of the model's homogenous output, or "leets". They are produced by means of labor, capital, and land, and it is difficult to imagine a world where buyers of single-family homes could obtain the required plots essentially for free. In areas such as Hong Kong, Manhattan, or Tokyo, land-augmenting technical progress (colloquially referred to as skyscrapers) increases land rents in the same way as labor-augmenting technical progress increases wage rates. Because land rents result from heterogeneous land qualities, there is no compelling reason why the land income share should vary systematically with population or output.

A second objection, due to Kim and Lee (1997) and Weizsäcker (2014: 48), holds that taxation or expropriation may invalidate the land theorem. To evaluate this critique, it should first be pointed out that reproducible capital consists mainly of buildings and other structures attached to the ground. These are as immobile as the land itself and face a similar political risk. Any tax or expropriation scheme which diminishes the returns of land and capital uniformly leaves the arbitrage condition (5) unaffected and preserves dynamic efficiency as long as the returns are not entirely taxed away, cf. Homburg (2014). Under the same provision, even a special tax on rent as analyzed by Feldstein (1977) or a property tax on land value does not change the above results; to see this one must only redefine the variable ρ_t as the after-tax rent.

Motivating public deficits by means of expropriation risks comes close to circular reasoning because governments do not seize their citizens' property at random. Historically, the main cause of expropriation is government over-indebtedness, irrespective of whether this over-indebtedness results from excessive wartime spending (Germany), excessive peacetime spending (Greece), or excessive bank aids (Cyprus). In this perspective, debt breaks are the right measure to strengthen property rights whereas compensatory fiscal policies of the Hansen type are like putting out a fire with gasoline—they increase the expropriation risk. In his critique of Hansen's proposals, Henry C. Simons (1942: 174) put this point vividly: „But the magnitude and rate of increase of internal debt is a measure of political instability and exposure to revolution ... Somewhere, sometime ahead, taxpayers or claimants of governmental dispensations will revolt against deprivations in the name of bondholders, especially as free spending and repudiation of all fiscal norms relax pressure against minority demands. As soon as such possibilities become discernible ahead—which may be soon or decades away—the fright of bondholders will create a revolutionary situation.”

To sum up this point: Compared with other assets, land is not subject to a higher expropriation risk. Historical experience suggests that this risk is even lower, especially when one contrasts land with sovereign debt. Because the general expropriation risk is not zero

but basically due to possible government over-indebtedness, the objective to foster property rights and to avoid government over-indebtedness constitutes an independent argument against public deficits.

6. Uncertainty and Diverse Interest Rates

A considerable literature on overaccumulation in stochastic growth models has evolved over the last decades. By and large, these models sustain the results obtained so far. But they also make an important distinction between risky and safe returns. To introduce uncertainty into the model, output becomes the product of F_t , which represents any of the above deterministic production functions, and strictly positive shocks that are realized at the start of each period:

$$(9) \quad Y_{t+1} = \theta_{t+1} F_t \quad \text{for } t=0, 1, 2, \dots$$

Any macroeconomic shock θ_{t+1} will generally affect all future states of the economy. Overaccumulation is now taken to mean that an alternative growth path exists which yields more consumption in some period and some state, and no less otherwise.

If the shock is verifiable, firms issue contingent bonds to finance investment, otherwise they issue shares. All hazards are shifted back to the households who face stochastic returns. Under the natural assumption that the households are risk averse, they accept government bonds yielding lower but *safe returns* $R_t^f = 1 + r_t^f$. The safe (or risk-free) interest rate r_t^f makes the households indifferent between risky and safe investments.

Example 5: Combining example 3 with equation (9), the time preference rate is set to $\beta=2.08$, and the macroeconomic shocks take realizations $\theta^d=0.7$ or $\theta^u=1.3$ with equal probabilities. At zero public debt, the economy follows a stationary stochastic process. As shown in appendix D, the risky interest rate averages +3 percent. But the safe interest rate averages -2 percent.

So tempting a scenario raises the question whether the government could play a sustainable Ponzi game. Or more specifically, do the model's efficiency properties depend on the risky rate or on the safe rate? Before answering these questions, a Ponzi scheme of the following type is considered: Starting with zero debt, the government incurs an initial deficit of 0.2 percent of GDP by issuing safe bonds and transferring the proceeds to the elderly. In each following period, the government preserves a primary balance, redeeming debt if the safe interest rate happens to be negative and raising additional debt if the safe rate is positive. Thus, the outstanding debt is given by $D_{t+1} = R_{t+1}^f D_t$ for all $t \geq 1$.

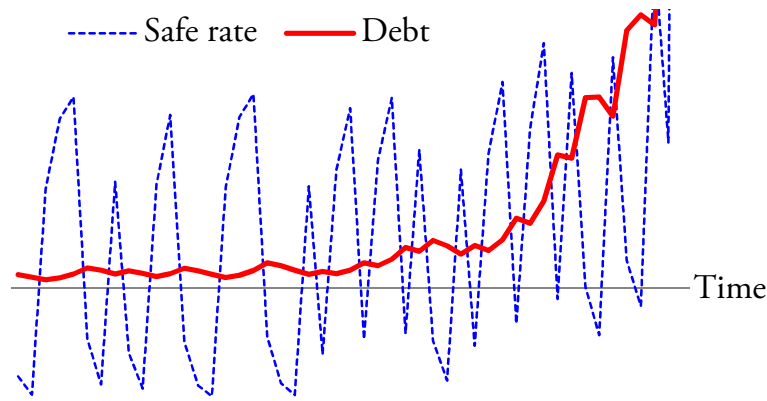


Figure 5: Failure of a Ponzi scheme.

The dotted line in figure 5 traces the safe interest rate which becomes repeatedly negative, perhaps lulling the public in a false sense of security and enabling the government to judge the risk of its policy as “contained”. However, the solid line shows what is going on in the background: Public debt climbs steadily and explodes shortly after the last period depicted. In order to prevent total economic collapse, the government must eventually repudiate its debt. While the concrete simulation displays only one of many possible outcomes, it is nevertheless true that the debt explodes with probability one—a clear warning that the technical term “safe rate” must not be understood literally.

It is accepted in the theoretical literature that an assessment of fiscal sustainability must take the risky rates into account rather than the safe rates, cf. Bohn (1995: 266). Reviewing the above proof of the efficiency condition makes it easy to see why: Due to the aggregate constraint $K_1 = S_1 - D_1$, the initial deficit reduces the capital stock by some $\varepsilon_1 > 0$ which equals D_1 if desired wealth is interest inelastic and may be somewhat smaller or greater under more general assumptions. With a concave production function, the decrease in the capital stock diminishes second-period output by $\varepsilon_2 \geq R_2 \varepsilon_1$. Crucially, the output loss involves the risky interest factor whose expected value coincides with the marginal productivity of capital. The decline in the capital stock perpetuates itself into the following periods until it exceeds total output, which will happen with probability one. In sum, only the risky interest rates represent an unbiased measure of expected output losses.

The safe interest rates, gaining so much attention in policy debates, are a delusive indicator of overaccumulation because they underestimate the expected output loss resulting from deficit spending, and the equity premium puzzle suggests that the differences between expected marginal productivities and safe interest rates are large. These observations may help understand what happened in the aftermath of the financial crises: Extreme government deficits impaired capital accumulation and induced output losses which showed up as sluggish growth later. Of course, productive government spending could foster prosperity in principle. But Stockman’s (2013: xiv) review of the decision making process underlying the American Recovery and Reinvestment Act, the large “stimulus program” of 2009, raises serious doubts regarding the rationality of such attempts: “Obama’s \$800 billion grab bag of consumer tax-cut handouts, business loopholes, money dumps to state and local governments, highway pork barrels, green energy giveaways, and hundreds more

was passed in twenty-one days with no deliberation and after an epic feeding frenzy among the K Street lobbies.”

Example 5 illustrates the central point that assessments of overaccumulation in stochastic economies must compare risky interest rates with growth rates, whereas the safe interest rates are immaterial. However, the literature has ascertained two results which hold under much more general assumptions: First, overaccumulation can be excluded under conditions similar to their deterministic counterparts, cf. Barbie, Hagedorn, and Kaul (2007). Second, the land theorem holds in the stochastic case, too: Land ensures that allocations are Pareto efficient, see Richter (1993) or Demange (2002). Therefore, all results for deterministic economies as exposed in the preceding sections carry over to a stochastic environment. From an applied perspective, this suffices to turn to an empirical assessment of dynamic efficiency.

7. A Test of Overaccumulation

In the foregoing sections a compelling theoretical case against overaccumulation has been made. It remains to be seen whether the data sustain the position that overaccumulation cannot occur in the presence of land. But there is a further reason motivating a look at the facts: While the above theory implies that the compound interest factor exceeds the compound growth factor in the limit, it does not exclude finite, perhaps extended, periods where the interest rate falls short of the growth rate. Richter (1993: 97) presents an example with first-order stochastic dominance of the interest factor over the growth factor, but this holds true only in special cases.

Proceeding from the observation that safe interest rates are unsuited for assessing dynamic efficiency, Abel et al. (1989) developed an alternative test: They define a national dividend as the excess of output over labor income and investment ($Y - wN - K$). Dynamic efficiency is said to hold if the ratio of this national dividend and the value of the market portfolio happens to be uniformly strictly positive. This is indeed true for all periods and countries studied by the authors. Nevertheless, there are three reasons why this alternative test appears less useful. First, the dividend criterion contains a mathematical flaw, as pointed out by Chattopadhyay (2008) who presents counterexamples. Second, the sufficiency part of the criterion is very demanding because it must be satisfied in *every* period, whereas the interest and growth criterion must hold only for compound rates calculated over the entire horizon. If the national dividend were negative in a single period but strictly positive otherwise, the dividend criterion would remain inconclusive while the interest and growth criterion would indicate dynamic efficiency. Third, the dividends specified by Abel et al. (1989) include items such as implicit labor income of sole proprietorships, land rents, monopoly profits, or imputed services of owner-occupied dwellings which are not capital income in the theoretical sense.

Hence, a different strategy is pursued here which involves only the directly observable interest and growth rates and rests on the following idea: Because real investments do not produce certain cash-flows, firms face a trade-off between interest tax shields on the one hand and costs of financial distress on the other, cf. Brealey, Myers and Allen (2014: 455). According to this trade-off theory of capital structure, firms will generally choose a mix of

equity and debt financing and, following an idea originally due to Stiglitz (1973: 32), may finance as much investment as possible through retained earnings and the rest by bonds or other forms of debt. Therefore, the interest rate on corporate bonds represents a lower estimate of the total return. One may object that this measure is too weak, corporate bonds being not nearly as risky as corporate shares. However, if the efficiency condition holds using corporate bond rates, then it will hold *a fortiori* using weighted average cost of capital (WACC) or similar measures which are difficult to obtain at the national level.

The following test specifies interest rates as Moody's Aaa corporate bond yields and compares them with nominal GDP growth rates. Since multiplying interest and growth factors by any inflation measure does not affect the ratios G_t/R_t , there is no need to calculate real rates first. Figure 6 shows the result for the U.S. The series starts shortly after the Volcker reflation with its skyrocketing interest rates. Interest rates did not stochastically dominate growth rates but were almost always higher, averaging at 7.0 percent while the growth rates averaged at 5.1 percent. Subject to an ergodicity assumption one has to make anyway (observed averages must converge to ensemble averages), the series clearly sustain the hypothesis that the U.S. economy follows a dynamically efficient growth path. It should be added that there was no single month in which the real risky rate (measured *ex post* using the consumer price index) became negative.

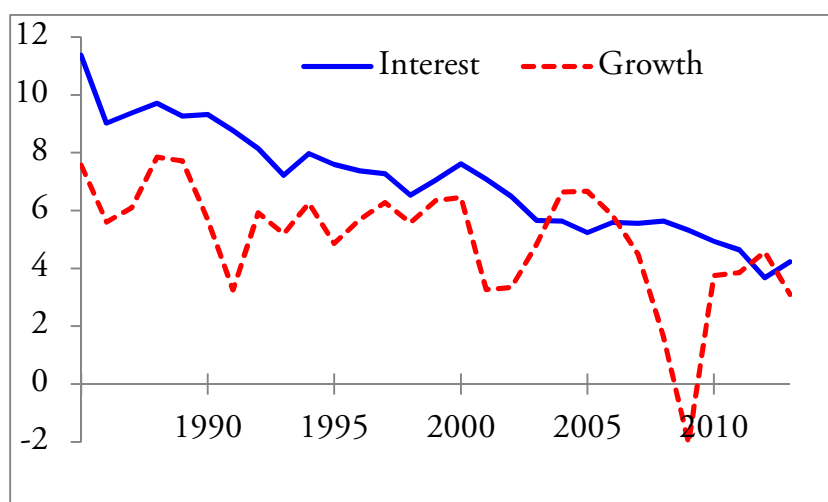


Figure 6: Nominal interest and growth rates in the U.S. (percent).

Figure 7 shows a similar pattern for the eurozone. Because European firms usually prefer loan financing to bond financing, the European Central Bank provides harmonized rates of loans to nonfinancial corporations rather than corporate bond rates; these series start in 2003. The average risky rate of the considered period was 4.2 percent, well above the average nominal growth rate of 2.6 percent. The U.S. and the European time series have an interesting feature in common: In both cases, the interest growth differential $r_t - g_t$ rose sharply during the Great Recession, making intertemporal budget constraints tighter and generating a number of private and sovereign defaults.

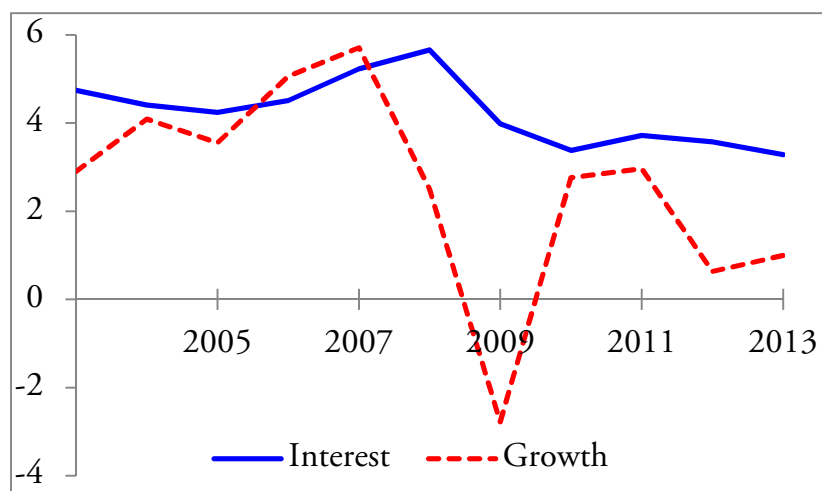


Figure 7: Nominal interest and growth rates in the eurozone (percent).

Other countries, including Japan, were also considered. An indication of overaccumulation was not found anywhere which confirms the theoretical results. The case for overaccumulation seems to rest on a mere confusion between safe overnight interest rates such as the Federal Funds Rate on the one hand and risky rates relevant for nonfinancial investment decisions on the other. Put in terms of macroeconomic principles, it cannot be stressed enough that the “ i ” in the investment function $I(i)$ is altogether different from the safe interest rate. Because prudent investors cannot be sure to realize the calculated cash-flows, they rely mostly on equity and long-term debt financing, and hence only the interest on corporate bonds and the much higher equity returns are relevant for their investment decisions. In modern economies there are only two groups of players that follow imprudent finance strategies without noteworthy equity and with heavy recourse to short-term rollovers—these are the banks, because of their implicit government guarantees, and the governments themselves.

As a caveat, the reported data refer to countries with considerable public debt. It would be desirable without doubt to conduct an empirical study for a debtless world, but this is impossible. However, the land theorem suggests that land values would be correspondingly higher in such a debtless world, still securing dynamic efficiency. And another conclusion is definite: Contrary to the secular stagnation proposals cited in the introduction, there is no need for *additional* public deficits in order to settle a fancied overaccumulation issue.

8. Conclusion

This paper has analyzed the overaccumulation issue theoretically and empirically. From both perspectives, the assertion that mature economies accumulate too much capital appears ill-conceived. The argument proceeded in four successive steps: First, overaccumulation is impossible in an economy with land. Second, bearing in mind that land values are of the same order of magnitude as capital stocks, an economy with land is not a theoretical curiosity but the reality in which we live. Third, regarding overaccumulation, only the risky interest rates matter because these are unbiased measures of expected output losses. The “zero lower bound” chitchat confounds risky interest rates with safe rates that are relevant for monetary policy but have nothing to do with real investment decisions. Fourth,

the risky rates exceed the growth rates consistently in all cases studied, which is a strong indication of dynamic efficiency. Thus the main conclusion reads that public deficits do not represent a free lunch but merely shift burdens into the future.

A final remark pertains to the Janus-faced nature of land: The arbitrage condition (5) relates interest rates and current land prices to expected future land prices, and recent experience from Asia, the U.S., Spain, and Ireland tells that such expectations can go seriously astray at times. Looked at in this way, land is both a long-run stabilizer and a short-run destabilizer of economic development.

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Appendix

The appendix provides derivations and describes the data. At the outset, a remark on depreciations seems apt. This paper follows Cass (1972) in assuming full depreciation whereas authors such as Diamond (1965) assume no depreciation. To see this is immaterial one defines a pure production function $\Phi_t(K_t)$ and sets $F_t(K_t) = \Phi_t(K_t) + (1 - \delta)K_t$, where $\delta \in [0; 1]$ represents the rate of depreciation. All proofs go through without modification, and in equilibrium the derivative of Φ_t equals $\delta + r_{t+1}$, the capital rental rate.

A. Three Efficiency Conditions

This section discusses three sufficient conditions for dynamic efficiency, shows their logical connections, and attaches an economic meaning to each. The conditions are also related to those invented by Cass (1972) and Zilcha (1990) who exclude permanent growth and work with the reciprocals of the expressions used here. Under their approach, the resulting sequences diverge and do not allow an economic interpretation, cf. Cass (1972: 210). Maintaining the exclusion of growth, Balasko and Shell (1980) reproduced Cass' results for exchange economies.

First Condition (Compound Factor): The compound interest factor exceeds the compound growth factor in the limit. This condition (3) was derived and motivated in section 2 and is copied here for easier reference:

$$(10) \quad \prod_{t=2}^T \frac{G_t}{R_t} \rightarrow 0.$$

Second Condition (Average Factor): The average interest factor exceeds the average growth factor in the limit. Understanding averages in the sense of geometric means yields

$$(11) \quad GM_T = \left(\prod_{t=2}^T \frac{G_t}{R_t} \right)^{\frac{1}{T-1}} \rightarrow \alpha_1 < 1.$$

Raising both sides to the power of $T-1$ shows that the second condition implies the first. The reverse is not true, as the harmonic sequence $\prod G_t/R_t = 1/(T-1)$ illustrates.

Third Condition (Total Output): The present value of all future outputs remains finite:

$$(12) \quad \sum_{t=2}^T \prod_{s=2}^t \frac{G_s}{R_s} \rightarrow \alpha_2 < \infty.$$

Multiplying by Y_1 and writing out the sum yields $Y_2/R_2 + Y_3/(R_2R_3) + \dots$ which is simply the present value of the entire output stream. If that present value remains finite, it is easy to prove the first welfare theorem along conventional lines, so that from an economic perspective this condition is the most fundamental. The third condition implies the first but the harmonic sequence $\prod G_s/R_s = 1/t$ illustrates again that the reverse is not true. Writing the third condition in the form

$$(13) \quad \sum_{T=2}^{\infty} (GM_T)^{T-1} < \infty$$

shows that it is implied by the second. (If $GM_\infty = \alpha_1 < 1$, then there exists $\delta \in (\alpha_1; 1)$ such that at most finitely many summands exceed or equal δ^{T-1} ; the infinite rest of the series being smaller than the convergent geometric series associated with δ^{T-1} .) Conversely, the third condition excludes a limit geometric mean above one.

Under strong technical assumptions, the third condition is also necessary for dynamic efficiency. More importantly, however, the third condition is fulfilled in an economy with land, as has been demonstrated in the proof of the land theorem, see (8). Therefore, in an economy with land one need not bother about necessary conditions or the borderline case $GM_\infty = 1$; the sufficient condition is met anyway.

B. Derivation of Figure 1

Each of a large number of identical households supplies labor inelastically, total labor supply being normalized to unity, and solves

$$(14) \quad \begin{aligned} & \max_{C_t^1, C_{t+1}^2, S_t} \beta \ln C_t^1 + \ln C_{t+1}^2 \\ & s.t. \quad i) C_t^1 + S_t = w_t, \\ & \quad \quad ii) C_{t+1}^2 = R_{t+1} S_t. \end{aligned}$$

Substituting from the constraints yields the unconstrained problem

$$(15) \quad \max_{C_t^1} \beta \ln C_t^1 + \ln R_{t+1}(w_t - C_t^1).$$

Differentiating gives $\beta/C_t^1 + 1/[R_{t+1}(w_t - C_t^1)](-R_{t+1})$, from which one obtains

$$(16) \quad C_t^1 = \frac{\beta w_t}{1 + \beta}, \quad C_{t+1}^2 = \frac{R_{t+1} w_t}{1 + \beta}, \quad S_t = \frac{w_t}{1 + \beta}.$$

Each of a large number of identical firms solves

$$(17) \quad \max_{N_{t+1}, K_t} 8 N_{t+1}^{3/4} K_t^{1/4} - w_{t+1} N_{t+1} - R_{t+1} K_t.$$

At the optimum, profit vanishes due to constant returns to scale, labor demand equals unity, and factor prices become

$$(18) \quad w_{t+1} = 6 K_t^{1/4}, \quad R_{t+1} = 2 K_t^{-3/4}.$$

A market equilibrium satisfies $K_t = S_t$. The capital stock K_t induces a wage rate w_{t+1} and a capital stock $K_{t+1} = S_{t+1}$. Substituting from (16) and (18) yields $K_{t+1} = 6 K_t^{1/4} / (1 + \beta)$. A stationary state is a non-trivial fixed point K of the mapping $K_t \rightarrow K_{t+1}$, implying

$$(19) \quad K = \left(\frac{6}{1 + \beta} \right)^{4/3} \quad \text{and} \quad R = \frac{1 + \beta}{3}.$$

C. Derivation of Figure 2

With household behavior completely unchanged, firms now solve

$$(20) \quad \max_{N_{t+1}, K_t, L} 6 N_{t+1}^{2/3} K_t^{1/6} L^{1/6} + q_{t+1} L - w_{t+1} N_{t+1} - R_{t+1} (K_t + q_t L).$$

At the optimum, profit vanishes due to constant returns to scale, the demands for labor and land equal unity, and prices become

$$(21) \quad w_{t+1} = 4K_t^{1/6}, \quad R_{t+1} = K_t^{-5/6}, \quad q_{t+1} = R_{t+1}q_t - K_t^{1/6}.$$

A market equilibrium satisfies $K_t = S_t - q_t$. The capital stock induces a wage rate w_{t+1} and a subsequent equilibrium condition $K_{t+1} = S_{t+1} - q_{t+1}$. Substituting from (16) and (21) yields

$$(22) \quad K_{t+1} = \frac{4K_t^{1/6}}{1+\beta} - q_{t+1}.$$

A stationary state is a non-trivial fixed point (K, q) of the mapping $(K_t, q_t) \rightarrow (K_{t+1}, q_{t+1})$. Using (21), land prices satisfy $q = K^{1/6}/(R-1)$. Putting this and R from (21) into (22), one obtains

$$(23) \quad K = \frac{4K^{1/6}}{1+\beta} - \frac{K^{1/6}}{K^{-5/6}-1} \Leftrightarrow K^{5/6} = \frac{4}{1+\beta} - \frac{K^{5/6}}{1-K^{5/6}}.$$

Setting $x = K^{5/6}$ and $a = 4/(1+\beta)$, the last equation can be written as $x^2 - (2+a)x + a = 0$ with solutions $x = 1 + a/2 \pm \sqrt{1 + (a/2)^2}$. As land prices are positive and finite, one has $R > 1$, $K < 1$ and $x < 1$; hence only the smaller of the two solutions is admissible. Substituting back yields $K = x^{6/5}$, $R = 1/x$ and $q = K/(1-x)$, where $x = 1 + 2/(1+\beta) - \sqrt{1 + 4/(1+\beta)^2}$.

D. Uncertainty

The macroeconomic shock is realized at the beginning of each period and determines, via (18), the actual values of the wage rate and the interest factor:

$$(24) \quad w_{t+1} = 6\theta_{t+1}K_t^{1/4}, \quad R_{t+1} = 2\theta_{t+1}K_t^{-3/4}.$$

Wages adjust to clear the labor market, and the firms pay contingent interest, or dividends, in accordance with the realized shock. In what follows, E_t denotes the mathematical expectation conditioned on all information available at t . Substituting the shock into the profit optimization problem (17) yields

$$(25) \quad \max_{N_{t+1}, K_t} E_t(8\theta_{t+1}N_{t+1}^{3/4}K_t^{1/4} - w_{t+1}N_{t+1} - R_{t+1}K_t).$$

Together with (24), one recognizes that uncertainty does not affect the firms' behavior; the shocks simply drop out. The households' optimization problem takes the following form, where desired total wealth has been replaced by capital and government debt:

$$(26) \quad \begin{aligned} & \max_{C_t^1, C_{t+1}^2, K_t, D_t} \beta \ln C_t^1 + E_t \ln C_{t+1}^2 \\ & s.t. \quad i) \quad C_t^1 + K_t + D_t = w_t, \\ & \quad \quad ii) \quad C_{t+1}^2 = R_{t+1}K_t + R_{t+1}^f D_t. \end{aligned}$$

The solutions for first-period consumption and desired wealth are the same as above, whereas second-period consumption depends on the realization of the shock. In order to induce households to accept the debt, the government must offer a safe interest rate such that the resulting portfolio, consisting of debt and capital, yields the same expected utility as a portfolio consisting of capital only:

$$(27a) \quad \frac{1}{2} \ln(K_t R_{t+1}^d + D_t R_{t+1}^f) + \frac{1}{2} \ln(K_t R_{t+1}^u + D_t R_{t+1}^f) = \frac{1}{2} \ln(K_t R_{t+1}^d) + \frac{1}{2} \ln(K_t R_{t+1}^u) ,$$

where R_{t+1}^d and R_{t+1}^u denote the interest factors induced by the respective realizations of the macroeconomic shock. In this formula, total desired wealth $S_t = D_t + K_t$ is determined by the previous capital stock and the resulting wage rate, while D_t is a policy instrument. The two variables determine both K_t and the stochastic interest factors R_{t+1}^d and R_{t+1}^u uniquely. Hence, (27) can be solved for the fixed interest factor as the only endogenous variable. Defining $x = K_t / (K_t + D_t)$ as the share of risky investment, one obtains

$$(27b) \quad (1-x)(R_{t+1}^f)^2 + x(R_{t+1}^d + R_{t+1}^u)R_{t+1}^f - (1+x)R_{t+1}^d R_{t+1}^u = 0 .$$

Evaluated at $x=1$, the equilibrium without debt, solving yields the harmonic mean

$$(27c) \quad R_{t+1}^f = \frac{2}{\frac{1}{R^d} + \frac{1}{R^u}} .$$

With strictly positive debt, however, the solutions satisfy

$$(27d) \quad R_{t+1}^f = Z + \sqrt{Z^2 + \frac{1+x}{1-x} R_{t+1}^d R_{t+1}^u} , \quad Z = \frac{x}{x-1} \frac{R_{t+1}^d + R_{t+1}^u}{2} < 0 ,$$

which is bounded from above by the geometric mean $R_{t+1}^f = \sqrt{R_{t+1}^d R_{t+1}^u}$. Using (24), the expected risky interest rate and the upper bound of the safe rate can be expressed as functions of the capital stock, demonstrating first-order stochastic dominance of the former over the latter:

$$(28) \quad E_t R_{t+1} = (\theta^d + \theta^u) K_t^{-3/4} = 2 K_t^{-3/4} \quad \text{and} \quad R_{t+1}^f = 2\sqrt{\theta^d \theta^u} K_t^{-3/4} = 1.91 K_t^{-3/4} .$$

Let stochastic sequences be represented by boldface letters such as $\mathbf{K} = (K_1, K_2, \dots)$ and let E represent unconditional expectations over all periods and states. $E \mathbf{K}$ equals 2.433, which is simply the capital stock of the stationary economy studied in example 3, inferred for $\beta = 2.08$ from formula (19). Substituting into (24) and (28) yields $E R = 1.03$ for the risky interest factor and $E R^f = 0.98$ for the upper bound of the safe interest factor, as mentioned in the text. While a safe rate of -2 percent induces the households to accept the debt, the government can still reduce this rate for any level $D_t < S_t$. Specifically, the government can sell the first marginal debt unit at a safe rate of only -6.6 percent, as can be inferred from (27d). Interestingly, the degree of uncertainty (the absolute values of θ^d and θ^u) affects only the safe rate and leaves all other expectations unaffected.

The example satisfies a condition invented by Zilcha (1991) which is sufficient but not necessary for dynamic efficiency in stationary stochastic economies, cf. Rangazas and Russell (2005), and Barbie and Kaul (2009):

$$(29) \quad E \ln \mathbf{G}/\mathbf{R} < 0 .$$

The term \mathbf{G}/\mathbf{R} is understood to mean $(G_2/R_2, G_3/R_3 \dots)$. To see that this condition is sufficient, one rewrites inequality (2) as:

$$(30) \quad \varepsilon_1 \leq Y_1 \exp\left(\ln \frac{G_2}{R_2} + \dots + \ln \frac{G_T}{R_T}\right) \quad \text{for } T = 2, 3, \dots$$

As G/R follows an ergodic process, the exponent converges to $(T-1)E \ln G/R$ almost surely, and condition (29) implies that any $\varepsilon_1 > 0$ would violate the last inequality. According to the familiar log-average identity, $\exp(E \ln G/R)$ represents the expected geometric mean of the ratio of the growth and interest factors. Hence, allocations in stationary stochastic economies are dynamically efficient if interest rates exceed growth rates on the average.

In deterministic economies, dynamic efficiency together with short-run Pareto efficiency implies Pareto efficiency. Such is not the case in stochastic economies. Whereas a condition similar to (12) suffices to rule out overaccumulation (Barbie, Hagedorn, and Kaul 2007: 573), there may still be room for gains from intergenerational risk-sharing. These gains, which are beyond the present paper's scope, can be seen as an artifact of the overlapping generations structure where just two generations meet at any time.

E. Data

Figure 3 was computed using the OECD database, stats.oecd.org. The items "fixed assets N11" and "land N211" from table 9B refer to the total economy. "Financial net worth SBF90" from table 710 (720 in case of Japan) refers to the sector general government. All are expressed as multiples of nominal GDP. Data refer to 2011 but to 2010 for Canada.

Figure 4 was computed from the Z.1 financial data provided by the Federal Reserve Board database www.federalreserve.gov/datadownload. "Capital" was specified as equipment plus residential and nonresidential structures at current cost. "Land" was obtained by subtracting the value of residential and nonresidential structures at current cost from the value of real estate at market values. The data encompass households and nonprofit organizations (table B.100), nonfinancial corporate business (table B.102) and nonfinancial noncorporate business (table B.103), they were divided by nominal GDP taken from table F.6.

Figure 6 was computed from the Z.1 data for nominal GDP and from the H.15 data available under <http://www.federalreserve.gov/releases/h15/data.htm> for interest rates, both provided by the Federal Reserve Board.

Figure 7 was computed from the time series MIR.M.U2.B.A20.A.R.A.2240.EUR.O (loans to nonfinancial corporations S11, annualized agreed rate, total original maturity), available at the ECB's Statistical Data Warehouse: <http://sdw.ecb.europa.eu>, yearly averages, and the time series B1GM (GDP at current prices for EA changing composition), provided by Eurostat: <http://epp.eurostat.ec.europa.eu>.