Credit Risk Modeling under Conditional Volatility

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Abstract

The accuracy of measuring credit risk directly decides on the interest on credit, which has to be paid when raising a credit, and the amount of capital to keep in reserve by a firm. The structural credit risk model proposed by Merton (1974) lays the groundwork for the assessment of a firm's credit risk by its default probability. Doubtlessly, the volatility of the firm's equity represents the most sensitive parameter influencing the default probability. By combining the Merton approach with conditional volatility models, we empirically examine in this article that the specification of conditional volatility affects the probability of default and therefor the credit rating. More precisely, we show on German stock market data that financial market data properties (i.e. asymmetric response of conditional volatility to return shocks and long-range dependencies within the conditional volatility) may not be neglected within the computation of credit risk. Moreover, the influence on the default probability by the type of conditional distribution is pointed out.

Keywords: Credit risk, Merton model, conditional volatility, default probability, stylized facts

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1. Introduction

The aim of credit rating of firms, which apply for a credit, consists in their classification in rating categories. The accurate measurement of credit risk is of prime importance for the entire economic sector and potentiates credit ratings and rating agencies: Creditors are interested in an adequate credit rating that reflects the debtors' reliability, while borrowing firms strive for a preferably low interest on credits and a small amount of capital to keep in reserve, both of which are determined by their credit risk.

For a long time the term *credit risk* featured only an abstract denotation. However, this changed since the enacting of the Basel II regulations issued by Basel Committee on Banking Supervision (2004) which mandatorily took effect in 2007 within the EU countries. One of the three pillars of Basel II addresses the maintenance of regulatory capital of credit institutes, between which in turn minimum capital requirements are imposed on a bank subject to its credit risk. Within the regulations it is ruled that corporate equity backing must depend on the probability of default of a firm. Thereby, credit risk becomes a quantifiable value which allows the evaluation of credit risk with quantitative methods.

The most popular approach to value credit risk in terms of probabilities of default involves the asset value model proposed by Merton (1974) which represents a generalization of the option pricing theory originated by Black and Scholes (1973) and Merton (1973). The Merton (1974) model was first commercially applied in an adjusted form by Moody's KMV which nowadays constitutes an industry standard tool for credit rating.

The probability of default commonly depends on a multiplicity of parameters. Among them the most sensitive parameter, which severely reacts to extreme shocks and therefor is in the main focus of investor's attention, is the volatility of stock price which directly affects the asset volatility and thereby also the probability of default. For this reason, it is of crucial interest to depict the stock volatility within the model framework in the most adequate way. This issue, however, was paid only minor attention so far, albeit there are articles that refer to the importance of the specification of volatility (see Leland (2006), Jacobs and Li (2008), Afik et al. (2012)).

The well-known *stylized facts* refer to empirical findings in financial time series and comprise (among others) volatility clustering and leptokurtosis of returns, a negative correlation between past returns and future volatilities (the so-called leverage effect) and long-range dependencies (see Sewell (2011) for a comprehensive overview about characteristics of financial series). The presence of stylized facts within stock market time series is undisputable and repeatedly proven even for German stock market data (see e.g. Corhay and Rad (1994), Sun et al. (2007)).

Several works exist which recognize the special role of volatility in credit risk valuation, but rather target to model the volatility as Itō stochastic process (see Heston (1993) for the most popular stochastic volatility approach and different extensions within the Merton framework such as Bu and Liao (2013)). Another strand of literature deals with implied volatilities, see e.g. Hull, Nelken and White (2004) where the parameters of the Merton model are estimated from options on the firm's stock.

However, while being considered when modeling stock market data, stylized facts are widely disregarded within the computation of credit risk. The main objective of this work is therefor to account for the existence of specific data characteristics by combining the Merton credit risk framework with conditional volatility models which were primary introduced by Engle (1982). By employing conditional volatility models which use fractional integration, we allow shocks to die out at a hyperbolical rate and take account for the possibility of long-range dependencies within the conditional volatility equation as well. Furthermore, we disclose that the disrespect of the leverage and long memory effect within the conditional volatility directly affects the credit rating of a firm. This in turn provides practical relevance for the consequential interest rate to be paid by the borrowing firm.

The remainder of this article is organized as follows. Section 2 presents Merton's structural approach to model corporate credit risk, defines all relevant variables and determining factors of the underlying model and illustrates a method to compute default probabilities. In Section 3 several conditional volatility models (the GARCH class of models) are introduced, which account for different stylized facts on financial market series. On the basis of German stock market data the introduced approaches are combined in Section 4 to compute default probabilities and to quantify the risk of neglecting properties of financial data. Section 5 concludes the article.

2. The Merton Credit Risk Model

Two approaches of credit risk modeling can be distinguished. The reduction approach on the one hand derives the credit risk directly from the market price of corporate bonds, where the point of firm's default can be considered as first jump of a Poisson process which (default) intensity is aligned to the given market values (see Duffie and Singleton (1994) for a more detailed overview of this model class). Robert Merton's (1974) credit risk model, on the other hand, which is grounded by the option pricing model by Black and Scholes (1973) and Merton (1973), marks the prominent of the structural model approach. The main issue of this approach lies in the capital structure of a firm and in particular in the development of the firm's asset. Consequently, the possible default of the considered firm happens endogenously and occurs if the firm's value falls behind a fixed boundary. Another advantage over the reduction approach, where the default is exogenous by design, is therefor the economic justification of default.

To introduce the Merton model, a firm is considered which capital structure contains an equity with a market value at time t of E_t. Moreover, the firm holds liabilities of constant face amount D, which consists only of a single debt taken up by a zero bond with debt maturity T. By assumption, without any priorities the entire amount of liabilities has to be released at T. [1] The firm defaults if the firm's asset value A_t

[1] In addition, some of the usual assumptions in financial modeling are imposed, such as the absence of transaction costs or taxes and a constant risk-free interest rate.

at maturity time $t = T$ is too small to compensate its liabilities, i.e. $A_T < D$. Within this setting it is assumed that the firm is conveyed to the creditors when the credit is raised, while the firm is transferred back to the holders if the asset value is sufficiently large to repay the liabilities at T.

Thus, the holders have a payoff function given by

$$\Lambda^H := \max\{0; A_T - D\}.$$

This is the same payoff structure as given by the long position of a European call option within the Black-Scholes model, so the equity value can be considered as a call option on the firm's asset value, $E(A_t, t)$. If the option is exercised by the firm holders, D is payed and debts are cleared, whereas in the Merton setup D is regarded as the Black-Scholes strike price. The firm's holders then earn $A_T - D$ for $A_T > D$ and zero otherwise which is equivalent to the non-exertion of the call. Since all standards of a European call option are satisfied, the Black-Scholes formula to determine the value of a European call option to describe the specified setup.[2] Let $\tau = T - t$ be the remaining time to maturity and $\Phi(\cdot)$ the $\mathcal{N}(0; 1)$ cdf. Then, according to Black-Scholes,

$$E(A_t, t) = A_t\, \Phi(v_1) - D\, \exp(-\mu_A\, \tau)\, \Phi(v_2) \tag{1}$$

depicts the equity value depending on t and the respective firm's asset A_t, where

$$v_1 = \frac{\ln\left(\frac{A_t}{D}\right) + \left(\mu_A + \frac{1}{2}\sigma_A^2\right)\tau}{\sigma_A\,\sqrt{\tau}} \tag{2}$$

and

$$v_2 = v_1 - \sigma_A\,\sqrt{\tau}. \tag{3}$$

The parameters $\mu_A \in \mathbb{R}$ and $\sigma_A > 0$ arise from the asset value process $\{A_t\}_{t\in\mathbb{R}_{\geq 0}}$ which follows (corresponding to Black-Scholes stock value) a Geometric Brownian Motion, solving the stochastic differential equation

$$dA_t = \mu_A\, A_t\, dt + \sigma_A\, A_t\, dW_t, \tag{4}$$

where $\{W_t\}_{t\in\mathbb{R}_{\geq 0}}$ is a standard Wiener process and μ_A depicts the expected return on assets. The diffusion parameter $\sigma_A > 0$ covers the dimension of volatility of the

[2] The situation from the creditors point of view determines a payoff of

$$\Lambda^C := \min\{D; A_T\} = D - \max\{0; D - A_T\},$$

i.e. D for $A_T > D$ or A_T if the firm defaults. If one takes a look at the latter term, it is quiet interesting that $\max\{0; D - A_T\}$ is a measure for the credit risk of the creditors. It is zero for the case of a non-defaulting firm and becomes $D - A_T$ for the case of default. As this depicts the payoff structure of a put option, the Black-Scholes formula for European put options can either be used to calculate the credit risk.

asset value. By Itō's Lemma the solution process for SDE (4) is given by

$$A_t = A_0 \exp\left((\mu_A - \frac{1}{2}\sigma_A^2)\, t + \sigma_A\, W_t \right).$$

The credit risk can be derived from the Black-Scholes framework. A key figure for the valuation of the creditor's risk is the probability of the firm's default (PD) which occurs if the credit cannot fully be repaid at T. If one takes a look at the Gaussian cdf $\Phi(v_2)$, it is obvious that this specifies the probability for full repayment, i.e. the firm does not default. Hence,

$$PD := P(A_T < D) = \Phi(-v_2) = \Phi\left(\frac{\ln\left(\frac{D}{A_t}\right) - \left(\mu_A - \frac{1}{2}\sigma_A^2\right)\tau}{\sigma_A\,\sqrt{\tau}} \right) \tag{5}$$

denotes the probability of default by time T, where $\frac{D}{A_t}$ is the debt financing ratio. Intuitively, increasing the debt financing ratio (thus meaning a higher amount of liabilities and a smaller asset value, resp.) leads to an increasing PD. Since the GBM A_t is log normal distributed, it follows $\ln(A_t)$ to be Gaussian. Thus, $(\mu - \frac{1}{2}\sigma^2)\tau$ depicts the time-dependent expected value of the asset value, while $\sigma_A\,\sqrt{\tau}$ is the time-dependent asset volatility, increasing the probability of default for a high value of σ_A.

Within the Black-Scholes framework, $E(A_t, t)$ depicts the option values to be computed, depending on observable stock price A_t. In contrast, the unobservable variable within the Merton approach is the asset value A_t (and thereby also its volatility σ_A), while the equity value E_t is known by the stock price here.

Since both variables are employed in calculating the PD (5), a system of equations depending on both variables needs to be solved prior to the computation of (5).

Using Itō's Lemma for the equity value $E(A_t, t)$ the equation

$$\sigma_E\, E_t = \frac{\partial E}{\partial A}\, A_t\, \sigma_A$$

holds (see Jones et al. (1984)), where σ_E is the instantaneous volatility of equity at time t. The derivative $\frac{\partial E}{\partial A}$ equals the European call option delta in the Black-Scholes framework. Thus,

$$\sigma_E = \Phi(v_1)\, \frac{A_t}{E_t}\, \sigma_A. \tag{6}$$

forms the first part of the system of equations. Moreover, the Black-Scholes type formula for the equity value given by (1), (2) and (3) is an equation in A_t and σ_A. Solving (1) (in conjunction with (2) and (3)) and (6) for A_t and σ_A, the unobservable values can be obtained to then compute the probability of default (5). To solve this high-grade nonlinear system of equations the parameters E_t, σ_E, μ_A and the remaining time to maturity τ are needed. Usually, the firm's stock price is used to model the equity value of the firm.

3. Conditional Volatility Modeling

The accurate modeling of the stock price volatility is of crucial relevance for the valuation of credit risk since high volatilities give rise to a high possibility of heavy amplitudes of the stock price process. Accounting for the stylized facts of financial time series, i.e. heteroskedastic volatilities along with volatility clustering, heavy tailed distributions of returns, asymmetric response of conditional volatility to return shocks (leverage effect) and the existence of long memory, the class of Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models should be employed to model the stock price volatility.

3.1. Symmetric and Asymmetric GARCH Models

The GARCH class of models originates Engle (1982) by introducing the feasibility of separate modeling of a process volatility, which is supposed to be a function of p past squared innovations, $\varepsilon_{t-1}^2, \ldots, \varepsilon_{t-p}^2$. Employing Engle's ARCH model, Bollerslev (1986) remarked that a high lag order p cannot be avoided to obtain a good fit. Generalizing the work of Engle (1982), Bollerslev (1986) then introduced the GARCH model which allows next to the past squared innovations the past variances to influence the instantaneous volatility.

Let $\{R_t\}_{t\in\mathbb{N}_0}$ be the mean process of a time series and assume $\{R_t\}$ to follow some ARMA(k,l) type process. Furthermore, let $\{\mathcal{F}_t\}_{t\in\mathbb{N}_0}$, $\mathcal{F}_t = \sigma(R_s, s \le t)$, be the filtration generated by $\{R_t\}$. The innovation process $\{\varepsilon_t\}_{t\in\mathbb{N}}$ then follows a conditional distribution,

$$\varepsilon_t | \mathcal{F}_{t-1} \sim iid(0, \sigma_t^2), \tag{7}$$

depending on the information gathered by the past observations of the mean process. The conditional volatility of the residual process is then given by

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \, \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \, \sigma_{t-j}^2, \tag{8}$$

representing the GARCH(p,q) model, where $\omega > 0$, $\alpha_i \ge 0$, $i = 1, \ldots, p$ and $\beta_j \ge 0$, $j = 1, \ldots, q$ are imposed to ensure positivity of the conditional variance. However, Nelson and Cao (1992) show that positivity of (8) can be ensured without the non-negativity restrictions of the coefficients. The GARCH model features the stylized fact of volatility clustering as high values of elapsed conditional volatilities increase the probability to observe a high present conditional volatility. Transforming the GARCH(p,q) into its ARCH(∞) representation it can easily be shown that an innovation observed infinitely long ago still influences the instantaneous variance by t by only include $p + q + 1$ model parameters. Bollerslev (1986) shows that (8) is weakly stationary for $\sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \beta_j < 1$.

Since the past innovations influence the current volatility by its squared value, both negative and positive innovations have the same influence on (8). However, Black (1976) remarks that negative innovations cause a higher influence on the conditional volatility than positive ones. This is commonly known as the leverage effect on financial markets which is reasoned by a higher risk of default seized by the stock owners after a decreasing stock price, as the liabilities D are constant and the ratio $\frac{D}{A_t}$ increases. This leads to a higher fluctuation of the stock price and a phase of high volatilities.

Ding, Engle and Granger (1993) generalize the GARCH model by accounting for the direction of impact of the innovations. The assumption of the conditional variance, i.e. the squared volatility, to be the best method of modeling the conditional volatility is renounced and replaced by the volatility to the power of $\delta \in \mathbb{R}_{\geq 0}$. The Asymptotic Power ARCH (APARCH) of order (p, q, γ, δ) is then expressed by eq. (7) and the corresponding asymmetric conditional volatility equation

$$\sigma_t^\delta = \omega + \sum_{i=1}^{p} \alpha_i \left(|\varepsilon_{t-i}| - \gamma_i \, \varepsilon_{t-i}\right)^\delta + \sum_{j=1}^{q} \beta_j \, \sigma_{t-j}^\delta. \tag{9}$$

The restrictions for the parameters α_i and β_j, $i = 1, \ldots, p, j = 1, \ldots, q$ are abided while $\gamma_i \in (-1; 1)\,, i = 1, \ldots, p$ is imposed on the leverage parameter again to ensure positivity of (9). Besides, $\delta > 0$ is required. For $\gamma_i > 0$ negative innovations have a higher influence on the volatility than positive innovations (leverage effect). The power parameter δ describes a Box-Cox transformation of the volatility σ_t. Note that the GARCH model is nested by the APARCH model for $\delta = 2$ and $\gamma_i = 0 \, \forall i$.

For $\delta = 2$ it is assumed that the conditional volatility can be depicted best by the second centralized moment of $\{\varepsilon_t\}$, without neglecting the leverage effect. This case is covered by the GJR-GARCH introduced by Glosten et al. (1993), which restricts $\delta = 2$ within the APARCH conditional volatility (9). All parameter restrictions stay the same as for the APARCH. Modeling a return series by GJR(p, q, γ), however, might rather be adequate if the innovations $\{\varepsilon_t\}$ follow a conditional Gaussian distribution. Duan et al. (2006) employ the GJR to represent the volatilities in option price models.

3.2. Long Memory GARCH Models

Another property belonging to the well-known stylized facts on financial markets comprises the existence of a long term structure of dependence, i.e. innovations which occurred way back in the past still have a significant impact on present values of the process.

Within the mean equation the ARFIMA(k, d, l) model by Granger and Joyeux (1980) accounts for the long term structure by introducing the memory parameter d which represents the degree of persistence. Here, d is no longer restricted to be a natural number, but can embrace the set of real numbers. However, Harris and Nguyen (2011) refer to lots of empirical evidence for a more slowly declining ACF of the past squared returns than a GARCH model, which is characterized by a geometrical decay of the ACF, could catch. Thus, modeling the long memory of the stock price only within the mean equation could not be sufficient as conditional volatilities are possibly influenced by past innovations as well affecting the instantaneous fluctuation of the stock price.

When generalizing the GARCH model to allow for long term dependencies within the conditional volatility equation, it is practical to rewrite the GARCH conditional volatility equation (8) by its ARMA($p, \max(p, q)$) in squares form [3]

$$(1 - \alpha(L) - \beta(L))\,\varepsilon_t^2 = \omega + (1 - \beta(L))\,(\sigma_t^2 - \varepsilon_t^2) \tag{10}$$

using the GARCH lag polynomial notation, where

$$\alpha(L) = \sum_{i=1}^{p} \alpha_i\, L^i \quad \text{and} \quad \beta(L) = \sum_{j=1}^{q} \beta_j\, L^j$$

as well as $L\sigma_t^2 = \sigma_{t-1}^2$ and $L\varepsilon_t^2 = \varepsilon_{t-1}$, resp. An alternative definition of the conditional variance of the GARCH (8) is then given by

$$\sigma_t^2 = \frac{\omega}{1 - \beta(L)} + \Theta(L)\varepsilon_t, \tag{11}$$

where $\Theta(L) := 1 - \frac{1-\alpha(L)-\beta(L)}{1-\beta(L)}$. Note that each of the models introduced in the following are firstly defined by the corresponding ARMA in squares representation for constructional reasons. Define the lag polynomial of GARCH coefficients

$$\varphi(L) = (1 - \alpha(L) - \beta(L))(1 - L)^{-d} \tag{12}$$

to obtain the Integrated GARCH (IGARCH) introduced by Engle and Bollerslev (1986) for $d = 1$ with

$$\varphi(L)\,(1 - L)\varepsilon_t^2 = \omega + (1 - \beta(L))(\sigma_t^2 - \varepsilon_t^2).$$

In contrast to GARCH, the IGARCH comprises the possibility of a unit root for $1 - \alpha(L) - \beta(L) = 0$. Nelson (1990) shows that the IGARCH unconditional volatility is infinite, while the first squared differences are stationary. Thus, the IGARCH covers infinite persistence which comprises, however, commonly no property of financial series.

Baillie et al. (1996) provide the Fractionally Integrated GARCH (FIGARCH) which generalizes the degree of integration for the squared innovations to real numbers, resulting in

$$\varphi(L)\,(1 - L)^d\varepsilon_t^2 = \omega + (1 - \beta(L))(\sigma_t^2 - \varepsilon_t^2) \qquad d \in \mathbb{R} \tag{13}$$

where $\varphi(L)$ is defined by (12) for $d \in \mathbb{R}$. By transposition of (13) and definition of

$$\tilde{\omega} = \frac{\omega}{1 - \beta(L)}$$
$$\psi(L) = 1 - \frac{\varphi(L)}{1 - \beta(L)}\,(1 - L)^d$$

[3] The order $\max(p, q)$ results from the dependence of the squared innovations from the GARCH coefficients.

the explicit form of the FIGARCH conditional volatility ends in

$$\sigma_t^2 = \tilde{\omega} + \psi(L)\,\varepsilon_t^2, \tag{14}$$

where $d \in [0;1]$ and $\tilde{\omega} > 0$ ensure positive values of conditional volatility. Further non-negativity restrictions are derived by Bollerslev and Mikkelsen (1996). Note that (14) depicts an ARCH(∞) representation with lag polynomial $\psi(L) = \sum_{i=1}^{\infty} \psi_i\, L^i$. For $d = 0$ and $d = 1$ the FIGARCH results in the GARCH and IGARCH, respectively.

Robinson (1991) uses the dissolved lag polynomial representation of $\psi(L)$ to show that the coefficients ψ_i for $d \in (0;1)$ decrease hyperbolically if $\forall i : \psi_i \geq 0$ holds. Baillie et al. (2007) remark that for the relevant interval of d the series is sufficiently flexible to allow for slower hyperbolic rates of decay of the ACF.

However, the unconditional variance of the FIGARCH

$$E[\varepsilon_t^2] = \frac{\tilde{\omega}}{1 - \psi(1)} \tag{15}$$

is infinite for values of $d \in (0;1)$. By developing the arguments of Nelson (1990) it is alleged by Baillie et al. (1996) that despite the lack of weakly stationarity the FIGARCH process is strongly stationary and ergodic. For a prove see Caporin (2002). Kazakevicius and Leipus (1999) formulate a necessary condition for weak stationarity in the existence of summable ψ_i coefficients.

It has to be remarked that the properties of d varying in the range of $[0;1]$ is contrary to the modeling of the mean equation with ARFIMA since for the FIGARCH memory becomes shorter when d is increasing. Consistently, the lower the value of d the longer the memory. Davidson (2004) refers this property to be counterintuitive as for the transition from $d \to 0$ to $d = 0$ memory jumps from infinite long memory to the short memory GARCH case and, respectively, by transition from $d \to 1$ to $d = 1$ from short memory to infinite persistence (IGARCH). The reason for this finding is caused by the lag operator $(1 - L)$ since it is connected to the squared residuals in the FIGARCH case (see (13)), while in the ARFIMA model the lag operator is tied to the process values.

Allowing again for asymmetric effects without neglecting long memory the features of the APARCH and the FIGARCH are combined within the Fractional Integrated Asymmetric Power ARCH (FIAPARCH) model developed by Tse (1998). The parameters $(\omega, p, d, q, \gamma, \delta)$ determine the model volatility which is given analogue to the ARMA in squares representation of the FIGARCH (13) by

$$\varphi(L)\,(1-L)^d\,(|\varepsilon_t| - \gamma\varepsilon_t)^\delta = \omega + (1 - \beta(L))\left((|\varepsilon_t| - \gamma\varepsilon_t)^\delta - \varepsilon_t^\delta\right). \tag{16}$$

In analogy to the FIGARCH the explicit form of the conditional volatility can be written by

$$\sigma_t^\delta = \tilde{\omega} + \psi(L)\ (|\varepsilon_t| - \gamma\varepsilon_t)\,,$$

where $\delta > 0$, $\forall i \in 1,\ldots,q : \gamma_i = \gamma \in (-1;1)$, $d \in [0;1]$, $\tilde{\omega} = \omega(1 - \beta(L))^{-1}$,

$\varphi(L) = (1 - \alpha(L) - \beta(L))(1 - L)^d$ and $\psi(L) = 1 - \left[\phi(L)(1 - L)^d(1 - \beta(L))^{-1}\right]$ still to be the summarized back-shifted ARCH(∞) coefficients. Values of d varying in $[0; 1]$ ensure hyperbolic decreasing ACFs and strong stationarity again (see Degiannakis (2004)). Correspondingly, weak stationarity is not achieved for $d \in (0; 1)$. The parameter choice $\gamma = 0$ an $\delta = 2$ results in the FIGARCH alternative. Note that the FIAPARCH representation is exclusively able to picture the most frequently arising stylized facts within a sole model: heavy tailed distribution of returns, volatility clustering, long memory and asymmetric impacts of random shocks. A proof of weak stationarity, however, fails to appear so far for the FIAPARCH as well.

Combining the advantages of weak stationarity of the GARCH on the one hand, and the ability of modeling long memory of the FIGARCH on the other hand, Davidson (2004) provides the Hyperbolic GARCH (HYGARCH) model. By introducing the HYGARCH parameter η to the lagged squared residuals through the linear combination $((1-\eta)+\eta\,(1-L)^d)\varepsilon_t^2$ the ARMA in squares representation of the FIGARCH (13) results in

$$\varphi(L)(1 + \eta[(1 - L)^d - 1])\varepsilon_t^2 = \omega + (1 - \beta(L))(\sigma_t^2 - \varepsilon_t^2).$$

Thus, the explicit form of the conditional variance of the HYGARCH(p,d,q,η) is given by

$$\sigma_t^2 = \tilde{\omega} + \Xi(L)\varepsilon_t^2, \tag{17}$$

where $d \in [0; 1]$, $\eta \in \mathbb{R}_{\geq 0}$, $\varphi(L) = (1 - \alpha(L) - \beta(L))(1 - L)^d$, $\Xi(L) = 1 - [\varphi(L)(1 + \eta[(1 - L)^d - 1])(1 - \beta(L))^{-1}]$ and $\tilde{\omega} = \omega(1 - \beta(L))^{-1}$. Analogue to the FIGARCH case, (17) represents the ARCH(∞) form of the HYGARCH, while $\Xi(L)\varepsilon_t^2$ expresses the infinite sum of the lagged squared residuals (with coefficients Ξ_j, $j = 1, \ldots$). Under certain parameter restrictions the HYGARCH achieves weak stationarity and therefor existence of variance.

Theorem. The HYGARCH provides weak stationarity if both $1 - \frac{\alpha(1)}{1-\beta(1)} > 0$ and $\eta \in [0; 1)$ holds.

Proof. Firstly, it is to show that HYGARCH can be decomposed into a GARCH and a FIGARCH fraction. In continuance of notation (see (11),(14) and (17))

$$\begin{aligned}
\Theta(L) &= 1 - \frac{\varphi(L)}{1 - \beta(L)} \\
\psi(L) &= 1 - \frac{\varphi(L)(1 - L)^d}{1 - \beta(L)} \\
\Xi(L) &= 1 - \frac{\phi(L)(1 + \eta((1 - L)^d - 1))}{1 - \beta(L)}
\end{aligned}$$

denote the ARCH(∞) lag polynomials for GARCH, FIGARCH and HYGARCH, respectively, where for $\Theta(L)$ $d = 0$ holds. Then it easily follows for $\Xi(L)$ by adding an absolute zero

$$\begin{aligned}\Xi(L) &= \eta - \eta\frac{\phi(L)(1-L)^d}{1-\beta(L)} + (1-\eta) - (1-\eta)\frac{\phi(L)}{1-\beta(L)} \\ &= \eta\left(1 - \frac{\phi(L)(1-L)^d}{1-\beta(L)}\right) + (1-\eta)\left(\frac{\phi(L)}{1-\beta(L)}\right) \\ &= \eta\,\psi(L) + (1-\eta)\,\Theta(L).\end{aligned}$$

Apparently, the bigger the value for η in this linear combination, the higher the influence of the long memory FIGARCH part and the less the short memory GARCH part.

Secondly, restrictions must be derived for which the process assures weak stationarity. Reminding of $E[\varepsilon_t] = 0\,\forall t$ and $Cov(\varepsilon_t, \varepsilon_{t-j}) = 0\,\forall t\,\forall j \in \mathbb{N}$ in the general case for the GARCH class of models only $E[\varepsilon_t^2] = \dfrac{\tilde{\omega}}{1-\Xi(1)} < \infty$ is left to prove. For this purpose consider

$$\Xi(1) = \sum_{i=1}^{\infty} \Xi_i = \eta\,\psi(1) + (1-\eta)\,\Theta(1)$$

and investigate the ARCH(∞) polynomials separately for covariance stationarity. Clearly, the GARCH polynomial provides weak stationarity if $\Theta(1) < 1$ is fulfilled (which is an alternative definition of the more common condition $\varphi(1) = 1 - \alpha(1) - \beta(1) > 0$ from the ARMA representation of GARCH). However, since FIGARCH is not able to provide weak stationarity $\psi(1) = 1$ for $d \in (0;1)$ must hold, see (15). Thus,

$$\eta + (1-\eta)\,\Theta(1) < 1$$

is fulfilled, if

$$\Theta(1) = 1 - \frac{1-\alpha(1)-\beta(1)}{1-\beta(1)} = \frac{\alpha(1)}{1-\beta(1)} < 1 \tag{18}$$

and $\eta \in (0;1)$ constitutes a linear combination as mean between GARCH and FIGARCH polynomial. Trivially, this is also true for $\eta = 0$ (GARCH case). Rewriting (18) the parameter restrictions for the HYGARCH to be weak stationary result in

$$1 - \frac{\alpha(1)}{1-\beta(1)} > 0 \qquad \text{and} \qquad \eta \in [0;1). \tag{19}$$

□

Conrad (2010) points out that a weak stationary HYGARCH under small modifications is possible to be obtained even for $\eta \geq 1$. Also note that an asymmetric version of HYGARCH, the HYAPARCH model, is provided by Dark (2006), but is of less practical relevance.

4. Computing Default Probabilities

4.1. Data Description and Estimation Procedure

In this section we want to bring together both the ideas of Merton's credit risk model and conditional volatility modeling with the GARCH class of models in order to compute probabilities of default (PD's) for a one-year horizon. We therefor consider daily stock data over a period from July 2002 to September 2007 of 24 firms which were part of German DAX30 at that time, i.e. we observe 1370 trading days for each of the firms (with the exception of Lanxess which stock market launch took place by February 2005, leaving only 695 observations here). Appendix A.1 provides the plots of the log return series. In contribution of better understanding the procedure can be abbreviated as follows: The first step comprises the estimation of different models of the GARCH class (GARCH, APARCH, GJR, FIGARCH, FIAPARCH, HYGARCH) for the log-differences of the stock price which represents the proportional equity value. The DGP which describes the data best is then selected by the Hannan-Quinn information criterion. Subsequently, data for the selected model are simulated over the relevant horizon of one year from which the volatility parameter is estimated. Again, this parameter is needed to solve the non-linear system of equations represented by (1) and (6) in order to finally compute the PD's for firm i given by (5),

$$PD_i = \Phi\left(\frac{\ln\left(\frac{D_i}{A_{t,i}}\right) - \left(\mu_A - \frac{1}{2}\sigma^2_{A,i}\right)\tau}{\sigma_{A,i}\sqrt{\tau}}\right). \tag{20}$$

Note that μ_A may not be mixed up with the risk-free interest rate r, but denotes the expected return on assets which has to be determined separately. Consistent with Campbell et al. (2008) we use a constant market risk premium $\mu_A = r + 0.06$, where $r = 0.04$ is the effective key interest rate set by ECB in June 2007. Several other approaches to determine μ_A exist, some of them using the CAPM model (see Afik et al. (2012) for an overview), while Bharath and Shumway (2008) set the expected return assets equal to stock return over the preceding year. The debt capital per share can be extracted from the annual business reports. However, it might fall short of considering only the short term debt as inauspicious developments could the firm require to serve long term credits preferentially. Most of recent studies use the KMV approach devised by Bohn and Crosbie (2003), where the default barrier is composed of the short term debt plus half of the long term debt, see e.g. Bharath and Shumway (2008), Campbell et al. (2008), Duffie et al. (2007).

4.2. Results

For the estimation of the AR-GARCH models, let $R_t = \ln\left(\frac{E_t}{E_{t-1}}\right)$ be the log return at time t of the stock prices E_t. The mean equation of all models estimated in the following are represented by a simple AR(1) process, $R_t = \varrho R_{t-1} + \varepsilon_t$, where $\varepsilon_t = \sigma_t \nu_t$ with $\nu_t \sim iid(0;1)\,\forall t$ and σ_t the conditional variance equation of the suitable model. The usage of AR(1) for the mean can describe the observed log returns well and is in line with many other work on modeling finance data with AR-GARCH (e.g. Ferenstein and Gasowski (2004)). Furthermore, in order to compare the effect on PD's resulting from the applied conditional distribution we employ both a Gaussian and a student-t distribution for all firms and models.

Different orders (p, q) for the GARCH part of all models were applied in the estimation process, but for the very most of cases the setting $p = q = 1$ outperforms all other combinations. Thus, only the models of GARCH order $(1, 1)$ with coefficients $\alpha := \alpha_1$ and $\beta := \beta_1$ are reported.

The full estimation results for the GARCH class of models both for assuming a Gaussian and a student-t conditional distribution can be found in Appendix A.2. It is not surprising that a simple GARCH model is selected for only one firm (this being the Siemens stock which is commonly known for its stability and insensitivity for cycles), since typical properties of financial data are suppressed though. For the selected models we mostly observe highly significance for those parameters that indicate for specific stylized facts, i.e. γ for the leverage effect (APARCH, GJR), d for long memory (FIGARCH) or both γ and d (FIAPARCH), every time the model features the effect in question. These results confirm that the well-known stylized facts as well have to be accounted when modeling the conditional variance of stock market data and not only within the mean. Notably, for the Gaussian conditional distribution the HYGARCH parameter η is not significantly different from 1 in nearly each case, meaning that the model falls back into the FIGARCH case which is nested for $\eta = 1$. Assuming the student-t conditional distribution η clearly fails to be located within the interval that assures weak stationarity (see (19)). Thus, the HYGARCH in general seems not to be adequate to model stock market data.

Table I provides the selected models and the corresponding PD's for each firm when assuming a Gaussian and a Student-t conditional distribution within the volatility equation, respectively. In the majority of cases the selected models for both conditional Gaussian and Student-t distribution are equal. For only nine firms the best performing model is different, with only a marginal discrepancy for two of these firms as APARCH and GJR measure essentially the same effect. In contrast, for only one case a rough deviance (APARCH vs. FIGARCH measuring different effects for Dt. Telekom) is observed. Note that the Student-t selected models always outnumber the Gaussian selected model by maximizing the HQIC which is in line with the findings by Corhay and Rad (1994).

In most cases the computed default probabilities are slightly higher for a student-t conditional distribution than for a Gaussian, which can especially be compared when the selected models for one and the same firm are equal. Under identical conditions otherwise, this finding appears to be intuitive when comparing Gaussian and heavy tailed innovations. For three firms we observe a higher PD for the Gaussian conditional distribution. It can also be derived from the results that for those models which feature long memory tend to yield higher values of PD (of course, under the assumption that equity quotas for two firms are nearly on an equal level, e.g. Henkel and Lanxess, Bayer and Infineon, Continental and RWE).

Firm	Sel. Model & PD *Gaussian*	*Student-t*	Firm	Sel. Model & PD *Gaussian*	*Student-t*
Adidas	FIAPARCH 0.00004	FIGARCH 0.00006	E.ON	FIGARCH 0.00098	FIAPARCH 0.00095
Allianz	FIAPARCH 0.00000	FIAPARCH 0.00000	Fresenius MedCare	FIGARCH 0.00008	FIAPARCH 0.00014
BASF	APARCH 0.00015	APARCH 0.00016	Henkel	FIAPARCH 0.00013	FIAPARCH 0.00014
Bayer	GJR 0.00005	GJR 0.00003	Infineon	FIGARCH 0.00007	FIGARCH 0.00011
BMW	FIGARCH 0.00075	FIGARCH 0.00084	Lanxess	GJR 0.00009	GJR 0.00009
Continental	FIAPARCH 0.00029	FIAPARCH 0.00037	Linde	GJR 0.00018	APARCH 0.00006
Daimler	FIGARCH 0.00032	FIGARCH 0.00032	RWE	GJR 0.00025	GJR 0.00031
Dt. Bank	FIAPARCH 0.00104	GJR 0.00117	SAP	FIGARCH 0.00000	FIAPARCH 0.00000
Dt. Börse	APARCH 0.00037	GJR 0.00243	Siemens	GARCH 0.00012	GARCH 0.00017
Dt. Lufthansa	FIAPARCH 0.00043	FIGARCH 0.00050	ThyssenKrupp	FIGARCH 0.00045	FIGARCH 0.00048
Dt. Post	FIGARCH 0.01880	FIGARCH 0.01933	TUI	FIGARCH 0.00047	FIGARCH 0.00048
Dt. Telekom	APARCH 0.00070	FIGARCH 0.00072	Volkswagen	FIGARCH 0.00052	FIGARCH 0.00050

Table I: Selected models and estimated PD's for DAX30 firms for Gaussian and Student-*t* conditional distribution, respectively.

The next question arising for consideration is whether there is an effect on PD's when not the best model (selected by HQIC) is used to model the conditional volatility, but a "wrong" model. For this purpose we employ the simple GARCH(1,1), insinuating to discount special stylized facts such as leverage and long memory effects, one of which is found in nearly all data. The comparison between the selected and the GARCH model is exemplifically elaborated for the assumption of a Gaussian conditional distribution. Table II provides the PD's computed both for the actual model and under the assumption of GARCH innovations and its corresponding one year credit ratings as awarded by Standard & Poor's.

For those firms for which an APARCH/GJR was selected by HQIC the PD's tend to be higher when the "wrong" GARCH is used to model the conditional volatility (i.e. BASF, Bayer, Dt. Börse, Dt. Telekom, Lanxess, Linde, RWE). This effect is rather reverse for the models which account for long memory, even if not as distinct as for those which cover asymmetric reaction. This tendency might be explained by the fact that fractionally integrated conditional volatility models do not feature weak stationarity and therefor are prone to be explosive, although the very most of the estimated models are very mildly explosive if at all.

Firm	PD & Rating *Selected Model*	PD & Rating *GARCH*	Firm	PD & Rating *Selected Model*	PD & Rating *GARCH*
Adidas	0.00004 AAA	0.00001 AAA	E.ON	0.00098 A-	0.00085 A-
Allianz	0.00000 AAA	0.00000 AAA	Fresenius MedCare	0.00008 AAA	0.00000 AAA
BASF	0.00015 AA+	0.00017 AA+	Henkel	0.00013 AA+	0.00005 AAA
Bayer	0.00005 AAA	0.00005 AAA	Infineon	0.00007 AAA	0.00001 AAA
BMW	0.00075 A	0.00069 A	Lanxess	0.00009 AAA	0.00010 AA+
Continental	0.00029 AA	0.00020 AA	Linde	0.00018 AA+	0.00027 AA
Daimler	0.00032 AA-	0.00034 AA-	RWE	0.00025 AA	0.00031 AA-
Dt. Bank	0.00104 A-	0.00099 A-	SAP	0.00000 AAA	0.00000 AAA
Dt. Börse	0.00037 AA-	0.00258 BBB	Siemens	0.00012 AA+	-
Dt. Lufthansa	0.00043 A+	0.00050 A	ThyssenKrupp	0.00045 A+	0.0023 AA
Dt. Post	0.01880 BB-	0.01506 BB	TUI	0.00047 A+	0.00037 AA-
Dt. Telekom	0.00070 A	0.00070 A	Volkswagen	0.00052 A	0.00058 A

Table II: Influence of "wrong" model on PD and S&P 1yr rating using Gaussian conditional distribution.

The impact resulting from the employment of the wrong model seems not to be decisive at first view. However, taking into consideration that highest graded credit ratings are awarded only within an PD interval of $[0.0\%; 0.1\%]$ and that a stock is already labeled to be speculative for a PD in excess of 0.94% (see Appendix A.3 for an overview), the consequence from neglecting occurrent effects in stock data becomes more evident. At least for nearly 40% of the firms the disregard of special characteristics of financial data entails a change of credit rating. Four of these show a positive chance of rating (Dt. Post, Henkel, ThyssenKrupp, TUI), while five firms are classified worse (Dt. Börse, Dt. Post, Lanxess, Linde, RWE). The degree of discrepancy yields one rating category each, with the exception of ThyssenKrupp (improvement of two categories) and Dt. Börse, for which the degradation of five rating categories is striking. Certainly, all of these results come off by means of the S&P rating categorization - using a different classification of credit rating would possibly bring out different rating migrations, as a result of which different firms could be affected.

For the sake of completeness the empirical examination also involved constant stock price volatilities estimated from an AR(1) process. All of the results, however, yield significantly higher volatilities than under the assumption of conditional volatility leading to higher PD's in consequence. This finding might be an explanation for the gap between the computed PD's and corresponding credit ratings and the actual rating of the firms in question, which tend to be worse than expectable under conditional volatility.

5. Conclusion

We combine the structural credit risk model proposed by Merton (1974) and the GARCH conditional volatility class of models to compute default probabilities in consideration of occurrent characteristics of stock market data. This can be achieved by employing conditional volatility models, which account for leverage effect and the existence of long memory, while credit risk is depicted by the probability of default of a firm subject to the Basel II regulations.

Applying this method to data on firms of German stock market, we thereby find strong evidence for the adequacy of separate conditional volatility modeling as nearly all data sets contain leverage effects and/or long memory. One considered conditional volatility model using fractional integration (HYGARCH), whose weak stationarity is proved previously, turns out to be inappropriate to model stock market data.

Computing one year default probabilities, slightly higher PD's result when assuming a conditional student-t distribution compared to a Gaussian conditional distribution. To derive implications regarding the risk of neglecting special stylized facts, we assume that simple GARCH models are preferred over the actually selected models and obtain distinct credit ratings for one and the same firm in a considerable number of cases. The main finding therefor comprises the fact that the occurrence of specific stylized facts must not only be regarded within the mean equation of stock price series when computing PD's, but within the conditional volatility as well.

Practical relevance arises directly from the high share of discrepant ratings resulting from the employment of an inferior model since credit ratings provide an indicating device for a firm's reliability and the consequential interest rate, which has to be paid out when raising a credit.

The computation of credit risk is a highly extensive topic as there are plenty of potential adjustable screws to rotate on. Along these lines, it would be reasonable to also implement conditional volatility within some of the large number of extensions of the Merton approach. E.g., the *first passage* class assumes a time dependent exogenous default barrier where default is possible to appear as stopping time before expiration (see Black and Cox (1976)), while Longstaff and Schwartz (1995) suggest the expected return to follow a stochastic process, to name but a few. Additionally, a more detailed empirical investigation which involves the influence of conditional volatility on mid and long term credit PD's would be important to determine the full credit risk a firm has to bear. The treatment of these issues would be an interesting task for future research.

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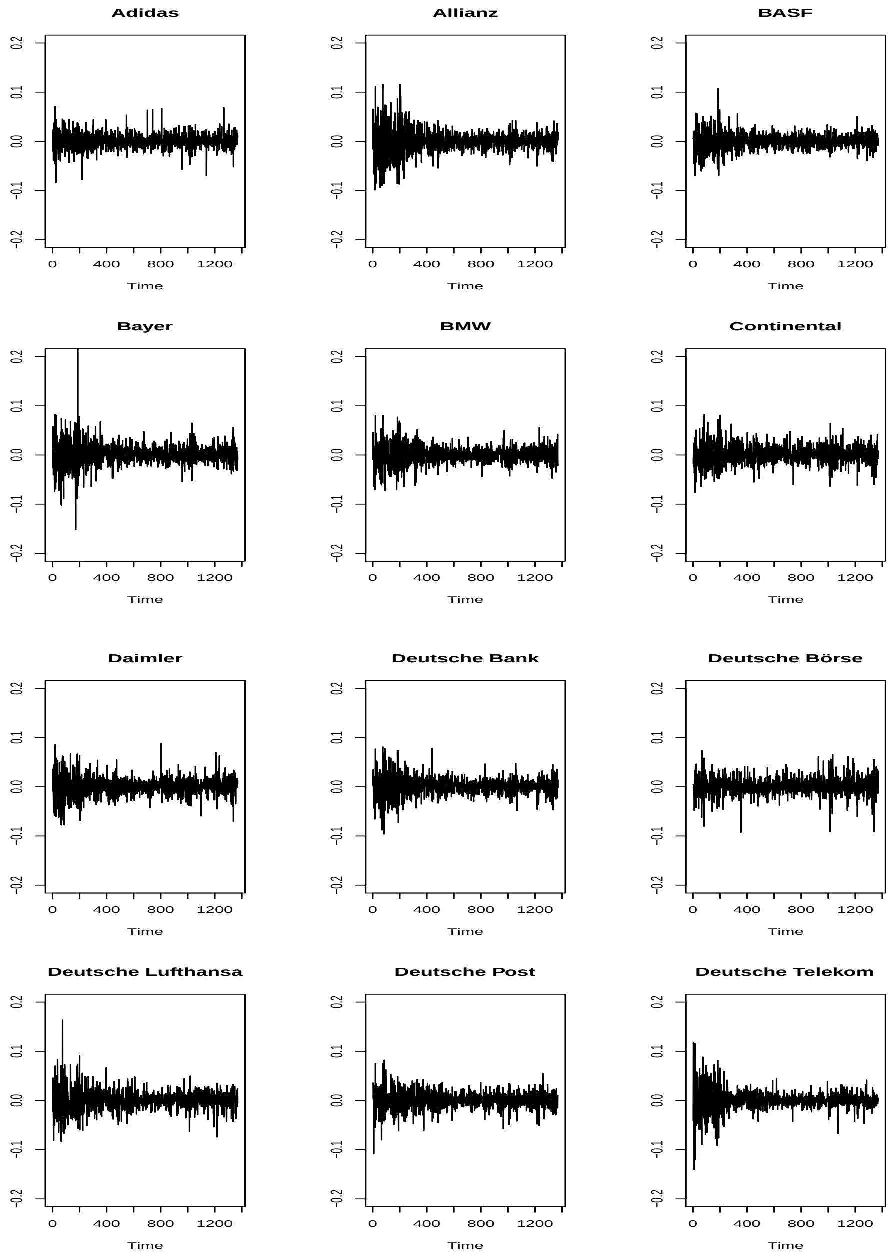
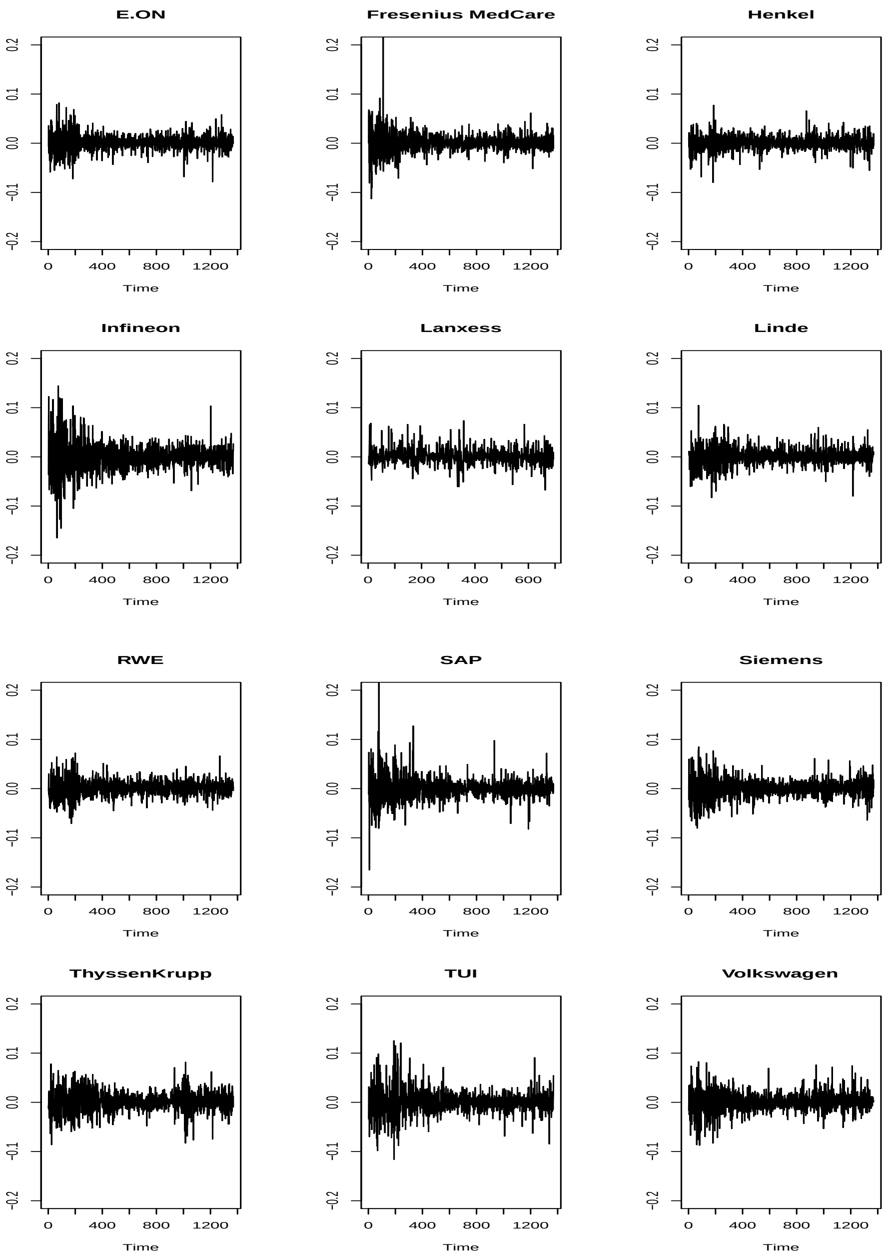
A. Appendix

A.1. Time Series Plots




E.ON
Fresenius MedCare
Henkel
Infineon
Lanxess
Linde
RWE
SAP
Siemens
ThyssenKrupp
TUI
Volkswagen
-0.2 -0.1 0.0 0.1 0.2
0 400 800 1200
0 200 400 600
Time

A.2. Estimation Results for GARCH model class

Description: - All estimations for the GARCH constants do have actually positive values with digits different from zero at least the from sixth position after decimal point on. - $(***)$, $(**)$, $(*)$ indicate significance of the coefficient to 1%, 5% and 10% level, respectively. - Testing $H_0 : \ln(\eta) = 0$ for the HYGARCH parameters. - Highest HQIC values written in bold indicating the corresponding selected model. - ncr: No convergence reached for this model.

A.2.1. Choose Gaussian conditional distribution

Adidas	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	0.0444	0.0471	0.0470	0.0433	0.0478	0.0433
ω	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
d	-	-	-	0.1385^{***}	0.2125^{***}	0.0012^{**}
α	0.0698^{*}	0.0694^{***}	0.0399	0.2409	0.1141	0.2186
β	0.8085^{***}	0.8658^{***}	0.8113^{***}	0.3072	0.2592	0.2480
γ	-	0.5291^{**}	0.0752^{*}	-	0.7262^{**}	-
δ	-	1.2496^{***}	-	-	0.8862^{***}	-
η	-	-	-	-	-	4.5396^{***}
HQIC	5.647	5.650	5.649	5.651	**5.656**	5.649

Allianz	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	0.0564^{**}	0.05668^{*}	0.0567^{**}	0.0579^{**}	0.0539^{*}	0.0567^{**}
ω	0.0000^{***}	0.0000^{***}	0.0000^{***}	0.0000^{***}	0.0000^{***}	0.0000^{***}
d	-	-	-	0.4345^{***}	0.2262^{***}	0.2126^{***}
α	0.0794^{***}	0.0754^{***}	0.0427^{***}	0.2612^{***}	0.1552	0.2645^{*}
β	0.9083^{***}	0.9047^{***}	0.9048^{***}	0.5815^{***}	0.2997^{*}	0.4361^{**}
γ	-	0.2452^{***}	0.0741^{***}	-	0.2092^{***}	-
δ	-	2.0000^{***}	-	-	2.5965^{***}	-
η	-	-	-	-	-	1.3569
HQIC	5.311	5.321	5.319	5.314	**5.324**	5.313

BASF	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	-0.0372^{*}	-0.03836	-0.0384	-0.0358	-0.0424	-0.0366
ω	0.0000^{***}	0.0000^{***}	0.0000^{***}	0.0000^{***}	0.0000^{***}	0.0000^{***}
d	-	-	-	0.3376^{***}	0.1377	0.1608
α	0.0674^{***}	0.0634^{***}	0.0214^{*}	0.2449^{**}	0.1559	0.1807
β	0.9164^{***}	0.9012^{***}	0.9013^{***}	0.04919^{***}	0.2387	0.3264
γ	-	0.4179^{**}	0.1060^{***}	-	0.2670^{***}	-
δ	-	2.0000^{***}	-	-	2.8267^{***}	-
η	-	-	-	-	-	1.4492
HQIC	5.784	**5.803**	5.802	5.783	5.780	5.781

Bayer	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	0.0145	0.0140	0.0234	0.0144		0.0140
ω	0.0000***	0.0000***	0.0000***	0.0000***		0.0000***
d	-	-	-	0.8023***		0.8584***
α	0.0766***	0.8584***	0.0284***	0.1287		0.0971
β	0.9181***	0.0971	0.9399***	0.8482***		0.8694***
γ	-	0.8694***	0.9917***	-		-
δ	-	0.0056	-	-		-
η	-	-	-	-	-	0.9944
HQIC	5.263	5.309	**5.321**	5.261	ncr	5.258

BMW	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	0.0149	0.0144	0.0152	0.0149	0.0155	0.0148
ω	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***
d	-	-	-	0.4911***	0.04720***	0.4467***
α	0.0557***	0.0482***	0.0408***	0.2703***	0.2747***	0.2902***
β	0.9348***	0.9351***	0.9339***	0.7111***	0.7008***	0.6966***
γ	-	0.1502*	0.0276*	-	0.1436*	-
δ	-	2.1652***	-	-	1.9367***	-
η	-	-	-	-	-	1.0223
HQIC	5.566	5.562	5.565	**5.568**	5.564	5.564

Continental	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	-0.0138	-0.0101	-0.0144	-0.0127	-0.0100	-0.0128
ω	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***
d	-	-	-	0.3435***	0.4705***	0.3713*
α	0.0727***	0.0731***	0.0313**	0.2195***	0.2692***	0.2157***
β	0.8976***	0.9086***	0.8957***	0.5021***	0.6980***	0.5164***
γ	-	0.5629***	0.0855***	-	0.7969*	-
δ	-	1.1405***	-	-	0.8736***	-
η	-	-	-	-	-	0.9740
HQIC	5.308	5.319	5.317	5.309	**5.324**	5.306

Daimler	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	0.0079	0.0093	0.0091	0.0115	0.0119	0.0112
ω	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***
d	-	-	-	0.3246***	0.3726***	0.5081*
α	0.0757***	0.0753***	0.0597***	0.1600*	0.1775**	0.1294
β	0.8932***	0.8953***	0.8951***	0.4633***	0.5237***	0.5736***
γ	-	0.0897	0.0260	-	0.0485	-
δ	-	1.8722***	-	-	1.7857***	-
η	-	-	-	-	-	0.9064*
HQIC	5.346	5.341	5.344	**5.348**	5.343	5.345

Deutsche Bank	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	0.0488*	0.0464*	0.0464*	0.0539*	0.0494*	0.0543*
ω	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***
d	-	-	-	0.4449***	0.4294***	0.5044**
α	0.0690***	0.0519***	0.0269**	0.2250***	0.2730***	0.2047**
β	0.9147***	0.9303***	0.9279***	0.6300***	0.6567***	0.6571***
γ	-	0.3240***	0.0608***	-	0.3168***	-
δ	-	1.9145***	-	-	1.7606***	-
η	-	-	-	-	-	0.9759
HQIC	5.536	5.540	5.542	5.539	**5.543**	5.536

Deutsche Börse	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	0.0588*	0.0661***	0.0684**	0.0567*		0.0580*
ω	0.0000***	0.0000***	0.0000***	0.0000***		0.0000***
d	-	-	-	0.2928***		0.8308***
α	0.1251***	0.1139***	0.0688***	0.2103*		0.0562
β	0.7798***	0.8237***	0.7783***	0.3875***		0.6565***
γ	-	0.7052***	0.1250***	-		-
δ	-	0.5524***	-	-		-
η	-	-	-	-	-	0.8561*
HQIC	5.426	**5.449**	5.431	5.418	ncr	5.420

Deutsche Lufthansa	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	0.0244	0.0277	0.0274	0.0380	0.0318	0.0359
ω	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***
d	-	-	-	0.3945***	0.3393***	0.0017***
α	0.0408***	0.0327*	0.0168*	0.3674***	0.3818***	0.4217
β	0.9524***	0.9555***	0.9514***	0.6236***	0.5984***	0.4618
γ	-	0.4438	0.0475***	-	0.2864***	-
δ	-	1.8952***	-	-	2.0171***	-
η	-	-	-	-	-	107.7916***
HQIC	5.291	5.296	5.298	5.295	**5.301**	5.297

Deutsche Post	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	-0.0254	-0.0234	-0.0261	-0.0312	-0.0324	-0.0302
ω	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***
d	-	-	-	0.4104***	0.3616***	0.2463
α	0.0405***	0.0471***	0.0449***	0.3249***	0.3434***	0.3618***
β	0.9495***	0.9730***	0.9503***	0.6701***	0.6499***	0.6321***
γ	-	-0.0334	-0.0076	-	-0.0393	-
δ	-	1.6654***	-	-	2.2019***	-
η	-	-	-	-	-	1.2246
HQIC	5.519	5.520	5.516	**5.523**	5.518	5.521

Deutsche Telekom	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	0.0303	0.0304	0.0304	0.0381	0.0258	0.0366
ω	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***
d	-	-	-	0.3770***	0.1476**	0.2645
α	0.0543***	0.0545***	0.0542***	0.2933***	0.3843***	0.3274***
β	0.9287***	0.9288***	0.9286***	0.6004***	0.4965***	0.5582***
γ	-	0.1733*	0.0006	-	-0.0127	-
δ	-	2.0304***	-	-	2.9276***	-
η	-	-	-	-	-	1.1417
HQIC	5.689	**5.697**	5.686	5.692	5.693	5.689

E.ON	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	-0.0386	-0.0398	-0.0398	-0.0422	-0.0411	-0.0401
ω	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***
d	-	-	-	0.8466***	0.9125***	0.9184***
α	0.0486***	0.0469***	0.0353***	0.2116***	0.1533*	0.1355*
β	0.9394***	0.9382***	0.9382***	0.9205***	0.9323***	0.9292***
γ	-	0.1428	0.0254	-	0.1481	-
δ	-	1.9979***	-	-	1.6992***	-
η	-	-	-	-	-	0.9882
HQIC	5.674	5.671	5.674	**5.675**	5.670	5.673

Fresenius MedCare	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	-0.0625**	-0.0476**	-0.0683**	-0.0570**	-0.0649**	-0.0694**
ω	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***
d	-	-	-	0.6206***	1.0000***	0.1875
α	0.0376***	0.0324***	0.0165*	0.4567***	0.1328**	0.7329***
β	0.9573***	0.9728***	0.9658***	0.8859***	0.9707***	0.8597***
γ	-	0.5021**	0.0301**	-	0.1689	-
δ	-	0.5307***	-	-	1.7987***	-
η	-	-	-	-	-	1.4977
HQIC	5.550	5.552	5.551	**5.5575**	5.557	5.556

Henkel	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	-0.0236	-0.0387	-0.0271	-0.0257	-0.0382	-0.0252
ω	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***
d	-	-	-	0.2815***	0.4018***	0.6001**
α	0.0722***	0.0620***	0.0109	0.4337***	0.3296***	0.3058**
β	0.8775***	0.9189***	0.8820***	0.6013***	0.6596***	0.7006***
γ	-	0.7606***	0.1060***	-	0.7775***	-
δ	-	0.8305***	-	-	0.8690***	-
η	-	-	-	-	-	0.8671*
HQIC	5.832	5.844	5.843	5.831	**5.848**	5.829

Infineon	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	0.0175	0.0167	0.0173	0.0126	0.0122	0.0156
ω	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***
d	-	-	-	0.4100***	0.3690***	0.1506
α	0.0654***	0.0612***	0.0557***	0.3743***	0.3928***	0.4395**
β	0.9188***	0.9182***	0.9186***	0.6666***	0.6495***	0.5637**
γ	-	0.0830	0.0199	-	0.0811	-
δ	-	2.1540***	-	-	2.1345***	-
η	-	-	-	-	-	1.5917
HQIC	4.793	4.788	4.790	**4.794**	4.789	4.791

Lanxess	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	0.0178	0.0021	0.0215	0.0132	0.0241	0.0142
ω	0.0000**	0.0000**	0.0000**	0.0000**	0.0000**	0.0000**
d	-	-	-	0.1393**	0.1387*	0.7904***
α	0.0638**	0.0648*	0.0138	0.0771	0.0433	0.0000
β	0.7682***	0.7622***	0.7634***	0.2021	0.1374	0.5181**
γ	-	0.1226	0.1364**	-	0.9261	-
δ	-	2.0011*	-	-	1.0149	-
η	-	-	-	-	-	0.6707
HQIC	5.168	5.172	**5.176**	5.155	5.160	5.155

Linde	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	-0.0421	-0.0362*	-0.0316	-0.0361		-0.0363
ω	0.0000***	0.0001***	0.0000***	0.0000***		0.0000***
d	-	-	-	0.2918***		0.0016***
α	0.0302***	0.0336***	0.0377**	0.5814***		0.8596***
β	0.9591***	0.9648***	0.9275***	0.7420***		0.9016***
γ	-	0.9787***	0.0941***	-		-
δ	-	0.5539***	-	-		-
η	-	-	-	-	-	96.6890***
HQIC	5.510	5.528	**5.529**	5.513	ncr	5.514

RWE	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	0.0033	0.0033	0.0018	0.0032	0.0021	0.0031
ω	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***
d	-	-	-	0.3428***	0.3551***	0.3135
α	0.0690***	0.0697***	0.0302*	0.5321***	0.4630***	0.5513***
β	0.9024***	0.8949***	0.8947***	0.7011***	0.6685***	0.7015***
γ	-	0.3392***	0.0830***	-	0.3106***	-
δ	-	1.7920***	-	-	1.6231***	-
η	-	-	-	-	-	1.0313
HQIC	5.604	5.610	**5.613**	5.607	5.612	5.604

SAP	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	-0.0321	-0.0227	-0.0240	-0.0191	-0.0183	-0.0191
ω	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***
d	-	-	-	0.5721***	0.5817***	0.5744***
α	0.1304***	0.1375***	0.0974***	0.0491	0.0698	0.0489
β	0.8553***	0.8578***	0.8537***	0.5691***	0.5818***	0.5704***
γ	-	0.1597***	0.0751**	-	0.1150*	-
δ	-	1.7742***	-	-	1.9080***	-
η	-	-	-	-	-	0.9986
HQIC	5.198	5.198	5.200	**5.209**	5.206	5.206

Siemens	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	0.0539*	0.0528*	0.0535*	0.0517*	0.0532*	0.0545*
ω	0.0000**	0.0000***	0.0000**	0.0000**	0.0000**	0.0000**
d	-	-	-	0.5085***	0.4089***	0.2450
α	0.0482***	0.0412***	0.0395***	0.2932***	0.3253***	0.3733***
β	0.9441***	0.9447***	0.9426***	0.7417***	0.6815***	0.6234***
γ	-	0.1135	0.0189	-	0.1453*	-
δ	-	2.2711***	-	-	2.1410***	-
η	-	-	-	-	-	1.2614
HQIC	**5.439**	5.435	5.438	5.435	5.432	5.433

ThyssenKrupp	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	0.0306	0.0323	0.0305	0.0257	0.0274	0.0266
ω	0.0000**	0.0000**	0.0000**	0.0000**	0.0000**	0.0000**
d	-	-	-	0.5620***	0.3783***	0.3779**
α	0.0543***	0.0339***	0.0548***	0.4197***	0.5222***	0.5242***
β	0.9383***	0.9437***	0.9386***	0.8248***	0.7650***	0.7882***
γ	-	-0.0492	-0.0014	-	-0.0051	-
δ	-	2.7987***	-	-	2.4798***	-
η	-	-	-	-	-	1.0964
HQIC	5.195	5.192	5.192	**5.202**	5.198	5.200

TUI	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	0.0178	0.0174	0.0178	0.0161	0.0160	0.0160
ω	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***
d	-	-	-	0.5141***	0.6959***	0.4964**
α	0.0520***	0.0668***	0.0482***	0.3375***	0.2468**	0.3458***
β	0.9397***	0.9358***	0.9404***	0.7648***	0.8401***	0.7589***
γ	-	0.0115	0.0060	-	0.0419***	-
δ	-	1.4548***	-	-	1.6153***	-
η	-	-	-	-	-	1.0059
HQIC	5.090	5.087	5.088	**5.092**	5.089	5.090

Volkswagen	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	0.0793^{***}	0.0785^{***}	0.0787^{***}	0.0802^{***}	0.0803^{***}	0.0798^{***}
ω	0.0000^{***}	0.0000^{***}	0.0000^{***}	0.0000^{***}	0.0000^{***}	0.0000^{***}
d	-	-	-	0.4202^{***}	0.4549^{***}	0.3594
α	0.0834^{***}	0.0819^{***}	0.0657^{***}	0.2594^{***}	0.2697^{***}	0.2762^{***}
β	0.8912^{***}	0.8879^{***}	0.8883^{***}	0.6135^{***}	0.6562^{***}	0.5850^{***}
γ	-	0.1165^{*}	0.0390^{*}	-	0.1160	-
δ	-	2.0795^{***}	-	-	1.9195^{***}	-
η	-	-	-	-	-	1.0451
HQIC	5.258	5.255	5.258	**5.262**	5.259	5.259

A.2.2. Choose Student-t conditional distribution

Adidas	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	0.0165	0.0220	0.0168	0.0227	0.0235	0.0188
ω	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
d	-	-	-	0.2920^{***}	0.4078^{***}	0.0029^{***}
α	0.0179^{***}	0.0658^{***}	0.0161^{**}	0.6714^{***}	0.3961^{***}	0.8998^{***}
β	0.9809^{***}	0.9262^{***}	0.9788^{***}	0.8003^{***}	0.6914^{***}	0.9528^{***}
γ	-	0.4548^{**}	0.0065	-	0.5873^{*}	-
δ	-	1.1189^{***}	-	-	0.8930^{**}	-
η	-	-	-	-	-	74.1804^{***}
HQIC	5.771	5.767	5.769	**5.772**	5.769	5.771

Allianz	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	0.0591^{**}	0.0600^{**}	0.0622^{**}	0.0598^{**}	0.0603^{**}	0.0588^{**}
ω	0.0000^{**}	0.0000^{**}	0.0000^{**}	0.0000^{**}	0.0000^{**}	0.0000^{**}
d	-	-	-	0.4740^{***}	0.2749^{***}	0.2392
α	0.0930^{***}	0.0706^{***}	0.0444^{**}	0.2338^{***}	0.1524	0.2342
β	0.8976^{***}	0.8900^{***}	0.8989^{***}	0.5940^{***}	0.3488^{*}	0.4316^{**}
γ	-	0.2245^{***}	0.0899^{***}	-	0.2506^{***}	-
δ	-	2.6010^{***}	-	-	2.4183^{***}	-
η	-	-	-	-	-	130744^{***}
HQIC	5.321	5.328	5.3293	5.322	**5.3295**	5.320

BASF	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	-0.0460^{*}	-0.0457^{*}	-0.0456^{*}	-0.0423	-0.0438	-0.0429
ω	0.0000^{**}	0.0000^{**}	0.0000^{**}	0.0000^{**}	0.0000^{**}	0.0000^{**}
d	-	-	-	0.3714^{***}	0.2588^{*}	0.1441
α	0.0700^{***}	0.0658^{***}	0.0199	0.2022	0.2298	0.0998
β	0.9167^{***}	0.9053^{***}	0.9053^{***}	0.4919^{***}	0.4256^{*}	0.2587
γ	-	0.4433^{*}	0.1143^{***}	-	0.3509^{**}	-
δ	-	2.0102^{***}	-	-	2.2460^{***}	-
η	-	-	-	-	-	14200^{***}
HQIC	5.802	**5.820**	5.818	5.801	5.813	5.800

Bayer	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	0.0215	0.0241	0.0258	0.0248		0.0252
ω	0.0000***	0.0000***	0.0000***	0.0000***		0.0000***
d	-	-	-	0.3955***		0.2525
α	0.0706***	0.0433***	0.0253***	0.2553**		0.2451
β	0.9156***	0.9497***	0.9435***	0.5602***		0.4531*
γ	-	0.9878***	0.9877***	-		-
δ	-	1.2341***	-	-		-
η	-	-	-	-	-	199.099***
HQIC	5.333	5.355	**5.360**	5.332	ncr	5.330

BMW	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	0.0178	0.0172	0.0175	0.0174	0.0158	0.0165
ω	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***
d	-	-	-	0.5223***	0.4680***	0.3607*
α	0.0541***	0.0488**	0.0377**	0.2715***	0.3010***	0.3452***
β	0.9400***	0.9402***	0.9388***	0.7321***	0.7107***	0.6834***
γ	-	0.1683	0.0311	-	0.1601	-
δ	-	2.0706***	-	-	2.0309***	-
η	-	-	-	-	-	2248.7***
HQIC	5.589	5.585	5.588	**5.590**	5.586	5.587

Continental	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	-0.0184	-0.0164	-0.0205	-0.0160	-0.0154	-0.0158
ω	0.0000**	0.0000**	0.0000**	0.0000**	0.0000**	0.0000**
d	-	-	-	0.3691***	0.5147***	0.3394
α	0.1007***	0.0976***	0.0501**	0.1349	0.2349***	0.1323
β	0.8635***	0.8840***	0.8667***	0.4259**	0.6724***	0.4044
γ	-	0.4398***	0.1032***	-	0.5496**	-
δ	-	1.2146***	-	-	1.0494***	-
η	-	-	-	-	-	416.089***
HQIC	5.347	5.352	5.352	5.348	**5.353**	5.344

Daimler	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	0.0083	0.0111	0.0093	0.0135	0.0137	0.0123
ω	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***
d	-	-	-	0.0478***	0.5919***	0.2371
α	0.0678***	0.0817***	0.0503***	0.1567**	0.1516*	0.1997*
β	0.9239***	0.9165***	0.9209***	0.6218***	0.7065***	0.4873***
γ	-	0.1698*	0.0397***	-	0.1412	-
δ	-	1.6028***	-	-	1.7251***	-
η	-	-	-	-	-	622.478***
HQIC	5.409	5.407	5.409	**5.411**	5.407	5.408

Deutsche Bank	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	0.0420*	0.0409	0.0407	0.0454*	0.0407	0.0447*
ω	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***
d	-	-	-	0.5479***	0.3717***	0.3891**
α	0.0724***	0.0473**	0.0164	0.1913***	0.3053***	0.2450***
β	0.9226***	0.9386***	0.9336***	0.6931***	0.6251***	0.6279***
γ	-	0.5424**	0.0901***	-	0.4510***	-
δ	-	1.8653***	-	-	2.0079***	-
η	-	-	-	-	-	3116.6***
HQIC	5.567	5.578	**5.580**	5.569	5.579	5.567

Deutsche Börse	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	0.0500*	0.0661*	0.0547*	0.0517*		0.0503*
ω	0.0000***	0.0000***	0.0000***	0.0000***		0.0000***
d	-	-	-	0.4106***		0.8750***
α	0.1744***	0.1331***	0.1019***	0.1503***		0.0453
β	0.7345***	0.8104***	0.7416***	0.3684***		0.06454***
γ	-	0.6114***	0.1556*	-		-
δ	-	0.5666**	-	-		-
η	-	-	-	-	-	102.044***
HQIC	5.510	5.513	**5.514**	5.505	ncr	5.505

Deutsche Lufthansa	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	0.0002	0.0021	0.0023	0.0052	0.0049	0.0027
ω	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***
d	-	-	-	0.4361***	0.4062***	0.0404
α	0.0788**	0.0981***	0.0560*	0.2653**	0.2703**	0.2892
β	0.9137***	0.9032***	0.9074***	0.5669***	0.5496***	0.3535
γ	-	0.2017**	0.0545*	-	0.1995**	-
δ	-	1.4724***	-	-	2.0163***	-
η	-	-	-	-	-	245.501***
HQIC	5.349	5.348	5.350	**5.352**	5.351	5.351

Deutsche Post	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	-0.0216	-0.0197	-0.0214	-0.0243	-0.0230	-0.0241
ω	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***
d	-	-	-	0.4656***	0.4985***	0.2335
α	0.0864***	0.0934***	0.0840***	0.3426***	0.3305***	0.4538***
β	0.8994***	0.9000***	0.8981***	0.6918***	0.7050***	0.6457***
γ	-	0.0323	0.0068	-	0.0527	-
δ	-	1.7285***	-	-	1.8657***	-
η	-	-	-	-	-	401.336***
HQIC	5.569	5.564	5.567	**5.5710**	5.566	5.569

Deutsche Telekom	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	-0.0026	-0.0010	-0.0011	0.0028	0.0037	0.0013
ω	0.0000^{***}	0.0000^{***}	0.0000^{***}	0.0000^{***}	0.0000^{***}	0.0000^{***}
d	-	-	-	0.4593^{***}	0.3564^{***}	0.1431
α	0.0711^{***}	0.0772^{***}	0.0592^{***}	0.2199^{**}	0.1998^{*}	0.2346
β	0.9241^{***}	0.9189^{***}	0.9200^{***}	0.6028^{***}	0.4863^{***}	0.4128^{*}
γ	-	0.1151	0.0328	-	0.1368	-
δ	-	1.9339^{***}	-	-	2.2804^{***}	-
η	-	-	-	-	-	79.0673^{***}
HQIC	5.800	5.797	5.799	**5.801**	5.798	5.800

E.ON	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	-0.0371	-0.0322	-0.0379	-0.0341	-0.0292	-0.0342
ω	0.0000^{***}	0.0000^{***}	0.0000^{***}	0.0000^{***}	0.0000^{***}	0.0000^{***}
d	-	-	-	0.4680^{***}	0.5283^{***}	0.5127^{**}
α	0.0658^{***}	0.0698^{***}	0.0341^{*}	0.3118^{***}	0.3029^{***}	0.2942^{**}
β	0.9196^{***}	0.9229^{***}	0.9149^{***}	0.6740^{***}	0.7227^{***}	0.6919^{***}
γ	-	0.5593^{**}	0.0683^{**}	-	0.7150^{***}	-
δ	-	1.2616^{***}	-	-	1.0080^{***}	-
η	-	-	-	-	-	176.355^{***}
HQIC	5.737	5.7381	5.7386	5.734	**5.7389**	5.731

Fresenius MedCare	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	-0.0761^{***}	-0.0760^{***}	-0.0769^{***}	-0.0745^{**}	-0.0736^{**}	-0.0799^{***}
ω	0.0000^{***}	0.0000^{***}	0.0000^{***}	0.0000^{***}	0.0000^{***}	0.0000^{***}
d	-	-	-	0.3781^{***}	0.3487^{**}	0.0227
α	0.0594^{***}	0.0720^{***}	0.0256^{***}	0.5982^{***}	0.6036^{***}	0.9799^{***}
β	0.9257^{***}	0.9266^{***}	0.9365^{***}	0.7729^{***}	0.7577^{***}	0.9867^{***}
γ	-	0.2835^{**}	0.0581^{***}	-	0.2367^{***}	-
δ	-	1.5118^{***}	-	-	2.0894^{***}	-
η	-	-	-	-	-	372.747^{***}
HQIC	5.635	5.638	5.640	5.6429	**5.643**	5.641

Henkel	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	-0.0474^{*}	-0.0484^{*}	-0.0502^{*}	-0.0494^{*}	-0.0484^{*}	-0.0494^{*}
ω	0.0000^{***}	0.0000^{***}	0.0000^{***}	0.0000^{***}	0.0000^{***}	0.0000^{***}
d	-	-	-	0.2807^{***}	0.3317^{***}	0.2600
α	0.0685^{***}	0.0629^{***}	0.0475^{*}	0.4159^{**}	0.3356^{***}	0.4219^{**}
β	0.8878^{***}	0.9005^{***}	0.8806^{***}	0.5762^{***}	0.5914^{***}	0.5706^{***}
γ	-	0.7066^{**}	0.1251^{***}	-	0.6949^{**}	-
δ	-	1.3280^{***}	-	-	1.2819^{***}	-
η	-	-	-	-	-	92.712^{***}
HQIC	5.919	5.924	5.923	5.919	**5.927**	5.916

Infineon	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	0.0283	0.0283	0.0283	0.0275	0.0282	0.0295
ω	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***
d	-	-	-	0.4922***	0.4306***	0.1979
α	0.0633***	0.0609***	0.0512***	0.3260***	0.3476***	0.4024***
β	0.9283***	0.9309***	0.9305***	0.7098***	0.6798***	0.5828***
γ	-	0.0875	0.0203	-	0.0803	-
δ	-	1.9860***	-	-	2.1650***	-
η	-	-	-	-	-	12510.2***
HQIC	4.822	4.817	4.820	**4.821**	4.816	4.819

Lanxess	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	-0.0183	-0.0179	-0.0179	-0.0261	-0.0210	-0.0236
ω	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
d	-	-	-	0.2222**	0.2904*	0.6730*
α	0.0577*	0.0645	0.0156	0.0258	0.3682	0.0000
β	0.8510***	0.8112***	0.8113***	0.2758	0.3682*	0.5025*
γ	-	0.5033***	0.1298*	-	0.5885	-
δ	-	0.8343	-	-	0.8722	-
η	-	-	-	-	-	146.028***
HQIC	5.222	5.226	**5.229**	5.218	5.218	5.216

Linde	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	-0.0479*	-0.0507*	-0.0390	-0.0460*	-0.0398*	-0.0436
ω	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***
d	-	-	-	0.3785***	0.3623***	0.0015***
α	0.0350**	0.0316***	0.0709***	0.4714***	0.3533***	0.9345***
β	0.9627***	0.9739***	0.9049***	0.7038***	0.6073***	0.9652***
γ	-	0.9999***	0.3480***	-	0.4850**	-
δ	-	0.7783***	-	-	1.6697***	-
η	-	-	-	-	-	74.896***
HQIC	5.607	**5.615**	5.613	5.607	5.614	5.611

RWE	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	0.0030	0.0021	0.0008	0.0045	0.0009	0.0045
ω	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***
d	-	-	-	0.3815***	0.3960***	0.2837
α	0.0619***	0.0700***	0.0343*	0.4646***	0.4178***	0.5166***
β	0.9200***	0.9064***	0.9059***	0.6870***	0.6676***	0.6761***
γ	-	0.2959**	0.0696**	-	0.2890**	-
δ	-	1.7333***	-	-	1.6355***	-
η	-	-	-	-	-	13436.1***
HQIC	5.621	5.623	**5.625**	5.622	5.624	5.619

SAP	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	-0.0130	-0.0099	-0.0093	-0.0148	0.0009	-0.0133
ω	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***
d	-	-	-	0.5119***	0.3960***	0.2888
α	0.0713***	0.0753***	0.0358*	0.2344***	0.4178***	0.2915***
β	0.9252***	0.9364***	0.9390***	0.6742***	0.6676***	0.5756***
γ	-	0.2592**	0.0465**	-	0.2890**	-
δ	-	1.3090***	-	-	1.6355***	-
η	-	-	-	-	-	61.6701***
HQIC	5.335	5.338	5.336	5.335	**5.339**	5.334

Siemens	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	0.0392	0.0382	0.0382	0.0409	0.0392	0.0409
ω	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***
d	-	-	-	0.7035***	0.4427**	0.3689
α	0.0509***	0.0388**	0.0378***	0.1883	0.3332***	0.3412***
β	0.9456***	0.9486***	0.9450***	0.8428***	0.7247***	0.7086***
γ	-	0.1470***	0.0269	-	0.1599*	-
δ	-	2.4089***	-	-	2.3055***	-
η	-	-	-	-	-	1327.03***
HQIC	**5.471**	5.468	5.470	5.469	5.466	5.466

ThyssenKrupp	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	0.0227	0.0228	0.0227	0.0195	0.0189	0.0199
ω	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***
d	-	-	-	0.7145***	0.5618**	0.4187*
α	0.0624***	0.0671***	0.0583***	0.3192*	0.4143***	0.4968***
β	0.9348***	0.9320***	0.9328***	0.8697***	0.8166***	0.7978***
γ	-	0.0547	0.0115	-	0.0437	-
δ	-	1.8896***	-	-	2.2043***	-
η	-	-	-	-	-	883.331
HQIC	5.231	5.226	5.229	**5.234**	5.229	5.232

TUI	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	0.0177	0.0148	0.0171	0.0139	0.0126	0.0159
ω	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***
d	-	-	-	0.5858***	0.6132**	0.3419
α	0.0514***	0.0620***	0.0352**	0.3005***	0.2911*	0.4107***
β	0.9467***	0.9378***	0.9532***	0.7942***	0.8084***	0.7228***
γ	-	0.1105	0.0204	-	0.0813	-
δ	-	1.4791***	-	-	1.9194***	-
η	-	-	-	-	-	96.3127***
HQIC	5.1707	5.167	5.169	**5.1708**	5.166	5.168

Volkswagen	GARCH	APARCH	GJR	FIGARCH	FIAPARCH	HYGARCH
ϱ	0.0646**	0.0639**	0.0639**	0.0658**	0.0655**	0.0658**
ω	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***
d	-	-	-	0.5301***	0.5890***	0.5504*
α	0.0919***	0.0910***	0.0660***	0.2232***	0.2057**	0.2147
β	0.8902***	0.8914***	0.8913***	0.6809***	0.7264***	0.6894***
γ	-	0.1499*	0.0538*	-	0.1760**	-
δ	-	1.9965***	-	-	1.8784***	-
η	-	-	-	-	-	396.629***
HQIC	5.3182	5.315	5.3180	**5.3183**	5.316	5.315

A.3. Standard & Poor's 1 Year Credit Ratings

Rating	**PD (in %)**	**Rating category**	**Rating**	**PD (in %)**	**Rating category**
AAA	<0,01	Prime	BB+	<0,94	Speculative
AA+	<0,02	High grade	BB	<1,55	
AA	<0,03		BB-	<2,50	
AA-	<0,04		B+	<4,08	Highly speculative
A+	<0,05	Upper medium grade	B	<6,75	
A	<0,08		B-	<10,88	
A-	<0,13		CCC	<17,75	Extremely speculative
BBB+	<0,22	Lower medium grade	CC	<29,35	
BBB	<0,36		C	>29,35	
BBB-	<0,58		D		In default