Long Memory, Breaks, and Trends: On the Sources of Persistence in Inflation Rates

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Abstract
The persistence of inflation rates is of major importance to central banks due to the fact that it determines the costs of monetary policy according to the Phillips curve. This article is motivated by newly available econometric methods which allow for a consistent estimation of the persistence parameter under low frequency contaminations and consistent break point estimation under long memory without a priori assumptions on the presence of breaks. In contrast to previous studies, we allow for smooth trends in addition to breaks as a source of spurious long memory. We support the finding of reduced memory parameters in monthly inflation rates of the G7 countries as well as spurious long memory, except for the US. Nevertheless, only a few breaks can be located. Instead, all countries exhibit significant trends at the 5% level with the exception of the US.

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1 Introduction

The question to which extend inflation time series follow a long-memory or rather a spurious long-memory process is still discussed. Spurious long memory exists when a time series with a nonlinear property, such as structural breaks in the mean or smooth trends, exhibits characteristics which can hardly be discriminated from a long-memory process. For example, both types of processes can generate an almost identical hyperbolic behavior of the autocorrelation function (cf. Granger and Hyung, 2004; Kuensch, 1986). Many empirical studies show that inflation rates are characterized by a mixture of long memory and breaks (cf. Bos et al., 1999; Kumar and Okimoto, 2007). Moreover, allowing for breaks typically reduces the degree of persistence. However, contradictory results have been presented about the number and location of break points in inflation rates.

This article is motivated by newly available econometric methods which allow for a consistent estimation of the persistence parameter under low frequency contaminations and consistent break point estimation under long memory without a priori assumptions on the presence of breaks. Moreover, the current empirical literature especially concentrates on abrupt structural breaks in inflation rates. Since the empirical literature focuses mainly on shifts, we extent the class of nonlinear models by considering smooth trends via a $SEMIFAR$ model.

The early literature in the 1990s focuses on the difference of $I(0)$ and $I(1)$ inflation processes. Instead of the aforementioned two specifications, many contributions conclude that the inflation rates are well described by a fractionally integrated process, such as an $ARFIMA(p,d,q)$ model, among them Backus and Zin (1993), Hassler and Wolters (1995), and Baum et al. (1999). In the following years the possibility of breaks or level shifts in inflation series has been studied mainly. This branch of literature stresses the aspect of spurious long memory in inflation rates. Bos et al. (1999) have been one of the first to consider exogenous level shifts in addition to the long memory behavior. Taking two level shifts into account, they reject the null hypothesis of no breaks for the G7 countries, except for Germany and Japan. Morana (2002) finds evidence of long memory and structural changes in European inflation rates. The empirical analysis of Gadea et al. (2004) shows a reduced memory parameter if they allow for endogenous structural changes, which can be interpreted as a first hint of spurious long memory. Hsu (2005) proposes a test for long memory in case of mean shifts and an estimation method for break points under long memory. However, the estimation method for the memory parameter is inconsistent under low frequency contaminations. His results show that the inflation rates of Italy and the US follow a long-memory process, whereas spurious long memory is indicated for Germany and Japan. Baillie and Kapetanios (2007) prove the presence of nonlinear and long memory components in monthly inflation rates for the majority of the G7 countries.

Beside changes in the mean, the second moment of a process gains more attention in the latest literature. Baillie et al. (1996) find strong persistence in the conditional variance in addition to the long memory behavior. Conrad and Karanasos (2005) also take persistent conditional heteroskedasticity into account by applying an $ARFIMA-FIGARCH$ model and find support for a dual long memory model for ten European countries.

Additionally, models incorporating also changes in the order of fractional integration have been proposed which can be interpreted as an effect of spurious long memory and indirectly support the argument of breaks in the mean. The empirical results of Kumar and Okimoto (2007) suggest a changing memory parameter d over time for the G7 countries, except for Japan. Charfeddine and Guégan (2012) examine the different behavior in various subsamples of US and French monthly inflation rates. They conclude that inflation data are best described by a mixture of long memory and switching processes and further suggest a model with time-varying memory parameter. Before considering changes in the persistence parameter and in the variance, Bos et al. (2014) concentrate also on changes in the mean. They combine an $ARFIMA$ model with a stochastic volatility model and find changes in the variance as well as in the memory parameter d for the US inflation rate. Hassler and Meller (2014) propose a test for determining the timing of multiple breaks in the memory parameter and find one significant break in the memory parameter of the US inflation rate.

The previous empirical findings are either based on a priori assumptions for the break point estimation or the time series are adjusted before determining the persistence due to the lack of estimation methods of the memory parameter under structural breaks in the mean (cf. Gadea et al., 2004; Charfeddine and Guégan, 2012). Gadea et al. (2004) criticize the absence of a test to distinguish between long memory and structural changes. Kumar and Okimoto (2007) highlight the need of formal testing and modeling procedures in the case of structural changes for future work.

The aforementioned lacks in the empirical literature motivate our work and we contribute to this literature by applying new econometric methods to distinguish between long memory and spurious long-memory processes. By employing the test of Qu (2011), we are able to differentiate between long memory and spurious long-memory processes, such as random level shifts or smoothly varying trends. Further, we consider semiparametric estimation procedures to determine the order of fractional integration, the local Whittle estimation approach and the modified method of Hou and Perron (2014). The modified version is robust to a variety of low frequency contaminations under long memory. Further, we combine the estimation method of Hou and Perron (2014) with the method of Lavielle and Moulines (2000) to determine the number and location of break points under long memory. Basically, the break points are estimated by ordinary least squares (*OLS*) like in Bai and Perron (1998, 2003), only the number of break points has to be determined using the information criterion developed by Lavielle and Moulines (2000). As an alternative to structural breaks, we apply a semiparametric fractional autoregressive model to estimate smooth trends.

Our results indicate a reduced memory parameter in the case of low frequency contaminations for the G7 countries. We reject the null hypothesis of true long memory, except for the US, and find almost no breaks, except for Japan. In contrast to other empirical studies which are often entirely based on abrupt structural breaks, we focus on smooth trends as another type of nonlinearity. The estimated trends provide a good approximation of the real data and capture spurious long memory more appropriately than structural breaks. With the exception of the US, all other countries exhibit significant trends to the 5% level. Therefore, we conclude that the inflation rates are best described by a smooth trend model with a reduced persistence parameter for Canada, France, Germany, Italy, Japan, and the UK. In contrast, the US inflation rate follows a long-memory process with no indication of nonlinearity, such as structural breaks or smooth trends.

The rest of the paper is organized as follows. In Section 2 we stress the importance of determining the correct degree of persistence by illustrating its effect on disinflation costs. Section 3 gives a brief overview of the different methods used in the empirical analysis. The results of the empirical study can be found in Section 4. Finally, Section 5 concludes.

2 Inflation Persistence and the Costs of Monetary Policy

Being one of the key variables in macroeconomics, inflation is linked to output or unemployment via the Phillips curve relation. This economic concept formulates the trade-off between two competing economic aims: price stability and stable markets. Decreasing inflation in order to guarantee price stability will always imply some costs in terms of output losses or higher unemployment. The level of these monetary policy costs depends on the persistence of inflation. The more persistent inflation rates are, the longer or the more aggressive the policy has to be, leading to higher welfare losses. Using a small scale economic model including a Phillips curve, we illustrate this effect of inflation persistence on the costs of monetary policy. Thus, we emphasize the importance of determining the degree of persistence inherent in inflation rates and further motivate our paper.

2.1 The Phillips Curve

In the existing literature there are some competing versions of the Phillips curve. A survey of the development of the Phillips curve literature can be found in Gordon (2011). Due to the fact that we are only interested in the effect of inflation persistence on the costs of monetary policy, we focus on the simplest form of the Phillips curve. It relates inflation to expected inflation $E\pi_t$, an economic variable like the output gap y_t, and an error term ε_t,

$$\pi_t = E\pi_t + \gamma y_t + \varepsilon_t. \tag{2.1}$$

According to Gordon (2011) the expectation term in Equation (2.1) can either be forward- or backward-looking or both.

Inserting lagged inflation for the expectation term, $E\pi_t = \pi_{t-1}$, yields the persistent Phillips curve. This model is backward-looking and thus produces the typical persistence of inflation rates. Although yielding good empirical fit, the model does not have any microeconomic foundation. The well known alternative is the New Keynesian Phillips curve, where the expectation term is given by $E\pi_t = E\pi_{t+1|t}$. In this model the expectations are purely forward-looking, resulting in a very flexible model. This is due to the fact that expectations in contrast to inflation rates can immediately respond to policy changes or shocks. However, a weakness of the New Keynesian Phillips curve is that it cannot fully capture the persistence of inflation. Moreover, Fuhrer (1995) shows that disinflations can be costless in this model framework which contradicts empirical findings. As a result, the empirical power of the New Keynesian Phillips curve is rather low. In order to reconcile these two approaches, Galí and Gertler (1999) consider the hybrid New Keynesian Phillips curve. This model includes backward- and forward-looking terms, so the expectation term is replaced by $E\pi_t = \beta_f E\pi_{t+1|t} + \beta_b \pi_{t-1}$. In fact, it is controversially discussed which term is more important and whether both are significant at all (cf. Galí and Gertler, 1999; Rudd and Whelan, 2005, 2007; Malikane, 2014). An alternative approach is mentioned by Fuhrer (1995). He points out that the flexible New Keynesian Phillips curve may be correct but that the expectations are persistent. We follow this argument and modify the New Keynesian Phillips curve. Instead of rational expectations we assume that expectations of future inflation rates are constructed by forecasting a fractionally integrated process with persistence parameter d ($FI(d)$),

$$(1-L)^d(\pi_t - \pi^*) = \varepsilon_t,$$

which has the $AR(\infty)$ representation

$$\pi_t = \pi^* + \sum_{j=0}^{\infty} w_j(\pi_{t-j} - \pi^*) + \varepsilon_t,$$

where ε_t forms a mean zero process and the autoregressive coefficients are given by $w_j = \Gamma(j-d)/(\Gamma(-d)\Gamma(j+1))$ (cf. Baillie, 1996).

We choose the $FI(d)$ model to compute our forecasts of the inflation series due to the fact that a stationary AR process exhibits not enough inflation persistence whereas a unit root process exhibits too much persistence. Instead, the $FI(d)$ specification in the context of inflation rates is supported by Hassler and Wolters (1995) and Baum et al. (1999).

The forecast of the inflation series using a $FI(d)$ process is calculated as the target rate π^* plus an infinitely long weighted sum of the current and past deviations of the inflation rate from its target,

$$\hat{\pi}_{t+1|t} = \pi^* + \sum_{j=0}^{\infty} w_j(\pi_{t-j} - \pi^*). \tag{2.2}$$

We approximate this sum by using 100 lags. As a result, the expectations are persistent since they depend on the memory parameter d via the weights w. The degree of persistence is determined by d.

2.2 Simulation of Disinflation Costs

In order to assess the costs of monetary policy, we define the following model framework according to Fuhrer (1995) and Fuhrer and Moore (1995). We consider a New Keynesian

Phillips curve,

$$\pi_t = E\pi_{t+1|t} + \gamma y_t,$$

with $E\pi_{t+1|t} = \hat{\pi}_{t+1|t}$ which is the forecast of the inflation series using a $FI(d)$ model. The monetary policy reaction function is given by

$$i_t - i_{t-1} = \alpha_\pi(\pi_t - \pi^*) + \alpha_y y_t.$$

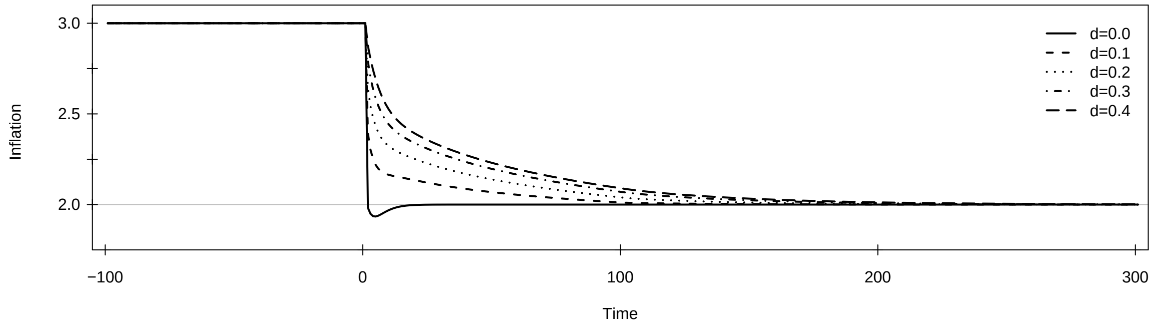
The higher the values of α_π and α_y, the more determined the central bank reacts to deviations of the inflation rate from the target rate π^* and to the output gap y_t, respectively by changing the nominal interest rate i_t. The Fisher equation describes the relation between real interest rate, nominal interest rate and inflation,

$$r_t = i_t - \pi_t.$$

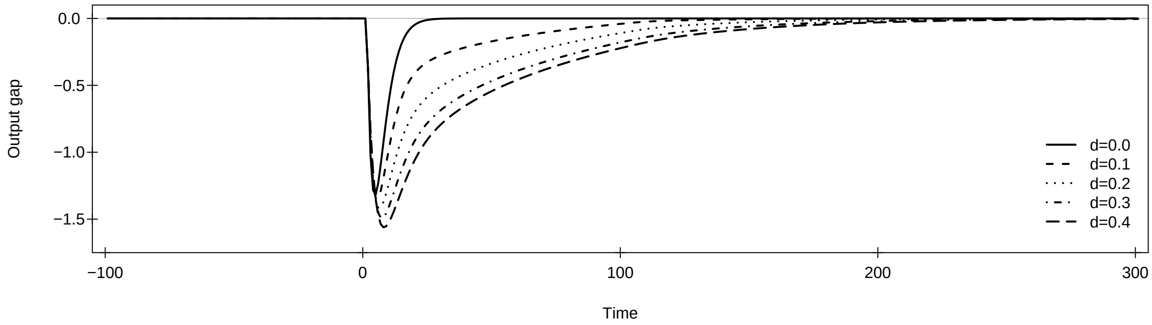
Finally, the transmission mechanism of the monetary policy to real output is given by

$$y_t = \lambda y_{t-1} + \beta r_{t-1}.$$

We assume that the economy is in the steady state, i.e. $\pi_t = \pi^* = 3\%$ and $y_t = 0$. Then, in period $t = 1$ the target inflation rate is decreased to $\pi^* = 2\%$. As a result, the current inflation rate exceeds the new target rate. Thus, the central bank reacts according to its policy reaction function and increases the nominal interest rate. The rising interest rates contract real output and the output gap becomes negative. In addition, the inflation expectations change, yielding the new inflation rate. Finally, the inflation rate converges to the new target rate and the output gap approaches zero from below (cf. Figure 2.1). The speed of convergence depends on the memory parameter d. Figure 2.1 illustrates the fact that with an increasing degree of persistence d, the inflation rate converges more slowly to the new target rate. Naturally, the time horizons are rather long in this example and we do not expect inflation to be as sluggish in reality. In fact, this simulation aims at demonstrating the influence of inflation persistence. For a large value of $d = 0.4$ (long-dashed line) the inflation rate needs more than twice as long as for a small value of $d = 0.1$ (dashed line) to reach the new target rate. In case of short memory $d = 0$ (solid line), the inflation expectations are completely flexible and drop below the new target rate to converge from below (cf. Fuhrer, 1995). Furthermore, the persistence determines how far the output falls under its potential and how long the output gap is negative. For a small degree of persistence of $d = 0.1$ (dashed line) the maximum absolute value of the output gap is about 1.3. If $d = 0.4$ (long-dashed line),



(a) Inflation rate



(b) Output gap

Figure 2.1: Simulation of a disinflation for different memory parameters d with $\gamma = 0.05$, $\alpha_\pi = 0.4$, $\alpha_y = 0.2$, $\lambda = 0.5$, and $\beta = -0.36$

the maximum absolute deviation from potential exceeds 1.5 and it takes almost twice as long to reach the potential again.

Figure 2.1 thus illustrates that the degree of persistence inherent in inflation crucially influences the costs of central bank policy measured as the shortfall of output below its potential when the expected inflation equals the forecast of a $FI(d)$ process. It is therefore important to determine the correct degree of persistence of inflation rates in order to be able to assess and compare the costs and the time until full implementation of different policy measures.

3 Econometric Methods

This section gives a brief overview of the different econometric methods which are used in our empirical analysis. First, we characterize the standard *ARFIMA* model and introduce the well known semiparametric local Whittle estimator. Then, we consider the test of Qu (2011) which distinguishes between long memory and spurious long memory. Further, we present the recently developed estimation method of Hou and Perron (2014) for the memory parameter under low frequency contaminations. The estimation method of Lavielle and Moulines (2000) to determine break points under long memory is described as one source of spurious long memory. As an alternative to abrupt structural changes we finally consider smooth trends fitted by a *SEMIFAR* model.

3.1 ARFIMA Model

The most popular model type to capture long-memory processes are *ARFIMA* models and, therefore, we start with a short introduction to this model class. Many contributions in the literature show that inflation rates are well described by an *ARFIMA* (p,d,q) process which has been developed by Granger and Joyeux (1980) and Hosking (1981). The *ARFIMA* model is defined as

$$\Phi(L)(1-L)^d x_t = \Theta(L)\epsilon_t,$$

with $\epsilon_t \sim iid(0,\sigma_\epsilon^2)$, $\Phi(L) = 1-\phi_1 L-\ldots-\phi_p L^p$ being the autoregressive polynomial and $\Theta(L) = 1-\theta_1 L-\ldots-\theta_q L^q$ the moving average polynomial with all roots lying outside the unit circle. The order of fractional integration is given by d, L denotes the backshift operator, and $(1-L)^d = \sum_{j=0}^{\infty} \frac{\Gamma(j-d)L^j}{\Gamma(-d)\Gamma(j+1)}$ with the Gamma function $\Gamma(z)$. The interval $0 < d < 0.5$ implies long memory with a hyperbolically decaying autocorrelation function and an unbounded spectral density function at frequencies close to the origin. For $d = 0$, x_t follows a short memory process with an exponentially decaying autocorrelation function and a bounded spectral density function for zero frequencies. The *ARFIMA* process is stationary for the interval $-0.5 < d < 0.5$ and mean reverting as well as invertible for $d < 1$.

3.2 Local Whittle Estimator

In the following, the semiparametric estimator of Kuensch (1987) and Robinson (1995) is presented since we are interested in the persistence of a series and whether the process exhibits long memory. The local Whittle method, also known as the Gaussian semiparametric estimation method, is often applied in the empirical literature. It is the typical reference procedure since it requires only mild assumptions, such as non-Gaussianity, and exhibits good asymptotic properties with a smaller variance compared to other semiparametric methods. Due to its aforementioned flexibility several authors extend this method, such as Hou and Perron (2014). As a semiparametric estimator the form of the spectral density is not fully modified and considers frequencies only close to the origin. The spectral density function of a stationary process x_t at frequency λ is given by

$$f(\lambda) \sim G\lambda^{-2d} \quad \text{as } \lambda \to 0+, \tag{3.1}$$

with G being a slowly varying function and $d \in (-0.5, 0.5)$. The periodogram is defined as

$$I(\lambda_j) = (2\pi T)^{-1} \left| \sum_{t=1}^{T} x_t \exp(i\lambda_j t) \right|^2, \tag{3.2}$$

with $\lambda_j = 2\pi j/T$, $j = 1, \ldots, \lfloor T/2 \rfloor$ are the Fourier frequencies and T being the sample size. The estimator is based on the local Whittle likelihood function

$$Q(G,d) = \frac{1}{m} \sum_{j=1}^{m} \left\{ \log G\lambda_j^{-2d} + \frac{I(\lambda_j)}{G\lambda_j^{-2d}} \right\}, \tag{3.3}$$

with the bandwidth parameter $m = \lfloor T^{\delta} \rfloor$ and $\delta = [\frac{1}{3}, \frac{4}{5}]$. Minimizing $Q(G,d)$ w.r.t. G leads to the profiled likelihood function

$$R(d) = \log \hat{G}(d) - \frac{2d}{m} \sum_{j=1}^{m} \log \lambda_j \quad \text{with} \quad \hat{G}(d) = \frac{1}{m} \sum_{j=1}^{m} \lambda_j^{2d} I(\lambda_j). \tag{3.4}$$

The local Whittle estimator is given by $\hat{d} = \arg\min_{d} R(d)$ and converges to a normal distribution according to $\sqrt{m}(\hat{d} - d) \xrightarrow{d} \mathcal{N}\left(0, \frac{1}{4}\right)$.

3.3 Test of Spurious Long Memory

Due to the fact that the local Whittle estimator cannot detect spurious long-memory processes, statistical tests can be applied to assess the properties of the time series. In general, several tests exist in the literature to differentiate between spurious and true long memory. Qu (2011) derives a spectral based test to distinguish between long memory and spurious long memory because the typical characteristics of a persistent process can also be a result of low frequency contaminations. Compared to other tests, the Qu test provides some advantages which justify our choice and simulation studies

stress the superior asymptotic properties (cf. Qu, 2011). The test is a score-type test that can handle multiple structural breaks and uses the properties of the local Whittle estimator with its mild assumptions. It is a flexible test as the alternative does not need to be specified, i.e. information about the form of the trend or the number of breaks under the alternative are not necessary.
The null hypothesis is described by a stationary long-memory process whereas under the alternative the process is contaminated by structural breaks or smooth trends. The general test idea is based on the different behavior of the fitted spectral density and the periodogram for frequencies close to the origin (cf. Perron and Qu, 2010). Taking the derivative of $R(d)$ w.r.t. d from Equation (3.4) results in

$$\frac{\partial R(d)}{\partial d} = \frac{2G^0}{\sqrt{m}\hat{G}(d)}\left(m^{-\frac{1}{2}}\sum_{j=1}^{m}\nu_j\left(\frac{I(\lambda_j)}{G^0\lambda_j^{-2d}}-1\right)\right) \text{ with } \nu_j = \log\lambda_j - \frac{1}{m}\sum_{j=1}^{m}\log\lambda_j. \tag{3.5}$$

In the rest of the paper the superscript 0 denotes the true value of a parameter. The test statistic takes the supremum of the term in the big braces of Equation (3.5) for different bandwidths $\lfloor mr \rfloor < m$, where $r \in [\epsilon, 1]$ with $\epsilon > 0$,

$$W = \sup_{r\in[\epsilon,1]}\left(\sum_{j=1}^{m}\nu_j^2\right)^{-\frac{1}{2}}\left|\sum_{j=1}^{\lfloor mr \rfloor}\nu_j\left(\frac{I(\lambda_j)}{G(\hat{d})\lambda_j^{-2\hat{d}}}-1\right)\right|.$$

Note, that the local Whittle estimator $\hat{d}$ is based on m bandwidth instead of $\lfloor mr \rfloor$ bandwidths as considered for the test statistic. Qu (2011) recommends to replace $m^{-\frac{1}{2}}$ from Equation (3.5) by $\left(\sum_{j=1}^{m}\nu_j^2\right)^{-\frac{1}{2}}$ for size correction and he introduces a small trimming parameter ϵ. For a small sample size with less than 500 observations the author suggests to set $\epsilon = 0.05$ and for a larger sample $\epsilon = 0.02$. Further, the test achieves good results in terms of size and power by taking a higher bandwidth parameter, such as $m = T^{0.7}$ (cf. Qu, 2011).

3.4 Modified Local Whittle Estimator

Consequentially, when the test of Qu (2011) rejects the null hypothesis this indicates spurious long memory, such as random level shifts or deterministic trends. In this case the local Whittle estimator becomes inconsistent. Therefore, Hou and Perron (2014) propose a consistent estimation method of the memory parameter under these types of low frequency contaminations. Comparing the estimation results of the original with the modified local Whittle method gives a first insight whether the time series follows a long-memory or rather a spurious long-memory process. The modified local Whittle estimation method considers the following underlying process

$$z_t = c + y_t + u_t,$$

with c being a constant, y_t is either a short or long-memory process with $d \in [0, 0.5)$, and u_t captures low frequency contaminations which are typically characterized as level shifts or as trend functions (cf. Hou and Perron, 2014). In case of the former process $u_t = \sum_{t=1}^{T} \pi_{T,t}\, \eta_t$ with $\eta_t \sim i.i.d.\ N(0, \sigma_\eta^2)$ and $\pi_{T,t}$ being an i.i.d. Bernoulli variable with the shift probability p/T. For the latter process the low frequency contamination term takes the form $u_t = g(t/T)$ with $g(\cdot)$ being a trend function on [0,1]. Adding the new term $G_u \lambda^{-2}/T$ to the standard form of Equation (3.1), a so called pseudo spectral density can be written as

$$f_z(\lambda) = G\lambda^{-2d} + G_u\lambda^{-2}/T = G(\lambda^{-2d} + (G_u/G)\lambda^{-2}/T) = G(\lambda^{-2d} + \theta\lambda^{-2}/T),$$

with θ being the signal to noise ratio. Replacing x_t by z_t in Equation (3.2) yields the periodogram $I_z(\lambda_j)$. The pseudo likelihood function is given by

$$Q(G, d, \theta) = \frac{1}{m} \sum_{j=1}^{m} \left\{ \log f_z(\lambda_j) + \frac{I_z(\lambda_j)}{f_z(\lambda_j)} \right\}. \tag{3.6}$$

Finally, the modified local Whittle estimator is defined as $\hat{d} = \arg\min_{d,\theta} R(d, \theta)$ with

$$R(d, \theta) = \log\left(\frac{1}{m} \sum_{j=1}^{m} \frac{I_z(\lambda_j)}{\lambda_j^{-2d} + \theta\lambda_j^{-2}/T} \right) + \frac{1}{m} \sum_{j=1}^{m} \log\left(\lambda_j^{-2d} + \theta\lambda_j^{-2}/T\right).$$

Hou and Perron (2014) suggest to set the bandwidth parameter greater than $m = T^{5/9}$, therefore, we consider higher bandwidths in our empirical study. In the absence of low frequency contamination the form of the pseudo likelihood function in Equation (3.6) reduces to the standard local Whittle likelihood function of Equation (3.3) by setting $f_z(\lambda) = f_y(\lambda) = G\lambda^{-2d}$. Hence, no asymptotic efficiency loss exists since the asymptotic properties remain identical to the standard local Whittle approach. In the presence of low frequency contaminations Hou and Perron (2014) show that their estimation method is characterized by the smallest bias and mean squared error compared to other estimation approaches.

3.5 Estimating Break Points

In the same way as the presence of structural breaks causes spurious long memory, structural break tests will spuriously indicate the presence of mean shifts if the true data generating process exhibits long memory (cf. Krämer and Sibbertsen, 2002). In order to resolve this issue, the number and location of breaks is assumed to be known a priori. However, if the true number of breaks differs from this assumption the resulting estimates of the memory parameter d will be inconsistent.

For a given number of breaks b, Lavielle and Moulines (2000) show that ordinary least squares (OLS) estimates of the break locations remain consistent under long memory. Therefore, the well known estimation methods of Bai and Perron (1998, 2003) can be applied. Only the model selection procedure for the number of breaks has to be modified. Denote the vector of break fractions by $s = (\tau_1, ..., \tau_b)'$, such that $\tau_0 = 0 < \tau_1 < ... < \tau_b < \tau_{b+1} = 1$ and the breakpoints are given by $\lfloor \tau_i T \rfloor$ for $i = 1, ..., b$. The vector of the means in each segment is given by $\mu = (\mu_1, ..., \mu_b)'$, so that in the case of b breaks the residual sum of squares is given by

$$\mathrm{RSS}(s, \mu) = \sum_{i=1}^{b+1} \sum_{t=\lfloor \tau_{i-1} T \rfloor + 1}^{\lfloor \tau_i T \rfloor} (x_t - \mu_i)^2.$$

The corresponding OLS estimate is

$$(\hat{s}(b),\ \hat{\mu}(b)) = \arg\min_{s,\mu} \mathrm{RSS}(s,\ \mu).$$

To consistently estimate the number of breaks in long-memory time series, Lavielle and Moulines (2000) suggest a modified model selection criterion with a penalty term that depends on the memory parameter d. The more persistent the series, the higher is the penalty term, resulting in a parsimonious selection of break points. In contrast, for antipersistent series a higher number of breaks is selected since the penalty term becomes negative. The number of breaks is therefore estimated by minimizing the modified BIC,

$$BIC_{LM} = \mathrm{RSS}(\hat{s}(b),\ \hat{\mu}(b)) + 4b \frac{\log T}{T^{1-2d}}.$$

If the memory parameter d was known, the number of breaks could be directly estimated by minimizing the BIC_{LM}. However, in absence of a robust estimate of d, this method remains infeasible. Combining the BIC_{LM} with the method of Hou and Perron (2014) enables us to conduct a semiparametric analysis of spurious long memory in inflation rates that does not require any a priori assumptions on the number or location of breaks.

3.6 SEMIFAR Model

Additional to abrupt changes, such as structural breaks, smooth trends are another potential source of spurious long memory. Since both low frequency contaminations have similar impacts on the persistence of a process, we also consider a trend model. A simplified version of the semiparametric fractional autoregressive ($SEMIFAR$) model introduced by Beran (1999) and further developed by Beran and Feng (2002a,b) is a combination of a fractional autoregressive model with a nonparametric trend and is defined as

$$(1-L)^d\{x_i - g(t_i)\} = \epsilon_i,$$

with $d \in (-0.5, 0.5)$, $t_i = (i/T)$ with $i \in \mathbb{Z}$, and a trend function g on [0,1]. We obtain this simplified version of the $SEMIFAR$ model by setting the maximum autoregressive order as well as the integration order equal to zero. The $SEMIFAR$ model combines deterministic trends, stochastic trends, short memory and long memory components. Further, it incorporates nonparametric modeling of the trend function and a simultaneous parametric procedure for the dependence structure (cf. Beran and Feng, 2002b). Fitting this simplified $SEMIFAR$ model requires a maximum likelihood estimation for the memory parameter and kernel smoothing for the trend function. We follow the approach of Beran and Feng (2002a) for estimating the trend function

$$\hat{g}(t) = \frac{1}{Th}\sum_{i=1}^{T} K\left(\frac{t-t_i}{h}\right)x_i,$$

where h is the optimal bandwidth of a symmetric positive second-order kernel K. According to Beran and Feng (2002a) estimating $\hat{g}$ depends on h. However, the optimal bandwidth h also depends on the aforementioned memory parameter that is estimated by maximum likelihood. This interdependence structure is solved by an iterative procedure which varies between kernel smoothing and parameter estimation. A variety of estimation approaches exists. In our empirical analysis we apply the data-driven algorithm of Beran and Feng (2002a) which is based on a full grid search with respect to d in the interval $[0, 0.5)$ in steps of 0.025. The trend function g is estimated nonparametrically using the uniform kernel $K(u) = \frac{1}{2}\mathbf{1}_{\{|u|\leq 1\}}$. The optimal bandwidth h for the kernel estimation depends, among others, on the second derivative of the trend function (cf. Beran, 1999). In order to estimate the second derivative, the following fourth order kernel is used $K(u) = \frac{105}{16}\left(-5x^4 + 6x^2 - 1\right)\mathbf{1}_{\{|u|\leq 1\}}$ (cf. Gasser et al., 1985).

4 Empirical Analysis

In this section we apply the previously introduced econometric methods to inflation data of the G7 countries in order to distinguish the influence of true and spurious long memory in inflation rates. We use month-on-month CPI and core CPI (excluding food and energy) data from 1970:1 until 2015:2 available from Thomson Reuters Datastream. Following Bos et al. (1999), Baillie et al. (1996), and Hassler and Wolters (1995) we define the inflation rates as

$$\pi_t = 100 \cdot (\log(P_t) - \log(P_{t-1}))\,.$$

Thus, our data set consists of 541 observations. We use the R package X13 for seasonal adjustment. Figure 4.1 illustrates the respective US inflation series as a typical example.

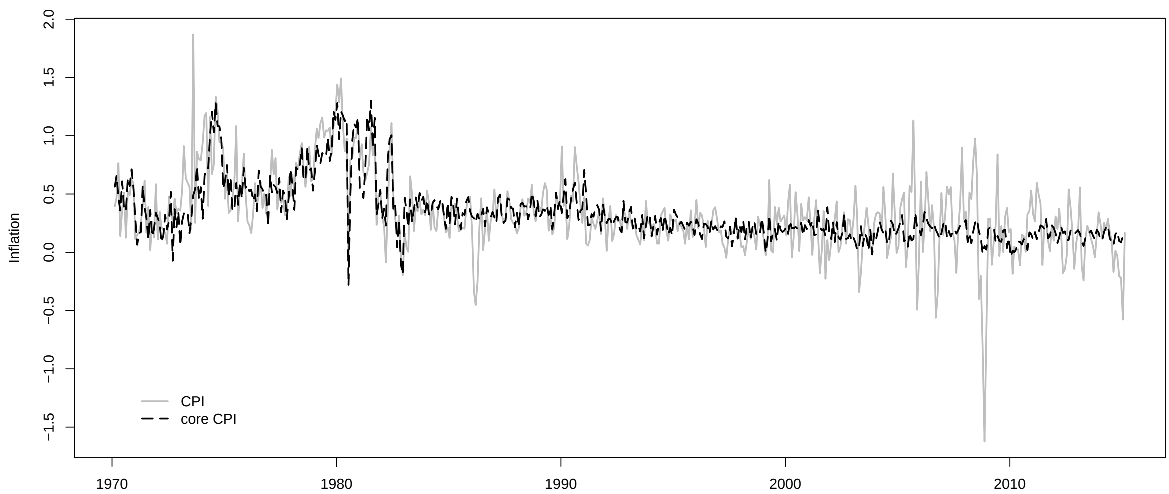


Figure 4.1: US inflation rates

The properties of the series change over time (cf. Cecchetti et al., 2007 and Bos et al., 2014). During the Great Inflation in the 1970s and early 1980s the inflation level is relatively high combined with a high volatility. At the beginning of the 1980s there is an overall decrease in the inflation level. In addition, the series appears to be more stable. This period is referred to as the Great Moderation. Until the end of the 1990s both inflation series exhibit a similar behavior, but later the CPI based inflation becomes more volatile. In contrast, the core CPI based inflation preserves the low volatility of the Great Moderation due to the fact that food and energy prices are excluded. The patterns of the Great Inflation and Great Moderation can be found for the other G7 countries as well. In the rest of this section we aim at finding a model that appropriately describes these features of inflation data. In a first step, we apply the local Whittle estimator to each series in order to check whether inflation exhibits long memory. The estimation results for different bandwidths are presented in Tables 4.1a and 4.1b.

δ	**0.70**	**0.72**	**0.74**	**0.76**	**0.78**	**0.80**
CAN	0.461	0.436	0.389	0.376	0.357	0.334
FRA	0.584	0.596	0.562	0.553	0.545	0.519
GBR	0.496	0.478	0.463	0.480	0.496	0.476
GER	0.400	0.410	0.369	0.342	0.349	0.321
ITA	0.577	0.562	0.551	0.559	0.557	0.558
JPN	0.443	0.431	0.393	0.359	0.361	0.352
USA	0.386	0.411	0.368	0.373	0.402	0.384

(a) CPI based inflation

δ	**0.70**	**0.72**	**0.74**	**0.76**	**0.78**	**0.80**
CAN	0.521	0.497	0.479	0.449	0.425	0.396
FRA	0.605	0.635	0.635	0.619	0.625	0.614
GBR	0.470	0.433	0.417	0.428	0.441	0.438
GER	0.443	0.448	0.427	0.380	0.370	0.346
ITA	0.617	0.573	0.595	0.597	0.519	0.519
JPN	0.519	0.508	0.475	0.426	0.428	0.413
USA	0.505	0.498	0.517	0.497	0.511	0.525

(b) Core CPI based inflation

Table 4.1: Local Whittle estimates for different bandwidths $m = \lfloor T^{\delta} \rfloor$

All estimates of d lie in the interval $[0.321, 0.596]$ for CPI and in the interval $[0.346, 0.635]$ for core CPI, respectively. This implies that the data can neither be properly described by an $ARMA$ nor by an $ARIMA$ process. For increasing bandwidths the estimates of the memory parameter decrease. Nevertheless, they appear to be significantly different from zero and thus, may best be modeled by an $ARFIMA$ process. For a large bandwidth with $\delta = 0.8$ most series exhibit stationary long memory. France, Italy, and the US (only for core CPI) yield a long memory parameter exceeding 0.5, meaning they are nonstationary but mean reverting. These results are supported by Hassler and Wolters (1995) who estimate the persistence parameters of the European G7 countries and the US to lie in the interval $[0.35; 0.50]$, where all countries exhibit stationary long memory except Italy. In addition, Baum et al. (1999) estimate the memory parameters of the G7 countries in the interval $[0.362; 0.563]$ for larger bandwidths where only the US and French inflation rates exhibit nonstationary long memory. These findings suggest that the inflation series of all G7 countries exhibit long memory.

However, as already mentioned there are some types of low frequency contaminations that can lead to spurious long memory (cf. Granger and Hyung, 2004; Kuensch, 1986). Therefore, we apply the Qu test in order to discriminate whether the large values of the local Whittle estimates are due to true long memory or are only spurious. Under the null hypothesis of the Qu test the series is a true long-memory process. If the test statistic exceeds the critical value, the series follows no longer a true long memory-process. The values of the test statistics depending on the bandwidth m are given in Tables 4.2a and 4.2b. As already mentioned in Section 3.3, bandwidths greater or equal to m with $\delta = 0.8$ are excluded by Qu (2011). Considering a significance level of 5%, we are able to reject the null hypothesis of true long memory for all countries except for the US and for a significance level of 1% for all countries except for the US, the UK (mostly for CPI), and Japan for smaller bandwidths (only for CPI). The results of the Qu test are remarkably stable across different bandwidths and significance levels and thus point out that the inflation series are not true long-memory processes except for the US.

δ	0.70	0.72	0.74	0.76	0.78
CAN	1.607	1.823	2.305	2.381	2.572
FRA	1.972	1.793	2.110	2.148	2.185
GBR	1.286	1.395	1.475	1.257	1.069
GER	1.797	1.677	2.022	2.263	2.131
ITA	1.686	1.715	1.761	1.610	1.571
JPN	1.353	1.410	1.706	2.018	1.916
USA	1.143	0.882	1.289	1.191	0.885

(a) CPI based inflation

δ	0.70	0.72	0.74	0.76	0.78
CAN	1.751	1.932	2.059	2.355	2.595
FRA	2.238	1.861	1.674	1.808	1.667
GBR	1.474	1.748	1.848	1.666	1.464
GER	2.276	2.184	2.347	2.806	2.855
ITA	1.701	2.081	1.748	1.661	2.596
JPN	1.577	1.629	1.916	2.405	2.301
USA	1.242	1.280	1.051	1.232	1.039

(b) Core CPI based inflation

Table 4.2: Test statistics of the Qu test for different bandwidths $m = \lfloor T^{\delta} \rfloor$. The critical values for $\epsilon = 0.02$ are 1.118, 1.252 and 1.517 for the respective significance levels of $\alpha = 10\%, 5\%, 1\%$ (cf. Qu, 2011).

Since the local Whittle estimator cannot successfully distinguish between true and spurious long memory, the estimates are inconsistent and exaggerate the degree of persistence. In order to capture the true degree of persistence, we apply the estimator of Hou and Perron (2014) since it can discriminate between true long memory and low frequency contaminations like random level shifts or smooth trends that yield spurious long memory.

Tables 4.3a and 4.3b show the reestimated long memory parameters if the modified local Whittle estimator is used. Comparing the results with the local Whittle estimates, the reduced d becomes obvious. This implies that at least some of the memory found by the local Whittle approach is spurious. Not depending on the bandwidth, all inflation series are stationary, but for some countries the memory parameter estimate is negative which can be interpreted as antipersistent memory. However, this is inconsistent with economic theory. Therefore, we restrict d to the interval $[0, 0.5)$ in the following computations. Limiting the interval of the modified local Whittle estimator does not affect the positive estimates, whereas all negative values are estimated equal to the lower bound of zero.

Our findings are supported by similar results in the literature. Allowing for breaks, Hsu (2005) hardly finds evidence for long memory in the inflation rates of Japan and Germany. This is in line with our findings of a negative d in case of Germany and a maximum d of 0.026 across bandwidths for Japan. Furthermore, Hsu (2005) shows that

Italy and the US exhibit a relatively high degree of persistence after controlling for two breaks. We identify the UK, Italy and the US as the countries with the most persistent inflation rates, especially if we focus on larger bandwidths. This result is also supported by Bos et al. (1999).

δ	0.70	0.72	0.74	0.76	0.78	0.80
CAN	0.021	0.016	-0.077	-0.042	-0.036	-0.046
FRA	-0.202	0.086	0.042	0.115	0.169	0.146
GBR	0.203	0.192	0.199	0.302	0.372	0.344
GER	-0.282	-0.080	-0.127	-0.129	-0.018	-0.036
ITA	0.116	0.144	0.189	0.286	0.329	0.372
JPN	-0.411	-0.089	-0.161	-0.207	-0.039	0.015
USA	0.063	0.240	0.132	0.196	0.302	0.278

(a) CPI based inflation

δ	0.70	0.72	0.74	0.76	0.78	0.80
CAN	-0.078	-0.064	-0.022	-0.048	-0.049	-0.076
FRA	-0.120	0.169	0.273	0.286	0.367	0.381
GBR	0.173	0.098	0.104	0.194	0.268	0.288
GER	-0.382	-0.198	-0.149	-0.216	-0.148	-0.140
ITA	0.237	0.141	0.292	0.351	0.142	0.200
JPN	-0.123	0.025	-0.016	-0.150	-0.006	0.026
USA	-0.136	0.035	0.298	0.266	0.364	0.426

(b) Core CPI based inflation

Table 4.3: Modified local Whittle estimates for different bandwidths $m = \lfloor T^{\delta} \rfloor$

So far both, the test decisions of the Qu test and the reduced memory parameter estimates of Hou and Perron (2014), point to the fact that the inflation rates are not true long-memory processes but contain some spurious long memory components. A well known explanation of this phenomenon is the existence of random level shifts in the series. Therefore, we locate break points in the data. Gadea et al. (2004) apply the method of Bai and Perron (1998, 2003) although they have found long memory in their data. This approach is not appropriate because the number of breaks is overestimated under long range dependence (cf. Lavielle and Moulines, 2000). Instead, we use the method of Lavielle and Moulines (2000) due to the fact that most series still exhibit long memory. The estimated number of break points are presented in Tables 4.4a and 4.4b.

δ	0.70	0.72	0.74	0.76	0.78	0.80
CAN	-	-	-	-	-	-
FRA	1	-	1	-	-	-
GBR	-	-	-	-	-	-
GER	-	-	-	-	-	-
ITA	-	-	-	-	-	-
JPN	1	1	1	1	1	1
USA	-	-	-	-	-	-

(a) CPI based inflation

δ	0.70	0.72	0.74	0.76	0.78	0.80
CAN	-	-	-	-	-	-
FRA	1	-	-	-	-	-
GBR	-	-	-	-	-	-
GER	-	-	-	-	-	-
ITA	-	-	-	-	-	-
JPN	1	1	1	1	1	1
USA	-	-	-	-	-	-

(b) Core CPI based inflation

Table 4.4: Number of break points for different bandwidths $m = \lfloor T^{\delta} \rfloor$

They show that we hardly find any break points at all. We locate one break for Japan independent of the bandwidth at the end of the second oil crisis in 1981 (for CPI) and in 1977 (for core CPI). These findings differ from the results in Bos et al. (1999). However, they only take exogenously fixed break dates into account and hence find two breaks for Canada, France, Italy, UK, and the US. Allowing for four breaks they also find breaks for Japan. Hsu (2005) reconsiders the empirical contribution of Bos et al. (1999) for the same countries but for an updated sample period. Since he provides an estimation method for the break points, the break dates are no longer exogenously determined. These two adjustments reduce the evidence of breaks in the G7 countries. Canada, France, Germany, Italy and the UK are still having two breaks, whereas the number of breaks reduces for the US and Japan to one and three breaks, respectively. However, Hsu (2005) selects the number of breaks using the information criterion of Lavielle and Moulines (2000) which depends on an estimate of the persistence parameter. In fact, consistent estimation of the memory parameter requires knowledge about the breaks. Therefore, Hsu (2005) needs a priori assumptions about the number of breaks and computes the memory parameter conditional on the break points. This explains the higher number of located break points in Hsu (2005) in contrast to our empirical study. Contradictory results to the aforementioned literature are given by Bos et al. (2014), who do not find any mean shifts in the US inflation rate. This finding is in line with our breakpoint estimates. All in all, our results differ from the majority of findings in the literature which is due to the fact that we provide a consistent estimation approach without any a priori assumptions about the number or location of break points in the inflation series. Apart from Japan and France (only for small bandwidths), there are no level shifts in the inflation series. However, we have found reduced memory parameters with the estimator of Hou and Perron (2014) and also the Qu test suggests that there is spurious long memory in all series (except for the US). This points to other low frequency contaminations different from abrupt level shifts. Therefore, we propose to account for smooth changes in the modeling process of inflation rates that may be captured by fitting a semiparametric fractional autoregressive ($SEMIFAR$) model instead of a random level shift model.

Figures 4.2 and 4.3 depict the respective trends in CPI and core CPI based inflation. For six out of seven countries a smooth trend (dashed line) can be detected in the inflation rates, whereas the US trend is a flat line.

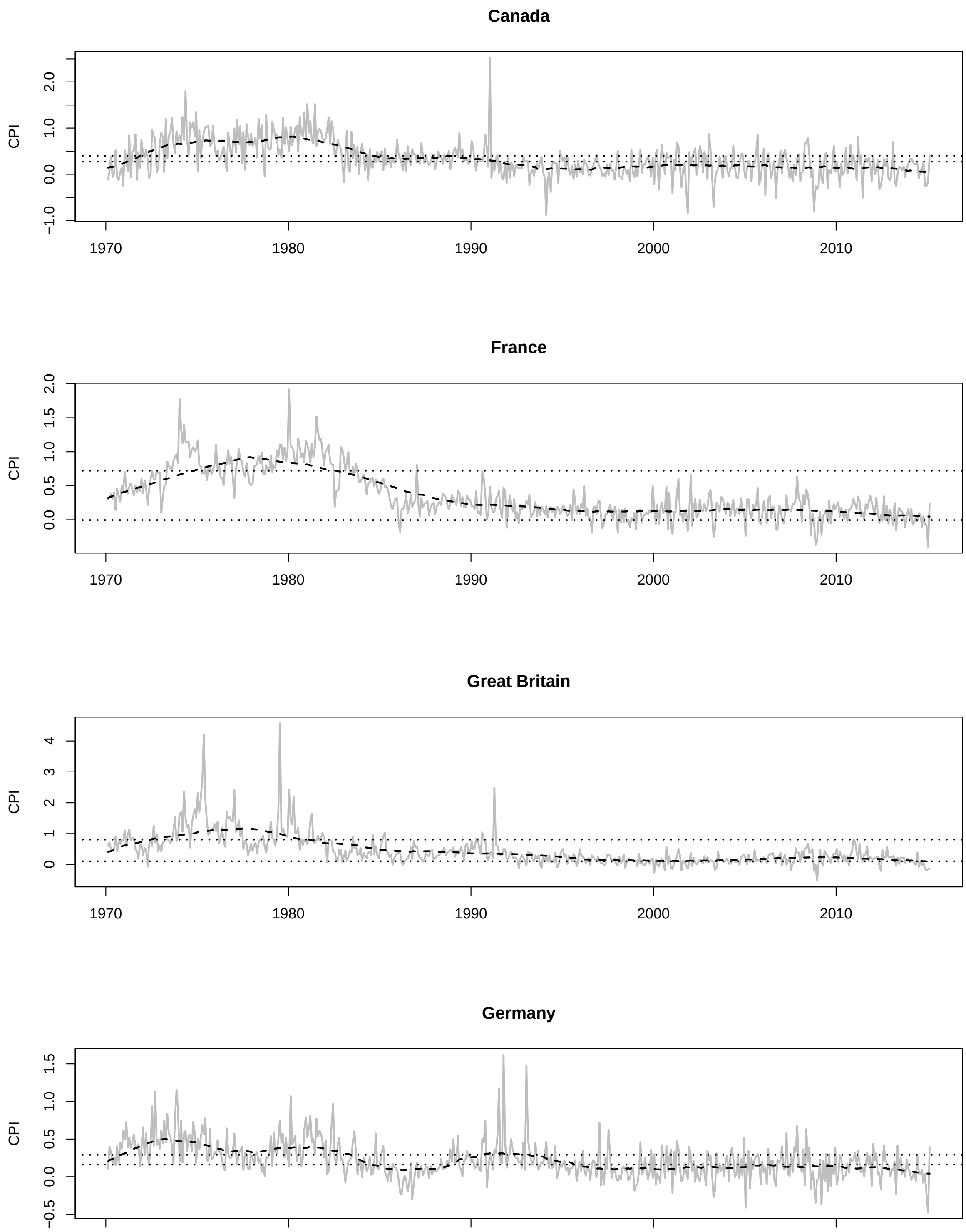


Figure 4.2: Smooth trends in CPI

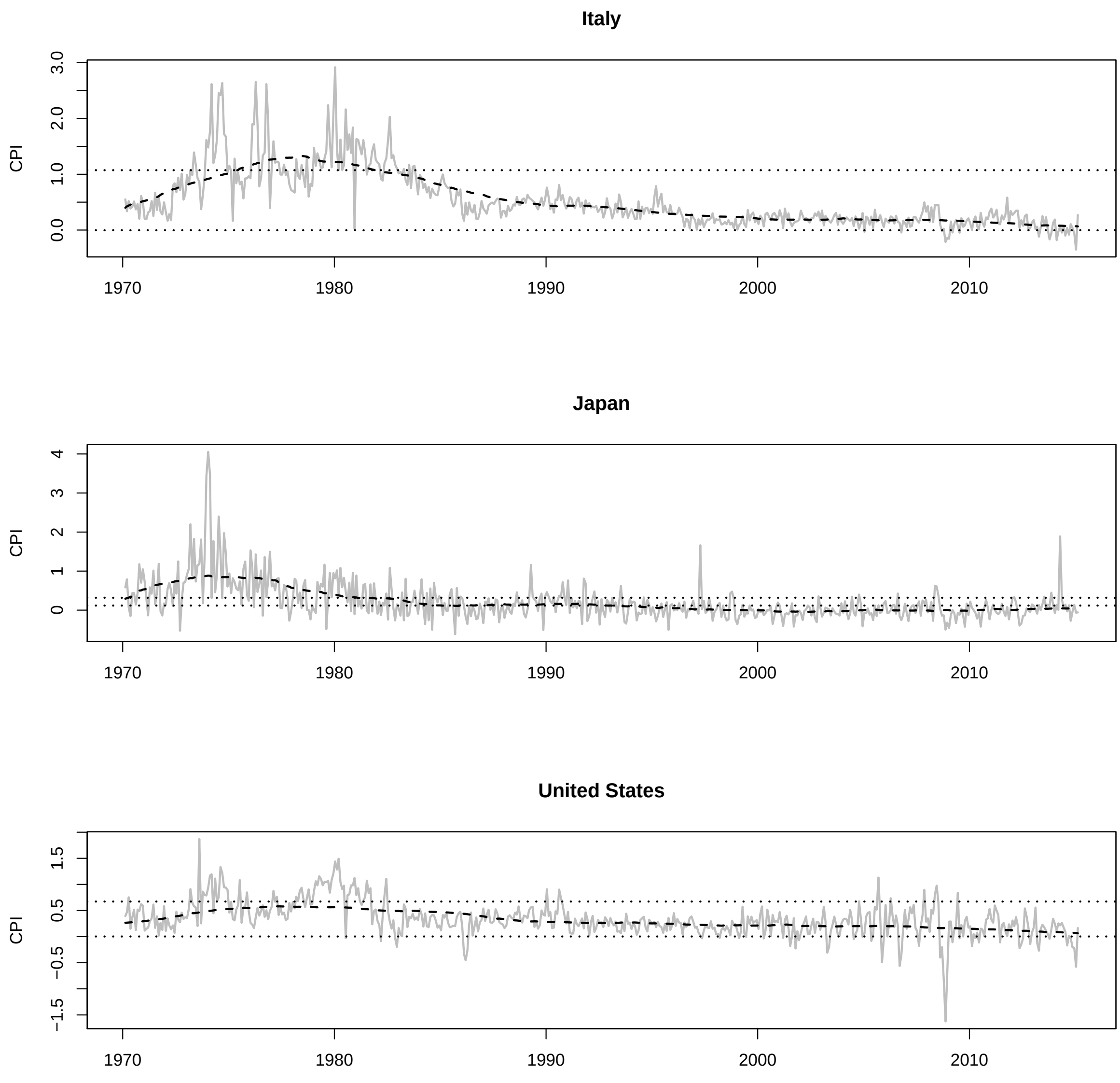


Figure 4.2: Smooth trends in CPI

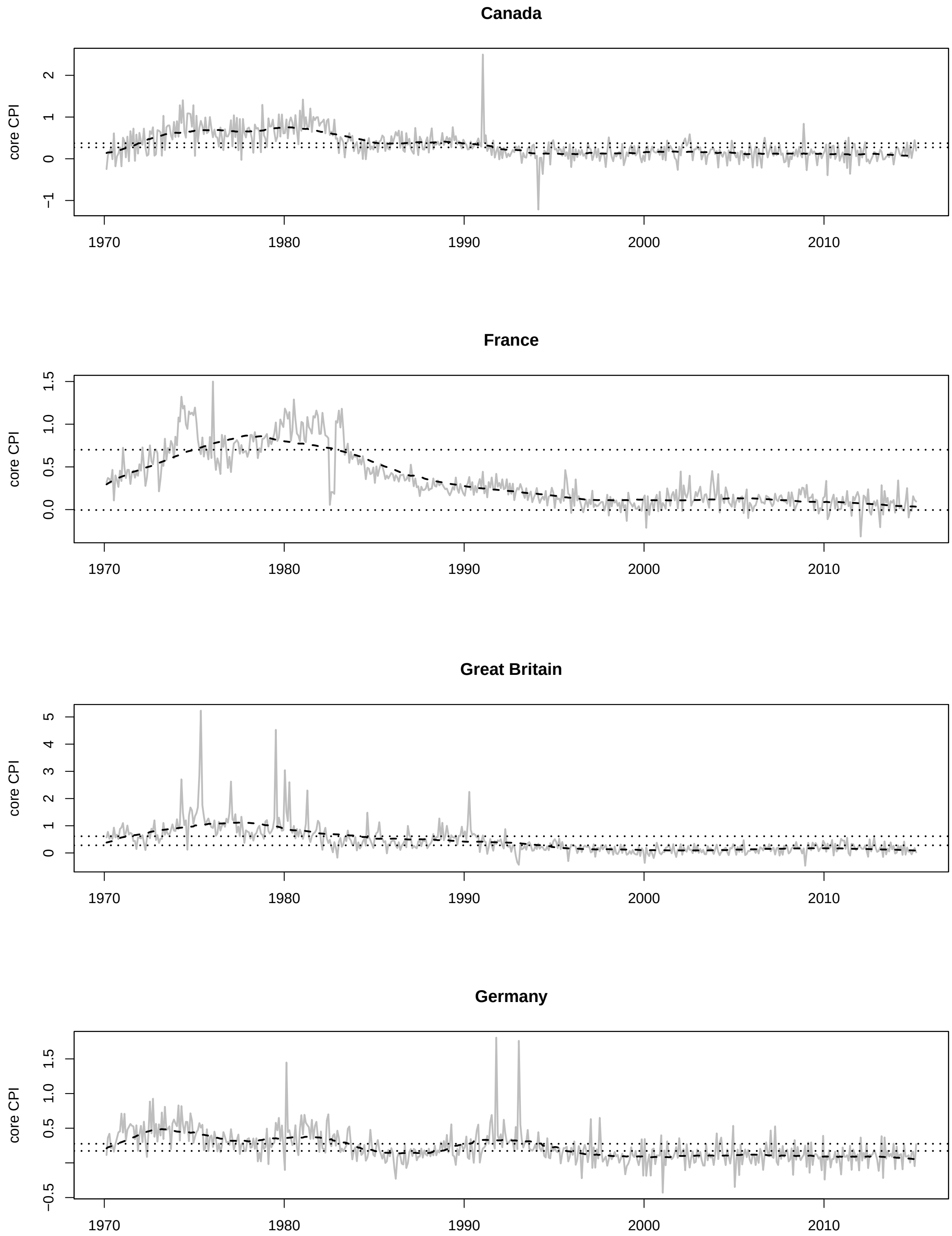


Figure 4.3: Smooth trends in core CPI

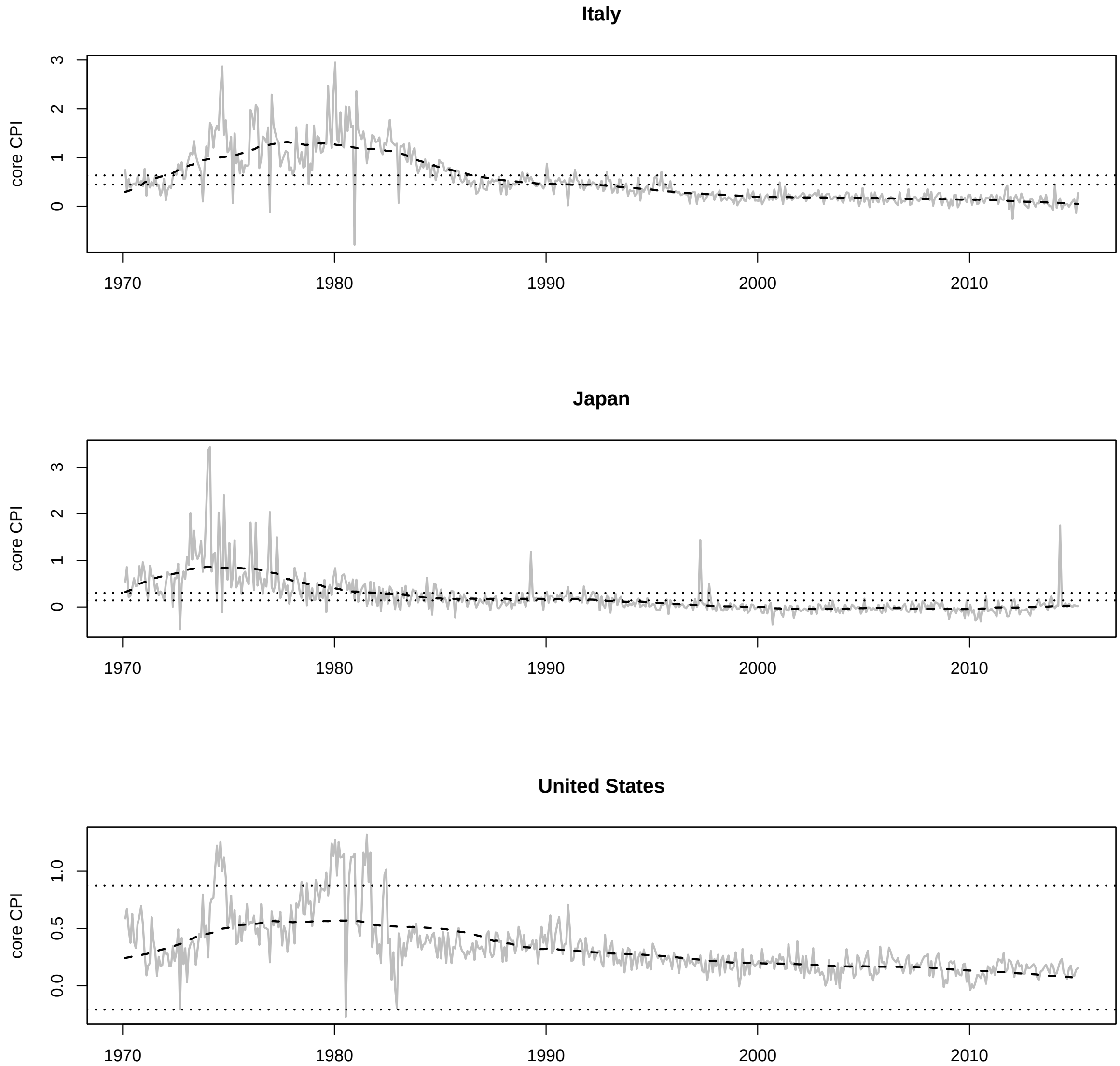


Figure 4.3: Smooth trends in core CPI

In order to check whether the trends are significant, we construct pointwise acceptance regions at a 5% significance level following Beran and Feng (2002b). The dotted lines illustrate the borders and the band within these lines the acceptance region of a constant trend. If the trend moves outside the band, it is significant. Our results show that for six out of seven countries the $SEMIFAR$ trend is significant. Only for the US, the trend lies within the acceptance region. However, this confirms our previous results that the US inflation follows a true long-memory process. For France, the UK (only for CPI), and Italy (only for CPI) we find a significant trend only in the time period of the Great Inflation.

Tables 4.5a and 4.5b give the corresponding estimated memory parameters of the $SEMIFAR$ estimation algorithm. In case of Canada and Germany the estimates are zero implying short memory. The other countries' inflation rates exhibit stationary long memory. The comparison of the $SEMIFAR$ estimates of d and the modified local Whittle estimates with $\delta = 0.8$ in Tables 4.3a and 4.3b emphasize the robustness of the results. Although, the values are not identical, the scale is the same.

CAN	FRA	GBR	GER	ITA	JPN	USA
0.000	0.350	0.350	0.000	0.400	0.125	0.350

(a) CPI based inflation

CAN	FRA	GBR	GER	ITA	JPN	USA
0.000	0.375	0.275	0.000	0.175	0.125	0.425

(b) Core CPI based inflation

Table 4.5: Estimated memory parameters of SEMIFAR models

The idea to fit a $SEMIFAR$ model to inflation data in order to capture other contaminations than level shifts is supported by the findings of Baillie and Kapetanios (2007) who apply tests in order to detect unspecified nonlinearities in addition to long memory components. They detect nonlinearity for all G7 countries except for Germany.

The application of $SEMIFAR$ models is not common in the inflation persistence literature. In fact, to our knowledge we are the first to fit $SEMIFAR$ models to inflation data. Therefore, on the one hand it may give further insights into the inflation behavior but on the other hand our results cannot be compared to previous findings of other authors. Nevertheless, we put our main findings of hardly any breaks but significant smooth trends into the context of the current literature. Recently, level shifts are considered as the main source of spurious long memory, whereas other low frequency contaminations like trends are not that prominent. Thus, there are many contributions supporting the existence of breaks in inflation rates. However, the number and location of the break points is not unambiguous. This phenomenon can be easily explained due to the fact that abrupt level shifts are a rough approximation of a smooth trend. Hence, the actual low frequency contaminations are smooth trends which are neglected but breaks give a first impression of the nonlinearities in the data.

We show that the persistence of inflation rates is overestimated if low frequency contaminations are not considered in the estimation procedure. This implies that the costs and the time until full implementation of monetary policy are upwardly biased. If we control for nonlinearities, the degree of persistence clearly decreases. Thus, policy actions do not have to be as aggressive and need less time until full implementation. To conclude, monetary policy is less costly by taking the correct type of nonlinearity into account.

5 Conclusion

We find a reduction in the memory parameter by allowing for low frequency contaminations in the G7 inflation rates and we reject the null hypothesis of true long memory, except for the US. In general, only a few breaks can be located and therefore we consider another type of nonlinearity, smooth trends. The estimated trends provide a good approximation of the real data and capture spurious long memory more appropriately than structural breaks. With the exception of the US, all other countries exhibit significant trends at the 5% level. This result is in line with the earlier finding of a higher memory parameter for the US and a non-rejection of the test for spurious long memory.

To conclude, we provide an analysis of different estimation methods without a priori assumptions in case of spurious long memory in inflation rates. Although the existing empirical literature focuses more on the possibility of structural breaks in inflation time series, we show that smooth trend models, such as $SEMIFAR$ models, are a promising alternative approach in order to capture the characteristics of inflation rates. Therefore, future research should be dedicated to the assessment of trends in inflation rates in addition to breaks.

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