Origins of Spurious Long Memory*

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Abstract

We consider a large class of structural change processes that generate spurious long memory. Among others, this class encompasses structural breaks as well as random level shift processes and smooth trends. The properties of these processes are studied based on a simple representation of their discrete Fourier transform. We find, that under very general conditions all of the models nested in this class generate poles in the periodogram at the zero frequency. These are of order $O(T)$, instead of the usual $O(T^{2d})$ for long memory processes and $O(T^2)$ for a random walk. This order arises whenever both the mean changes and sample fractions at which they occur are non-degenerate, asymptotically.

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1 Introduction

Long memory has been defined by an hyperbolic decay of the autocorrelation function or equivalently by the order of a pole at the origin of the spectral density of a time series. Since Künsch (1986) and Bhattacharya et al. (1983) it has been discussed that this behavior can be artificially induced by deterministic trends such as slowly decaying trends as in Künsch (1986) or Bhattacharya et al. (1983) or structural breaks as in Parke (1999), Granger and Teräsvirta (1999), Gourieroux and Jasiak (2001) or Diebold and Inoue (2001), among many others. All these processes share the property that they mimic the autocorrelation structure of a long-memory process but none of them is self-similar or fulfills a functional central limit theorem in the sense that the partial sums of these processes converge to a fractional Brownian motion. Therefore, these processes share some but not all properties with true long-memory processes. These type of processes are well known to be called spurious long memory.

Typically, contributions on spurious long memory study either the properties of a specific estimator under some form of structural change, or they study specific processes and the properties of the autocorrelation functions that they generate.

For example, Bhattacharya et al. (1983) show that a slowly decaying trend biases the rescaled range statistic as an estimator for the memory parameter. Similarly, Granger and Teräsvirta (1999) and Granger and Hyung (2004) show that breaks in the mean can lead to a bias of the GPH estimator towards indicating long-range dependencies. Gourieroux and Jasiak (2001) show that breaks in the mean of a time series can also lead to biased estimates of the covariance structure of the process again falsely indicating long memory.

Other than the before mentioned papers Künsch (1986), Parke (1999) and Diebold and Inoue (2001) construct diverse processes which do share the autocorrelation structure with a long-memory time series at least in finite samples. Künsch (1986) considers monotonic deterministic trends. Parke (1999) constructs a random level shift process where the shift probability is derived from a heavy tailed distribution. A similar process is considered in Mikosch et al. (2002). This process has the same autocorrelation function as a long memory process. However, Davidson and Sibbertsen (2005) show that its partial sums converge to a Levy motion with independent increments not exhibiting long memory. Only cross-sectional aggregation of these processes generates long memory, asymptotically.

Diebold and Inoue (2001) construct several types of random level shift processes all sharing the property that the expected number of shifts grows more slowly than the sample size. They show that these processes have a similar autocorrelation function to a long-memory process.

In the more recent literature Qu and Perron (2007) derived the order of the pole in the periodogram of long memory processes under "rare shift asymptotics" which means that the expected number of shifts is finite even though the sample size grows to infinity. They show that the periodogram of this process is of order $O(\lambda^{-2}T^{-1})$. Qu (2011) extends this result to smooth trends, but is only able to establish the order as an upper bound. Similar results are obtained by Iacone (2010). McCloskey and Perron (2013) also establish this order for deterministic shifts and a class of fractional trends. These findings are of huge practical importance, since estimators such as those of McCloskey and Perron (2013) and Hou and Perron (2014) and test like that of Qu (2011) use them to detect spurious long memory and to separate true and spurious memory.

However, all of these findings are based on specific data generating processes on specific estimators so that they allow only limited generalizations. Here, we contribute to this literature by considering a versatile structural change model that nests all of the processes discussed above. We derive a simple representation of the discrete Fourier transform of these processes and use these to study the behavior of their periodograms. This allows us to establish conditions under which the order derived by Qu and Perron (2007), Qu (2011) and McCloskey and Perron (2013) arises. That is whenever there are mean changes at sample fractions that are non-degenerate asymptotically. Furthermore, we show that the established order is exact.
The rest of the paper is organized as follows. Section 2 gives a short discussion of the orders of poles for some typical processes. Section 3 discusses our structural change model and gives some first results on its DFT and the properties of its components. In Section 4 we use these results to establish the orders of the DFT and the periodogram of our model. The relationship of these results to other findings in the literature is discussed in Section 5. Finally, Section 6 concludes.

2 Orders of the Periodogram

We say a stochastic process exhibits long memory if the spectral density behaves near the origin as

$$f(\lambda) \sim G\lambda^{-2d}$$

for $\lambda \to 0_+$, where $0 < d < 1/2$ is the memory parameter. We are interested in the behavior of the periodogram at the Fourier frequencies $\lambda_j = (2\pi j)/T$, local to the origin as $\lambda_j \to 0_+$, or equivalently $j/T \to 0$. Therefore, for fixed j, the periodogram of a true long memory process is

$$I(\lambda_j) = O_P\left(\lambda_j^{-2d}\right) = O_P\left(\left(\frac{2\pi j}{T}\right)^{-2d}\right) = O_P\left(T^{2d}\right).$$

Therefore, the periodogram at the Fourier frequency $j = \lfloor T^{\epsilon} \rfloor$ for $\epsilon \in [0,1)$ grows at rate $T^{2d(1-\epsilon)}$ as the sample size increases.

Similarly, the periodogram of a random walk is of order

$$I(\lambda_j) = \mathcal{O}_P(\lambda_j^{-2}) = \mathcal{O}_P(T^2),$$

for fixed j. The random level shift process of Qu and Perron (2007) and the structural break cases in McCloskey and Perron (2013), on the other hand, are found to generate periodograms that are of order

$$\mathcal{O}_P(\lambda_j^{-2}T^{-1}) = \mathcal{O}_P(T), \tag{1}$$

which is faster than that of a stationary long memory process, but more slowly than a unit root process. For $j = \lfloor T^{\epsilon} \rfloor$ this is $\mathcal{O}(T^{1-2\epsilon})$.
In general, we are interested in component models of the form

$$y_t = \mu_t + x_t,$$

where x_t is a zero mean time series process that possibly has long memory and μ_t is a time-varying mean that can be stochastic or deterministic and $t = 1, ..., T$. Both x_t and μ_t are $q \times 1$ vector valued processes.

From the results above, it is clear to see that it depends on the position parameter ϵ, whether the periodogram is dominated by the structural change component that is $\mathcal{O}(T^{1-2\epsilon})$ or the long memory component that is $T^{2d(1-\epsilon)}$. This is the property used by the procedures of Qu and Perron (2007), Qu (2011), McCloskey and Perron (2013), Hou and Perron (2014) and Sibbertsen et al. (2015). Therefore, the applicability of these procedures depends on whether the order in (1) is specific to the DGPs considered or whether it generalizes to other structural change processes.

3 A General Model of Structural Change and its Fourier Transform

In this paper our focus is on the mean change process μ_t. Define $\mu = T^{-1}\sum_{t=1}^{T}\mu_t$. Then μ_t can be expressed as

$$\mu_t = \mu_0 + \sum_{k=1}^{K} \Delta\mu_k \mathbb{I}(t \geqslant T_k), \tag{2}$$

where μ_0 is the initial offset of the first mean from the sample mean $\mu_0 = \mu_1 - \mu$. The variable K is the number of observations for which there is a structural change in the mean relative to the previous observation and T_k is the point in time at which the k-th change occurs. If we set $T_0 = 0$, then we can also express μ as $\mu = \sum_{k=0}^{K}\mu_k(T_{k+1} - T_k)/T$. Consider the versatility of this representation. For deterministic structural breaks this representation is obvious and commonly used, albeit μ_0 usually represents the mean of

the initial observations and not the offset from the time mean.
If the changes $\Delta\mu_k$ and the breakpoints T_k are stochastic, then the process nests a random level shift process. Even a random walk is nested in (2), if $K = T$, $T_k = t$ for all $k = t$ and the $\Delta\mu_k$ are a martingale difference sequence.
Similarly, deterministic trends can be represented as in (2). Deterministic trends are usually modeled as functions $h(s)$ on $[0,1]$. Consider $s_t = t/T$ for $t = 1,...,T$, then we have $h(s_t) = h(0) + \sum_{k=1}^{t}(h(k/T) - h((k-1)/T))$ and obviously $\mu_t = h(s_t)$ if $\Delta\mu_k = (h(k/T) - h((k-1)/T))$, $K = T$, and $T_k = 1,2,...,T$. Restrictions on $h(s)$ - such as Lipschitz-continuity - correspond to restrictions on the $\Delta\mu_k$.

To study the spectral behavior of (2), denote the discrete Fourier transform of z_t by $w_z(\lambda) = \frac{1}{\sqrt{2\pi T}}\sum_{t=1}^{T} z_t e^{i\lambda t}$. Then we have the following result for μ_t.

Lemma 1. *The discrete Fourier transform of the process in (2) can be represented as*

$$w_\mu(\lambda_j) = -\frac{1}{\sqrt{2\pi T}}\sum_{k=1}^{K}\Delta\mu_k D_{T_k}(\lambda_j),$$

where $D_{T_k}(\lambda) = \sum_{t=1}^{T_k} e^{i\lambda t}$ is a version of the Dirichlet kernel.

Proof.

$$\begin{aligned}
w_\mu(\lambda) &= \frac{1}{\sqrt{2\pi T}}\sum_{t=1}^{T}\mu_t e^{i\lambda t} \\
&= \frac{1}{\sqrt{2\pi T}}\sum_{t=1}^{T}\left\{\mu_0 + \sum_{k=1}^{K}\mathbb{I}(t \geqslant T_k)\Delta\mu_k\right\}e^{i\lambda t} \\
&= \frac{1}{\sqrt{2\pi T}}\left\{\mu_0\sum_{t=1}^{T}e^{i\lambda t} + \sum_{t=1}^{T}\sum_{k=1}^{K}\mathbb{I}(t \geqslant T_k)\Delta\mu_k e^{i\lambda t}\right\} \\
&= \frac{1}{\sqrt{2\pi T}}\left\{\mu_0\sum_{t=1}^{T}e^{i\lambda t} + \sum_{k=1}^{K}\Delta\mu_k\sum_{t=1}^{T}\mathbb{I}(t \geqslant T_k)e^{i\lambda t}\right\}.
\end{aligned}$$

Here,

$$\begin{aligned}
\sum_{t=1}^{T}\mathbb{I}(t \geqslant T_k)e^{i\lambda t} &= \sum_{t=T_k}^{T} e^{i\lambda t} \\
&= \sum_{t=1}^{T}e^{i\lambda t} - \sum_{t=1}^{T_k}e^{i\lambda t} \\
&= D_T(\lambda) - D_{T_k}(\lambda).
\end{aligned}$$

Therefore,

$$\begin{aligned}
w_\mu(\lambda) &= \frac{1}{\sqrt{2\pi T}} \left\{ D_T(\lambda)\mu_0 + \sum_{k=1}^{K} \Delta\mu_k \left[D_T(\lambda) - D_{T_k}(\lambda) \right] \right\} \\
&= \frac{1}{\sqrt{2\pi T}} \left\{ D_T(\lambda)\mu_0 + \sum_{k=1}^{K} \Delta\mu_k D_T(\lambda) - \sum_{k=1}^{K} \Delta\mu_k D_{T_k}(\lambda) \right\} \\
&= \frac{1}{\sqrt{2\pi T}} \left\{ \left[\mu_0 + \sum_{k=1}^{K} \Delta\mu_k \right] D_T(\lambda) - \sum_{k=1}^{K} \Delta\mu_k D_{T_k}(\lambda) \right\}.
\end{aligned}$$

Furthermore, we have

$$D_T(\lambda) = \frac{e^{i(T+1)\lambda} - e^{i\lambda}}{e^{i\lambda} - 1} = e^{i(T-1)\lambda/2} \frac{\sin(T\lambda/2)}{\sin(\lambda/2)}. \tag{3}$$

Note that $\lambda_j T = 2\pi j$, $e^{i(T+1)\lambda} = e^{i\lambda T} e^{i\lambda}$ and $e^{i2\pi j} = \cos(2\pi j) + i\sin(2\pi j) = \cos(2\pi) + i\sin(2\pi) = 1$. Therefore,

$$D_T(\lambda_j) = \frac{e^{i\lambda} - e^{i\lambda}}{e^{i\lambda} - 1} = 0.$$

□

Lemma 1 shows that the DFT of a structural change process can be represented as the sum of changes of the mean - each weighted by a Dirichlet kernel that depends on the location of the mean change.
It is well known that the DFT of a constant is zero at the Fourier frequencies λ_j. This is the mechanism behind the self-centering property of the periodogram and the last step in the proof of Lemma 1. However, the DFT of an indicator function is non-zero at the Fourier frequencies.

We now derive the order of the Dirichlet kernel.

Lemma 2. *As $T \to \infty$, we have for $T_k(T)/T \to \delta_k$ and $0 < \delta_k < 1$, $D_{T_k}(\lambda_j) = T(1 + i\delta_k \pi j)\delta_k$.*

Proof. From the second expression in (3) we can write:

$$\begin{aligned}
D_{T_k}(\lambda_j) &= \frac{exp(i(T_k - 1)\lambda_j/2)\sin(T_k\lambda_j/2)}{\sin(\lambda_j/2)} = \frac{exp(i\frac{T_k - 1}{T}\pi j)\sin(\frac{T_k}{T}\pi j)}{\sin(\frac{\pi j}{T})} \\
&= \frac{exp(i\delta_k\pi j)\sin(\delta_k\pi j)}{\sin(\frac{\pi j}{T})} + o_P(1) \\
&= \frac{(1 + i\delta_k\pi j)\sin(\delta_k\pi j)}{\sin(\frac{\pi j}{T})} + o_P(1) \\
&= T(1 + i\delta_k\pi j)\delta_k,
\end{aligned}$$

where the third line is obtained from the Laurent series of the exponential function at zero and the fourth line is from the Laurent series of the sine functions. □

Clearly, from Lemma 2, both the real and the imaginary parts of the Dirichlet kernel have order $O(T)$ and the order is exact. The lemma covers the typical case of a structural change after some fixed proportion of the sample. This includes random level shift processes with a rare shift asymptotic where the expected number of shifts is finite as well as deterministic shifts.

4 The Order of Discrete Fourier Transforms and Periodograms

Based on the representation result from Lemma 1 and the results on the order of the Dirichlet kernel in Lemma 2, we can now derive the order of the DFT.

Theorem 1. *For $T \to \infty$, we have if $E[K] < \infty$, $1 > T_k/T = \delta_k > 0$, and $\exists\ \Delta\mu_k$ such that $\lim_{T\to\infty} \Delta\mu_k > 0$, then $Re(w_\mu(\lambda_j)) = O_P(\sqrt{T})$ and $Im(w_\mu(\lambda_j)) = O_P(\sqrt{T})$.*

Proof. From Lemma 1,

$$w_\mu(\lambda_j) = -\frac{1}{\sqrt{2\pi T}} \sum_{k=1}^{K} \Delta\mu_k D_{T_k}(\lambda_j),$$

and from Lemma 2, we have $D_{T_k} = O_P(T)$, such that

$$w_\mu(\lambda_j) = O(T^{-1/2}) \sum_{k=1}^{K} O_P(T) = O_P(T^{1/2}),$$

for both the real and imaginary part. □

The condition $E[K] < \infty$, as $T \to \infty$, in Theorem 1 implies that μ_t is not ergodic. If $K \to \infty$ and T_k/T is vanishing asymptotically and not fixed, then the summands in $\sum_{k=1}^{K} \Delta\mu_k D_{T_k}(\lambda_j)$ might cancel out asymptotically. This would for example be the case for a standard Markov-switching autoregressive model with fixed transition probabilities. Since the orders in Lemma 2 are exact, so are those in Theorem 1. Obviously, the stochastic rate becomes deterministic, if μ_t is deterministic.
For finite K it is obvious from Lemma 1 that the DFT will have the order of the Dirichlet kernel when there is a $\Delta\mu_k > 0$. However, in the case of a Lipschitz continuous trend, we have $|h(r) - h(s)| \leqslant \tilde{K}|r - s|$. Therefore, at successive observations, t and $t-1$, we have $\Delta\mu_t \leqslant \tilde{K}/T \to \infty$. Therefore, the behavior of the DFT is not obvious. This situation is covered in the following Theorem.

Theorem 2. *Let $h(s)$ be (piecewise) Lipschitz-continuous on $[0,1]$. Furthermore, let $h(s) \neq h(r)$ for some $s \neq r$, where both $s, r \in (0,1)$. Then $w_\mu(\lambda) = O(\sqrt{T})$.*

Proof. Assume without loss of generality, that $s > r$, and let $T_s = sT$ and $T_r = rT$. Then, by summation-by-parts,

$$S(r,s,T,\lambda_j) = \sum_{t=T_r}^{T_s} \Delta\mu_t D_t(\lambda_j) = \mu_{T_s} D_{T_s}(\lambda_j) - \mu_{T_r} D_{T_r}(\lambda_j) - \sum_{t=T_r+1}^{T_s} \left(D_t(\lambda_j) - D_{t-1}(\lambda_j)\right). \quad (4)$$

Defining $\delta = \mu_{T_s} - \mu_{T_r}$, we have

$$= \mu_{T_r}(D_{T_s}(\lambda_j) - D_{T_r}(\lambda_j)) + \delta D_{T_s}(\lambda_j) - \sum_{t=T_r+1}^{T_s} \left(D_t(\lambda_j) - D_{t-1}(\lambda_j)\right). \quad (5)$$

From Lemma 2, we have $D_N(\lambda_j) = N + i\pi j(N/T)^2 T$. Therefore,

$$\begin{aligned} D_t(\lambda_j) - D_{t-1}(\lambda_j) &= t + i(t/T)^2 \pi jT - ((t-1) + i((t-1)/T)^2 \pi jT) \\ &= 1 + \frac{i\pi j}{T}(t^2 - (t-1)^2) \\ &= 1 + \frac{2i\pi j}{T}(t + 1/2). \\ \text{and} \quad D_{T_s}(\lambda_j) - D_{T_r}(\lambda_j) &= sT - rT + i\pi jT(s^2 - r^2). \end{aligned}$$

Using this, we can rewrite

$$Re(S(r,s,T,\lambda_j)) = T\mu_{T_s}(s-r) + \delta sT - \sum_{t=T_r+1}^{T_s} \mu_{t-1}. \quad (6)$$

Obviously, each term in (6) is $O(T)$, unless $\mu_t = \mu$ for all $t = 1,...,T$. In this case $\delta = 0$ and $\sum_{t=T_r+1}^{T_s} \mu_{t-1} = \mu T(s-r)$, such that $Re(S(r,s,T,\lambda_j)) = 0$. If μ_t is not constant, the sum can no further be simplified and $Re(S(r,s,T,\lambda_j)) = O(T)$ is exact.

Similarly, we have for the imaginary part,

$$Im(S(r,s,T,\lambda_j)) = \mu_{T_s}\pi jT(s^2 - r^2) + \delta s^2 \pi jT - \frac{2\pi j}{T}\left\{\sum_{t=T_r+1}^{T_s} t\mu_{t-1} + 1/2 \sum_{t=T_r+1}^{T_s} \mu_{t-1}\right\}. \quad (7)$$

Again, for constant μ_t, we have $\delta = 0$ and

$$\sum_{t=T_r+1}^{T_s} t = \frac{(sT)^2 - sT}{2} - \frac{(rT+1)^2 - (rT+1)}{2}$$

as well as

$$\sum_{t=T_r+1}^{T_s} 1 = sT - (rT+1).$$

So that the imaginary part is $O(1)$, asymptotically. For non-constant μ_t, (7) can not be simplified any further and again, the order $O(T)$ for $S(r,s,T,\lambda_j)$ is exact.

Finally, the order of the DFT follows from

$$w_{\mu}(\lambda_j) = -\frac{S(0, r, T, \lambda_j) + S(r, s, T, \lambda_j) + S(s, 1, T, \lambda_j)}{\sqrt{2\pi T}},$$

where the first and last summand converge to zero, if the mean in these segments remains constant. In case of mean changes in other segments then $[r, s]$, the same arguments apply for each additional segment.
To deal with jumps between segments, we can express the DFT as the sum of the segments that can be treated as in (4) and the sum of the jumps between the segments for which Theorem 1 applies. □

Theorem 2 gives the exact order of the periodogram of Lipschitz continuous functions. Previous results such as that of Qu (2011) only established $O(\sqrt{T})$ as an upper bound. We therefore find that trends in fact have the same effect on the order of the pole as deterministic or stochastic structural breaks, if the condition is fulfilled that the function is non-constant between two non-degenerate sample fractions.

From Theorems 1 and 2, it is straightforward to establish the order of the periodogram of the mean change model in (2).

Corollary 1. *For every random process that can be represented as in (2) with an asymptotically finite number of shifts at non-degenerate sample fractions and for every piecewise Lipschitz-continuous process with change between non-degenerate sample fractions s and r the periodogram is of order $O_P(T)$.*

Proof. From $I_{\mu}(\lambda_j) = w_{\mu}(\lambda_j) w_{\mu}^{*}(\lambda_j)$, the order follows directly from Theorems 1 and 2, as the square of the order of the DFT. □

If the structural change processes considered in Corollary 1 are deterministic, then the order is deterministic as well.

5 Relationship to other Results

In this section we discuss some of the processes considered in the literature as spurious long memory which are covered by our findings. In our Theorem 1 we cover the random level shift process with rare shifts asymptotics as considered by Qu and Perron (2007), Qu (2011) and McCloskey and Perron (2013). Obviously the periodogram of these processes has a pole at the origin with a divergence rate of $O_P(T)$. Remembering that long memory processes have a divergence rate of $O_P(\lambda^{-2d}) = O_P(T^{2d})$, this would imply a memory parameter of $d = 1/2$. A fractionally integrated process with $d = 0.5$ would be degenerate with asymptotic autocorrelation function equal to 1 for all lags.
Theorem 2 covers the case of smooth trends as well as deterministic structural breaks. Deterministic structural breaks are the null model in the tests of Berkes et al. (2006)

and Yau and Davis (2012) against spurious long memory. Our Theorem shows that deterministic structural breaks have the same rate of divergence at the pole of the periodogram.

Note that Theorem 1 does not impose any condition on the nature of the shift process except that it generates an asymptotically finite number of shifts at non-degenerate sample fractions and the size of the shifts is non-negligible, asymptotically. The shift sizes can be serially correlated, break probabilities can be non-constant and depend on exogenous variables and so on. Therefore, this is a considerable generalization relative to previous results that were specifically based on non-stationary random level shift processes.

An example that is covered by Theorem 1 is the Markov switching autoregressive processes in which the probabilities to switch from one regime to any of the others are $O(T^{-1})$.

6 Conclusion

In this note we derive simple representations for the discrete Fourier transform of structural change models as well as deterministic trends and study their properties.

While many of the contributions on spurious long memory that are discussed in Section 1 present processes that have periodograms of order $O(T^{2d})$, our results show that structural change at non-degenerate break fractions always implies poles in the periodogram that are of order $O(T)$.

Therefore, methods such as those of Qu and Perron (2007), Qu (2011), McCloskey and Perron (2013), Hou and Perron (2014) and Sibbertsen et al. (2015) that rely on this order are widely applicable.

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