The Memory of Volatility

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Abstract

The focus of the volatility literature on forecasting and the predominance of the conceptually simpler HAR model over long memory stochastic volatility models has led to the fact that the actual degree of memory estimates has rarely been considered. Estimates in the literature range roughly between 0.4 and 0.6 - that is from the higher stationary to the lower non-stationary region. This difference, however, has important practical implications - such as the existence or non-existence of the fourth moment of the return distribution. Inference on the memory order is complicated by the presence of measurement error in realized volatility and the potential of spurious long memory. In this paper we provide a comprehensive analysis of the memory in variances of international stock indices and exchange rates. On the one hand, we find that the variance of exchange rates is subject to spurious long memory and the true memory parameter is in the higher stationary range. Stock index variances, on the other hand, are free of low frequency contaminations and the memory is in the lower non-stationary range. These results are obtained using state of the art local Whittle methods that allow consistent estimation in presence of perturbations or low frequency contaminations.

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1 Introduction

Modeling and forecasting asset volatility is one of the central topics of financial econometrics. While the early literature has focused on short memory GARCH models, today it is well established that financial market volatility typically exhibits long memory. Standard models that capture the long-memory feature are, for example, ARCH(∞) and LARCH models (see Giraitis et al. (2007), Giraitis et al. (2009)), as well as stochastic volatility models that make use of ARFIMA processes. The conceptually simpler HAR model of Corsi (2009) can also approximate long memory by using a regression with overlapping averages of past volatilities. While in the HAR model the actual degree of memory remains unknown, the other models provide estimates of the memory parameter d.

However, a problem arises when the volatility series are contaminated by level shifts or deterministic trends, known as low frequency contaminations. In this case standard estimation methods for d are positively biased.

The issue is that both long memory and mean shifts generate similar time series features such as significant autocorrelations at large lags or a pole in the periodogram at Fourier frequencies local to zero (cf. for example Diebold and Inoue (2001), Granger and Hyung (2004), or Mikosch and Stărică (2004)). If long memory is falsely detected in a short-memory time series subject to low frequency contaminations, it is referred to as 'spurious long memory'. However, recently several methods have been proposed that allow for robust estimation of d under these circumstances (see Iacone (2010), McCloskey and Perron (2013), Hou and Perron (2014)).

Another issue frequently discussed for volatility series is the effect of perturbations. Deo and Hurvich (2001) and Arteche (2004) show that standard estimation methods are negatively biased if a noisy volatility proxy is used. Local Whittle based methods that reduce this bias are proposed of Hurvich et al. (2005) and Frederiksen et al. (2012), among others.

Several studies have estimated the memory parameter in index- and exchange rate variance. However, most of them use standard estimation methods that do not account for the issues discussed above. The estimates achieved are roughly in the range of $0.4 < d < 0.6$, so in the higher stationary or in the lower non-stationary region (c.f. Andersen et al. (2003), Hurvich and Ray (2003), Martens et al. (2009), among others). It is an important question whether $d > 0.5$ since the features of the underlying processes are substantially different. In particular, if $d > 0.5$ the variance of the variance series is infinite, so that the kurtosis of the returns does not exist. To see this, denote the continuously compounded asset returns by r_t and assume that they are mean zero with conditional heteroscedasticity of the form

$$r_t = \sigma_t \eta_t,$$

where σ_t denotes the volatility at day t and it is assumed that $\eta_t \overset{iid}{\sim} (0,1)$ with finite kurtosis K_η. Then the return kurtosis (K_r) can be decomposed into the kurtosis of the volatility process (K_σ) and that of the innovation sequence as follows

$$K_r = \frac{E(r_t^4)}{E(r_t^2)^2} = \frac{E(\sigma_t^4)}{E(\sigma_t^2)^2}\frac{E(\eta_t^4)}{E(\eta_t^2)^2} = K_\sigma K_\eta.$$

If $d > 0.5$, we have

$$Var(\sigma_t^2) = E[\sigma_t^4] - E[\sigma_t]^4 = \infty$$

with $E[\sigma_t]^4 = E[\sigma_t^2]^2 < \infty$. This implies

$$K_r = \left(\frac{Var(\sigma_t^2)^2}{E[\sigma_t^2]^2} + 1\right) K_\eta = \infty.$$

Here, we provide a comprehensive analysis of the memory of a wide range of international stock indices and exchange rates using recently published robust estimation methods. We find that the variance of exchange rates is in the higher stationary range while the variance of stock indices is in the lower non-stationary range. Additionally, we find that exchange rates are likely to be subject to low frequency contaminations which bias standard estimation methods upwards, whereas the stock index variances are free of spurious long memory.
The rest of the paper is structured as follows. In Section 2 we review the methodological issues associated with the estimation of long memory in realized volatility time series. This motivates the use of robust estimation methods reviewed in Section 3. Here, we also provide a Monte Carlo simulation that analyzes the performance of the robust estimation methods if there is potential for both - low frequency contaminations and perturbations. Section 4 contains our empirical contribution that analyzes the memory parameters of a large set of international stock indices and exchange rates. Finally, Section 5 concludes.

2 The Effect of Perturbations and Level Shifts

ARCH-type models usually assume that the daily variance σ_t^2 is some function of the past squared returns, so that $\sigma_t^2 = h(r_{t-1}, r_{t-2}, ...)$. On the contrary, the stochastic volatility literature usually assumes that the log-variance $\log \sigma_t^2$ is a function of the lagged returns as well as an additional innovation sequence ε_t that is specific to the volatility process

$$\log \sigma_t^2 = g(r_{t-1}, r_{t-2}, ..., \varepsilon_t, \varepsilon_{t-1}, ...).$$

To fit these models, one either has to rely on complicated unobserved component models or it is necessary to employ a proxy for the unobserved volatility process σ_t.
Since high frequency data has become widely available, it has become standard practice to use realized variance as a proxy. Realized variance was popularized (among others) by Andersen et al. (2001) and Andersen et al. (2003). Recent examples of long memory models for realized variance include Deo et al. (2006), Martens et al. (2009), and Chiriac and Voev (2011).
Let the log-price p_t of an asset be observed at regular intervals - N times per trading day - and denote the i-th intraday log-return by $r_{i,t} = p_{i,t} - p_{i-1,t}$, then the realized variance is given by

$$z_t = \sum_{i=1}^{N} r_{i,t}^2,$$

so that $z_t = \sigma_t^2(1 + w_t)$, for some error sequence w_t and therefore

$$\log z_t = \log \sigma_t^2 + \log(1 + w_t) \approx \log \sigma_t^2 + w_t, \tag{1}$$

for small w_t. It is clear from (1) that z_t can be regarded as a perturbed version of the underlying volatility process. The influence of this estimation error in the volatility proxy on the accuracy of the estimated memory parameter is an important topic in the long memory stochastic volatility literature.
Note however, that Barndorff-Nielsen and Shephard (2002) show that *plim* $z_t = \sigma_t^2$, as $N \to \infty$, so that $w_t \to 0$. Therefore, z_t is a relatively precise estimate of σ_t^2 and it is sometimes treated as if it was a direct observation of the variance process. Nevertheless, a careful analysis of the memory of volatilities as intended here should take these effects into account.
Another issue that has to be taken into account is the possible presence of low frequency contaminations such as level shifts or deterministic trends. Especially log-squared returns have been a prominent example in the literature on spurious long memory, from early contributions such as Granger and Ding (1996), Mikosch and Stărică (2004), or Granger and Hyung (2004), to more recent contributions such as Lu and Perron (2010), or Xu and Perron (2014).

We therefore consider the following model for the realized variance

$$z_t = c + y_t + u_{T,t} + w_t, \tag{2}$$

where the variance process z_t consists of a short- or long-memory process y_t, a constant c, a so called low frequency contamination $u_{T,t}$ (e.g. a level shift process or trend), and the additive, mean zero, short memory noise term w_t with variance $\sigma_w^2 < \infty$.
The low frequency contamination $u_{T,t}$ is assumed to be a random level shift process as given by

$$u_{T,t} = \sum_{t=1}^{T} \delta_{T,t}, \quad \text{where} \quad \delta_{T,t} = \pi_{T,t}\xi_t, \tag{3}$$

with $\xi_t \sim N(0, \sigma_\xi^2)$ and $\pi_{T,t} \overset{iid}{\sim} B(p/T, 1)$, for $p \geq 0$. Here, $\pi_{T,t}$ and ξ_t are mutually independent and they are also independent of y_t and w_t. To estimate the unknown memory parameter d in applications, it is common to use the local Whittle estimator of Künsch (1987) and Robinson (1995a). Compared to ARFIMA models this semiparametric approach has the advantage that it is consistent irrespective of the form of the short run dynamics. Furthermore, the asymptotic variance of the local Whittle estimator is smaller than that of the log-periodogram estimator of Geweke and Porter-Hudak (1983) and Robinson (1995b). The discussion in this paper is therefore focused on the local Whittle estimator that is discussed in detail in the next section.
In absence of low frequency contaminations in model (2) - that is if $u_{t,T} = 0$ for all $t = 1, ..., T$ - Arteche (2004) shows that the local Whittle estimator is biased downwards. Bias corrected versions of the estimator have been proposed, among others, by Hurvich et al. (2005) and Frederiksen et al. (2012).
Similarly, in absence of perturbations in (2) - that is when $w_t = 0$ for all $t = 1, ..., T$ - Perron and Qu (2010) and McCloskey and Perron (2013) show that the periodogram of z_t can be decomposed into

$$\begin{aligned} I_{z,T}(\lambda_j) &= I_{y,T}(\lambda_j) + I_{u,T}(\lambda_j) + I_{yu,T}(\lambda_j) \\ &= \frac{1}{2\pi T}\left|\sum_{t=1}^{T} y_t e^{i\lambda_j t}\right|^2 + \frac{1}{2\pi T}\left|\sum_{t=1}^{T} u_{T,t} e^{i\lambda_j t}\right|^2 \\ &+ \frac{2}{2\pi T}\sum_{t=1}^{T}\sum_{s=1}^{T} y_t u_{T,t} \cos(\lambda_j(t-s)). \end{aligned}$$

For $\lambda_j = o(1)$ they show that

$$I_{z,T} = O_p\left(\frac{1}{\lambda_j^{2d}}\right) + O_p\left(\frac{1}{T\lambda_j^2}\right) + O_p\left(\frac{1}{\sqrt{T}\lambda_j^{1+d}}\right). \tag{4}$$

It follows that the part corresponding to the random level shift process $u_{T,t}$ dominates for $j = o(T^{(1-2d)/(2-2d)})$ while the part corresponding to the short- or long-memory process y_t dominates for $jT^{(2d-1)/(2-2d)} \to \infty$.

Therefore, local Whittle estimates are biased upwards especially when small bandwidths are used. This results in the aforementioned effect of spurious long memory.

For the model in (2), it is therefore well established that there are potential effects that cause both upwards bias as well as downwards bias in the estimated memory parameters if standard methods such as the local Whittle estimator are used.

3 Robust Long Memory Estimation

As argued in the previous section, it is likely that volatility measures are perturbed (even though the perturbation is less pronounced when the realized variance is used) and subject to low frequency contaminations. Therefore, robust methods against these issues have to be used to estimate d.

The spectral density function $f_z(\lambda)$ of the perturbed volatility measure process under low frequency contaminations in (2) at frequency λ is given by

$$f_z(\lambda) = \phi_y(\lambda)\lambda^{-2d} + \phi_w(\lambda) + \phi_u(\lambda)\lambda^{-2}/T \tag{5}$$

where ϕ_a with $a \in \{y, w, u\}$ corresponds to the spectral density of the short run components in y_t, w_t, and $u_{t,T}$. All local Whittle estimation methods are based on the local log-likelihood as given by

$$R_a(d,\theta) = \log \widehat{G}_a - \frac{2d}{m-l+1}\sum_{j=1}^{m}\log\lambda_j + \frac{1}{m}\sum_{j=1}^{m}\log(g_a), \tag{6}$$

where $\widehat{G}_a$ approximates the spectral density local to zero, $m = \lfloor T^b \rfloor$ is the bandwidth, l is a trimming parameter which is equal to one except for the trimmed local Whittle estimator, $\lambda_j = (2\pi j/T)$ are the Fourier frequencies, g_a is a function that controls for perturbations and/or low frequency contaminations, and $a \in \{LW, LPWN, mLW, tLW\}$.

Depending on whether perturbations, low frequency contaminations, or both are present in the volatility series, the spectral density in (5) has to be approximated differently local to zero. Therefore, several estimators can be derived.

For the standard local Whittle estimator it is assumed that there are no perturbations and low frequency contaminations. Hence, the spectral density of the series z_t in (5) is approximated by a constant G. It follows that

$$\widehat{G}_{LW} = \frac{1}{m}\sum_{j=1}^{m} \lambda_j^{2d} I_z(\lambda_j) \quad \text{and} \quad g_{LW}(d,\theta,\lambda) = 0,$$

where the periodogram $I_z(\lambda)$ is given by $I_z(\lambda) = (2\pi T)^{-1}|\sum_{t=1}^{T} z_t e^{it\lambda}|^2$. The standard local Whittle estimator suggested by Künsch (1987) is then given by

$$\widehat{d}_{LW} = \arg\min_{d} R_{LW}(d). \tag{7}$$

For $1/m + m/T \to \infty$, as $T \to \infty$ and $d \in (-0.5, 0.5)$ consistency of $\widehat{d}_{LW}$ is shown by Robinson (1995a). Under strengthened assumptions (especially on the bandwidth choice) it is also shown that $\sqrt{m}(\widehat{d}_{LW} - d) \overset{a}{\sim} N(0, 1/4)$. Velasco (1999) extends these results and shows that the local Whittle estimator is consistent for $d \in (-0.5, 1]$ and asymptotically normal for $d \in (-0.5, 0.75]$.

In the case where the volatility measure exhibits short memory dynamics and perturbations, the local polynomial Whittle with noise (LPWN) estimator of Frederiksen et al. (2012) can be applied. They extend the idea of Andrews and Sun (2004) who approximate the spectral density local to zero by a polynomial instead of a constant to reduce the finite sample bias of the local Whittle estimator. Frederiksen et al. (2012) add an additional polynomial to approximate the spectrum $\phi_w(\lambda)$ of the perturbation in (5). Other approaches that try to approximate the perturbation by a constant rather than a polynomial are, for example, made by Hurvich and Ray (2003) who proposed the local polynomial Whittle estimator with noise (LWN). This estimator is nested in the LPWN estimator if the polynomials are chosen of order zero.

Precisely, Frederiksen et al. (2012) fit the following two polynomials

$$\begin{aligned} \log\phi_y(\lambda) &\simeq \log G + h_y(\theta_y, \lambda) \\ \log\phi_w(\lambda) &\simeq \log G + \log\theta_p + h_w(\theta_w, \lambda). \end{aligned}$$

to approximate the logarithms of $\phi_y(\lambda)$ and $\phi_w(\lambda)$ in (5). Here $\theta = (\theta_y', \theta_\rho, \theta_w')'$, $\theta_\rho = \phi_w(0)/\phi_y(0)$ is the long-run signal-to-noise ratio, $h_a(\theta_a, \lambda) = \sum_{r=1}^{R_a} \theta_{a,r}\lambda^{2r}$, and $a \in \{y, w\}$.

Therefore,

$$\widehat{G}_{LPWN} = \frac{1}{m}\sum_{j=1}^{m} \frac{\lambda_j^{2d} I_z(\lambda_j)}{g_{LPWN}(d,\theta,\lambda_j)}$$

$$\text{and} \quad g_{LPWN}(d,\theta,\lambda) = \exp(h_y(\theta_y,\lambda)) + \theta_\rho \lambda^{2d} \exp(h_w(\theta_w,\lambda)).$$

The estimator is given by

$$(\widehat{d}_{LPWN}, \widehat{\theta}) = \underset{d\in[d_1,d_2],\theta\in\Theta}{\arg\min}\ R_{LPWN}(d,\theta),$$

where $0 < d_1 < d_2 < 1$ is assumed to be stationary, and Θ is a compact and convex set in $\mathbb{R}^{R_y} \times (0,\infty) \times \mathbb{R}^{R_w}$. For a properly chosen m and $d \in (0,1)$ Frederiksen et al. (2012) show in their Theorem 2 under some regularity conditions that the memory estimator is consistent. They further show for $d \in (0,0.75)$ that it converges in distribution to the normal distribution when perturbations are present. Compared with the local Whittle estimator the asymptotic variance increases by a multiplicative constant, but the bias through perturbation and short memory dynamics is reduced.

In case that the volatility series is subject to level shifts or other low frequency contaminations, the trimmed local Whittle (tLW) estimator of Iacone (2010) and the modified local Whittle (mLW) estimator of Hou and Perron (2014) can be applied.

The idea of the trimmed local Whittle estimator of Iacone (2010) is to use a trimming of the l lowest frequencies where the contaminations have their biggest effect on the spectral density of the series according to (4). Therefore, we obtain

$$\widehat{G}_{tLW} = \frac{1}{m-l+1}\sum_{j=1}^{m} \lambda_j^{2d} I_z(\lambda_j) \quad \text{and} \quad g_{tLW}(d,\theta,\lambda) = 0,$$

where $1 \le l < m \le \lfloor T/2 \rfloor$, so that

$$\widehat{d}_{tLW} = \arg\min_{d} R_{tLW}(d).$$

In case $l = 1$ the estimator is reduced to the standard local Whittle. Under suitable assumptions on the bandwidth and trimming parameter Iacone (2010) shows consistency and asymptotic normality of $\widehat{d}_{tLW}$ for $d \in (0,0.5)$. Its asymptotic variance is the same as that of the local Whittle estimator.

The modified local Whittle estimator uses another approach to achieve consistent estimates of d under low frequency contaminations. It adds an additional term to account for $\phi_u(\lambda)\lambda^{-2}/T$ - the influence of the low frequency contamination in the spectral density function of the variance z_t. Hou and Perron (2014) also provide an additional extension of the modified local Whittle estimator to account for both the perturbation and the low frequency contamination. In this case they approximate the spectral density $\phi_w(\lambda)$ of the perturbation by a constant term following the approach of Hurvich et al. (2005). Denoting $\theta = (\theta_w, \theta_u)'$ as the signal-to-noise ratios of the perturbation and the low frequency contamination the estimator uses

$$\widehat{G}_{mLW} = \frac{1}{m}\sum_{j=1}^{m}\frac{I_z(\lambda_j)}{g_{mLW}(d,\theta,\lambda_j)} \quad \text{and} \quad g_{mLW}(d,\theta,\lambda) = (\lambda^{-2d} + \theta_w + \theta_u\lambda^{-2}/T)$$

and is given by

$$(\widehat{d}_{mLW}, \widehat{\theta}) = \arg\min_{d,\theta} R_{mLW}(d,\theta).$$

If $\theta_w = 0$, we have the modified local Whittle estimator (mLW) and if $\theta_w \neq 0$ we have the modified local Whittle plus noise estimator (mLWN). For $\theta_w = 0$, a properly chosen m which needs to be larger than $T^{5/9}$ and $d \in (0, 0.5)$, Hou and Perron (2014) show consistency and under strengthened assumptions asymptotic normality of the estimator. The estimator possesses the same asymptotic variance as the local Whittle estimator. The consistency of these methods for $d > 0.5$ is addressed in our simulations below.
A prominent test to distinguish true from spurious long memory is the Lagrange multiplier-type test of Qu (2011). Its null hypothesis incorporates all second-order stationary short- or long-memory processes. Under the alternative, the process is contaminated by some low frequency contamination, for example a random level shift as given in (3). The test statistic uses the difference between the rates given in (4) and it is based on the local Whittle likelihood function. It is given by

$$W = \sup_{r\in[\epsilon,1]}\left(\sum_{j=1}^{m}\nu_j^2\right)^{-1/2}\left|\sum_{j=1}^{[mr]}\nu_j\left(\frac{I_z(\lambda_j)}{G(\hat{d}_{LW})\lambda_j^{-2\hat{d}_{LW}}} - 1\right)\right|, \tag{8}$$

with $\nu_j = \log\lambda_j - (1/m)\sum_{j=1}^{m}\log\lambda_j$, and a small trimming parameter ϵ. Qu (2011) derives the consistency and the limiting distribution of (8) for $d \in (0, 0.5)$. Sibbertsen et al. (2017) show via simulations that the test also works in the low non-stationary range for d. Qu (2011) reports critical values for $\epsilon \in \{0.02, 0.05\}$, where the first value is recommended for sample sizes $T > 500$. It is further recommended to use $m = \lfloor T^{0.7}\rfloor$ frequency ordinates. The test of Qu (2011) has several desirable properties such as not requiring Gaussianity, allowing for conditional heteroskedasticity, not requiring a precise specification of the low frequency contamination due to its score-type nature and displaying high finite sample power results compared to competing tests (c.f. Leccadito et al. (2015)).

		Bias						standard deviation					
	d	0.4			0.6			0.4			0.6		
	σ_w/σ_η	0	1	2	0	1	2	0	1	2	0	1	2
LW	0	0.00	0.08	0.17	0.00	0.04	0.10	0.03	0.06	0.09	0.03	0.05	0.07
	0.1	0.00	0.08	0.17	0.00	0.04	0.10	0.03	0.06	0.09	0.03	0.05	0.07
	0.25	-0.01	0.07	0.16	-0.02	0.03	0.09	0.03	0.06	0.09	0.03	0.05	0.07
	0.5	-0.03	0.05	0.14	-0.06	-0.02	0.05	0.03	0.06	0.09	0.02	0.05	0.07
mLW	0	-0.01	-0.01	-0.01	-0.02	-0.01	0.02	0.02	0.03	0.04	0.02	0.04	0.07
	0.1	-0.01	-0.01	-0.01	-0.03	-0.02	0.01	0.02	0.03	0.04	0.02	0.04	0.07
	0.25	-0.03	-0.03	-0.03	-0.08	-0.08	-0.08	0.02	0.03	0.04	0.04	0.05	0.08
	0.5	-0.07	-0.07	-0.08	-0.20	-0.22	-0.24	0.02	0.03	0.04	0.06	0.06	0.07
tLW	0	-0.01	0.01	0.04	-0.01	0.01	0.04	0.03	0.04	0.05	0.03	0.04	0.05
	0.1	-0.01	0.01	0.04	-0.01	0.00	0.03	0.03	0.04	0.05	0.03	0.04	0.05
	0.25	-0.02	-0.01	0.03	-0.05	-0.04	0.00	0.03	0.03	0.05	0.03	0.04	0.05
	0.5	-0.06	-0.05	-0.02	-0.15	-0.14	-0.10	0.03	0.03	0.05	0.04	0.04	0.06
LWN	0	0.02	0.12	0.23	0.01	0.06	0.12	0.03	0.08	0.10	0.03	0.05	0.07
	0.1	0.02	0.12	0.23	0.01	0.06	0.12	0.03	0.08	0.10	0.03	0.05	0.07
	0.25	0.01	0.12	0.23	0.01	0.06	0.13	0.04	0.08	0.11	0.04	0.06	0.08
	0.5	0.01	0.13	0.24	0.01	0.06	0.13	0.04	0.09	0.11	0.04	0.06	0.08
mLWN	0	0.01	0.03	0.11	0.01	0.05	0.11	0.03	0.07	0.15	0.03	0.06	0.08
	0.1	0.01	0.03	0.11	0.01	0.04	0.11	0.03	0.07	0.15	0.03	0.06	0.09
	0.25	0.00	0.03	0.11	0.00	0.03	0.10	0.04	0.07	0.16	0.05	0.07	0.10
	0.5	-0.01	0.02	0.11	-0.01	0.03	0.11	0.05	0.09	0.17	0.06	0.09	0.11

Table 1: Bias and standard deviation of the long-memory estimators.

To evaluate the finite sample performance of the estimators discussed above in the situation we are facing in our empirical application, we conduct a small Monte Carlo simulation. The data generating process (DGP) is based on (2), where y_t is a fractionally integrated process of order d, w_t is white noise with variance σ_w^2, $p = 5$, and $T = 4000$, which approximately mirrors the sample sizes in our empirical application. Since the variance of the perturbations can be expected to be small, we set $\sigma_w \in \{0, 1/10, 1/4, 1/2\}$, whereas y_t is scaled so that the variance of the process is one. Furthermore, we set $\sigma_\eta^2 \in \{0, 1, 2\}$, since we do not have any a priori knowledge about the magnitude of potential mean shifts. The bandwidth parameters are chosen as in the empirical application following the recommendations of the authors who proposed the respective methods. For the local Whittle estimator we set $m = \lfloor T^{0.7} \rfloor$.

The results reported in Table 1 are based on 5000 Monte Carlo replications. Starting with the local Whittle estimator, we can observe the expected result that there is a positive bias if level shift components are present and the perturbations cause only a slight negative bias due to their moderate scale. It is worth noting that the bias of the local Whittle estimator is smaller for $d = 0.6$ than for $d = 0.4$, so that all the estimated ds are in the range between 0.5 and 0.7.

Turning to the mLW estimator of Hou and Perron (2014), we observe that the estimator successfully mitigates the bias caused by the level shift components. However, with increasing magnitude of the perturbation, the estimator suffers a strong negative bias - much stronger than the original local Whittle estimator.

Similar results hold true for the tLW estimator of Iacone (2010), but the magnitude of the perturbation bias is considerably smaller than that of the mLW estimator.

The LWN estimator behaves similarly, but in the contrary direction. It successfully mitigates the downward bias caused by perturbations, but it suffers from a stronger upward bias in case of level shift components than the local Whittle estimator.

Finally, the mLWN estimator seems to be mitigating the perturbation bias, but it does not control the spurious long memory bias to its full extend.

The results of the mLW and tLW estimators for $d = 0.6$ show that the consistency extends to the lower non-stationary region - exactly like that of the LW and LPWN.

With regard to the variation of the estimators, we can observe that all methods become increasingly variable as the influence of the level shifts increases. The mLW estimator turns out to have slightly less variance than the tLW estimator of Iacone (2010) for $d = 0.4$, but higher variance for $d = 0.6$. The LWN estimator is more variable than the LW estimator and the mLWN estimator is extremely volatile in presence of level shifts in a stationary long-memory sequence with $d = 0.4$.

4 The Memory of Realized Volatility

We consider daily realized variances of 41 major stock indices and 10 nominal exchange rates relative to the US Dollar. The data for the indices is obtained from the 'Oxford-Man Institute's realised library' and was compiled by Heber et al. (2009). The series start between 1996 and 2000 and they end on 9 June 2017. An overview of the symbols is given in Table 5 and a summary of the start and end dates as well as the length of the series is given in Table 7 in the appendix.

To construct similar series for the exchange rates, we use 5-minute returns obtained from the Thomson Reuters Tick History database. The data is cleaned following the recommendations of Barndorff-Nielsen et al. (2009) to account for the typical high frequency data quality issues. Similar to the procedure of Heber et al. (2009), some additional manual edits are made so that the data is suitable for statistical inference. Since there is no market closure for exchange rates the log-realized variance is calculated based on all 5-minute log-returns within each 24-hour period. As for the indices, an overview of the meaning of the symbols is given in Table 6 and starting dates of the resulting series and the number of observations are given in Table 8. The last observation of all exchange rate series is from 31 January 2017.

The results of the different long-memory estimators applied to the log-realized variances $log\ z_t$ of the indices are given in Tables 2 and 3. Starting with the local Whittle estimates, we observe that they tend to decrease as the bandwidth increases from $m = \lfloor T^{0.6} \rfloor$ to $m = \lfloor T^{0.8} \rfloor$. Even though this decrease has a magnitude of up to 0.2 for some of the series, it is moderate for the majority of them. Nevertheless, this could be seen as an indication for low frequency contaminations in the respective series. The intuition behind this is given in Section 2: the long-memory component of a contaminated series dominates the low frequency contamination at higher frequencies such that the positive bias of the estimated d decreases if a larger bandwidth is chosen.

Nearly all point estimates are within the lower non-stationary region between 0.5 and 0.6 and the vast majority of asymptotic confidence intervals are completely in the non-stationary region as well. If the impact of perturbations and low frequency contaminations is low, this is strong evidence that the memory of the indices is larger than 0.5.

Turning to the LPWN estimates, we first observe that the estimates are very stable across the different specifications of the estimator. The level of the estimates tends to be higher than that of the local Whittle estimates. Most of them are in the range between 0.6 and 0.7. This further supports the previous finding that the index variances possess non-stationary long memory and points to the fact that the measurement error in the log-realized variance still has a magnitude so that it causes downward bias in the local Whittle estimator.

The right hand side of Tables 2 and 3 shows the results of the mLW and tLW estimators of Hou and Perron (2014) and Iacone (2010) as well as the mLWN estimator. In all cases the mLW estimates are smaller than the local Whittle estimates and in some series the memory drops by a considerable amount. The same holds true for the tLW estimator of Iacone (2010), but the reduction in memory compared to the local Whittle estimates is of a smaller magnitude. This could be seen as evidence for low frequency contaminations in the variances. However, the results of the mLWN estimator are more in line with those of the LPWN estimators and the test of Qu (2011) fails to reject the null hypothesis of true long memory for the vast majority of index series. At the 5% significance level the test of Qu (2011) only rejects for GSPTSE, HSI, MSCIBE, MSCIDE, and MSCIJP.

Having in mind that the mLW and tLW estimators are severely downward biased in the presence of moderate perturbations, we therefore conclude that there is no evidence for spurious long memory. Hence, the LPWN estimator is most suitable to give the best

	$LW_{0.6}$	$LW_{0.7}$	$LW_{0.8}$	LWN	LPWN(1,0)	LPWN(0,1)	LPWN(1,1)	mLW	mLWN	tLW	$Qu_{0.75}$
AEX	0.625	0.635	0.577	0.659	0.661	0.670	0.669	0.568	0.658	0.567	0.575
	(0.542 0.709)	(0.581 0.688)	(0.543 0.612)	(0.624 0.694)	(0.626 0.696)	(0.636 0.705)	(0.634 0.704)	(0.533 0.602)	(0.623,0.693)	(0.506,0.627)	
AORD	0.553	0.586	0.460	0.650	0.611	0.617	0.611	0.344	0.648	0.444	1.061
	(0.469 0.637)	(0.532 0.640)	(0.426 0.495)	(0.615 0.685)	(0.576 0.646)	(0.582 0.652)	(0.576 0.646)	(0.309 0.379)	(0.613,0.683)	(0.383,0.505)	
BVSP	0.549	0.521	0.475	0.561	0.530	0.522	0.530	0.462	0.561	0.484	0.555
	(0.464 0.633)	(0.467 0.575)	(0.440 0.510)	(0.526 0.597)	(0.495 0.565)	(0.487 0.558)	(0.495 0.565)	(0.426 0.497)	(0.526,0.597)	(0.423,0.545)	
DJI	0.622	0.586	0.534	0.624	0.594	0.572	0.594	0.488	0.624	0.539	0.610
	(0.538 0.706)	(0.533 0.640)	(0.499 0.569)	(0.589 0.659)	(0.559 0.629)	(0.537 0.607)	(0.559 0.629)	(0.453 0.523)	(0.589,0.659)	(0.479,0.600)	
FCHI	0.605	0.624	0.564	0.643	0.646	0.656	0.655	0.518	0.642	0.550	0.616
	(0.521 0.688)	(0.571 0.678)	(0.529 0.599)	(0.609 0.678)	(0.611 0.681)	(0.621 0.691)	(0.620 0.689)	(0.483 0.552)	(0.607,0.677)	(0.490,0.610)	
FTSE	0.650	0.640	0.569	0.672	0.677	0.691	0.691	0.487	0.671	0.544	0.677
	(0.566 0.733)	(0.586 0.694)	(0.534 0.604)	(0.637 0.707)	(0.643 0.712)	(0.656 0.726)	(0.656 0.726)	(0.452 0.522)	(0.636,0.706)	(0.483,0.604)	
FTSEMIB	0.600	0.607	0.547	0.632	0.626	0.631	0.627	0.522	0.631	0.548	0.557
	(0.517 0.684)	(0.553 0.660)	(0.512 0.582)	(0.597 0.667)	(0.591 0.661)	(0.596 0.666)	(0.592 0.662)	(0.488 0.557)	(0.596,0.665)	(0.487,0.609)	
GDAXI	0.653	0.637	0.564	0.663	0.667	0.728	0.728	0.471	0.662	0.499	0.722
	(0.569 0.736)	(0.584 0.691)	(0.529 0.598)	(0.629 0.698)	(0.632 0.702)	(0.693 0.762)	(0.694 0.763)	(0.436 0.506)	(0.627,0.697)	(0.438,0.559)	
GSPTSE	0.603	0.565	0.490	0.642	0.647	0.639	0.651	0.375	0.593	0.451	**1.264***
	(0.514 0.691)	(0.509 0.622)	(0.453 0.527)	(0.605 0.679)	(0.610 0.684)	(0.602 0.676)	(0.614 0.689)	(0.338 0.412)	(0.556,0.630)	(0.387,0.515)	
HSI	0.640	0.557	0.503	0.640	0.646	0.649	0.649	0.384	0.638	0.446	**1.461***
	(0.554 0.726)	(0.502 0.613)	(0.467 0.539)	(0.604 0.676)	(0.610 0.682)	(0.613 0.685)	(0.613 0.686)	(0.348 0.420)	(0.602,0.674)	(0.383,0.509)	
IBEX	0.593	0.596	0.545	0.611	0.617	0.632	0.631	0.510	0.576	0.525	0.858
	(0.509 0.677)	(0.542 0.649)	(0.510 0.580)	(0.576 0.645)	(0.583 0.652)	(0.597 0.666)	(0.597 0.666)	(0.475 0.544)	(0.541,0.611)	(0.465,0.586)	
IXIC	0.644	0.598	0.552	0.628	0.623	0.579	0.625	0.493	0.625	0.542	0.535
	(0.560 0.728)	(0.545 0.652)	(0.517 0.587)	(0.593 0.662)	(0.588 0.657)	(0.544 0.614)	(0.590 0.660)	(0.458 0.527)	(0.590,0.660)	(0.481,0.603)	
KS11	0.692	0.622	0.548	0.686	0.691	0.718	0.743	0.407	0.684	0.510	**1.181**
	(0.607 0.776)	(0.567 0.676)	(0.512 0.583)	(0.651 0.721)	(0.656 0.726)	(0.683 0.753)	(0.708 0.778)	(0.372 0.442)	(0.649,0.720)	(0.448,0.571)	
MIB30	0.608	0.613	0.550	0.662	0.656	0.661	0.657	0.510	0.661	0.562	0.640
	(0.516 0.700)	(0.553 0.673)	(0.511 0.590)	(0.623 0.702)	(0.617 0.695)	(0.622 0.700)	(0.618 0.696)	(0.471 0.549)	(0.622,0.700)	(0.458,0.596)	
MIBTEL	0.667	0.622	0.532	0.735	0.686	0.693	0.687	0.343	0.735	0.536	1.002
	(0.562 0.773)	(0.552 0.692)	(0.486 0.579)	(0.689 0.782)	(0.640 0.733)	(0.646 0.739)	(0.640 0.733)	(0.296 0.389)	(0.689,0.782)	(0.493,0.630)	
MID	0.712	0.637	0.564	0.727	0.730	0.754	0.754	0.373	0.726	0.530	**1.160**
	(0.620 0.804)	(0.577 0.697)	(0.525 0.603)	(0.687 0.766)	(0.691 0.770)	(0.714 0.793)	(0.714 0.793)	(0.333 0.412)	(0.687,0.765)	(0.453,0.618)	
MSCIAU	0.659	0.606	0.499	0.731	0.698	0.701	0.698	0.297	0.728	0.470	0.721
	(0.555 0.762)	(0.538 0.674)	(0.454 0.545)	(0.685 0.776)	(0.653 0.743)	(0.655 0.746)	(0.653 0.743)	(0.252 0.343)	(0.682,0.773)	(0.460,0.599)	
MSCIBE	0.730	0.656	0.536	0.805	0.738	0.733	0.790	0.160	0.804	0.501	**1.349***
	(0.628 0.832)	(0.589 0.723)	(0.492 0.581)	(0.760 0.849)	(0.693 0.782)	(0.688 0.777)	(0.745 0.834)	(0.116 0.204)	(0.759,0.848)	(0.389,0.550)	
MSCIBR	0.629	0.567	0.501	0.677	0.670	0.671	0.675	0.358	0.675	0.503	0.628
	(0.511 0.747)	(0.488 0.645)	(0.448 0.555)	(0.624 0.730)	(0.617 0.723)	(0.618 0.724)	(0.622 0.729)	(0.304 0.411)	(0.622,0.728)	(0.422,0.580)	
MSCICA	0.681	0.614	0.503	0.765	0.680	0.683	0.676	0.215	0.763	0.512	0.636
	(0.572 0.789)	(0.543 0.686)	(0.454 0.551)	(0.716 0.813)	(0.632 0.728)	(0.635 0.731)	(0.628 0.724)	(0.167 0.263)	(0.715,0.811)	(0.407,0.599)	
MSCICH	0.730	0.676	0.565	0.782	0.724	0.720	0.719	0.263	0.782	0.546	1.103
	(0.629 0.832)	(0.609 0.743)	(0.521 0.610)	(0.738 0.827)	(0.680 0.768)	(0.675 0.764)	(0.675 0.764)	(0.218 0.307)	(0.737,0.826)	(0.426,0.598)	

Table 2: Estimated long-memory coefficients of the log-realized variances for different indices. Theoretical confidence intervals are given in brackets below. For the Qu-test bold-faced values indicate significance at the nominal 10% level; an additional * (**) indicates significance at the nominal 5% (1%) level.

estimate of the true memory of the variances of the stock indices under consideration. We therefore find, that the memory of stock index variance is non-stationary. Most stock index variances display memory parameters in the range between 0.6 and 0.7, which is far in the non-stationary region.

In Table 4 we report the results for the realized variance of exchange rates. Here, we observe some major differences compared to the results for the variances of indices, discussed above. First of all, the local Whittle estimates decrease heavily when the bandwidth increases. Again, this can be seen as an indication of low frequency contaminations by the same arguments as discussed above. The assertion that the exchange rate variances exhibit spurious long memory is further supported by the fact that both the mLW estimator of Hou and Perron (2014) and the tLW estimator of Iacone (2010) are reduced compared to the local Whittle estimates. But most importantly, the Qu (2011)

	$LW_{0.6}$	$LW_{0.7}$	$LW_{0.8}$	LWN	LPWN(1,0)	LPWN(0,1)	LPWN(1,1)	mLW	mLWN	tLW	$Qu_{0.75}$
MSCIDE	0.680	0.639	0.521	0.781	0.695	0.670	0.680	0.191	0.780	0.493	**1.468***
	(0.578 0.782)	(0.573 0.706)	(0.476 0.565)	(0.737 0.826)	(0.650 0.739)	(0.626 0.715)	(0.635 0.724)	(0.147 0.236)	(0.736,0.825)	(0.467,0.625)	
MSCIES	0.640	0.623	0.519	0.716	0.653	0.656	0.651	0.359	0.714	0.524	0.616
	(0.539 0.742)	(0.556 0.690)	(0.474 0.563)	(0.672 0.761)	(0.608 0.697)	(0.611 0.700)	(0.606 0.695)	(0.314 0.403)	(0.669,0.758)	(0.414,0.572)	
MSCIFR	0.656	0.619	0.522	0.761	0.680	0.645	0.680	0.237	0.761	0.537	1.054
	(0.554 0.758)	(0.553 0.686)	(0.477 0.566)	(0.717 0.806)	(0.635 0.724)	(0.601 0.690)	(0.635 0.724)	(0.193 0.281)	(0.717,0.806)	(0.445,0.603)	
MSCIGB	0.693	0.643	0.544	0.773	0.697	0.687	0.676	0.246	0.772	0.542	1.013
	(0.591 0.794)	(0.576 0.710)	(0.499 0.588)	(0.729 0.817)	(0.652 0.741)	(0.643 0.731)	(0.631 0.720)	(0.202 0.290)	(0.728,0.816)	(0.458,0.615)	
MSCIIT	0.633	0.628	0.532	0.728	0.674	0.680	0.673	0.352	0.727	0.532	1.113
	(0.532 0.735)	(0.562 0.695)	(0.488 0.577)	(0.684 0.773)	(0.630 0.718)	(0.635 0.724)	(0.629 0.718)	(0.307 0.396)	(0.683,0.772)	(0.463,0.621)	
MSCIJP	0.685	0.568	0.514	0.619	0.626	0.691	0.696	0.437	0.594	0.455	**1.372***
	(0.581 0.790)	(0.499 0.637)	(0.468 0.560)	(0.573 0.665)	(0.580 0.672)	(0.645 0.737)	(0.650 0.742)	(0.391 0.483)	(0.548,0.640)	(0.454,0.611)	
MSCIKR	0.695	0.625	0.534	0.691	0.638	0.639	0.625	0.407	0.689	0.512	0.785
	(0.591 0.800)	(0.556 0.694)	(0.488 0.580)	(0.645 0.737)	(0.592 0.684)	(0.593 0.685)	(0.579 0.671)	(0.361 0.453)	(0.643,0.735)	(0.373,0.536)	
MSCIMX	0.669	0.603	0.494	0.768	0.705	0.709	0.714	0.184	0.767	0.491	0.967
	(0.552 0.786)	(0.525 0.681)	(0.441 0.547)	(0.715 0.821)	(0.652 0.758)	(0.656 0.762)	(0.661 0.767)	(0.132 0.237)	(0.715,0.820)	(0.431,0.593)	
MSCINL	0.672	-0.635	0.534	0.772	0.704	0.631	0.704	0.208	0.772	0.574	0.729
	(0.571 0.774)	(0.569 0.702)	(0.490 0.579)	(0.728 0.817)	(0.660 0.749)	(0.586 0.675)	(0.660 0.749)	(0.163 0.252)	(0.727,0.816)	(0.396,0.585)	
MSCIWO	0.600	0.528	0.445	0.661	0.582	0.582	0.591	0.291	0.658	0.468	0.482
	(0.494 0.707)	(0.457 0.599)	(0.398 0.493)	(0.614 0.708)	(0.534 0.629)	(0.535 0.630)	(0.543 0.638)	(0.243 0.338)	(0.610,0.705)	(0.495,0.652)	
MXX	0.575	0.503	0.441	0.601	0.576	0.576	0.624	0.343	0.599	0.418	0.848
	(0.491 0.659)	(0.449 0.557)	(0.406 0.476)	(0.566 0.636)	(0.541 0.611)	(0.541 0.611)	(0.589 0.659)	(0.308 0.378)	(0.564,0.634)	(0.357,0.479)	
N2252	0.618	0.557	0.514	0.589	0.598	0.636	0.636	0.477	0.588	0.504	0.949
	(0.533 0.703)	(0.502 0.611)	(0.478 0.549)	(0.554 0.625)	(0.563 0.633)	(0.601 0.672)	(0.601 0.672)	(0.442 0.513)	(0.553,0.624)	(0.442,0.565)	
NSEI	0.572	0.515	0.497	0.537	0.549	0.601	0.617	0.470	0.465	0.455	0.622
	(0.484 0.660)	(0.458 0.572)	(0.460 0.534)	(0.500 0.574)	(0.512 0.586)	(0.564 0.638)	(0.580 0.654)	(0.433 0.507)	(0.428,0.502)	(0.390,0.520)	
RUA	0.646	0.583	0.539	0.663	0.664	0.667	0.665	0.440	0.662	0.541	0.534
	(0.554 0.738)	(0.524 0.643)	(0.499 0.578)	(0.623 0.702)	(0.624 0.703)	(0.627 0.706)	(0.626 0.704)	(0.400 0.479)	(0.622,0.701)	(0.385,0.552)	
RUI	0.644	0.580	0.537	0.657	0.660	0.665	0.664	0.444	0.657	0.539	0.520
	(0.551 0.736)	(0.521 0.640)	(0.498 0.577)	(0.618 0.697)	(0.621 0.700)	(0.626 0.705)	(0.625 0.703)	(0.405 0.484)	(0.618,0.697)	(0.472,0.611)	
RUT2	0.549	0.551	0.501	0.574	0.549	0.527	0.549	0.471	0.573	0.494	0.658
	(0.465 0.633)	(0.497 0.605)	(0.466 0.536)	(0.539 0.609)	(0.515 0.584)	(0.492 0.562)	(0.515 0.584)	(0.436 0.506)	(0.538,0.608)	(0.434,0.555)	
SPTSE	0.681	0.600	0.508	0.742	0.660	0.643	0.652	0.275	0.742	0.485	**1.169**
	(0.581 0.781)	(0.534 0.665)	(0.464 0.551)	(0.698 0.785)	(0.617 0.704)	(0.599 0.687)	(0.608 0.696)	(0.231 0.318)	(0.698,0.785)	(0.470,0.608)	
SPX2	0.623	0.586	0.548	0.610	0.591	0.573	0.568	0.534	0.608	0.546	0.562
	(0.539 0.707)	(0.532 0.640)	(0.514 0.583)	(0.575 0.645)	(0.556 0.626)	(0.538 0.608)	(0.533 0.603)	(0.499 0.569)	(0.573,0.643)	(0.485,0.607)	
SSMI	0.656	0.671	0.590	0.694	0.695	0.707	0.705	0.581	0.693	0.573	**1.119**
	(0.572 0.740)	(0.617 0.725)	(0.555 0.625)	(0.659 0.729)	(0.660 0.730)	(0.672 0.742)	(0.670 0.740)	(0.546 0.616)	(0.659,0.728)	(0.512,0.634)	
STOXX50E	0.591	0.594	0.527	0.613	0.618	0.637	0.637	0.480	0.611	0.494	0.828
	(0.507 0.675)	(0.540 0.647)	(0.492 0.562)	(0.578 0.647)	(0.584 0.653)	(0.603 0.672)	(0.602 0.672)	(0.445 0.514)	(0.576,0.646)	(0.433,0.554)	

Table 3: Estimated long-memory coefficients of the log-realized variances for different indices. Theoretical confidence intervals are given in brackets below. For the Qu-test bold-faced values indicate significance at the nominal 10% level; an additional * (**) indicates significance at the nominal 5% (1%) level.

test rejects strongly for all currencies considered. This is clear evidence for the presence of spurious long memory. Only the mLWN estimator does not show evidence for a lower degree of memory. However, the results show a very high variability and we know from the simulations in the previous section that the estimator fails to control the spurious long memory bias, if the level shift component is large. Finally, the LPWN estimates are much higher compared to the local Whittle estimates, which is also consistent with the observation that the LPWN estimators have a larger spurious long memory bias than the standard local Whittle estimator, as shown in our simulations.

If we now consider the mLW and tLW estimators that are most likely to give consistent estimates in this setup, we observe that all estimates are in the stationary region. In particular the estimator of Iacone (2010) gives estimates that lie consistently between 0.3 and 0.4. The results for the Hou and Perron (2014) estimator are a bit more variable and are found to be in the range between 0.05 and 0.4.

	$LW_{0.6}$	$LW_{0.7}$	$LW_{0.8}$	LWN	LPWN(1,0)	LPWN(0,1)	LPWN(1,1)	mLW	mLWN	tLW	$Qu_{0.75}$
AUD	0.648	0.558	0.404	0.782	0.625	0.507	0.889	0.051	0.782	0.367	**2.765****
	(0.575 0.722)	(0.512 0.605)	(0.375 0.433)	(0.753 0.811)	(0.596 0.655)	(0.477 0.536)	(0.859 0.918)	(0.022 0.080)	(0.753,0.811)	(0.317,0.417)	
BRL	0.615	0.554	0.465	0.635	0.641	0.630	0.640	0.329	0.634	0.400	**1.924****
	(0.530 0.699)	(0.500 0.608)	(0.430 0.500)	(0.600 0.670)	(0.606 0.677)	(0.595 0.665)	(0.605 0.675)	(0.293 0.364)	(0.599,0.670)	(0.339,0.461)	
CAD	0.676	0.560	0.438	0.766	0.693	0.680	0.838	0.140	0.765	0.335	**3.447****
	(0.603 0.749)	(0.514 0.606)	(0.408 0.467)	(0.737 0.795)	(0.663 0.722)	(0.651 0.709)	(0.809 0.867)	(0.111 0.170)	(0.735,0.794)	(0.285,0.385)	
CHF	0.586	0.522	0.406	0.674	0.601	0.597	0.678	0.222	0.673	0.384	**2.542****
	(0.513 0.659)	(0.476 0.568)	(0.377 0.435)	(0.645 0.703)	(0.572 0.630)	(0.567 0.626)	(0.649 0.707)	(0.192 0.251)	(0.644,0.703)	(0.334,0.434)	
EUR	0.619	0.508	0.372	0.765	0.646	0.602	0.794	0.090	0.764	0.316	**3.132****
	(0.544 0.695)	(0.460 0.555)	(0.342 0.403)	(0.734 0.795)	(0.616 0.677)	(0.571 0.632)	(0.764 0.825)	(0.059 0.120)	(0.733,0.794)	(0.264,0.369)	
GBP	0.662	0.566	0.406	0.824	0.713	0.676	0.835	0.023	0.823	0.355	**3.388****
	(0.590 0.735)	(0.520 0.611)	(0.377 0.435)	(0.794 0.853)	(0.684 0.742)	(0.647 0.705)	(0.806 0.864)	(-0.006 0.052)	(0.794,0.852)	(0.305,0.405)	
INR	0.517	0.498	0.445	0.539	0.549	0.596	0.626	0.375	0.380	0.384	**2.259****
	(0.436 0.597)	(0.447 0.550)	(0.412 0.478)	(0.506 0.572)	(0.516 0.582)	(0.563 0.629)	(0.593 0.659)	(0.342 0.408)	(0.347,0.413)	(0.327,0.441)	
JPY	0.561	0.462	0.416	0.558	0.499	0.462	0.499	0.342	0.555	0.405	**1.466***
	(0.488 0.633)	(0.417 0.508)	(0.387 0.445)	(0.529 0.587)	(0.470 0.528)	(0.433 0.491)	(0.470 0.528)	(0.313 0.371)	(0.526,0.584)	(0.356,0.455)	
RUB	0.708	0.567	0.503	0.708	0.707	0.852	0.919	0.286	0.703	0.334	**2.908***
	(0.617 0.798)	(0.509 0.626)	(0.464 0.541)	(0.669 0.746)	(0.669 0.746)	(0.813 0.890)	(0.881 0.957)	(0.248 0.324)	(0.664,0.741)	(0.267,0.401)	
ZAR	0.758	0.679	0.538	0.842	0.751	0.726	0.886	0.113	0.842	0.481	**3.178****
	(0.682 0.833)	(0.631 0.727)	(0.508 0.569)	(0.811 0.872)	(0.721 0.782)	(0.696 0.757)	(0.856 0.916)	(0.083 0.143)	(0.811,0.872)	(0.429,0.534)	

Table 4: Estimated long-memory coefficients of the log-realized variances for different exchange rates. Theoretical confidence intervals are given in brackets below. For the Qu-test bold-faced values indicate significance at the nominal 10% level; an additional * (**) indicates significance at the nominal 5% (1%) level.

To ensure the robustness of our findings, we consider a variety of alternative specifications. The results of these exercises are given in the appendix. First, we consider realized kernels instead of realized variances for the indices. Realized kernels are a measure of variance that is more robust to market microstructure effects. The results are given in Tables 9 and 10 in the appendix. Furthermore, in Tables 11 and 12, we construct confidence intervals for the local Whittle estimator using the frequency domain bootstrap procedure of Arteche and Orbe (2016). Finally, we apply the trimmed log-periodogram regression of McCloskey and Perron (2013) as an alternative to estimate the memory robust to spurious long memory. These results can be found in Tables 13 and 14. The results of all these analyses are remarkably similar to those presented here, which highlights the robustness of our findings.

Considering these results, we are able to establish a number of key findings. First of all, there is a considerable difference between the behavior of stock index variances and exchange rate variances. The stock index variances exhibit true long memory in the nonstationary range between 0.6 and 0.7. In contrast to that, the exchange rate variances show clear signs of spurious long memory and the true long memory of the series are only around 0.3.

5 Conclusion

In Section 2 we discuss the effect of measurement error and level shifts on estimates of the memory parameter in log-realized variances using the local Whittle estimator of Künsch (1987) and Robinson (1995a). In the recent literature a large number of new local Whittle estimators has been proposed that are robust to these effects, most importantly those of Hurvich et al. (2005) and Frederiksen et al. (2012), Iacone (2010) and Hou and Perron (2014). These are discussed in Section 3, where we also conduct a simulation study to evaluate the performance of these methods if both of these complications are incurred at the same time. We find that, while the estimators are successful in mitigating the bias they are build to address, they become more vulnerable to the bias they do not account for. That means the LPWN estimator has a larger bias due to spurious long memory than the standard local Whittle estimator and the modified and trimmed local Whittle estimators have a larger bias in presence of perturbations. In our empirical application we are able to establish some new stylized facts about the memory in realized variances. Considering a wide range of stock indices, we find that the index variances are true long-memory processes with a memory parameter between 0.6 and 0.7, which is in the non-stationary range. As discussed in the introduction, this means that long memory stochastic volatility models are able to reproduce the finding that the kurtosis of stock market returns is infinite. Exchange rate variances, however, exhibit spurious long memory and the true memory parameters are between 0.3 and 0.4, which is far in the stationary region.

6 Appendix

AEX	AEX Index	MSCIDE	MSCI Germany
AORD	All Ordinaries	MSCIES	MSCI Spain
BVSP	Bovespa Index	MSCIFR	MSCI France
DJI	Dow Jones Industrials	MSCIGB	MSCI UK
FCHI	CAC 40	MSCIIT	MSCI Italy
FTSE	FTSE 100	MSCIJP	MSCI Japan
FTSEMIB	FTSE MIB	MSCIKR	MSCI South Korea
GDAXI	German DAX	MSCIMX	MSCI Mexico
GSPTSE	S&P/TSX Composite Index	MSCINL	MSCI Netherlands
HSI	Hang Seng	MSCIWO	MSCI World
IBEX	Spanish IBEX	MXX	IPC Mexico
IXIC	Nasdaq 100	N2252	Nikkei 250
KS11	KOSPI Composite Index	NSEI	S&P CNX Nifty
MIB30	Milan MIB 30	RUA	Russell 3000
MIBTEL	Italian MIBTEL	RUI	Russell 1000
MID	S&P 400 Midcap	RUT2	Russell 2000
MSCIAU	MSCI Australia	SPTSE	S&P TSE
MSCIBE	MSCI Belgium	SPX	S&P 500
MSCIBR	MSCI Brazil	SSMI	Swiss Market Index
MSCICA	MSCI Canada	STOXX50E	Euro STOXX 50
MSCICH	MSCI Switzerland		

Table 5: Identification codes of the indices.

AUD	USD/Australian Dollar
BRL	USD/Brazilian Real
CAD	USD/Canadian Dollar
CHF	USD/Swiss Franc
EUR	USD/Euro
GBP	USD/British Pound
INR	USD/Indian Rupee
JPY	USD/Japanese Yen
RUB	USD/Russian Rouble
ZAR	USD/South African Rand

Table 6: Identification codes of the exchange rates.

Symbol	start date	obs	Symbol	start date	obs
AEX	03-01-2000	4440	MSCIDE	02-07-1999	2427
AORD	04-01-2000	4358	MSCIES	02-07-1999	2441
BVSP	03-01-2000	4266	MSCIFR	02-07-1999	2416
DJI	03-01-2000	4360	MSCIGB	09-06-1999	2448
FCHI	03-01-2000	4441	MSCIIT	02-07-1999	2443
FTSE	04-01-2000	4379	MSCIJP	05-12-1999	2430
FTSEMIB	03-01-2000	4398	MSCIKR	06-12-1999	2231
GDAXI	03-01-2000	4413	MSCIMX	07-10-2002	2253
GSPTSE	02-05-2002	3772	MSCINL	02-07-1999	1602
HSI	03-01-2000	4026	MSCIWO	12-02-2001	2447
IBEX	03-01-2000	4406	MXX	03-01-2000	4361
IXIC	03-01-2000	4362	N2252	04-01-2000	4225
KS11	04-01-2000	4290	NSEI	06-01-2000	3780
MIB30	03-01-1996	3261	RUA	03-01-1996	2091
MIBTEL	04-07-2000	3289	RUI	03-01-1996	3262
MID	03-01-1996	2176	RUT	03-01-2000	4359
MSCIAU	05-12-1999	3258	SPTSE	04-01-1999	3262
MSCIBE	02-07-1999	2314	SPX	03-01-2000	4357
MSCIBR	07-10-2002	2435	SSMI	04-01-2000	4364
MSCICA	13-02-2001	1577	STOXX50E	03-01-2000	4417
MSCICH	10-06-1999	2003			

Table 7: Starting dates and available observations of the indices.

Symbol	start date	obs
AUD	01-01-1996	6700
BRL	27-10-2000	4311
CAD	02-01-1996	6754
CHF	02-01-1996	6781
EUR	05-05-1998	6090
GBP	01-01-1996	6856
INR	01-01-1998	5001
JPY	02-01-1996	6866
RUB	06-01-2005	3483
ZAR	15-02-1996	6087

Table 8: Starting dates and available observations of the exchange rates.

	$LW_{0.6}$	$LW_{0.7}$	$LW_{0.8}$	LWN	LPWN(1,0)	LPWN(0,1)	LPWN(1,1)	mLW	mLWN	tLW	$Qu_{0.75}$
AEX	0.618	0.630	0.576	0.654	0.652	0.656	0.654	0.566	0.654	0.573	0.561
	(0.535 0.702)	(0.576 0.683)	(0.541 0.610)	(0.619 0.689)	(0.618 0.687)	(0.621 0.691)	(0.619 0.688)	(0.531 0.601)	(0.619,0.689)	(0.513,0.634)	
AORD	0.555	0.593	0.471	0.647	0.613	0.619	0.613	0.364	0.645	0.458	1.003
	(0.471 0.639)	(0.539 0.647)	(0.436 0.506)	(0.612 0.682)	(0.578 0.648)	(0.584 0.654)	(0.578 0.648)	(0.329 0.399)	(0.610,0.680)	(0.398,0.519)	
BVSP	0.548	0.522	0.475	0.566	0.532	0.530	0.532	0.458	0.566	0.482	0.521
	(0.464 0.633)	(0.468 0.577)	(0.440 0.510)	(0.531 0.601)	(0.496 0.567)	(0.495 0.565)	(0.496 0.567)	(0.423 0.493)	(0.531,0.601)	(0.421,0.544)	
DJI	0.631	0.605	0.561	0.631	0.610	0.590	0.610	0.548	0.630	0.568	0.730
	(0.547 0.715)	(0.551 0.659)	(0.526 0.596)	(0.596 0.666)	(0.576 0.645)	(0.555 0.624)	(0.576 0.645)	(0.513 0.583)	(0.595,0.665)	(0.507,0.629)	
FCHI	0.608	0.623	0.553	0.645	0.648	0.659	0.658	0.494	0.644	0.536	0.668
	(0.525 0.692)	(0.569 0.676)	(0.518 0.587)	(0.610 0.680)	(0.613 0.683)	(0.624 0.693)	(0.623 0.692)	(0.459 0.528)	(0.609,0.678)	(0.476,0.597)	
FTSE	0.650	0.650	0.591	0.673	0.674	0.671	0.671	0.578	0.671	0.588	0.663
	(0.566 0.733)	(0.596 0.704)	(0.557 0.626)	(0.638 0.707)	(0.639 0.709)	(0.636 0.706)	(0.636 0.706)	(0.543 0.613)	(0.636,0.706)	(0.527,0.649)	
FTSEMIB	0.597	0.624	0.555	0.643	0.622	0.630	0.622	0.532	0.642	0.565	0.581
	(0.513 0.680)	(0.570 0.677)	(0.520 0.589)	(0.608 0.678)	(0.588 0.657)	(0.595 0.665)	(0.588 0.657)	(0.497 0.566)	(0.607,0.676)	(0.504,0.625)	
GDAXI	0.649	0.643	0.573	0.668	0.673	0.715	0.715	0.487	0.667	0.524	0.714
	(0.566 0.733)	(0.590 0.697)	(0.538 0.607)	(0.634 0.703)	(0.638 0.707)	(0.680 0.749)	(0.681 0.750)	(0.452 0.522)	(0.633,0.702)	(0.464,0.585)	
GSPTSE	0.612	0.578	0.509	0.640	0.648	0.653	0.653	0.404	0.563	0.462	**1.281***
	(0.524 0.701)	(0.521 0.635)	(0.472 0.546)	(0.603 0.677)	(0.611 0.685)	(0.616 0.690)	(0.616 0.690)	(0.367 0.441)	(0.526,0.600)	(0.398,0.527)	
HSI	0.641	0.561	0.522	0.616	0.624	0.669	0.701	0.433	0.503	0.460	**1.261***
	(0.555 0.727)	(0.506 0.617)	(0.486 0.558)	(0.580 0.652)	(0.588 0.660)	(0.633 0.705)	(0.665 0.737)	(0.397 0.470)	(0.467,0.539)	(0.397,0.523)	
IBEX	0.593	0.596	0.552	0.608	0.615	0.624	0.624	0.518	0.604	0.533	0.763
	(0.509 0.676)	(0.542 0.650)	(0.517 0.586)	(0.574 0.643)	(0.581 0.650)	(0.589 0.659)	(0.589 0.659)	(0.483 0.553)	(0.570,0.639)	(0.473,0.594)	
IXIC	0.657	0.608	0.570	0.634	0.625	0.594	0.603	0.545	0.584	0.562	0.578
	(0.573 0.741)	(0.554 0.661)	(0.535 0.605)	(0.599 0.669)	(0.590 0.660)	(0.559 0.629)	(0.568 0.638)	(0.510 0.580)	(0.549,0.619)	(0.501,0.623)	
KS11	0.699	0.632	0.558	0.680	0.684	0.735	0.739	0.430	0.678	0.518	**1.420***
	(0.614 0.783)	(0.578 0.686)	(0.523 0.594)	(0.644 0.715)	(0.649 0.719)	(0.699 0.770)	(0.704 0.774)	(0.395 0.465)	(0.643,0.713)	(0.457,0.579)	
MIB30	0.601	0.605	0.530	0.671	0.651	0.657	0.650	0.444	0.670	0.541	0.653
	(0.509 0.694)	(0.545 0.665)	(0.491 0.569)	(0.632 0.711)	(0.612 0.690)	(0.618 0.696)	(0.611 0.690)	(0.405 0.483)	(0.631,0.709)	(0.466,0.604)	
MIBTEL	0.659	0.611	0.513	0.737	0.686	0.693	0.686	0.297	0.736	0.515	1.038
	(0.554 0.765)	(0.541 0.681)	(0.466 0.559)	(0.691 0.784)	(0.640 0.733)	(0.647 0.740)	(0.639 0.732)	(0.251 0.344)	(0.689,0.782)	(0.472,0.610)	
MID	0.713	0.634	0.560	0.729	0.733	0.756	0.756	0.362	0.728	0.524	**1.204**
	(0.621 0.806)	(0.575 0.694)	(0.521 0.600)	(0.690 0.768)	(0.693 0.772)	(0.717 0.796)	(0.716 0.795)	(0.323 0.401)	(0.689,0.768)	(0.433,0.598)	
MSCIAU	0.638	0.588	0.476	0.727	0.684	0.687	0.683	0.265	0.724	0.445	0.696
	(0.535 0.742)	(0.520 0.656)	(0.431 0.521)	(0.682 0.773)	(0.639 0.729)	(0.642 0.732)	(0.638 0.728)	(0.220 0.311)	(0.679,0.770)	(0.455,0.593)	
MSCIBE	0.715	0.631	0.511	0.793	0.731	0.731	0.757	0.154	0.792	0.467	**1.240**
	(0.613 0.817)	(0.564 0.698)	(0.466 0.555)	(0.749 0.837)	(0.687 0.776)	(0.686 0.775)	(0.712 0.801)	(0.109 0.198)	(0.747,0.836)	(0.364,0.525)	
MSCIBR	0.603	0.548	0.490	0.644	0.649	0.654	0.653	0.382	0.641	0.481	0.447
	(0.485 0.720)	(0.469 0.626)	(0.437 0.543)	(0.590 0.697)	(0.596 0.703)	(0.601 0.707)	(0.600 0.706)	(0.329 0.435)	(0.588,0.694)	(0.388,0.546)	
MSCICA	0.660	0.598	0.483	0.749	0.673	0.677	0.670	0.223	0.747	0.484	0.641
	(0.551 0.769)	(0.526 0.670)	(0.435 0.531)	(0.701 0.797)	(0.624 0.721)	(0.629 0.726)	(0.622 0.718)	(0.175 0.271)	(0.699,0.795)	(0.385,0.577)	
MSCICH	0.706	0.646	0.537	0.772	0.720	0.722	0.743	0.237	0.771	0.505	**1.125**
	(0.604 0.807)	(0.579 0.713)	(0.492 0.581)	(0.727 0.816)	(0.675 0.764)	(0.677 0.766)	(0.698 0.787)	(0.192 0.281)	(0.727,0.815)	(0.398,0.570)	

Table 9: Estimated long-memory coefficients of the log-realized kernels. Theoretical confidence intervals are given in brackets below. For the Qu-test bold-faced values indicate significance at the nominal 10% level; an additional * (**) indicates significance at the nominal 5% (1%) level.

	$LW_{0.6}$	$LW_{0.7}$	$LW_{0.8}$	LWN	LPWN(1,0)	LPWN(0,1)	LPWN(1,1)	mLW	mLWN	tLW	$Qu_{0.75}$
MSCIDE	0.666	0.630	0.502	0.779	0.711	0.707	0.761	0.179	0.778	0.443	0.678
	(0.564 0.768)	(0.563 0.696)	(0.457 0.546)	(0.734 0.823)	(0.666 0.755)	(0.663 0.752)	(0.717 0.806)	(0.135 0.224)	(0.733,0.822)	(0.426,0.584)	
MSCIES	0.628	0.604	0.499	0.706	0.654	0.657	0.653	0.339	0.703	0.488	0.673
	(0.526 0.730)	(0.537 0.671)	(0.454 0.543)	(0.662 0.751)	(0.609 0.698)	(0.613 0.702)	(0.609 0.698)	(0.295 0.384)	(0.658,0.747)	(0.364,0.522)	
MSCIFR	0.649	0.614	0.501	0.767	0.679	0.675	0.726	0.194	0.766	0.490	**1.302***
	(0.548 0.751)	(0.548 0.681)	(0.457 0.546)	(0.723 0.811)	(0.635 0.724)	(0.631 0.720)	(0.682 0.770)	(0.150 0.239)	(0.722,0.810)	(0.409,0.568)	
MSCIGB	0.674	0.629	0.524	0.765	0.696	0.696	0.720	0.237	0.764	0.514	1.113
	(0.572 0.776)	(0.562 0.696)	(0.480 0.568)	(0.720 0.809)	(0.651 0.740)	(0.652 0.741)	(0.676 0.765)	(0.193 0.281)	(0.719,0.808)	(0.411,0.568)	
MSCIIT	0.625	0.609	0.510	0.725	0.672	0.677	0.675	0.315	0.723	0.496	**1.260***
	(0.524 0.727)	(0.542 0.676)	(0.466 0.555)	(0.680 0.769)	(0.628 0.717)	(0.633 0.722)	(0.631 0.720)	(0.271 0.360)	(0.679,0.767)	(0.436,0.593)	
MSCIJP	0.673	0.568	0.508	0.628	0.634	0.682	0.686	0.457	0.624	0.453	**1.362***
	(0.568 0.777)	(0.499 0.637)	(0.462 0.554)	(0.582 0.674)	(0.588 0.680)	(0.636 0.728)	(0.640 0.732)	(0.411 0.503)	(0.578,0.670)	(0.417,0.575)	
MSCIKR	0.678	0.602	0.519	0.673	0.621	0.605	0.601	0.400	0.671	0.494	0.925
	(0.574 0.782)	(0.533 0.671)	(0.473 0.564)	(0.628 0.719)	(0.575 0.667)	(0.559 0.651)	(0.555 0.647)	(0.354 0.446)	(0.625,0.716)	(0.371,0.534)	
MSCIMX	0.640	0.576	0.470	0.748	0.669	0.673	0.678	0.189	0.747	0.464	0.930
	(0.523 0.757)	(0.499 0.654)	(0.417 0.523)	(0.696 0.801)	(0.616 0.722)	(0.620 0.725)	(0.626 0.731)	(0.136 0.242)	(0.695,0.800)	(0.414,0.575)	
MSCINL	0.662	-0.621	0.522	0.755	0.678	0.641	0.678	0.252	0.755	0.532	0.825
	(0.561 0.764)	(0.555 0.688)	(0.478 0.566)	(0.711 0.800)	(0.634 0.722)	(0.597 0.685)	(0.634 0.722)	(0.208 0.296)	(0.710,0.799)	(0.370,0.559)	
MSCIWO	0.596	0.526	0.428	0.675	0.599	0.600	0.597	0.247	0.672	0.432	0.540
	(0.490 0.703)	(0.455 0.597)	(0.381 0.475)	(0.628 0.722)	(0.552 0.646)	(0.553 0.647)	(0.550 0.644)	(0.200 0.294)	(0.625,0.719)	(0.454,0.611)	
MXX	0.585	0.522	0.447	0.620	0.629	0.625	0.634	0.329	0.618	0.404	0.692
	(0.501 0.669)	(0.468 0.576)	(0.412 0.482)	(0.585 0.655)	(0.594 0.664)	(0.590 0.660)	(0.599 0.669)	(0.294 0.364)	(0.583,0.653)	(0.343,0.465)	
N2252	0.627	0.552	0.512	0.579	0.588	0.643	0.643	0.498	0.577	0.496	1.005
	(0.542 0.712)	(0.498 0.607)	(0.477 0.548)	(0.543 0.614)	(0.552 0.623)	(0.607 0.678)	(0.607 0.678)	(0.462 0.533)	(0.542,0.613)	(0.434,0.558)	
NSEI	0.579	0.518	0.523	0.539	0.552	0.596	0.611	0.500	0.500	0.492	0.628
	(0.492 0.667)	(0.461 0.575)	(0.486 0.560)	(0.502 0.576)	(0.515 0.589)	(0.559 0.633)	(0.574 0.648)	(0.463 0.537)	(0.462,0.537)	(0.427,0.557)	
RUA	0.648	0.582	0.540	0.659	0.664	0.669	0.668	0.444	0.659	0.539	0.558
	(0.556 0.741)	(0.523 0.642)	(0.501 0.579)	(0.620 0.699)	(0.625 0.703)	(0.630 0.709)	(0.629 0.707)	(0.404 0.483)	(0.620,0.699)	(0.348,0.516)	
RUI	0.646	0.580	0.538	0.655	0.660	0.668	0.667	0.446	0.655	0.536	0.555
	(0.554 0.738)	(0.520 0.640)	(0.498 0.577)	(0.616 0.694)	(0.621 0.700)	(0.628 0.707)	(0.627 0.706)	(0.406 0.485)	(0.616,0.694)	(0.470,0.608)	
RUT	0.567	0.564	0.514	0.589	0.563	0.540	0.563	0.479	0.587	0.506	0.654
	(0.483 0.651)	(0.510 0.618)	(0.479 0.549)	(0.554 0.624)	(0.529 0.598)	(0.505 0.575)	(0.529 0.598)	(0.444 0.514)	(0.552,0.622)	(0.445,0.567)	
SPTSE	0.672	0.589	0.503	0.729	0.653	0.633	0.641	0.290	0.728	0.479	0.909
	(0.572 0.772)	(0.523 0.655)	(0.459 0.547)	(0.685 0.773)	(0.609 0.697)	(0.589 0.676)	(0.597 0.685)	(0.246 0.333)	(0.684,0.772)	(0.467,0.606)	
SPX	0.628	0.596	0.564	0.618	0.603	0.588	0.581	0.550	0.617	0.567	0.480
	(0.544 0.712)	(0.542 0.650)	(0.529 0.599)	(0.583 0.653)	(0.568 0.638)	(0.553 0.623)	(0.546 0.616)	(0.516 0.585)	(0.582,0.652)	(0.506,0.628)	
SSMI	0.658	0.688	0.615	0.706	0.705	0.714	0.712	0.560	0.706	0.611	0.861
	(0.574 0.742)	(0.634 0.741)	(0.580 0.650)	(0.671 0.741)	(0.670 0.740)	(0.679 0.749)	(0.677 0.747)	(0.525 0.595)	(0.671,0.741)	(0.550,0.672)	
STOXX50E	0.598	0.594	0.533	0.614	0.619	0.638	0.639	0.484	0.612	0.503	0.678
	(0.514 0.681)	(0.541 0.648)	(0.499 0.568)	(0.579 0.648)	(0.585 0.654)	(0.604 0.673)	(0.604 0.673)	(0.450 0.519)	(0.577,0.647)	(0.443,0.564)	

Table 10: Estimated long-memory coefficients of the log-realized kernels. Theoretical confidence intervals are given in brackets below. For the Qu-test bold-faced values indicate significance at the nominal 10% level; an additional * (**) indicates significance at the nominal 5% (1%) level.

	$LW_{0.6}$	$LW_{0.7}$	$LW_{0.8}$		$LW_{0.6}$	$LW_{0.7}$	$LW_{0.8}$
AEX	0.625	0.635	0.577	MSCIDE	0.680	0.639	0.521
	(0.525 0.721)	(0.580 0.693)	(0.540 0.613)		(0.616 0.818)	(0.607 0.742)	(0.525 0.612)
AORD	0.553	0.586	0.460	MSCIES	0.640	0.623	0.519
	(0.466 0.629)	(0.526 0.635)	(0.424 0.498)		(0.583 0.788)	(0.574 0.711)	(0.463 0.573)
BVSP	0.549	0.521	0.475	MSCIFR	0.656	0.619	0.522
	(0.450 0.626)	(0.466 0.587)	(0.440 0.509)		(0.531 0.732)	(0.555 0.683)	(0.460 0.571)
DJI	0.622	0.586	0.534	MSCIGB	0.693	0.643	0.544
	(0.535 0.696)	(0.527 0.637)	(0.502 0.569)		(0.533 0.763)	(0.546 0.688)	(0.466 0.573)
FCHI	0.605	0.624	0.564	MSCIIT	0.633	0.628	0.532
	(0.526 0.681)	(0.569 0.676)	(0.530 0.599)		(0.596 0.792)	(0.570 0.713)	(0.493 0.593)
FTSE	0.650	0.640	0.569	MSCIJP	0.685	0.568	0.514
	(0.559 0.730)	(0.579 0.693)	(0.533 0.606)		(0.541 0.728)	(0.560 0.691)	(0.479 0.581)
FTSEMIB	0.600	0.607	0.547	MSCIKR	0.695	0.625	0.534
	(0.512 0.686)	(0.549 0.656)	(0.510 0.584)		(0.571 0.794)	(0.488 0.641)	(0.466 0.560)
GDAXI	0.653	0.637	0.564	MSCIMX	0.669	0.603	0.494
	(0.544 0.757)	(0.573 0.697)	(0.523 0.601)		(0.596 0.794)	(0.555 0.687)	(0.492 0.581)
GSPTSE	0.603	0.565	0.490	MSCINL	0.672	-0.635	0.534
	(0.508 0.681)	(0.506 0.621)	(0.452 0.528)		(0.553 0.777)	(0.523 0.675)	(0.436 0.550)
HSI	0.640	0.557	0.503	MSCIWO	0.600	0.528	0.445
	(0.546 0.724)	(0.500 0.604)	(0.467 0.541)		(0.562 0.765)	(0.573 0.695)	(0.480 0.590)
IBEX	0.593	0.596	0.545	MXX	0.575	0.503	0.441
	(0.498 0.668)	(0.542 0.651)	(0.504 0.576)		(0.506 0.661)	(0.448 0.558)	(0.406 0.470)
IXIC	0.644	0.598	0.552	N2252	0.618	0.557	0.514
	(0.564 0.708)	(0.545 0.648)	(0.516 0.586)		(0.509 0.701)	(0.497 0.609)	(0.474 0.549)
KS11	0.692	0.622	0.548	NSEI	0.572	0.515	0.497
	(0.604 0.767)	(0.570 0.672)	(0.510 0.582)		(0.475 0.656)	(0.456 0.567)	(0.464 0.533)
MIB30	0.608	0.613	0.550	RUA	0.646	0.583	0.539
	(0.537 0.703)	(0.511 0.633)	(0.491 0.567)		(0.489 0.729)	(0.429 0.615)	(0.381 0.511)
MIBTEL	0.667	0.622	0.532	RUI	0.644	0.580	0.537
	(0.509 0.681)	(0.551 0.670)	(0.512 0.589)		(0.558 0.722)	(0.517 0.641)	(0.502 0.575)
MID	0.712	0.637	0.564	RUT	0.549	0.551	0.501
	(0.571 0.746)	(0.558 0.683)	(0.475 0.582)		(0.459 0.625)	(0.497 0.603)	(0.468 0.534)
MSCIAU	0.659	0.606	0.499	SPTSE	0.681	0.600	0.508
	(0.626 0.802)	(0.574 0.701)	(0.526 0.603)		(0.554 0.719)	(0.517 0.634)	(0.500 0.571)
MSCIBE	0.730	0.656	0.536	SPX	0.623	0.586	0.548
	(0.547 0.741)	(0.540 0.672)	(0.452 0.544)		(0.540 0.693)	(0.528 0.638)	(0.516 0.581)
MSCIBR	0.629	0.567	0.501	SSMI	0.656	0.671	0.590
	(0.636 0.823)	(0.594 0.714)	(0.490 0.577)		(0.561 0.742)	(0.618 0.718)	(0.558 0.624)
MSCICA	0.681	0.614	0.503	STOXX50E	0.591	0.594	0.527
	(0.552 0.753)	(0.489 0.642)	(0.448 0.557)		(0.498 0.685)	(0.541 0.644)	(0.491 0.560)
MSCICH	0.730	0.676	0.565				
	(0.542 0.782)	(0.524 0.689)	(0.444 0.574)				

Table 11: Estimated long-memory coefficients of the log-realized variances. Confidence intervals using the bootstrap procedure of Arteche and Orbe (2016) are given in brackets below.

	$LW_{0.6}$	$LW_{0.7}$	$LW_{0.8}$		$LW_{0.6}$	$LW_{0.7}$	$LW_{0.8}$
AEX	0.618	0.630	0.576	MSCIDE	0.680	0.639	0.521
	(0.512 0.711)	(0.573 0.687)	(0.536 0.611)		(0.616 0.818)	(0.607 0.742)	(0.525 0.612)
AORD	0.555	0.593	0.471	MSCIES	0.640	0.623	0.519
	(0.464 0.635)	(0.534 0.647)	(0.436 0.507)		(0.583 0.788)	(0.574 0.711)	(0.463 0.573)
BVSP	0.548	0.522	0.475	MSCIFR	0.656	0.619	0.522
	(0.459 0.620)	(0.468 0.577)	(0.436 0.510)		(0.531 0.732)	(0.555 0.683)	(0.460 0.571)
DJI	0.631	0.605	0.561	MSCIGB	0.693	0.643	0.544
	(0.539 0.702)	(0.548 0.655)	(0.525 0.599)		(0.533 0.763)	(0.546 0.688)	(0.466 0.573)
FCHI	0.608	0.623	0.553	MSCIIT	0.633	0.628	0.532
	(0.518 0.692)	(0.569 0.676)	(0.515 0.587)		(0.596 0.792)	(0.570 0.713)	(0.493 0.593)
FTSE	0.650	0.650	0.591	MSCIJP	0.685	0.568	0.514
	(0.552 0.728)	(0.590 0.701)	(0.556 0.627)		(0.541 0.728)	(0.560 0.691)	(0.479 0.581)
FTSEMIB	0.597	0.624	0.555	MSCIKR	0.695	0.625	0.534
	(0.509 0.683)	(0.565 0.672)	(0.519 0.590)		(0.571 0.794)	(0.488 0.641)	(0.466 0.560)
GDAXI	0.649	0.643	0.573	MSCIMX	0.669	0.603	0.494
	(0.53 0.763)	(0.577 0.702)	(0.533 0.612)		(0.596 0.794)	(0.555 0.687)	(0.492 0.581)
GSPTSE	0.612	0.578	0.509	MSCINL	0.672	-0.635	0.534
	(0.508 0.688)	(0.522 0.630)	(0.470 0.545)		(0.553 0.777)	(0.523 0.675)	(0.436 0.550)
HSI	0.641	0.561	0.522	MSCIWO	0.600	0.528	0.445
	(0.543 0.715)	(0.509 0.609)	(0.486 0.556)		(0.562 0.765)	(0.573 0.695)	(0.480 0.590)
IBEX	0.593	0.596	0.552	MXX	0.575	0.503	0.441
	(0.493 0.675)	(0.544 0.646)	(0.514 0.587)		(0.506 0.661)	(0.448 0.558)	(0.406 0.470)
IXIC	0.657	0.608	0.570	N2252	0.618	0.557	0.514
	(0.577 0.722)	(0.552 0.659)	(0.536 0.602)		(0.509 0.701)	(0.497 0.609)	(0.474 0.549)
KS11	0.699	0.632	0.558	NSEI	0.572	0.515	0.497
	(0.608 0.777)	(0.576 0.679)	(0.526 0.591)		(0.475 0.656)	(0.456 0.567)	(0.464 0.533)
MIB30	0.601	0.605	0.530	RUA	0.646	0.583	0.539
	(0.495 0.676)	(0.542 0.661)	(0.489 0.564)		(0.489 0.729)	(0.429 0.615)	(0.381 0.511)
MIBTEL	0.659	0.611	0.513	RUI	0.644	0.580	0.537
	(0.560 0.738)	(0.545 0.672)	(0.458 0.557)		(0.558 0.722)	(0.517 0.641)	(0.502 0.575)
MID	0.713	0.634	0.560	RUT	0.549	0.551	0.501
	(0.624 0.796)	(0.566 0.693)	(0.518 0.597)		(0.459 0.625)	(0.497 0.603)	(0.468 0.534)
MSCIAU	0.638	0.588	0.476	SPTSE	0.681	0.600	0.508
	(0.535 0.727)	(0.521 0.646)	(0.430 0.518)		(0.554 0.719)	(0.517 0.634)	(0.500 0.571)
MSCIBE	0.715	0.631	0.511	SPX	0.623	0.586	0.548
	(0.607 0.814)	(0.564 0.695)	(0.462 0.553)		(0.540 0.693)	(0.528 0.638)	(0.516 0.581)
MSCIBR	0.603	0.548	0.490	SSMI	0.656	0.671	0.590
	(0.526 0.713)	(0.471 0.627)	(0.436 0.548)		(0.561 0.742)	(0.618 0.718)	(0.558 0.624)
MSCICA	0.660	0.598	0.483	STOXX50E	0.591	0.594	0.527
	(0.521 0.771)	(0.517 0.670)	(0.423 0.544)		(0.498 0.685)	(0.541 0.644)	(0.491 0.560)
MSCICH	0.706	0.646	0.537				
	(0.607 0.793)	(0.582 0.708)	(0.494 0.583)				

Table 12: Estimated long-memory coefficients of the log-realized kernels. Confidence intervals using the bootstrap procedure of Arteche and Orbe (2016) are given in brackets below.

AEX	0.570	MSCIDE	0.510
	(0.525, 0.614)		(0.537, 0.651)
AORD	0.433	MSCIES	0.507
	(0.385, 0.481)		(0.452, 0.569)
BVSP	0.467	MSCIFR	0.541
	(0.420, 0.514)		(0.448, 0.566)
DJI	0.559	MSCIGB	0.601
	(0.514, 0.604)		(0.484, 0.597)
FCHI	0.569	MSCIIT	0.557
	(0.525, 0.614)		(0.544, 0.658)
FTSE	0.554	MSCIJP	0.538
	(0.510, 0.599)		(0.500, 0.614)
FTSEMIB	0.557	MSCIKR	0.566
	(0.512, 0.601)		(0.478, 0.597)
GDAXI	0.573	MSCIMX	0.509
	(0.529, 0.618)		(0.507, 0.625)
GSPTSE	0.458	MSCINL	0.563
	(0.409, 0.508)		(0.438, 0.579)
HSI	0.503	MSCIWO	0.401
	(0.455, 0.550)		(0.506, 0.620)
IBEX	0.557	MXX	0.444
	(0.513, 0.602)		(0.397, 0.491)
IXIC	0.575	N2252	0.487
	(0.531, 0.620)		(0.440, 0.534)
KS11	0.541	NSEI	0.478
	(0.496, 0.586)		(0.429, 0.527)
MIB30	0.578	RUA	0.569
	(0.522, 0.623)		(0.332, 0.469)
MIBTEL	0.548	RUI	0.567
	(0.528, 0.629)		(0.518, 0.619)
MID	0.579	RUT2	0.506
	(0.488, 0.608)		(0.460, 0.552)
MSCIAU	0.487	SPTSE	0.513
	(0.528, 0.629)		(0.517, 0.618)
MSCIBE	0.545	SPX2	0.564
	(0.427, 0.547)		(0.520, 0.609)
MSCIBR	0.481	SSMI	0.596
	(0.488, 0.602)		(0.551, 0.641)
MSCICA	0.503	STOXX50E	0.526
	(0.408, 0.553)		(0.481, 0.570)
MSCICH	0.594		
	(0.439, 0.567)		

Table 13: Estimated long-memory coefficients of the log-realized variances applying the estimator of McCloskey and Perron (2013). Theoretical confidence intervals are given in brackets below.

AUD	0.406
	(0.366, 0.447)
BRL	0.441
	(0.394, 0.489)
CAD	0.390
	(0.349, 0.431)
CHF	0.352
	(0.310, 0.394)
EUR	0.260
	(0.211, 0.309)
GBP	0.313
	(0.269, 0.357)
INR	0.433
	(0.388, 0.478)
JPY	0.401
	(0.361, 0.441)
RUB	0.423
	(0.370, 0.476)
ZAR	0.506
	(0.467, 0.546)

Table 14: Estimated long-memory coefficients of the log-realized variances applying the estimator of McCloskey and Perron (2013). Theoretical confidence intervals are given in brackets below.

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