Age- and Sex-Specific Fertility in Germany until the Year 2040 – The Impact of International Migration

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Abstract

Good Forecasts for future fertility developments are of high importance in political planning, especially regarding measures in social insurance. Fertility is the main driver of demographic change, since small fertility rates lead to a shrinking population and together with decreasing mortality rates to an aging of the population structure. Which means an increasing stock of elder people, who have to be financed by less people in the working ages. Parametric time series models based on a quasi-three principal component model are fitted to the age- and sex-specific fertility rates (*ASSFR*). Age-specific migration, represented by a migration index derived from a previous principal component analysis (*PCA*), is used as a predictor variable to take into account its impact on fertility. Simulations of Wiener processes are used for estimating the future distributions of each ASSFR as well as the Total Fertility Rate (*TFR*). The forecast shows, ceteris paribus, a further increase in the TFR, with increases in the ASSFR for older women and decreasing ones for younger females. A test based on squared residuals shows that the model gives better forecast accuracy than the most commonly used methods in Germany. The modeling approach performs better than common projection and forecast methods in Germany while integrating the often discussed link between migration and fertility into a forecasting model. Next to the detailed and stochastic quantification of age-specific fertility it includes the gender of newborns, which allows for easy implementation into regular population updating through a stochastic cohort-component model.

Keywords: Statistical Demography, Forecasting, Fertility and International Migration, Principal Component Analysis, Time Series Analysis

JEL-classification: C22; C38; C53; J11; J13

1 Introduction

The future development of fertility is of great importance in many areas, particularly regarding planning of future needs for political intervention in family politics as well as migration politics aiming at people in the working ages. A shrinking younger population in the working ages, which is the long-term result of small fertility rates, along with decreasing mortality rates leads to a shift toward an older population structure in which relatively less people in working ages shoulder a higher financial burden in the social insurance for relatively more elderly people. The fertility level in a country is commonly represented by the Total Fertility Rate (*TFR*), which is the sum of the age-specific fertility rates (*ASFR*) over all ages during a specific year. Therefore, the TFR might be interpreted as the number of children a woman on average would bear during her reproductive phase, given the present ASFR remained constant in future. The TFR in most parts of Europe for decades has been well below the replacement level fertility, which is quantified at almost 2.1 for these regions. Although births show no immediate effect on the financial balance of the social insurance, they are the most important factor in the long run. Migration only in a small part is able to support the financial system and mostly gives no immediate positive effect. People migrating often need years of acclimatization and integration into a new society before giving productive input into the system, especially those coming from very different cultures with relatively low educational levels. Forecasts of future fertility is therefore an important informational ground for political decision-making. Political decisions on pension fund reforms must consider the future course of fertility in society and the resulting structure of the population. Family politics must try to address political measures aiming at an increase of the TFR.

This contribution aims to propose an innovative forecasting approach for the future course of the age- and sex-specific fertility rates (*ASSFR*) in Germany. The model works with statistical methods like the principal component analysis (*PCA*) and time series analysis (*TSA*). The methodology allows for the integration of the correlations among the ASSFR as well as the autocorrelations among each set of ASSFR. Simulations of Wiener processes allow for stochastic quantification of the future course of the ASSFR through predictive intervals (*PI*). The trajectories of the ASSFR may be cumulated to stochastic forecasts of the TFR as well, which will be illustrated at 90%-PI.

Since there is some controversy regarding the impact of international migration flows on the TFR, which have increased in the last years, the model proposed here measures the effect of the migration level using a migration index weighting migration by age, sex and nationality. This index has been derived in an earlier work within a PCA framework as well, which will be described in Section 4. Some important work dealing with the links between migration and fertility in Germany are mentioned in Section 3. A selection of results from the forecasting procedure is presented in Section 5, as well as a first test of the method based on the mean absolute percentage error (*MAPE*) comparing the ex-ante forecast to the ex-post TFR. The test shows a better performance of the model proposed here relative to the most common predictions in Germany by the Federal Statistical Office (*Destatis*) and by the Lee-Carter model. Furthermore, the model divides the stochastic estimation between births of males and females, which has the advantage that no further estimation of the genders is necessary in the context of future population forecasting by sex and age in a cohort-component framework. The model quantifies the risk in the sex ratio of the newborns simultaneously.

2 Common Fertility Forecasts and Projections for Germany

Common future projections for fertility in Germany are often based on deterministic scenario analyses. Germany's federal statistical office *Destatis* assumes a constant TFR. The underlying assumption is that the ASFR will be decreasing for younger women under 30 years of age, whereas those losses are balanced by an increasing ASFR for older women. This effect is also known as the Timing Effect. These assumptions are concluded from the survey data from the 2008 and 2012 microcensi on childlessness and the number of children the mothers conceive in combination with historical trends of the final numbers of children for older cohorts. The derived trends between these two points in time indicate the trends mentioned above. In an alternate scenario, which the authors classify as realistic, but without any scientific proof, the authors assume a slight increase in the TFR. This would result from a bigger increase in the ASFR of women over 30 simultaneously to constant fertility rates for younger women. In this case we would not only see the Timing, but also a Quantum Effect resulting in an increasing TFR. The TFR in this scenario will increase until the year 2028 to a level of 1.6 and remain constant on this level afterward (Statistisches Bundesamt 2015). While these assumptions might be well-based on reasonable arguments, they still appear too restrictive for deriving long-term trends until the year 2060. Furthermore, the high range of possible scenarios is neither identified nor quantified with individual probabilities.

The United Nations (*UN*) propose a Bayesian Hierarchical Model (BHM) for stochastic projection of the TFR by Raftery and Alkema. It is a world model made up of 158 countries, which are classified into one of the three possible cases of high fertility countries (Phase I), countries finding themselves in the transition from high to low fertility country (Phase II), and finally low fertility countries (Phase III). Germany belongs to the latter class, since its TFR is under the replacement level since the early 1970s (Vanella 2016). The TFR for Germany is assumed to slowly recover and converge toward 2.1 in the long-run, which is modeled by an autoregressive model of order one [*AR(1)*]. The quinquennial TFR has been stochastically simulated 60,000 times with Markov Chain Monte Carlo methods to identify the median scenarios with PI until the year 2100. In the median scenario, the TFR in Germany is expected to exceed 1.6 until the mid-21st century and to exceed 1.7 by the end of the century (United Nations 2015 and 2017; Raftery et al. 2014; Alkema et al. 2011). The trajectories for the TFR are thereafter distributed over the reproductive ages leading to trajectories of the ASFR. These schedules are weighted averages of the past experience of low fertility countries and Germany's latest own historical development in age-specific fertility. It is assumed that the fertility schedule will converge long-term toward a global age-specific fertility schedule (Ševčíková et al. 2015). The UN model has some nice features. It quantifies uncertainty by stochastic simulations while including national as well as international trends into the analysis. One might wonder though, whether the mathematical assumptions in the model are too restrictive, which would assume a global convergence of international fertility trends. Similar points might be made about the

ASFR. The model proposed in this paper takes correlations among age groups into account without imposing too strict assumptions on their future behavior.

Alders et al. (2007) try to combine the strengths of quantitative and qualitative models performing forecasts of the TFR for 18 European countries, Germany as well, through time series models. For Germany they used data from 1950-2000 to estimate a Generalized Autoregressive Conditional Heteroskedasticity (*GARCH*) model resulting in point forecasts as well as 80%-PI resulting from 3,000 trajectories until the year 2050. Given the estimated TFR, age schedules are used for the estimation of the ASFR. The plausibility of the results of the quantitative forecasts are qualitatively assessed by two fertility experts. Although the technique appears to take all necessities into account, the estimated 80%-PI for the TFR in Germany is between 0.88 and 2.21 which appears too wide to return valuable policy implications. Another caveat is the assumption of an age schedule, which can't include the Tempo Effect sufficiently.

Alho (1990) proposes indirect estimation of the TFR through forecasting the average ASFR. He constrains the average ASFR to an upper as well as a lower bound through a modified logistic transformation. Lee (1993) proposes a fertility index for indirect estimation of the ASFR, which he derives from a PCA for the ASFR. He integrates Alho's transformation into his forecast model to constrain the TFR to certain bounds. To include uncertainty into the forecast, Lee applies a simple Autoregressive Moving Average Model [ARMA(1,1)], from which he simulates 1,000 trajectories for the fertility index. This way, approximate 95%-PI for the TFR can be derived. Härdle and Myšičková apply Lee's model for a forecast for the TFR in Germany until 2060 with PI (Härdle and Myšičková 2009). Their forecast has some major flaws. First, they assume the TFR in the mean to remain constant on its last observed level, thus ignoring the current fertility trends. Secondly, the PI are way too narrow and have an illogical structure, since they become wider for the first few periods but stagnate on that level thereafter. Realistically, the risk in future predictions becomes bigger for more distant points in future. Thirdly, the restriction to one simple fertility index completely ignores the uncertainty associated with the remaining principal components (*PC*), which leads to an overall underestimation of future risks. Fuchs et al. (2017) use a quite similar approach for forecasting the labor force in Germany until 2060. They distinguish between nationals and foreigners, thus including the possible effects of international migration on fertility in their model. Hyndman and Ullah (2007) propose a robust adjustment to Lee's model, which is insensitive to past outliers due to extreme events, e.g. wars or epidemics. Deschermeier (2015) applies this approach for a forecast of the population in Germany until the year 2030. Vanella (2016) proposes a three PC model on ASFR for forecasting the ASFR and the TFR in Germany until the year 2040. That contribution has three weaknesses. First, it covers only the first three PC and ignores the variance in the remaining PC, although the first three PC already explained about 99% of the variation in the ASFR. So this error is small. Secondly, the long-term trends in the second and third PC is assumed to continue in the future, which might overestimate the TFR somewhat. Thirdly, as Fuchs et al. (2017) point out, the approach doesn't differ between Germans and migrants. This paper will elaborate upon Vanella's (2016) work, including the effects of international migration into the analysis. Furthermore, Vanella's (2017) proposed approach for dealing with the estimation error of the future variance will be applied in this paper for a more accurate estimation of the TFR-PI.

3 Effects of International Migration on Fertility in Germany

There are five major hypotheses about the fertility behavior of migrants. First, the socialization hypotheses reveals that upbringing and cultural background are the main criteria for the reproductive behavior of a female. Secondly, the adaptation hypotheses states that migrant's reproductive behavior adapt relatively fast to the natives'. Thirdly, the selectivity hypotheses stresses a selection bias between the out-migrants and the overall population at the country of origin. Therefore migrants moving to a low-fertility country would mostly be a selection of the population in the origin country, who plan to have a small number of children. Fourthly, the disruption hypothesis assumes a very low fertility level of the migrants in the first period following the migration process due to, e.g. the stressful situation or the uncertainty in that situation. After this first period, a reproduction at a normal or even higher level in comparison to the native population can be expected (Hervitz 1985). Finally, the interrelation of events hypothesis indicates high fertility rates directly after migration, because this includes partners that migrate because of marriage or through family re-union. These events lead to high birthrates shortly after migration (Milewski 2007). Let's consider the example of the strongest groups of migrants in Germany originating from Turkey, which traditionally have a relatively high TFR over 2 (The World Bank 2017). The socialization hypothesis would assume migrants coming from Turkey to Germany to retain their reproductive behavior constant after migration, therefore having a constant positive effect on the TFR in Germany. Following the adaptation hypothesis, the Turkish immigrants would quite rapidly lower their TFR within a couple of years toward the low German fertility level. The selectivity hypothesis would state that the migrants coming from Turkey would be a selection among the Turkish population, which, due to quite similar socioeconomic factors, have a similar reproductive behavior to the German population, therefore there would be no need to adapt either way. The disruption effect as well as the interrelation of events are straightforward and therefore do not need any further explanation at this point.

Milewski (2007) studies data from the Socio-Economic Panel (*SOEP*), which includes first and second generation migrants of Turkish, Italian, Spanish, Greek and former Yugoslavian descent relative to West German females with a focus on their first births, if there are any. The survey data spans the years 1984-2004. Milewski tests a variety of Generalized Linear Models (*GLM*) including variables on the time since migration as well as duration of marriage before birth, own birth cohort, age, education, employment status in addition to their heritage and data on the spouses. Basically, the analysis identifies positive effects of fresh migration as well as marriage on fertility. The study results that a higher education and female employment have negative effects on fertility. Nationality only has a positive effect for the Turkish migrants, even in the second migratory generation. The results show evidence for the correctness of the interrelation of events as well as the adaption hypotheses, rejecting the disruption hypothesis. Especially fertility up to one year after migration is elevated, which is a crucial result for the present contribution. A similar study with the same data and a similar method subsequently is conducted for second and third children. In this case Milewski (2010) finds out, that there is a significantly higher probability for mothers of Turkish descent to bear a second or third child after migrating to Germany than for the Germans or women of other nationalities in the study. This effect is even significant for second generation migrants from Turkey. She interprets this as some evidence for the correctness of the socialization hypothesis. Schmid and Kohls (2011) run a variety of Linear Models (LM) with survey data from the RAM 2006/2007 survey conducted by the German Federal Office for Migration and Refugees (*BAMF*) for Turkish, former Yugoslavian, Italian, Greek and Polish women. The results show significant differences in the reproductive behavior among the different groups, which basically stems from a combination of educational and religious factors as well as the mental connection of the immigrants to their country of origin compared to Germany. Muslims and people with a relatively low educational level have a significantly higher number of children. Since these two variables are strongly correlated in the data set, they cannot be separated from each other by the method used in that study. Nevertheless, the study underlines the importance of separate analyses of the different migrant groups with respect to their reproductive behavior. The study includes time variables like birth cohort or time since arrival in Germany as well, but the time intervals chosen do not give the opportunity to clearly conclude, which of the five hypotheses of fertility behavior for the migrants applies. Stichnoth and Yeter (2013) run a set of Panel data models with micro census data for migrants in Germany. They identify three groups of migrants in Germany:

- Generation 1: People who have directly migrated to Germany at age 15 or older
- Generation 1.5: People who have directly migrated to Germany, but before the age of 15
- Generation 2: Children of direct migrants

Stichnoth and Yeter estimate the effects of age and education, taking into account the current fertility level in the country of origin (represented by the TFR) on the number of children born to females for different model specifications. They conclude that the fertility norms of reproduction in the country of origin have a statistically significant positive influence on the females' reproductive behavior in Germany. This result applies for all three strata, not just for the first generation of migrants, although the effect becomes weaker for later generations, since the influence of the German culture weighs heavier with each generation. Moreover, the negative effect of higher educational levels on the birth counts can be confirmed. The results of the study by Stichnoth and Yeter give some further evidence for the reliability of the socialization in combination with the adaptation hypothesis.

To the author's knowledge, there doesn't exist many studies considering migration or ethnicity effects for forecasting fertility. Pang and McElroy (2014) propose time series forecasting of the log-TFR for five different ethnic groups in the U.S. From this they derive ASFR assuming an age schedule of fertility. They fail to quantify the uncertainty, since they just fit two Autoregressive Integrated Moving Average (*ARIMA*) specifications each to the ethnic groups. Fuchs et al. (2017) forecast ASFR and the TFR of German and foreign mothers in Germany separately with a time series model based on a PCA, illustrating PI as well, although the PI appear too narrow. In the following section, a forecast approach taking the effect of migration in fertility into account is developed.

4 Data and Modeling Approach

In this section, a forecast framework for ASFR in Germany will be proposed, which includes the effect of international migration which basically builds on the model proposed by Vanella (2016), using the results in Vanella and Deschermeier (2017). This will forecast net migration flows for Germany until the year 2040 by age, sex and group of nationalities. They identify seven groups of nationalities[1] which have different degrees of freedom of movement in Germany and the European Union as well as speech barriers. As mentioned in section 2, there are big differences in reproductive behaviors between different cultures. While the TFR in Germany (ignoring the nationality) overall was about 1.5 in 2015, it was e.g. around 2.1 in Turkey,

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[1] Germans, foreigners with citizenship from the European Union or Schengen-countries, people with nationalities from third countries in Europe, Africans, Asians, overseas nationals (the Americas or Oceania) and others, which might be people without or with unknown nationality.

3.1 in Israel and 1.2 in Korea (The World Bank 2017). Given the effects the fertility level in a migrants' country of origin has on their reproductive behavior in Germany, as shown in section 2, it might be appropriate to consider the migration effect in fertility forecasting as well. It is therefore not only important to differ between Germans and foreigners, but to consider the cultural backgrounds of the foreigners further. The studies mentioned earlier show evidence that a mixture of the socialization and the adaption hypotheses apply for migrants and their descendants in Germany. The goal of this study is to take this into account in a forecast setting. This paper makes use of data provided by Destatis on migration, fertility and population for estimation of a regression model including migration as a regressor for fertility. A caveat is that the migration data for Germany is only valid since the 1990s. Using older data would create a bias, since data before the re-unification cannot be used to forecast the migration schemes within the current regime. Since time series models are quite data consuming and long past time series are needed for long future forecasting, it is not possible to estimate the long-term effects of international migration on the fertility development statistically. Therefore, the socialization hypothesis cannot be integrated into a fertility forecast model for Germany, since we would need at least historical time series for the last two generations, rendering no historical data for long-term forecasting.[2] The model proposed in this paper will therefore assume the correctness of the adaptation hypothesis, which basically has been validated by Milewski (2007 and 2010) as well as by Stichnoth and Yeter (2013), as shown in Section 3. The impact of international migration for the last year on the current fertility will be tested, building on Milewski's result of high fertility directly after migration, which hint to the application of the interrelation of events hypothesis.

[2] One degree more in an ARIMA-model means a loss of historical data for one year.

First, the ASSFR are calculated. For this, the end of the year female population by year-of-age between 1990 and 2015 was downloaded as well as the age-specific birth numbers by sex of the newborn for each year from 1991 to 2015 (Statistisches Bundesamt 2017 and 2017a). The ASFR in year t are calculated by dividing the age-specific births in year t by the mean female population in year t in the respective age. Since only annual population data is available, the mean of the respective sub-population between the end of year t-1 and the end of year t is taken as the denominator of the ratio. The approach taken in this paper is calculating the ASSFR through dividing the number of live-births by age of mother and sex of the baby by the respective mean female population in that age annually. Since age-specific births for mothers over 49 years of age are aggregated into one group, the births in the past data is estimated by geometric extrapolation. The results of these extrapolations are judged mathematically as well as graphically and give plausible estimates of the ASSFR. This way, the base data set for mothers aged 14-52 years and both genders is derived. In the next step, a PCA is performed.[3] Using log- or logit-transformation of the ASFR or the TFR is very popular in the literature, because these transformations ensure future forecasts or projections to stay between certain limits, as was explained in section 2. A standard logit-transform leads to values in the interval (0;1) for the original variable, a log-transform has no upper bound, but makes sure the variable of interest stays positive. The author decides on using a square root transformation. It is a valid transformation, since it is a monotonous and continuous link function. Basically, it has quite similar features to the log-transformation, but a square root transformation allows the original variable to become zero, too. This makes sense for ASFR, because it may very well be

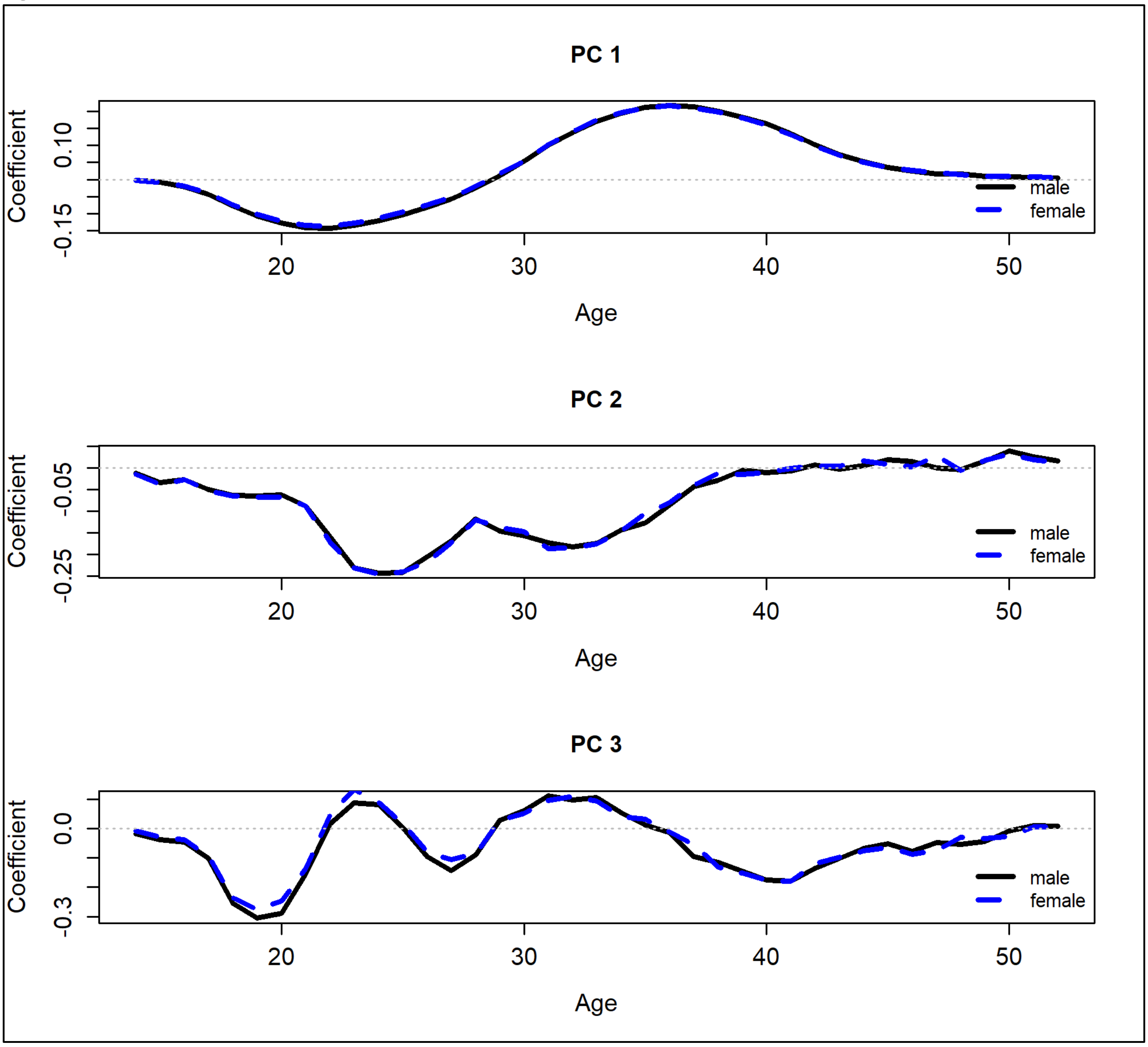
[3] For further reading, the method is explained in detail and for age-specific demographic rates by Vanella (2017a).

that in some period there won’t be any births for very young or relatively old females. Mathematically the transformation is

$$C = F^{1/2} \times E \qquad \textbf{(1)}$$

with F being the 25x78-Matrix of the ASSFR (25 years in the rows, 39 years of age for both sexes in the columns), E being the 78x78-Matrix of the loadings (each column is one eigenvector) and C meaning the theoretical time series of the principal components (a 25x78-Matrix).

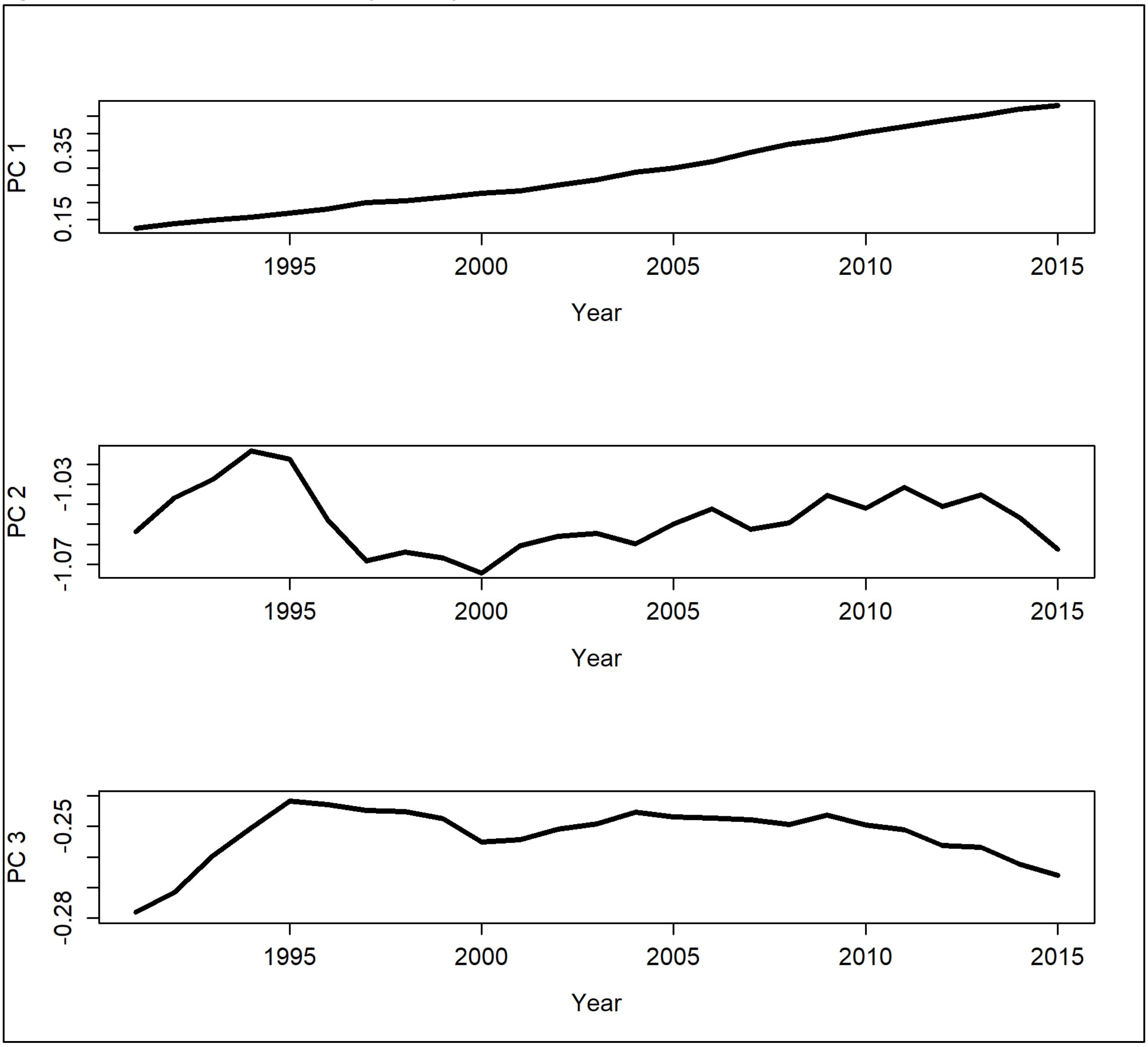
Figure 1. Correlations between ASSFR and PC



Source: Own calculation and design

Figure 1 illustrates the loadings of the first three PC. The first PC has relatively high negative values for younger women under 30 years of age and becomes highly positive for women over 30. This can be identified as the Timing Effect. Increasing values for PC 1 show a strong trend toward shifting births from younger ages to later points in life. Therefore, for the rest of the paper PC 1 will be addressed as the *Timing Index*. PC 2 is generally negative under 40 years of age, it is therefore overall negatively correlated with the fertility level, so this PC shows the Quantum Effect in fertility. From now on this will be referred to as the *Quantum Index.*

Figure 2. Theoretical course of Principal Components 1-3

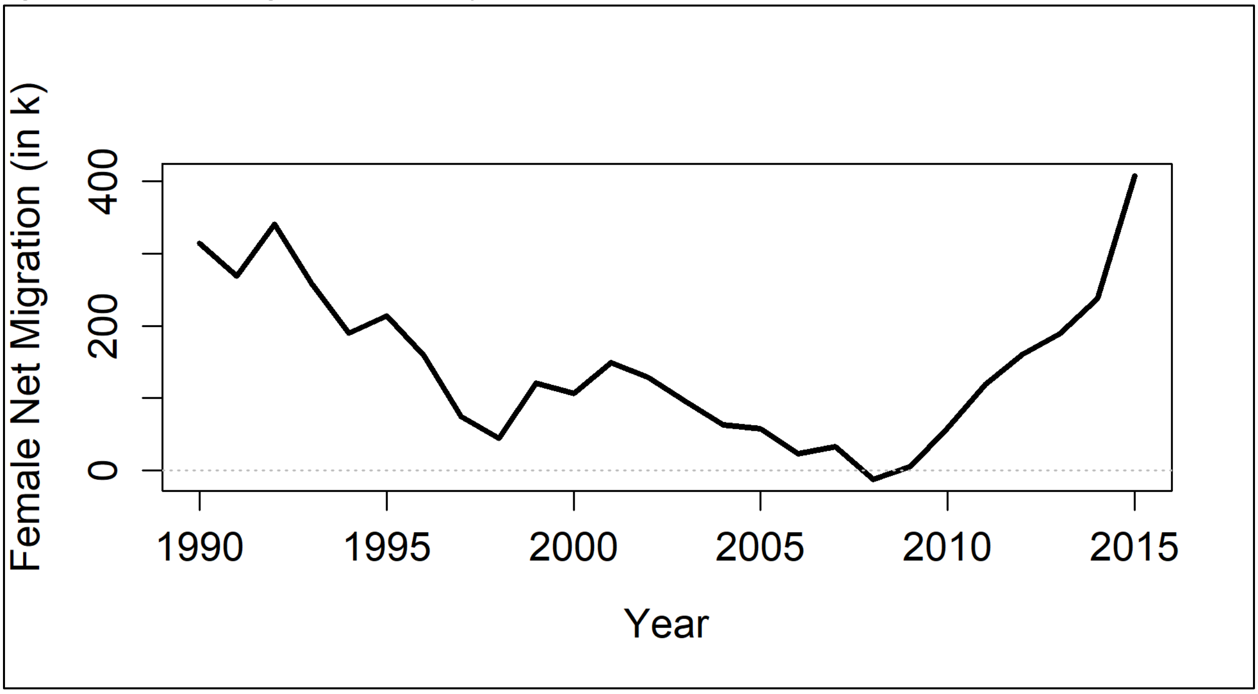


Source: Own calculation and design

PC 3 is much harder to interpret and needs to be set into context with its historical course, which is plotted in Figure 2, next to the Timing and the Quantum Index.

It is noticeable, that PC 3 has big negative values during the first half of the 1990s and has a strong negative trend since 2008. Whereas in between 1990 and 2008 it is relatively stable, although with a slight negative tendency. Taking a look at the female net migration from 1990 to 2015 (Figure 3), we see that there is some similarity between the curves. This leads to the conclusion that PC 3 might be somehow connected to international migration.

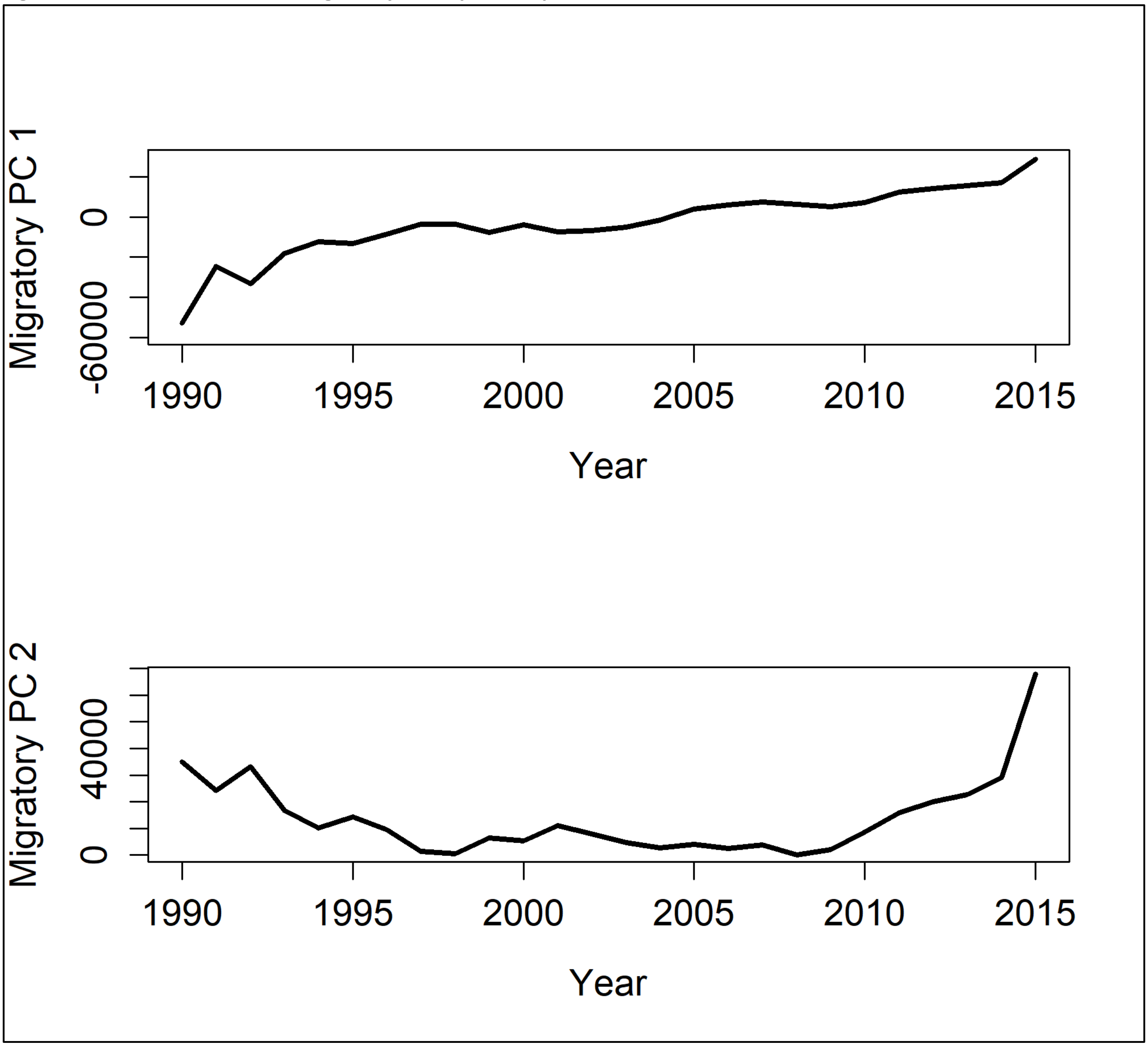
Figure 3. Female Net Migration in Germany



Source: Own design based on Statistisches Bundesamt (2017b)

Vanella and Deschermeier (2017) estimate the age- and sex-specific net migration by groups of nationality in Germany with the merge of two data sets by Destatis. In that paper, the first principal component estimates migration due to labor market exchange of Germans through certain groups of foreigners. The second principal component measures the general migration level. It is tested whether PC 3 identified here has some kind of statistical correlation to the two migratory principal components identified by Vanella and Deschermeier. Figure 4 illustrates the theoretical historical development of the two principal components identified there.

Figure 4. Historical Course of Migratory Principal Components



Source: Own calculation and design

Graphically, the first PC in the migration model does not appear to be associated with PC 3, which is approved by statistical tests. The second migratory principal component might instead have some negative correlation to PC 3. Considering the time between conception and birth of a child is about nine months, it makes sense inserting some time lag into the model.

Due to the restrictions of very short time series available only for the migration data, the correctness of the interrelation as well as the adaptation hypotheses are assumed for the model. These assumptions can be aggregated into the assumption that fertility is elevated by migration in the year after the migration took place and after that will not have any further influence on the reproductive behavior. This is a very strict assumption, but it is necessary due to the short time series available only at this point. Linear regression shows that the migratory principal component 2 is statistically highly significantly associated with PC 3 at a p-value around 0.000001. Therefore, this PC will be named as the *Migration Index* for the remainder of this paper.

The course of the Timing Index is clear, as it increases monotonically with time during the time period considered here. The curvature of the curve is nevertheless interesting. Before 2006 it is basically convex and shifts to a concave curvature at this point in time. So the Timing Effect is generally an increasing trend over time, but flattens a bit for the last decade, which leads to the qualitative conclusion, that the shift of births from younger to higher ages converges toward some equilibrium. The Quantum Effect has stronger underlying stochasticity and it is therefore hard to assign trends to. The future development of the three indices will be estimated in the following section, after some statistical inference.

5 Results

Based on the results from Section 4, parametric models are fitted to the first three PC, which will be used for indirect forecasting of the ASSFR until the Year 2040. The remaining 75 PC in the system will be assumed as white noise processes. This is a necessary assumption for forecasting. Since the Tempo Index, the Quantum Index and the Migration Index already explain about 99.45% of the variation in the ASSFR, the error resulting from the white noise assumption can be ignored.

The Tempo Index appears to increase progressively until 2006, has an inflection point there and increases further, but digresses after that. Such a development may be estimated by a logistic model, which is also often used in growth theory. The forecast model for the Tempo Index is estimated in two steps. First, a parametric logistic model is used for estimating the long-term trend by Ordinary Least Squares (*OLS*) regression. The parameter of the inverse logistic function is estimated by a nonlinear least squares (*NLS*) algorithm. The remaining stochastic part is fitted by a Box-Jenkins model. The identification procedures for the best ARIMA-orders are in detail described in Vanella (2017a). The tests conclude the fit of a random walk process to the stochastic part of the Tempo Index.

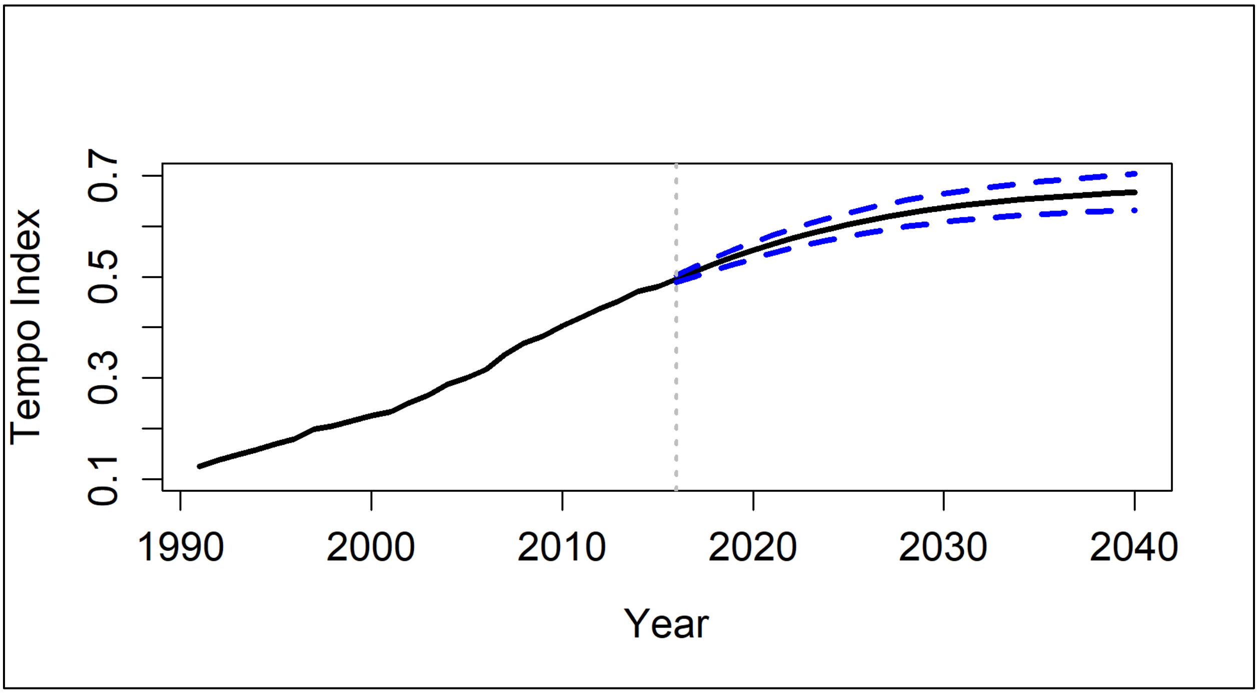
Equation **(2)** shows the forecast model for the Tempo Index *t*:

$$t_\tau = 0.070393 + 0.6184511 logit^{-1}\left(\frac{\tau}{8.243274}\right) + \omega_{\tau-1} + \xi_\tau \qquad \textbf{(2)}$$

where $\omega_\tau = \omega_{\tau-1} + \xi_\tau$ and $\xi_\tau \sim \mathcal{N}(0, 0.0042959^2) \forall \tau$. $\tau = 0$ denotes the year 2009. The forecast based on this model is plotted in Figure 5 with a 90%-PI.

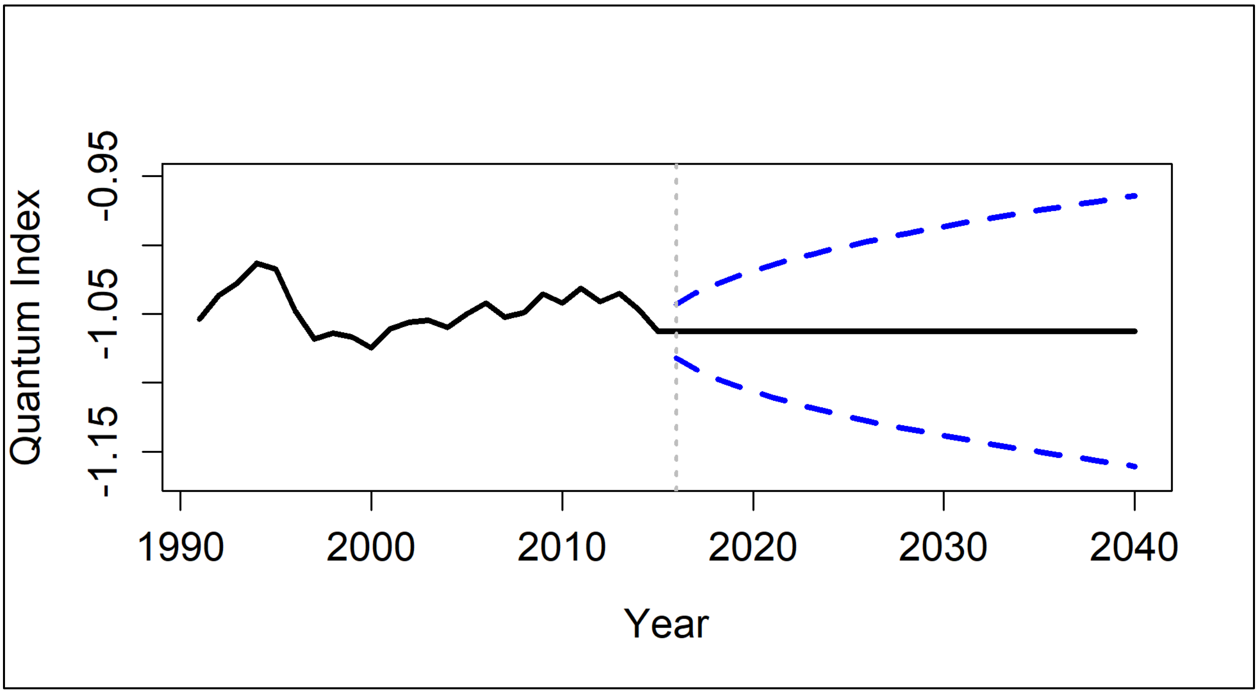
We see that we can expect an increasing future Tempo Index, although at a decreasing pace. The forecast is quite accurate, which can be understood by the relatively narrow PI.

Figure 5. Forecast for Tempo Index



Source: Own calculation and design

Figure 6. Forecast for Quantum Index



Source: Own calculation and design

The Quantum Index has no clear long-term trend. There appears to be a general positive trend since the late 1990s, which may be associated with appropriate political measures. This cannot be validated statistically at this point due to the short time horizon. The easiest assumption, due to our lack of knowledge on the matter, is that the Quantum Index in the mean will behave like a Random Walk process:

$$q_y = q_{y-1} + \varepsilon_y \qquad \textbf{(3)}$$

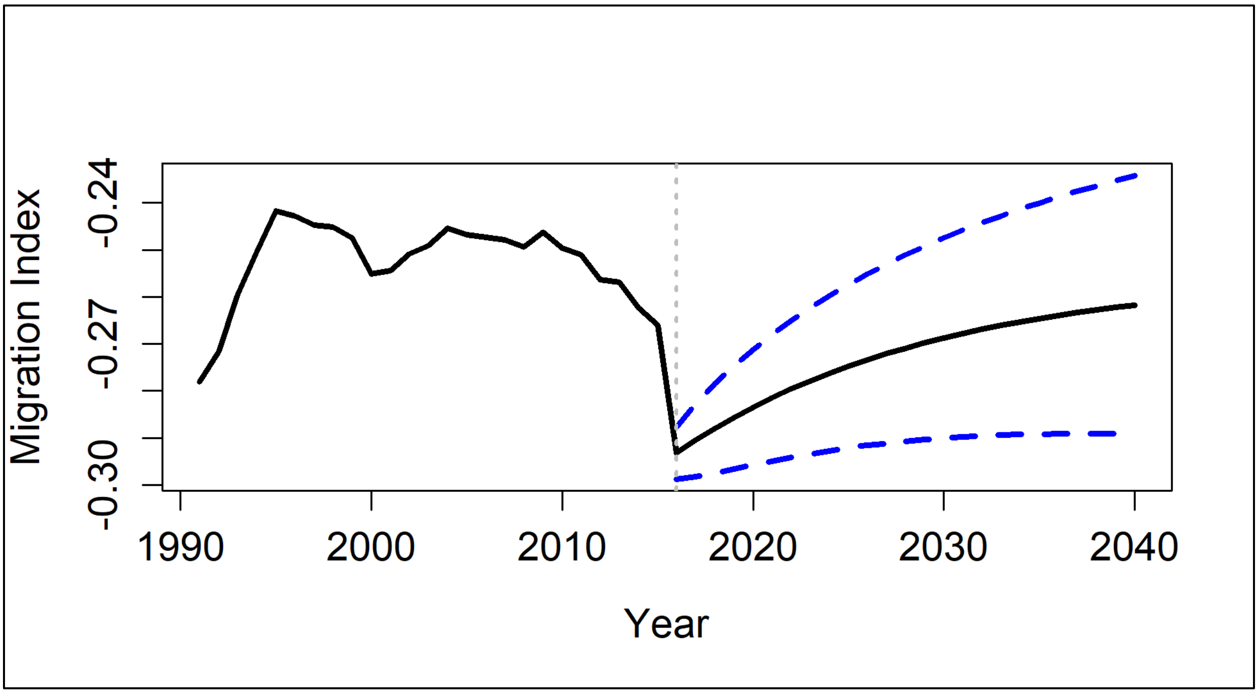
with $\varepsilon_y \sim \mathcal{N}(0, 0.0116954^2) \forall y$. Figure 6 illustrates the resulting forecast.

The first step in the modeling of the Migration Index is the OLS regression on the migratory PC 2 (*Mig)* lagged by one period. The advantage of taking an index composed of age-specific net migration numbers instead of using e.g. the net migration is that we can estimate the effect of migration of certain age groups on fertility by age of the mother. The time series model for this is

$$m_y = -0.244487 - 0.0000007 Mig_{y-1} + \alpha_{y-1} + \nu_y \qquad \textbf{(4)}$$

where $\alpha_y = \alpha_{y-1} + \nu_y$ and $\nu_y \sim \mathcal{N}(0, 0.0046753^2) \forall y$. We must consider the uncertainty in the migration. Since the risks in Mig are uncorrelated to the ones in ν, these two risks may be simulated independent from each other. The different risks are simulated 10,000 times as Wiener processes. The result is plotted in Figure 7.

Figure 7. Forecast for Migration Index



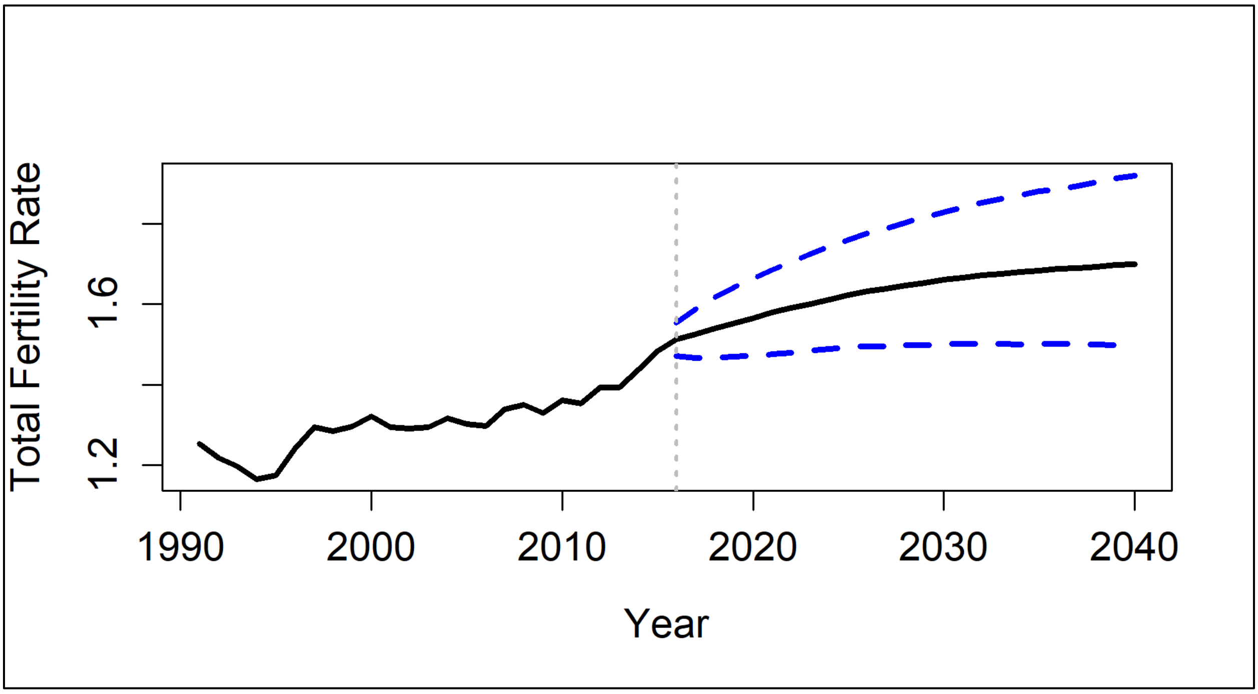
Source: Own calculation and design

Simulation of Wiener processes for the remaining PC as well gives 10,000 simulated future paths for all PC, which can transformed back into simulation results for the ASSFR by reversing **(1)**:

$$\boldsymbol{\Phi} = (\boldsymbol{K} \times \boldsymbol{E}^{-1})^2 \quad \textbf{(5)}$$

with $\boldsymbol{\Phi}$ denoting the matrix of the simulated ASSFR until 2040, and $\boldsymbol{K}$ being the matrix of the simulated PC until 2040. Out of the simulated time series we can extract the empirical quantiles to estimate the PI for the ASSFR. These may be used to forecast births by sex in a cohort-component model. Since age-specific results would be too much for the illustration in this paper, the ASSFR forecasts are just accumulated into a forecast for the TFR, which is illustrated in Figure 8.

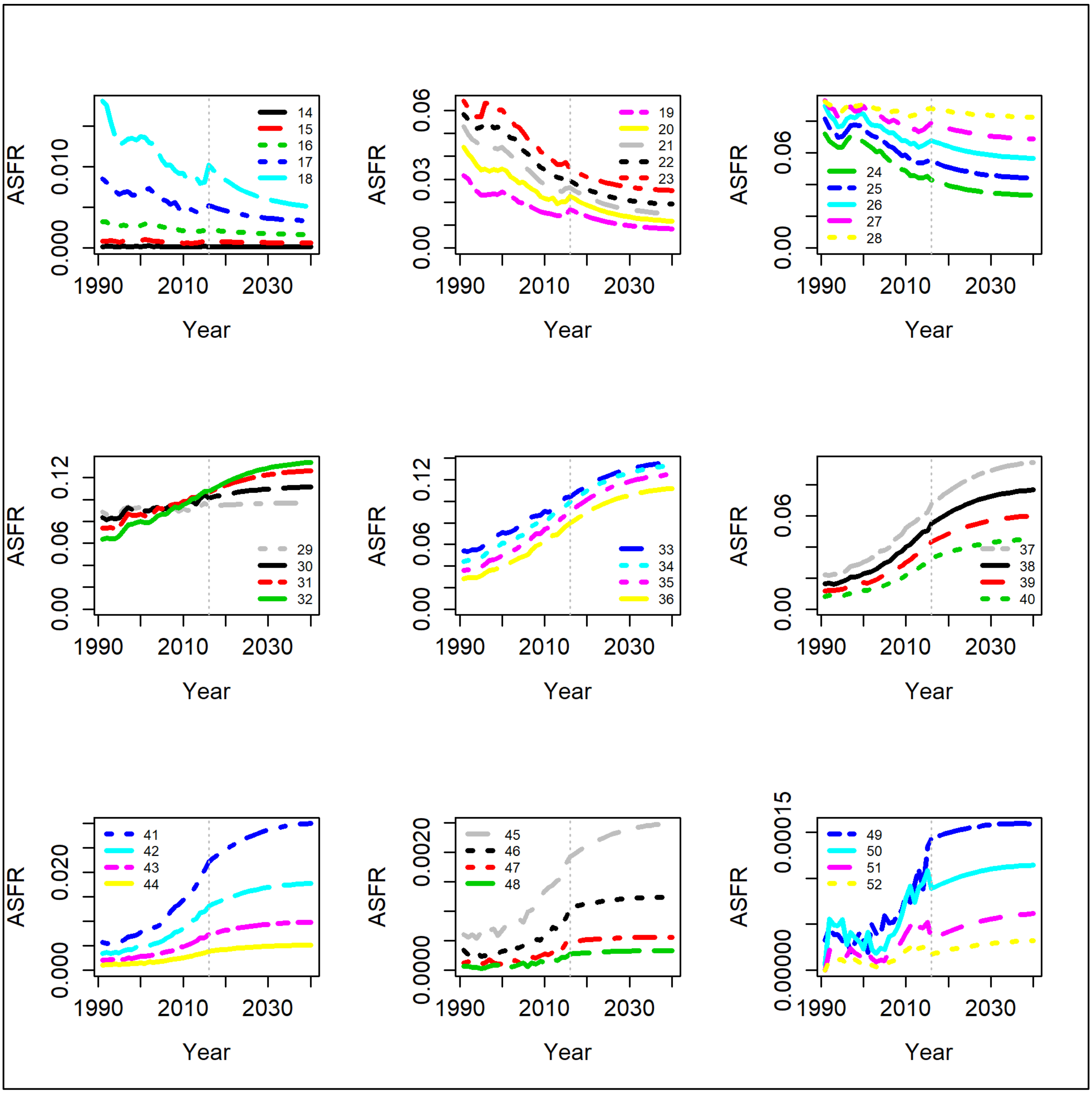
Figure 8. Forecast for TFR



Source: Own calculation and design

We see a probable further increase in the TFR until the year 2040 at a mostly degressive rate. The mean TFR will be around 1.699 by then with a 90%-PI [1.494; 1.918]. So a decrease in the TFR is very improbable, the probability to reach the general replacement-level fertility of 2.1 in 2040 is very small also estimated at about 0.25%. The result shows that the alternative scenario in the projection from Destatis as well as the UN projection are quite realistic, whereas the often used Random Walk Hypothesis (a constant TFR) is highly improbable to take place. Figure 9 gives some general information about the future development of the age-specific fertility with the mean ASFR for females aged 14-52. To keep it simple, the accumulated ASFR for both sexes are only shown. Since it is expected that the development of the ASSFR for both sexes are almost equal, there is no big problem. The differentiation into male and female births becomes important for forecasting future population.

Figure 9. Past and expected future ASFR

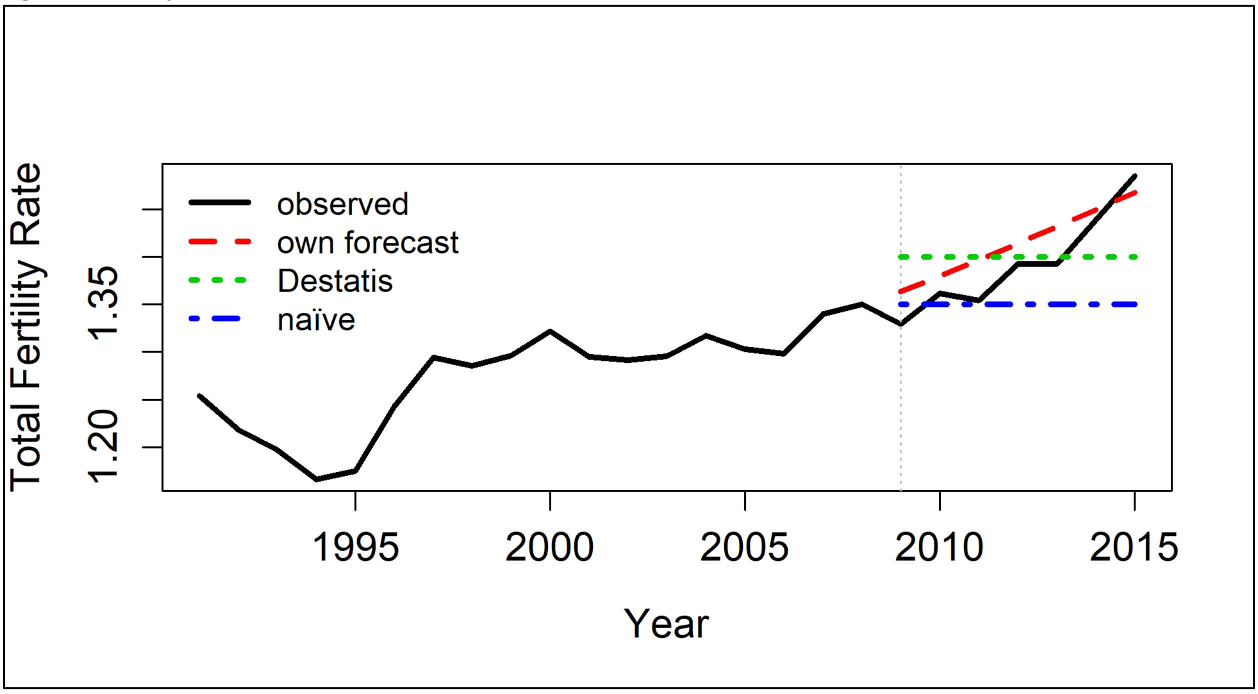


Source: Own calculation and design

We see the general future trend of further decreasing fertility for females under 28 years of age. The ASFR for 28- as well as 29-year-olds are expected to remain relatively constant for the near future, whereas fertility rates for females aged 30 and older will probably increase further, which may be explained by the Timing Effect on the one hand and higher fertility in general due to better health standards through the monotonically increasing life expectancy.

To evaluate the forecast method developed here, the MAPE of a theoretical forecast for the TFR for the years 2009-2015 was considered, based on the data for 1991-2008. The time period is chosen to make it comparable to the Destatis method. The forecast for these years is compared to the projected TFR in the 12. Coordinated Population Projection by Destatis, which has the same starting point of projection. Furthermore, the results are compared to the often (e.g. by Härdle and Myšičková 2009) assumed Random Walk behavior of the TFR, which might be also called a naïve forecast, due to Alho (1990). The results for the different techniques are illustrated in Figure 10.

Figure 10. Comparison of different Forecast Models



Source: Own calculation and design

Figure 10 suggests the forecast model developed in this paper does not only predict the direction of the future development of the TFR correctly, whereas the other forecasts and projections mostly assume a constant future development. The error in the forecast is measured by the MAPE for the period under study. The MAPE for the TFR in year t defined as: (Alho and Spencer 2005)

$$\frac{100}{n}\sum_{t=1}^{n}\left|\frac{TFR_t - \hat{TFR}_t}{TFR_t}\right| \quad \textbf{(6)}$$

Where n is the number of observations, TFR_t the ex-post observed TFR in period t and $\hat{TFR}_t$ denotes the ex-ante forecast for the TFR in period t.

The MAPE for the own forecast is at about 1.9%, considerably smaller than for the Lee-Carter model (3.4%) or the projection by Destatis (3%). Therefore, the forecast model developed here appears to give better fertility predictions than the common approaches, although at a necessarily small sample size.

6 Conclusion, Limitations and Outlook

This contribution has proposed a new approach for forecasting fertility in Germany by the age of the mother and the sex of the newborn through a principal component time series model on a stochastic basis. Next to the inclusion of auto- and cross-correlation effects in the ASSFR time series, the effect of international migration on the reproductive level was also measured and integrated into the forecast model. The model predicted all age- and sex-specific fertility rates as well as the TFR for Germany until the year 2040 with 90%-PI through stochastic simulation of Wiener processes. The results showed a further increase of the TFR, which disproofs the common assumption of a constant future TFR. The forecast reveals a future continuation of the Timing Effect in child-bearing. Although an increase in the TFR may be expected with a high probability, reaching the replacement-level fertility will most likely not happen. The quality of the model was assessed with a test based on the mean average squared error between the forecast and the ex-post observed TFR. The test showed that the accuracy of the model was clearly higher than for the in Germany most commonly used approaches, although it needs to be stressed that the test period was necessarily short due to the short time series available.

Although the strengths of the modeling approach appear obvious, it has two flaws. First, since migration was also used as a predictor, the historical time series which are used are very short at 26 years, which leads to relatively high uncertainty in the forecast for a forecast horizon of 25 years, as it was chosen here. Secondly, the model does not consider major structural changes in the future time series, which are not observed in the historical time series, like landslide measures in family politics. This is a common problem of objective quantitative forecast models. This flaw is accepted to retain an objective modeling procedure.

The model is well-designed for regular updates, since new information might be plugged into the model with ease. The dependence of fertility levels on the migration level leads to some uncertainty in the forecasting of the ASSFR, since migration itself is very hard to forecast because of its high vulnerability toward political, social, economic and environmental conditions in Germany and especially in the countries of origin by the migrants. This effect is not too big though, since the migration effect, represented by the Migration Index identified through the PCA, is estimated at just about 0.6% of the total variation in the ASSFR.

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