

# The Long Memory of Equity Volatility: International Evidence\*

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## Abstract

This paper examines long memory volatility in international stock markets. We show that long memory volatility is widespread in eighty-two countries and that the degree of memory can be related to macroeconomic variables such as inflation, unemployment rates, interest rates or stability of a country measured by jumps. The relationships hold both in the time-series and the cross-sectional dimension. We also find that developed countries possess longer memory in volatility than emerging and frontier countries.

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# I Introduction

In this paper we investigate the long memory in stock market volatility for a large number of countries. We first show that long memory volatility is prevalent in almost every international equity index. We then exploit the cross-sectional and time-series variation of the memory parameter to identify the sources of long memory in volatility. We find that long memory volatility can be related to macroeconomic variables in both the time-series and the cross-sectional dimension. On the one hand, longer memory is related to lower unemployment and lower interest rates for the majority of countries. On the other hand, longer memory is found to be related to more developed and stable countries.

We shed new light on long memory in volatility by exploiting and combining the methodologies of three strands of literature. First, we extend the current research, which only focuses on major economies and large firms by investigating eighty-two international countries including both developed and emerging countries. Second, we allow for a time-varying degree of long memory. Third, long memory so far has only been analyzed in the time-series dimension not in the cross-sectional. We closely investigate possible macroeconomic fundamentals which may explain the degree of long memory both in the time-series and cross-sectional dimension.

We find that 94% of the international countries possess long memory in volatility with an average memory parameter of 0.27, which is statistically significant.<sup>1</sup> In the time-series dimension, longer memory can be related to lower interest rates. In the cross-sectional dimension, higher memory parameter estimates can be related to economically stronger, i.e. developed countries. In contrast, lower memory parameter estimates are associated with emerging and frontier countries. Further, countries with higher interest rates, higher

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<sup>1</sup>This value presents a cross-sectional means using the GPH estimator and a bandwidth parameter of  $m = n^{0.5}$ .

unemployment rates and fewer jumps possess shorter memory in volatility. We verify our memory estimates by showing that volatility in countries with higher memory parameters are more predictable than in countries with low memory parameters.

Long memory properties have been investigated in the dynamics of both stock returns and volatility. Typically, the autoregressive fractionally integrated moving average (ARFIMA) model by [Granger & Joyeux \(1980\)](#), [Granger \(1981\)](#) and [Hosking \(1981\)](#) and the fractionally integrated generalized autoregressive conditional heteroskedasticity (FIGARCH) model introduced by [Baillie et al. \(1996\)](#) are used and shown to provide better forecasts than the short memory ARMA and GARCH models.

Several studies investigate the long memory of returns and volatility both in the U.S. stock market and in international stock markets. [Bollerslev & Mikkelsen \(1996\)](#) and [Ding & Granger \(1996\)](#) show that the conditional variance and absolute returns of the S&P 500 index possess long memory, respectively. Both papers rely on the FIGARCH model. [Breidt et al. \(1998\)](#) also find long memory in the variance of equally weighted and value-weighted CRSP stock market index returns by fitting a long memory stochastic volatility model and relying on the ARFIMA model. [Lobato & Savin \(1998\)](#) investigate long memory properties of the U.S. stock market index and thirty individual stock returns in the U.S. They apply a semiparametric test to returns, squared and absolute returns and find that squared returns exhibit long memory properties while the levels of returns do not. [Sadique & Silvapulle \(2001\)](#) and [Henry \(2002\)](#) consider the long memory property of various international stock indices including Germany, Japan, Korea, Malaysia, New Zealand, Singapore, Taiwan and the U.S. [Sadique & Silvapulle \(2001\)](#) rely on both the modified rescaled range tests and the GPH estimator while [Henry \(2002\)](#) relies on both parametric and semiparametric estimation methods including the GPH estimator, the es-

estimator of [Robinson \(1994\)](#) and the ARFIMA model. [Kasman et al. \(2009\)](#) show evidence of long memory dynamics in both the conditional mean and variance for eight Central and Eastern European countries' stock markets and also rely on the both semiparametric (GPH) and parametric (ARFIMA, FIGARCH and HYGARCH) estimation procedures. While long memory has been investigated extensively both in the U.S. and international stock markets, the works so far have mainly focus on the detection of long memory. We contribute to the existing literature by largely extending the sample of countries to eighty-two and examining the cross-sectional variation of long memory across countries and its link to macroeconomic variables. [Nguyen et al. \(2017\)](#) investigate the cross-sectional variation of long memory in volatility at the firm level. They provide evidence of long memory in volatility for the cross-section of U.S. stocks and find a negative price for long memory volatility.

The rest of the paper is organized as follows. Section II describes our data set and estimation procedure for long memory. Section III investigates long memory in the cross-section of countries. Section IV presents robustness tests. Section V concludes.

## II Data and Methodology

### A Data

The data used for our analyses come from various sources. For our international stock index data we follow [Pukthuanthong & Roll \(2015\)](#) and include eighty-two countries for which we obtain the data from Datastream.<sup>2</sup> If available, we rely on daily observations of

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<sup>2</sup>Table 8 in the Online Appendix presents an overview of the countries, the selected indices and the sample period.

the total return indices which include the dividends, and use the price index otherwise.<sup>3</sup>

The sample covers the period from December 1964 until December 2015.<sup>4</sup>

For each country we obtain country-specific macroeconomic variables from the Global Financial Database. We include the real gross domestic product (GDP), the consumer price index (CPI), unemployment, short maturity and long maturity interest rates.<sup>5</sup> Most of the short maturity yields are 3-month treasury bills and most of the long maturity yields are 10-year government bonds. Hence from now on we refer to them as treasury bills (Tbill) and government bonds (Gov.Bonds). Both are given in percentage form per annum. The Real GDP data is obtained in U.S. dollar currency converted using exchange rates from the Global Financial Database.<sup>6</sup>

## B Semiparametric Estimation of Long Memory

In our empirical analysis we work with the two most popular estimators, which are the GPH estimator and the Local Whittle estimator.

[Geweke & Porter-Hudak \(1983\)](#) introduce an estimator which is based on the log-periodogram. A linear regression is employed to the spectral density relying on the first  $m$  periodogram ordinates. Empirically, the spectral density of a stationary process  $X_t$  is estimated by the periodogram:

$$I_X(\lambda_j) = \frac{1}{2\pi N} \left| \sum_{t=1}^N X_t e^{-it\lambda} \right|^2, \quad t = 1, \dots, N \quad (1)$$

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<sup>3</sup>Prices are cleaned of outliers by removing observations which deviate by more than 10 standard deviations from the median using a rolling window of 50 observations ([Barndorff-Nielsen et al., 2009](#)).

<sup>4</sup>For Bangladesh, Slovenia and Zimbabwe, the last available observations are from April 2013, October 2010 and October 2006, respectively.

<sup>5</sup>The data for the U.S. is supplemented by data provided by Amit Goyal (website: <http://www.hec.unil.ch/agoyal/>) and FRED.

<sup>6</sup>Unfortunately, the Global Financial Database does not cover our complete sample of countries with macroeconomic variables. GDP data is available for seventy-two countries, inflation data is available for eighty countries, unemployment data is available for sixty-nine countries, treasury bill rates are available for seventy-eight countries and government bond rates are available for seventy-three countries.

where the periodogram is not affected by centering of the time series for Fourier frequencies  $\lambda_j = 2\pi j/N$  ( $j = 1, \dots, [(N-1)/2]$ ). The negative slope coefficient  $\beta_1$  in the regression presents the estimator:

$$\log(I(\lambda_j)) = \beta_0 + \beta_1 \log[4\sin^2(\lambda_j/2)] + \epsilon_j, \quad j = 1, \dots, m \quad (2)$$

The asymptotic standard errors for the long memory parameter can be obtained from the asymptotic distribution, which is derived by [Robinson \(1995b\)](#) under mild conditions ( $m \rightarrow \infty, N \rightarrow \infty, \frac{m}{N} \rightarrow 0$ ):

$$\sqrt{m}(\hat{d} - d) \xrightarrow{d} N\left(0, \frac{\pi^2}{24}\right) \quad (3)$$

The choice of the bandwidth parameter  $m$  results into a bias–variance trade-off. If the  $m$  is chosen too low and hence too close to the origin, an increased variance is the result, while a  $m$  chosen too high and hence too far from the origin leads to bias.

In the following empirical analyses, we focus on the GPH estimator and the bandwidth  $m = N^{0.5}$  following the existing literature ([Geweke & Porter-Hudak, 1983](#); [Diebold & Rudebusch, 1989](#); [Hurvich & Deo, 1999](#); [Henry, 2002](#)).<sup>7</sup> Results with alternative bandwidth choices and the Local Whittle estimator are reported in the Section IV.

We refer to  $d$  as the memory parameter and differentiate between three cases: A time series has short memory if  $d = 0$ . A time series has negative memory or is anti-persistent if  $d < 0$ . A time series has long memory if  $0 < d < 1$  where it is non-stationary if  $0.5 < d < 1$ .

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<sup>7</sup>Typically, empirical researches rely on this bandwidth choice since it is robust against short-range dependencies in the data.

# III Long Memory Volatility in International Equity Markets

In this section we provide evidence of long memory volatility in the cross-section of eighty-two countries. First, we show that long memory volatility is prevalent in most countries but that the memory parameter varies across countries in Section III.A. Section III.B refers long memory to predictability and Section III.C relates the memory parameter to macroeconomic variables in the time-series dimension. Section III.D relates the memory parameter to macroeconomic variables in the cross-section of countries and separately investigates the memory in developed and emerging countries.

## A Descriptive Statistics

We apply the GPH estimator to the time series of squared returns for the selected eighty-two countries. Table 1 provides summary statistics for the memory parameter  $d$ . The mean memory parameter over the eighty-two countries is 0.27 and the mean standard deviation is 0.13. If the time series exhibit short memory, the mean should be approximately zero. The average t-statistic of 3.95 suggests that long memory is present in volatility. In fact, 87% of the parameters are positive and statistically significant at the 5% level or lower. Further, the 5% to 95% quantiles suggest that most parameters lie in the interval  $(0, 0.5)$ . We find that 94% of the countries exhibit long memory in volatility, where  $0 < d < 0.5$ , while 4% show anti-persistence and 2% show non-stationary long memory in volatility. We hence conclude that most international stock markets exhibit long memory in volatility. These results extend the current literature which focuses on the U.S. and some major countries like Japan or the U.K. ([Cheung & Lai, 1995](#); [Sadique](#)

& Silvapulle, 2001; Henry, 2002).

The countries with the highest memory parameter are Taiwan, Finland and Kuwait, while countries with the lowest memory parameter are Bahrain and Egypt. Figure 1 displays the estimates for the eighty-two countries. The G-7 countries, representing the major advanced economies and those making the largest percentage of global wealth, do not possess the longest or shortest memory. But six of the seven major economies have a memory parameter higher than 0.3 while the ten countries with the shortest memory are all “frontier” countries.<sup>8</sup> In the following we closely investigate potential drivers of the memory parameter.

## B Long Memory and Predictability

Typically, long memory time series are described as highly persistent time series, for which the autocorrelation function is decaying at a hyperbolic rate rather than an exponential rate as for short memory processes. Intuitively, the higher persistence of the time series can be linked to higher predictability or lower uncertainty. In this section, we empirically show the link between long memory and predictability for the volatility of the stock indices.

At the same time, this exercise presents a validity check for our long memory estimates. A higher memory parameter should be associated with higher forecasting performance, if our memory estimates are correct and not biased by the quality of the data or spurious long memory.

We run monthly predictability regressions of the realized volatility for each country separately both in-sample and out-of-sample. We obtain monthly realized volatility obser-

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<sup>8</sup>Even though the beginning of the sample period varies across the countries, the memory parameters are comparable. In our empirical analysis we also consider the same sample size for all countries, which delivers qualitatively similar results.



vations by summing squared daily returns within each month (Bollerslev et al., 2014). We rely on the state of the art (Heterogeneous) Autoregressive models of Realized Volatility (HAR-RV) following Corsi (2009).<sup>9</sup> The independent variables are lagged observations of the realized volatility and we consider five different specifications by including the volatility from the previous month (HAR(1)), six months (HAR(2)), one year (HAR(3)), two years (HAR(4)) and 5 years (HAR(5)):

$$HAR(1) : RV_{t+1}^M = \alpha + \beta RV_t^M + \epsilon_{t+1} \quad (4)$$

$$HAR(2) : RV_{t+1}^M = \alpha + \beta RV_t^M + \beta RV_t^{6M} + \epsilon_{t+1} \quad (5)$$

$$HAR(3) : RV_{t+1}^M = \alpha + \beta RV_t^M + \beta RV_t^{6M} + \beta RV_t^{1Y} + \epsilon_{t+1} \quad (6)$$

$$HAR(4) : RV_{t+1}^M = \alpha + \beta RV_t^M + \beta RV_t^{6M} + \beta RV_t^{1Y} + \beta RV_t^{2Y} + \epsilon_{t+1} \quad (7)$$

$$HAR(5) : RV_{t+1}^M = \alpha + \beta RV_t^M + \beta RV_t^{6M} + \beta RV_t^{1Y} + \beta RV_t^{2Y} + \beta RV_t^{5Y} + \epsilon_{t+1} \quad (8)$$

The multiperiod volatilities are normalized sums of the one-month realized volatilities.

The six-months' realized volatility is exemplarily given by:

$$RV_t^{6M} = \frac{1}{6}(RV_t^M + RV_{t-1}^M + \dots + RV_{t-5}^M) \quad (9)$$

The models are able to mimic the behavior of long memory processes and exhibit strong forecasting performance, despite the simplicity of both the model and the estimation. We form tertile portfolios by sorting the cross-section of country stock market indices by the memory parameter. We then compute the average adjusted  $R^2$ , t-statistic, F-statistic and out-of-sample  $R_{OOS}^2$  for each tertile portfolio.<sup>10</sup>

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<sup>9</sup>We also considered simple Autoregressive models including the lags 1, 6, 12, 24 and 60, leading to qualitatively similar results.

<sup>10</sup>We report t-statistics of the slope coefficient for HAR(1) and F-statistics for the joint significance of the slope coefficients for the remaining models.

The results are reported in Table 2. Panel A shows the adjusted  $R^2$  of the in-sample predictability regressions. There is a strictly monotonic pattern of explanatory power, which is increasing in the memory parameter. This is further supported by the increasing t-statistics and F-statistics in Panel B. Countries with higher memory parameters have stronger explanatory power and the predictor variables are more statistically significant than countries with shorter memory in volatility. Lastly, in Panel C, the  $R_{OOS}^2$  also show that the out-of-sample forecasting performance of long memory countries is stronger than short memory countries. There is a strictly monotonic pattern for the short horizon model, HAR(1), which diminishes when including more lags. A graphical illustration of the results is reported in Figure 2.

We thus show that the degree of memory in volatility is a proxy for predictability. At the same time this exercise validates our estimation approach of memory. Our results are true for both in-sample and out-of-sample, while we allow for various model specifications including short memory processes and long memory mimicking processes.

## C Time Variation of Long Memory Volatility

We first investigate the temporal variation of the memory parameter for the individual countries and their relationships with macroeconomic variables. For this purpose, we allow for a time-varying memory parameter. We estimate the memory parameter by applying the GPH estimator at a monthly frequency to a rolling window of five years of daily return data. We start with a separate analysis of the U.S. and consider the complete cross-section in a second step.

## 1 Evidence from the U.S.

Each month we regress the memory parameter of the U.S. on the following macroeconomic variables: inflation proxied by changes in Consumer Price Index (Inflation), log Unemployment rate (Unemployment), treasury bill rates (Tbill), government bond rates (Gov.Bonds), gross domestic product growth (GDP) and an indicator function for the recession (Recession) that represents periods of expansion and recession defined by the National Bureau of Economic Research (NBER):

$$d_{U.S.,t} = \alpha_{U.S.} + \beta_{U.S.} X_{U.S.,t} + \epsilon_t \quad (10)$$

where  $d_t$  stands for the memory parameter at time  $t$ ,  $X_t$  contains one or more of the macroeconomic variables and  $\epsilon_t$  is the error term.<sup>11</sup> All time series are at monthly frequency except for the GDP, which is quarterly.<sup>12</sup> Table 3 reports the results. Our interpretations refer to the terms predictability, uncertainty and low memory parameters interchangeably.

We find that inflation proxied by the changes in the CPI has a negative relationship with the degree of long memory, which is statistically significant at the 10% level (Model 1). However, the explanatory power is rather low for inflation rates with an adjusted  $R^2$  of 0.8%. Economically, the negative sign of the coefficient implies that in times of lower inflation, the memory of U.S. market volatility is rather longer. [Ball \(1992\)](#) argues that inflation is expected to be kept low by authorities when it is low. When inflation is

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<sup>11</sup>Since our memory estimates  $d_t$  rely on rolling window estimates, one might argue that there is barely temporal variation in our estimates. If this is true, this should work against our empirical analysis and we should not find any significant drivers of the memory parameter, but we do. In addition, we repeat the analysis relying on smaller rolling windows using 12 months of daily return data. The results are qualitatively similar.

<sup>12</sup>We follow [Bloom \(2009\)](#) and detrend the time series using the Hodrick–Prescott filter with  $\lambda = 129,600$ .

high, on the other hand, there is a high degree of uncertainty since policymakers face the trade-off between deflation and the resulting recession. This uncertainty can be related to unpredictability in the U.S. market in general but more importantly also in the U.S. stock market. This argument is supported by [Fischer & Modigliani \(1978\)](#), who suggest that higher inflation rates cause governments to announce unrealistic stabilization programs which leads to uncertainty for market prices. The lower predictability in times of high inflation is reflected by the shorter memory.

The unemployment rate impacts the memory parameter positively and is statistically significant at the 5% level (Model 2). The adjusted  $R^2$  is of similar magnitude when including the inflation as a regressor with a value of only 1.17%. [Veronesi \(1999\)](#) shows that good news in bad times (and bad news in good times) is generally related to increased uncertainty. Similarly, [Boyd et al. \(2005\)](#) argue that the impact of unemployment for stocks depends on the business cycle but the economy is usually in an expansion phase. Hence, the average relationship of higher unemployment and higher uncertainty is consistent with the lower predictability proxied by shorter memory in volatility.

Both the short- and long- term interest rates given by Tbill and Gov.Bonds have a negative impact on the memory parameter which is statistically significant at the 1% level. The adjusted  $R^2$  are the highest with values of 24.53% and 36.30%, respectively. A large literature has researched the impact of interest rates on real activity. Typically high interest rates play a key role in (inflation) stabilization programs for the government in order to decrease inflation rates. As discussed above, high inflation rates are related to lower predictability. The lower predictability given by lower memory parameters coupled with higher interest rates can be confirmed from our regression analysis for the U.S.

Similar to inflation, GDP has a negative coefficient, but it is statistically insignificant.

The same is true for the recession indicator as defined by the NBER, which does not help explain the memory parameter.<sup>13</sup> Intuitively, one would expect recessions to be associated with low memory parameters due to the high uncertainty and low predictability in these times.

We also conduct regressions including all variables. Model 7 is a multiple regression without GDP at a monthly frequency while Model 8 is a multiple regression including GDP at a quarterly frequency. While the signs and the significance of Unemployment and Gov.Bonds in Model 7 are similar to the univariate regressions, the adjusted  $R^2$  increase to remarkable magnitudes of 41.81% and 62.71% for Model 7 and 8, respectively. In summary, the direction of the relationships between the memory parameter and macroeconomic variables makes sense economically and the variables jointly have high explanatory power for the memory parameter.

## 2 Evidence from the Complete Cross-Section

We repeat the analysis from above and estimate the same regression as Equation (10) for each of the countries individually. For overview purposes we do not report the same output as Table 3 for each country but report median estimates for the cross-section, the percentage of countries for which we find a negative (positive) and statistically significant coefficient and the average t-statistic and adjusted  $R^2$  across all countries. The results are presented in Table 4.

Overall, the median values deliver the same results for the entire cross-section as for the U.S. All macroeconomic variables except for unemployment have a negative impact on the memory parameter for the cross-section. Nonetheless, only for Tbill and Gov.Bonds

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<sup>13</sup>Note that there are much fewer observations for the regression including the GDP and hence plausibly less power due to the quarterly frequency, while the recession variable is just a dummy variable.

we find strong statistical evidence. For 63% (55%) of the countries, Tbill (Gov.Bonds) shows a negative and statistical significant relationship with the memory parameter, which is consistent with our results for the U.S. This is supported by average t-statistics above 8 and the highest adj.  $R^2$  value of 20% (19%).

For the remaining macroeconomic variables, we do not find any consistent pattern across countries. Both the explanatory power and the statistical significance of the slope coefficients are relatively low, where the  $R^2$  vary between 1% and 4%.

Using the kitchensink regression, excluding or including the GDP increases the adjusted  $R^2$  to 37% and 37%, respectively, indicating that the macroeconomic variables jointly have explanatory power for the memory parameter. While the sign of inflation, unemployment, interest rates, GDP and Recession are generally consistent with the analysis of the U.S., it is not true for the complete cross-section (proportion is less than 100%) and not statistically significant for many countries.

## D Cross-Sectional Variation and Macroeconomic Variables

Instead of investigating the temporal relationship between the long memory parameter and the macroeconomic variables for each country separately, we now examine the complete cross-section over the sample period. We employ two different approaches relying on either portfolio sorts or cross-sectional regressions. Since we are interested in country-specific variables, we exclude the recession dummy variable. Instead, we include a measure of stability directly obtained from the return time series: Jumps. Intuitively, a stable country should exhibit fewer stock market jumps. We apply the common jump test proposed by [Barndorff-Nielsen & Shephard \(2006\)](#).<sup>14</sup> The test relies on the bipower varia-

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<sup>14</sup>[Pukthuanthong & Roll \(2015\)](#) show, with the help of simulations using different jump size and frequency, that this test is preferable compared to the ones proposed by [Jiang & Oomen \(2008\)](#), [Lee & Mykland \(2008\)](#) and [Jacod & Todorov \(2009\)](#).

tion, which decomposes the quadratic variation into its part due to continuous movements and a jump part. The jump test statistic is given by:

$$BNS_t = \frac{(\pi/2)B_t - S_t}{\sqrt{((\pi^2/4) + \pi - 5)(\pi/2)^2 Q_t}} \quad (11)$$

$$Q_t = \frac{1}{K_t - 3} \sum_{k=4}^{K_t} |r_{t,k}| |r_{t,k-1}| |r_{t,k-2}| |r_{t,k-3}| \quad (12)$$

$$S_t = \frac{1}{K_t} \sum_{k=1}^{K_t} r_{t,k}^2 \quad (13)$$

$$B_t = \frac{1}{K_t - 1} \sum_{k=2}^{K_t} |r_{t,k}| |r_{t,k-1}| \quad (14)$$

where  $K_t$  is the number of observations over the examined period,  $r_{t,k}$  is the  $k$ th daily observation over the examined period  $t$  and  $BNS_t$  is normally distributed under the null. We rely on two measures of jumps. First, we compute the BNS jump statistic for each month and country using a pool of daily returns following [Pukthuanthong & Roll \(2015\)](#). The first measure is given by the jump statistic for each month. Our second measure presents an indicator function which shows whether the current month exhibits a statistically significant jump at a 5% significance level.

Each month, we sort the countries by their memory parameter and form tertile portfolios where the countries with the lowest memory parameter are in the first tertile and countries with the highest memory parameter are in the third tertile. We then compare averages of macroeconomic variables for the tertile portfolios. Table 5 reports average inflation, unemployment, treasury bill rates, government bond rates, GDP and jump measures for the tertile portfolios.<sup>15</sup> There is a monotonic pattern in all of the tertile portfolios (except for GDP) which are increasing or decreasing with the memory param-

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<sup>15</sup>Looking at the cross-section of countries, one might argue that GDP per capita is a more appropriate measure of comparison than GDP. Our main results rely on real GDP but we also repeated the analysis using GDP per capita, which leads to qualitatively similar results.

eter. We find that the unemployment and government bond rates are lower for countries with long memory. The average spread of the high minus low (LMS) portfolio, which holds the country indices with the longest memory and writes the country indices with the shortest memory, is statistically significant with t-statistics of  $-3.09$  and  $-3.25$ , respectively. This stands in contrast of our time-series analysis. While unemployment has a positive impact on the memory parameter in the time-series dimension for most countries, it has a negative impact on the memory parameter in the cross-sectional dimension. Moreover, countries with higher memory parameters have statistically significantly fewer jumps according to both the BNS statistic and the indicator function.<sup>16</sup> Lastly, countries with long memory show higher GDP growth than countries with short memory, which is weakly statistically significant (t-statistic of 1.85).

We also conduct cross-sectional regressions of the memory parameter by estimating the following regression:

$$d_{i,t} = \alpha_{i,t} + \beta_{i,t}X_{i,t} + \epsilon_{i,t} \quad (15)$$

where  $d_i$  is the memory parameter of country  $i$ ,  $X_i$  contains one or more macroeconomic variables and  $\epsilon_i$  is the error term. Table 6 reports the average coefficient estimates. The slope coefficients of Unemployment, Tbill and Gov.Bonds are all negative and statistically significant at the 1% level while the BNS coefficient is positive and statistically significant (1%) as well. For inflation and GDP, we do not find any significant relationship. The results are generally consistent with our sorting exercise.<sup>17</sup>

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<sup>16</sup>The BNS statistic is generally negative and falls below  $-1.96$  if there is a significant (5%) jump, hence lower statistics indicate more significant jumps.

<sup>17</sup>We also conduct panel regressions and find qualitatively similar results. The slope coefficients of Unemployment, Tbill and Gov.Bonds are negative and statistically significant at the 1% level while the BNS coefficient is positive and statistically significant as well. We account for both fixed effects and heteroskedasticity in the regression. Detailed results are reported in Table 9 of the Online Appendix.



Our results suggest that countries with stable economies possess longer memory volatility compared to less stable countries. Intuitively, a stable country should hence exhibit fewer jumps as well. Long-term interest rates as proxied by government bonds can also be related to the stability of a country. These tend to be lower in safer countries. Since the value of money might be unpredictable in unstable environments, people prefer to spend their money, which is counteracted with higher interest rates by the government. The U.S. has an average short term interest rate of 5.36% over the sample period compared to Brazil (22.60%), Romania (45%) and Turkey (45%).

We directly test whether developed countries possess longer memory than undeveloped countries. In the following we do not rely on proxies for the economic strength of a country, such as macroeconomic variables, but we use existing specifications. We differentiate between Organisation for Economic Co-operation and Development (OECD) countries and emerging countries as defined by Thomson Reuters Tickhistory (TRTH). We also differentiate between developed, emerging and frontier countries, as defined by the classification of Morgan Stanley Capital International (MSCI). We estimate the following cross-sectional regression:

$$d_i = \alpha_i + \beta_i D_i + \epsilon_i \tag{16}$$

where  $d_i$  is the memory parameter of country  $i$ ,  $D_i$  is a dummy variable indicating whether a country is part of group of countries and  $\epsilon_i$  is the error term. If frontier countries have a shorter memory than developed countries, the coefficient is expected to be negative and statistically significant.

We run three distinct analyses. First, we estimate the memory parameter over the complete sample from 1964 until 2015, resulting in a cross-sectional regression with eighty-

two observations. Since the classification of MSCI and the inclusion in the OECD group has changed within our sample period, one could argue that the first analysis leads to biased results. We hence repeat the same analysis, but estimate the memory parameter only for the most recent eight years for the period from 2008 until 2015. Lastly, we use the time series of memory parameters from the previous sections estimated from rolling windows and estimate the cross-sectional regression in each month. The regression equation is then modified as:

$$d_{i,t} = \alpha_{i,t} + \beta_{i,t}D_{i,t} + \epsilon_{i,t} \quad (17)$$

We are interested in the temporal variation of the slope coefficient  $\beta_{i,t}$  and report time-series averages for these.

The results for the three analyses are presented in Table 7 in Panel A, B and C, respectively. We can confirm the presumption that economically stronger countries have higher memory parameters than weaker countries for the period from 1964 until 2015 in Panel A. This holds true for both definitions of either TRTH or MSCI. OECD and developed countries exhibit a higher memory parameter which is statistically significant at the 5% level while emerging (TRTH) and frontier countries possess a shorter memory in volatility, which is also statistically significant at the 5% level or lower. The adjusted  $R^2$  vary from 1.43% to 16.36%. The results remain qualitatively similar when considering the subsample from 2008 until 2015 in Panel B. OECD and developed countries possess statistically higher memory parameters while emerging (TRTH) and frontier countries possess statistically shorter memory in volatility. Lastly, the time series averages of the slope coefficients deliver the same message. All coefficients are statistically significant at the 5% level or lower, and exhibit the same signs as for the other two analyses.

An economically strong country tends to be more stable and less sensitive to sudden shocks. Therefore, it is intuitive that stock market volatility in these countries will be more persistent. We can relate the memory of a country to its economic importance proxied by classifications such as OECD, MSCI or continents.

## IV Robustness

In this section we run various robustness tests including alternative long memory estimates and predictive regressions. All results are reported in the Online Appendix.

### A Estimation of the Memory Parameter

For our main analysis we follow the existing literature and choose the ad hoc bandwidth parameter of  $m = N^{0.5}$ . We repeat the exercises using a bandwidth parameter of  $m = N^{0.6}$  and  $m = N^{0.7}$ . Further, we apply the GPH estimator to absolute returns rather than squared returns as in our main analysis (Bollerslev & Wright, 2000). Lastly, we follow another commonly used approach to estimate long memory, the Local Whittle estimator. The Local Whittle estimator is obtained by minimizing the following objective function:

$$\hat{d}_{LW} = \arg \min_{d \in \theta} \left[ \log \left( \frac{1}{m} \sum_{j=1}^m \frac{I(\lambda_j)}{\lambda_j^{2d}} \right) - \frac{2d}{m} \sum_{j=1}^m \log \lambda_j \right], \quad \theta \subseteq (-0.5, 0.5) \quad (18)$$

where  $m$  is restricted to  $m < \frac{N}{2}$ . The originally proposed estimator by Whittle (1951) presents an approximate maximum likelihood approach, which is extended by the Local Whittle estimator. Under mild assumptions similar to those for the GPH estimator,

Robinson (1995a) derives the asymptotic distribution:

$$\sqrt{m}(\hat{d}_{LW} - d_0) \xrightarrow{d} N\left(0, \frac{1}{4}\right) \quad (19)$$

Table 10 reports the time-series regression of the memory parameter on macroeconomic variables for the U.S. The table presents results based on the four alternative memory estimators in Panel A, B, C and D, respectively. Even though the magnitudes of the slope coefficients slightly differ, the relationship between the variables and the memory parameter remains qualitatively similar. Generally, inflation, short and long interest rates have a negative impact on the memory parameter while unemployment has a positive relationship with the memory parameter.<sup>18</sup> The adjusted  $R^2$  vary from 0%–41%, 0%–63%, 0%–34% and 0%–52% in the univariate regressions for the four alternative estimators, respectively. For comparison, the adjusted  $R^2$  varies from 0%–36% in our main analysis using the GPH estimator and  $m = N^{0.5}$ .

Table 11 compares the memory parameter in developed and emerging countries for the alternative memory estimators. OECD countries and developed (MSCI) countries have statistically significantly higher memory parameters while emerging countries (TRTH) and frontier countries have statistically significantly shorter memory in volatility for all four estimators. The adjusted  $R^2$  vary from 1%–16%, 1%–23%, 2%–16% and 0%–8% in the univariate regressions for the four estimators, respectively. For comparison, the adjusted  $R^2$  varies from 1%–16% in our main analysis using the GPH estimator and  $m = N^{0.5}$ .

Table 12 investigates the average macroeconomic variables of tertile portfolios sorted

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<sup>18</sup>There is one exception. Unemployment has a negative and statistically significant impact on the memory parameter when using the bandwidth of  $m = N^{0.7}$ .

by the memory parameter. Countries with higher memory parameters exhibit fewer jumps (higher BNS and lower BNS-I) and show lower government bond rates. This result is true and statistically significant for all four estimators. Additionally, countries with a higher memory parameter have lower unemployment rates, which is statistically significant for three of the four estimators.

## B Predictive Regressions

In Section III.D, we investigate the contemporaneous relationship between the memory parameter and macroeconomic variables' cross-section of countries. It is argued in the literature that changes in macroeconomic variables do not directly impact the real economy and the stock market, but it takes several months or more. [Paye \(2012\)](#) investigates the predictability of stock return volatility by multiple macroeconomic variables including up to two lags while [Engle et al. \(2013\)](#) show that macroeconomic fundamentals are important for both short- and long-horizon forecasting of stock market volatility. We hence repeat our time-series analysis but investigate a lagged relationship rather than a contemporaneous one for the U.S. Equation (10) is modified as follows:

$$d_{U.S.,t} = \alpha_{U.S.} + \beta_{U.S.} X_{U.S.,t-h} + \epsilon_t \quad (20)$$

considering lags from one quarter, half a year and one year ( $h = 1, 2, 4$ ).<sup>19</sup> Table 13 presents the results for the three horizons in the three panels. Consistent with our main results, we find that inflation, short and long interest rates and GDP have a negative impact on the memory parameter while unemployment has a positive relationship with the memory parameter. The relationship between GDP and the memory parameter di-

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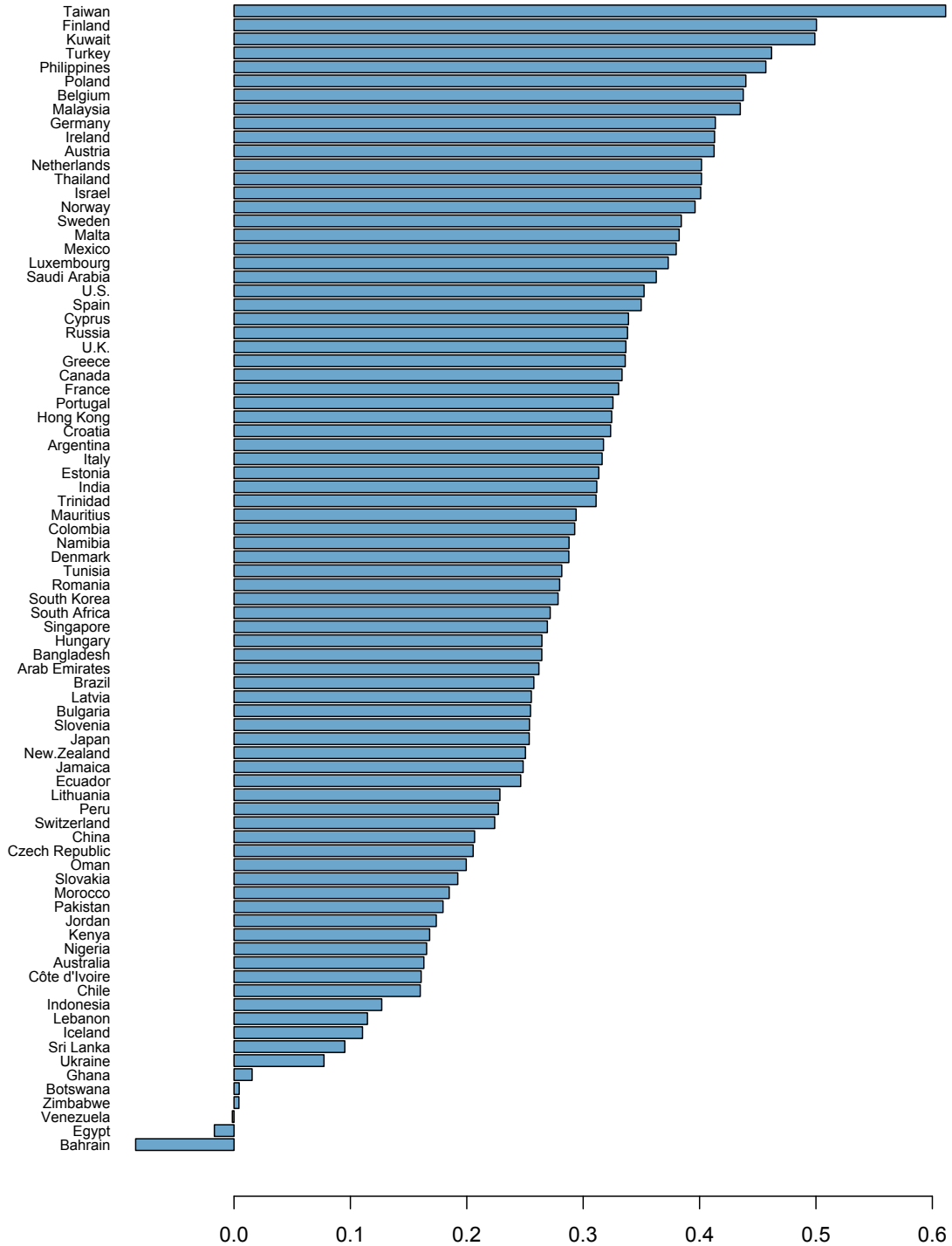
<sup>19</sup>We conduct this analysis in quarterly frequency because GDP data is only available at this frequency.

minishes for longer horizons and the slope coefficient is no longer statistically significant. The adjusted  $R^2$  varies between 0% and 39% for the univariate regressions. Hence, the relationship between memory and macroeconomic variables found in our main contemporaneous analysis persists into the future for up to one year.

## V Conclusion

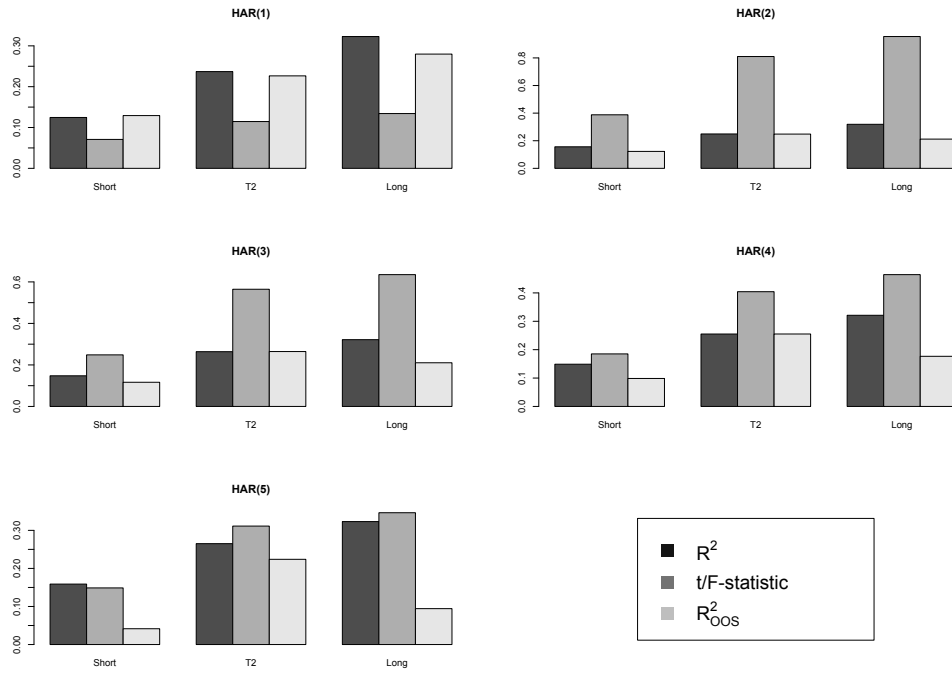
In this paper we shed new light on long memory in the volatility of international equity markets. With the help of portfolio sorts and cross-sectional regressions, we demonstrate how the memory parameter of a country stock index volatility can be explained by country-specific macroeconomic variables such as inflation, unemployment rates, interest rates and jumps. We show that macroeconomic variables help explain the memory parameter, both in the time-series and the cross-sectional dimension. Following the existing literature, we provide economically reasonable explanations for the sign of the relationships. In addition, classifications such as OECD, developed, emerging or frontier countries also matter for the memory parameter. More developed countries possess a higher memory parameter while frontier and emerging countries possess a shorter memory in volatility. Our results are robust against various variations of the examined models.

Figure 1: Memory Estimates of International Countries



This figure shows the memory parameter estimates applying the GPH estimator and a bandwidth parameter of  $m = N^{0.5}$  to the eighty-two countries for the period from January 1964 until December 2015.

Figure 2: Predictability of Tertile Portfolios



This figure reports adjusted  $R^2$ , t-statistics, F-statistics and  $R^2_{OOS}$  for tertile portfolios of the cross-section of countries. For a better presentation, the test statistics are all divided by 100.



Table 1: Summary Statistics

This table presents the summary statistics for the long memory volatility of international countries. The memory parameter is estimated with the GPH estimator and a bandwidth parameter of  $m = N^{0.5}$ . Obs. in column (1) stands for the number of observations, SD stands for the standard deviation, column (2) reports selected quantiles; t-statistic in column (3) reports the mean t-statistic, Sign. at 5% reports the proportion of significant long memory estimates, while the remainder of column (3) reports the proportion of the memory parameter being in a certain interval.

Descriptive		Quantiles		Memory	
Obs.	82	5%	0.01	t-statistic	3.95
Mean	0.27	25%	0.20	Sign. at 5%	0.87
SD	0.13	Median	0.28	-0.5<d<0.0	0.04
Skewness	-0.41	75%	0.35	0.0<d<0.5	0.94
Kurtosis	0.28	95%	0.46	0.5<d<1.0	0.02

Table 2: Long Memory and Predictability – Cross-Section of Countries

This table reports the results predictive regressions. We estimate the proposed HAR models by simple linear regressions including the previous 1, 6, 12, 24 and 60 observations. We form tertile portfolios where countries with the lowest memory parameter are in the first tertile and countries with the highest memory parameter are in the third tertile. The memory parameter is estimated with the GPH estimator and a bandwidth parameter of  $m = N^{0.5}$ . We report average adjusted  $R^2$  in Panel A, average t-statistics and F-statistics in Panel B and out-of-sample  $R^2$  in Panel C.

	T1	T2	T3
<i>Panel A: Adjusted <math>R^2</math></i>			
HAR(1)	0.1246	0.2370	0.3229
HAR(2)	0.1560	0.2491	0.3190
HAR(3)	0.1476	0.2638	0.3217
HAR(4)	0.1488	0.2552	0.3212
HAR(5)	0.1588	0.2651	0.3230
<i>Panel B: T-statistic/F-statistic</i>			
HAR(1)	7.0841	11.4621	13.4188
HAR(2)	38.7906	81.0082	95.4979
HAR(3)	24.8456	56.4617	63.5065
HAR(4)	18.5080	40.4269	46.4415
HAR(5)	14.8762	31.1230	34.6305
<i>Panel C: <math>R^2_{OOS}</math></i>			
HAR(1)	0.1292	0.2265	0.2798
HAR(2)	0.1227	0.2482	0.2118
HAR(3)	0.1165	0.2645	0.2104
HAR(4)	0.0986	0.2552	0.1766
HAR(5)	0.0415	0.2239	0.0943

Table 3: Long Memory of the U.S.

This table presents the coefficients from the regressions of the memory parameter on macroeconomic variables for the U.S. for the period from 1964 until 2015. The regressors are the inflation, the log unemployment, the treasury bill and the government bond rates and the GDP growth. Recession is the indicator function that represents periods of expansion and recession defined by the NBER. All the macroeconomic variables are monthly except for GDP, hence Model 5 and Model 8 are on a quarterly basis. The memory parameter is estimated with the GPH estimator and a bandwidth parameter of  $m = N^{0.5}$  applied to squared returns. Stars indicate significance of the mean differences: \* significant at  $p < 0.10$ ; \*\*  $p < 0.05$ ; \*\*\*  $p < 0.01$ .

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8
(Intercept)	0.4244*** (0.0142)	0.4070*** (0.0114)	0.5356*** (0.0161)	0.7847*** (0.0299)	0.4352*** (0.0275)	0.4125*** (0.0121)	0.9302*** (0.0434)	1.0393*** (0.0566)
Inflation	-7.9649* (4.2795)						3.9341 (3.3853)	2.1536 (4.9511)
Unemployment		0.2143** (0.0998)					0.7310*** (0.1290)	0.7641*** (0.0925)
Tbill			-0.0452*** (0.0045)				0.0501*** (0.0115)	0.0930*** (0.0150)
Gov.Bonds				-0.0711*** (0.0054)			-0.1283*** (0.0137)	-0.1719*** (0.0178)
GDP					-5.0221 (3.1868)			1.4630 (2.7594)
Recession						-0.0344 (0.0363)	0.0270 (0.0286)	-0.0084 (0.0533)
adj. $R^2$	0.0080	0.0117	0.2453	0.3630	0.0145	-0.0003	0.4181	0.6271

Table 4: Long Memory of the Cross-Section of Countries

This table presents the statistics from the regressions of the memory parameter on the macroeconomic variables for eighty-two countries for the period from 1964 until 2015. The regressors are the inflation, the log unemployment, treasury bill and the government bond rates, and the GDP growth. Recession is the indicator function that represents periods of expansion and recession defined by the NBER. The memory parameter is estimated with the GPH estimator and a bandwidth parameter of  $m = N^{0.5}$ . The first row reports the median of the coefficients over the cross-section. The second (third) row reports the percentage of countries for which the slope is negative (positive) and statistically significant at a 5% level. The fourth row reports the average absolute t-statistic across all countries and the fifth row reports the average adjusted  $R^2$  over all countries.

	Inflation	Unemployment	Tbill	Gov.Bonds	GDP	Recession	KS ex. GDP	KS
Median	-0.15	0.07	-0.01	-0.02	-0.05	-0.01		
$\beta < 0$ (significant)	6.49%	18.97%	62.69%	55.00%	2.50%	18.99%		
$\beta > 0$ (significant)	3.90%	24.14%	23.88%	21.67%	0.00%	13.92%		
t-statistic	0.97	2.16	8.04	8.02	0.81	1.61		
Adj. $R^2$	0.01	0.04	0.20	0.19	0.01	0.02	0.37	0.37

Table 5: International Portfolio Sorts

This table presents the average macroeconomic variables of the tertile portfolios sorted by the memory parameter. The investigated countries are the eighty-two following [Pukthuanthong & Roll \(2015\)](#) over the period from 1964 until 2015. Long memory is estimated with the GPH estimator and a bandwidth parameter of  $m = N^{0.5}$ . The column *LMS* reports the difference of the third and first portfolio with t-statistics in squared brackets.

	T1	T2	T3	T3-T1 (LMS)	
Inflation	0.0039	0.0034	0.0034	-0.0005	[-1.2397]
Unemployment	7.7295	7.3664	6.9280	-0.8015	[-3.0940]
Tbill	12.0172	10.5784	9.5123	-2.5048	[-1.0116]
Gov.Bonds	9.8846	8.5284	7.7230	-2.1616	[-3.2466]
GDP	0.0034	0.0033	0.0067	0.0034	[1.8528]
BNS	-3.9505	-0.3542	-0.2565	3.6940	[2.0753]
BNS-I	0.0843	0.0299	0.0180	-0.0662	[-4.5159]

Table 6: Cross-Sectional Regressions

This table presents results from the cross-sectional regressions. The dependent variable is the memory parameter for each country and the regressors are the inflation, the log unemployment, treasury bill and government bond rates, GDP growth and jumps measured by BNS. The investigated countries are the eighty-two following [Pukthuanthong & Roll \(2015\)](#) over the period from 1964 until 2015. Long memory is estimated with the GPH estimator and a bandwidth parameter of  $m = N^{0.5}$ . We report time-series averages and standard errors in parentheses below. Stars indicate significance of the mean differences: \* significant at  $p < 0.10$ ; \*\*  $p < 0.05$ ; \*\*\*  $p < 0.01$ .

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7
Intercept	0.0036*** (0.0009)	8.3460*** (0.0663)	11.9472*** (0.4604)	10.6636*** (0.1186)	0.0015 (0.0042)	-4.3354*** (0.8375)	0.2287*** (0.0197)
Inflation	-0.0003 (0.0017)						-0.1006 (0.4048)
Unemployment		-3.7159*** (0.1661)					-0.0008 (0.0011)
Tbill			-4.3856*** (1.3340)				-0.0047** (0.0021)
Gov.Bonds				-5.4660*** (0.3698)			0.0068** (0.0030)
GDP					-0.0086 (0.0088)		
BNS						10.1832*** (2.0815)	0.0308*** (0.0055)

Table 7: Long Memory in Developed and Emerging Countries

This table presents the cross-sectional regressions of the memory estimates on the dummy variables. The memory parameter is estimated with the GPH estimator and a bandwidth parameter of  $m = N^{0.5}$ . The investigated countries are the eighty-two following [Pukthuanthong & Roll \(2015\)](#) over the period from 1964 until 2015 in Panel A. Panel B investigates the subperiod from 2008 until 2015. OECD, Emerging, Developed and Frontier indicate whether a country is part of the OECD group, an emerging, developed or a frontier country according to the definition of Thomson Reuters Tickhistory (TRTH) or Morgan Stanley Capital International (MSCI). We repeat the estimation of the memory parameter at a monthly frequency relying on rolling windows of five years of daily observations. Each month we run the same cross-sectional regression as in Panel A and B and report the time-series averages of the coefficients in Panel C with the standard errors in parentheses below. We also report the average of the adjusted  $R^2$  over the sample period. Stars indicate significance of the mean differences: \* significant at  $p < 0.10$ ; \*\* $p < 0.05$ ; \*\*\* $p < 0.01$ .

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
<i>Panel A: 1964-2015</i>						
(Intercept)	0.2444*** (0.0170)	0.3250*** (0.0246)	0.2472*** (0.0160)	0.2609*** (0.0167)	0.3115*** (0.0160)	0.2428*** (0.0388)
OECD (TRTH)	0.0836** (0.0286)					
Emerging (TRTH)		-0.0748** (0.0298)				
Developed (MSCI)			0.0953** (0.0302)			0.0997** (0.0457)
Emerging (MSCI)				0.0466 (0.0316)		0.0646 (0.0457)
Frontier (MSCI)					-0.1142*** (0.0278)	-0.0455 (0.0448)
adj. $R^2$	0.0853	0.0616	0.0996	0.0143	0.1636	0.1919
<i>Panel B: 2008-2015</i>						
(Intercept)	0.3608*** (0.0268)	0.5255*** (0.0386)	0.3548*** (0.0237)	0.4279*** (0.0275)	0.4496*** (0.0277)	0.2177*** (0.0584)
OECD (TRTH)	0.1675*** (0.0448)					
Emerging (TRTH)		-0.1542** (0.0468)				
Developed (MSCI)			0.2324*** (0.0446)			0.3694*** (0.0689)
Emerging (MSCI)				-0.0252 (0.0516)		0.1850** (0.0689)
Frontier (MSCI)					-0.0898* (0.0489)	0.1420** (0.0677)
adj. $R^2$	0.1396	0.1098	0.2466	-0.0096	0.0288	0.2936
<i>Panel C: Time-Series Averages</i>						
Coefficient	0.0455*** (0.0048)	-0.0120** (0.0054)	0.0402*** (0.0067)	0.0363*** (0.0038)	-0.0552*** (0.0065)	
adj. $R^2$	0.0518	0.0551	0.0947	0.0125	0.0463	

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# The Long Memory of Equity Volatility: International Evidence

August 10, 2017

Online Appendix

Table 8: Overview of Country Sample

This table presents the eighty-two countries and their availability from Datastream. We rely on a common currency, the U.S. dollar, for all values. We work with either the total return index (“RI”) or the pure price index (“PI”).

Country	Datastream	Availability	Index Identification	Datastream Mnemonic	Country	Datastream	Availability	Index Identification	Datastream Mnemonic
Argentina	2-Aug-93	31-Dec-15	ARGENTINA Merval	ARGMerv(PI)~U\$	Lithuania	31-Dec-99	31-Dec-15	OMX VILNIUS (OMXV)	LNVILSE(RI)~U\$
Australia	1-Jan-73	31-Dec-15	AUSTRALIA-DS Market	TOTMAU\$(RI)	Luxembourg	2-Jan-92	31-Dec-15	LUXEMBURG-DS Market	TOTMKLX(RI)
Austria	1-Jan-73	31-Dec-15	AUSTRIA-DS Market	TOTMKOE(PI)~U\$	Malaysia	2-Jan-80	31-Dec-15	KLCI COMPOSITE	KLPCOMP(PI)~U\$
Bahrain	31-Dec-99	31-Dec-15	DOW JONES BAHRAIN	DJBAHR\$(PI)	Malta	27-Dec-95	31-Dec-15	MALTA SE MSE -	MALTAIX(PI)~U\$
Bangladesh	1-Jan-90	1-Apr-13	BANGLADESH SE ALL SHARE	BDTALSH(PI)~U\$	Mauritius	29-Dec-95	31-Dec-15	S&P/IFCF M MAURITIUS	IFFMMAL(PI)~U\$
Belgium	1-Jan-73	31-Dec-15	BELGIUM-DS Market	TOTMKBG(RI)~U\$	Mexico	4-Jan-88	31-Dec-15	MEXICO IPC (BOLSA)	MXIPC35(PI)~U\$
Botswana	29-Dec-95	31-Dec-15	S&P/IFCF M BOTSWAO.	IFFMBOL(PI)~U\$	Morocco	31-Dec-87	31-Dec-15	MOROCCO SE CFG25	MDCFG25(PI)~U\$
Brazil	7-Apr-83	31-Dec-15	BRAZIL BOVESPA	BRBOVES(RI)~U\$	Namibia	31-Jan-00	31-Dec-15	S&P/IFCF M NAMBIA	IFFMNAL(PI)~U\$
Bulgaria	20-Oct-00	31-Dec-15	BSE SOFIX	BSSOFIX(PI)~U\$	Netherlands	1-Jan-73	31-Dec-15	NETHERLAND-DS Market	TOTMKNL(RI)~U\$
Canada	31-Dec-64	31-Dec-15	S&P/TSX COMPOSITE INDEX	TTOCOMP(PI)~U\$	New Zealand	4-Jan-88	31-Dec-15	NEW ZEALAND-DS Market	TOTMNZ\$(RI)
Chile	2-Jan-87	31-Dec-15	CHILE GENERAL (IGPA)	IGPAGEN(PI)~U\$	Nigeria	30-Jun-95	31-Dec-15	S&P/IFCG D NIGERIA	IFGDNGL(PI)~U\$
China	3-Apr-91	31-Dec-15	SHENZHEN SE COMPOSITE	CHZCOMP(PI)~U\$	Norway	2-Jan-80	31-Dec-15	NORWAY-DS Market	TOTMNV\$(RI)
Colombia	10-Mar-92	31-Dec-15	COLOMBIA-DS Market	TOTMKCB(RI)~U\$	Oman	22-Oct-96	31-Dec-15	OMAN MUSCAT SECURITIES MKT.	OMANMSM(PI)~U\$
Côte d'Ivoire	29-Dec-95	31-Dec-15	S&P/IFCF M CÔTE D'IVOIRE	IFFMCIL(RI)~U\$	Pakistan	30-Dec-88	31-Dec-15	KARACHI SE 100	PKSE100(PI)~U\$
Croatia	2-Jan-97	31-Dec-15	CROATIA CROBEX	CTCROBE(PI)~U\$	Peru	2-Jan-91	31-Dec-15	LIMA SE GENERAL(IGBL)	PEGENRL(PI)~U\$
Cyprus	3-Sep-04	31-Dec-15	CYPRUS GENERAL	CYPMAPM(PI)~U\$	Philippines	2-Jan-86	31-Dec-15	PHILIPPINE SE I(PSEi)	PSECOMP(PI)~U\$
Czech Republic	9-Nov-93	31-Dec-15	CZECH REP.-DS NON-FINCIAL	TOTLICZ(RI)~U\$	Poland	16-Apr-91	31-Dec-15	WARSAW GENERALINDEX	POLWIG(RI)~U\$
Denmark	31-Dec-69	31-Dec-15	MSCI DENMARK	MSDNMKL(RI)~U\$	Portugal	5-Jan-88	31-Dec-15	PORTUGAL PSI GENERAL	POPSIGN(PI)~U\$
Ecuador	2-Aug-93	31-Dec-15	ECUADOR ECU (U\$)	ECUECUI(PI)	Romania	19-Sep-97	31-Dec-15	ROMANIA BET (L)	RMBETRL(PI)~U\$
Egypt	2-Jan-95	31-Dec-15	EGYPT HERMES FINANCIAL	EGHFINC(PI)~U\$	Russia	1-Sep-95	31-Dec-15	RUSSIA RTS INDEX	RSRTSIN(PI)~U\$
Estonia	3-Jun-96	31-Dec-15	OMX TALLINN (OMXT)	ESTALSE(PI)~U\$	Saudi Arabia	31-Dec-97	31-Dec-15	S&P/IFCG D SAUDI ARABIA	IFGDSB\$(RI)
Finland	2-Jan-91	31-Dec-15	OMX HELSINKI (OMXH)	HEXINDX(RI)~U\$	Singapore	1-Jan-73	31-Dec-15	SINGAPORE-DS Market EX TMT	TOTXTSG(RI)~U\$
France	1-Jan-73	31-Dec-15	FRANCE-DS Market	TOTMKFR(RI)~U\$	Slovakia	14-Sep-93	31-Dec-15	SLOVAKIA SAX 16	SXSAX16(PI)~U\$
Germany	31-Dec-64	31-Dec-15	DAX 30 PERFORMANCE	DAXINDX(RI)~U\$	Slovenia	31-Dec-93	14-Oct-10	SLOVENIAN EXCH. STOCK (SBI)	SLOESBI(PI)~U\$
Ghana	29-Dec-95	31-Dec-15	S&P/IFCF M GHA0.	IFFMGHL(PI)~U\$	South Africa	1-Jan-73	31-Dec-15	SOUTH AFRICA-DS Market	TOTMSA\$(RI)
Greece	26-Jan-06	31-Dec-15	ATHEX COMPOSITE	GRAGENL(RI)~U\$	South Korea	31-Dec-74	31-Dec-15	KOREA SE COMPOSITE (KOSPI)	KORCOMP(PI)~U\$
Hong Kong	2-Jan-90	31-Dec-15	HANG SENG	HNGKNGI(RI)~U\$	Spain	2-Jan-74	31-Dec-15	MADRID SE GENERAL	MADRIDI(PI)~U\$
Hungary	2-Jan-91	31-Dec-15	BUDAPEST (BUX)	BUXINDX(PI)~U\$	Sri Lanka	2-Jan-85	31-Dec-15	COLOMBO SE ALLSHARE	SRALLSH(PI)~U\$
Iceland	31-Dec-92	31-Dec-15	OMX ICELAND ALLSHARE	ICEXALL(PI)~U\$	Sweden	28-Dec-79	31-Dec-15	OMX STOCKHOLM (OMXS)	SWSEALI(PI)~U\$
India	2-Jan-87	31-Dec-15	INDIA BSE (100) NATIONAL	IBOMBSE(PI)~U\$	Switzerland	1-Jan-73	31-Dec-15	SWITZ-DS Market	TOTMKSW(RI)~U\$
Indonesia	2-Apr-90	31-Dec-15	INDONESIA-DS Market	TOTMKID(RI)~U\$	Taiwan	31-Dec-84	31-Dec-15	TAIWAN SE WEIGHTED	TAIWGHT(PI)~U\$
Ireland	1-Jan-73	31-Dec-15	IRELAND-DS Market	TOTMIR\$(RI)	Thailand	2-Jan-87	31-Dec-15	THAILAND-DS Market	TOTMTH\$(RI)
Israel	23-Apr-87	31-Dec-15	ISRAEL TA 100	ISTA100(PI)~U\$	Trinidad	29-Dec-95	31-Dec-15	S&P/IFCF M TRINIDAD & TOBAGO	IFFMTTL(PI)~U\$
Italy	1-Jan-73	31-Dec-15	ITALY-DS Market	TOTMIT\$(RI)	Tunisia	31-Dec-97	31-Dec-15	TUNISIA TUNINDEX	TUTUNIN(PI)~U\$
Jamaica	29-Dec-95	31-Dec-15	S&P/IFCF M JAMAICA	IFFMJAL(PI)~U\$	Turkey	4-Jan-88	31-Dec-15	ISE TIOL 100	TRKISTB(PI)~U\$
Japan	1-Jan-73	31-Dec-15	TOPIX	TOKYOSE(RI)~U\$	Ukraine	30-Jan-98	31-Dec-15	S&P/IFCF M UKRAINE	IFFMURL(PI)~U\$
Jordan	21-Nov-88	31-Dec-15	AMMAN SE FINANCIAL Market	AMMANFM(PI)~U\$	Utd. Arab	1-June-05	31-Dec-15	MSCI UAE	MSUAE\$
Kenya	11-Jan-90	31-Dec-15	KENYA NAIROBI SE	NSEINDX(PI)~U\$	United Kingdom	1-Jan-65	31-Dec-15	UK-DS Market	TOTMUK\$(RI)
Kuwait	28-Dec-94	31-Dec-15	KUWAIT KIC GENERAL	KWKICGN(PI)~U\$	United States	4-Jan-68	31-Dec-15	S&P 500 COMPOSITE	S&PCOMP(RI)~U\$
Latvia	3-Jan-00	31-Dec-15	OMX RIGA (OMXR)	RIGSEIN(RI)~U\$	Venezuela	2-Jan-90	31-Dec-15	VENEZUELA-DS Market	TOTMVE\$(RI)
Lebanon	31-Jan-00	31-Dec-15	S&P/IFCF M LEBANON	IFFMLEL(PI)~U\$	Zimbabwe	6-Apr-88	6-Oct-06	ZIMBABWE INDUSTRIALS	ZIMINDS(PI)

Table 9: Long Memory for the Cross-Section of Countries – Panel Regression

This table presents the statistics from the panel regressions of the memory parameter on macroeconomic variables for eighty-two countries for the period from 1964 until 2015. The regressors are the inflation, the log unemployment, treasury bill and the government bond rates, and the GDP growth. Recession is the indicator function that represents periods of expansion and recession defined by the NBER and BNS presents the [Barndorff-Nielsen et al. \(2009\)](#) jump test statistic. The memory parameter is estimated with the GPH estimator and a bandwidth parameter of  $m = N^{0.5}$ . Stars indicate significance of the mean differences: \* significant at  $p < 0.10$ ; \*\* $p < 0.05$ ; \*\*\* $p < 0.01$ .

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8
Inflation	-0.0027 (0.0227)						-0.0425 (0.0927)	-0.0680 (0.1472)
Unemployment		-0.0057*** (0.0002)					-0.0014*** (0.0003)	-0.0267 (0.0530)
Tbill			-0.0003*** (0.0001)				-0.0008 (0.0007)	-0.0024 (0.0015)
Gov				-0.0046*** (0.0003)			-0.0078*** (0.0008)	-0.0070*** (0.0014)
GDP					-0.0138 (0.0304)			-0.1210* (0.0706)
BNS						0.0001** (0.0000)	0.0009*** (0.0004)	0.0034*** (0.0010)

Table 10: Long Memory of the U.S. – Alternative Long Memory Estimates

This table presents the coefficients from the regressions of the memory parameter on the macroeconomic variables for the U.S. for the period from 1964 until 2015. The regressors are the inflation, the log unemployment, treasury bills and government bond rates and GDP growth. Recession is the indicator function that represents periods of expansion and recession defined by the NBER. All macroeconomic variables are monthly except for GDP, hence Model 5 and Model 8 are on a quarterly basis. Long memory is estimated with the GPH estimator and a bandwidth choice of  $m = N^{0.6}$  and  $m = N^{0.7}$  in Panel A and B, respectively. The GPH estimator is applied to absolute returns and a bandwidth of  $m = N^{0.5}$  in Panel C and Panel D shows results relying on the LW estimator and  $m = N^{0.5}$ . Stars indicate significance of the mean differences: \* significant at  $p < 0.10$ ; \*\* $p < 0.05$ ; \*\*\* $p < 0.01$ .

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8
<i>Panel A: GPH estimator (<math>m = N^{0.6}</math>)</i>								
(Intercept)	0.4373*** (0.0176)	0.4148*** (0.0142)	0.5999*** (0.0189)	0.9142*** (0.0358)	0.4136*** (0.0244)	0.4117*** (0.0151)	0.9819*** (0.0527)	1.1624*** (0.0604)
Inflation	-10.3997* (5.3207)						5.3654 (4.1047)	2.0343 (5.2583)
Unemployment		0.2539** (0.1242)					0.5740*** (0.1564)	1.1672*** (0.1170)
Tbill			-0.0652*** (0.0053)				0.0240* (0.0139)	0.0975*** (0.0158)
Gov.Bonds				-0.0940*** (0.0064)			-0.1246*** (0.0166)	-0.1951*** (0.0184)
GDP					-1.3916 (1.0635)			2.9567*** (0.8567)
Recession						0.0454 (0.0451)	0.1032** (0.0346)	0.0575 (0.0425)
adj. $R^2$	0.0092	0.0104	0.3308	0.4108	0.0070	0.0000	0.4472	0.7193
<i>Panel B: GPH estimator (<math>m = N^{0.7}</math>)</i>								
(Intercept)	0.2889*** (0.0089)	0.2790*** (0.0072)	0.3745*** (0.0094)	0.5912*** (0.0143)	0.2772*** (0.0127)	0.2793*** (0.0077)	0.5961*** (0.0215)	0.6656*** (0.0323)
Inflation	-5.6173** (2.6987)						2.5356 (1.6747)	1.9762 (2.8152)
Unemployment		-0.1573** (0.0628)					-0.0811 (0.0638)	0.2652*** (0.0626)
Tbill			-0.0345*** (0.0026)				0.0024 (0.0057)	0.0264** (0.0085)
Gov.Bonds				-0.0592*** (0.0026)			-0.0624*** (0.0068)	-0.0881*** (0.0098)
GDP					0.4833 (0.5524)			1.2870** (0.4587)
Recession						-0.0132 (0.0229)	0.0083 (0.0141)	-0.0060 (0.0228)
adj. $R^2$	0.0108	0.0170	0.3587	0.6349	-0.0023	-0.0022	0.6429	0.6990

Long Memory of the U.S. – Alternative Long Memory Estimates Continued

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8
<i>Panel C: GPH estimator (absolute returns; <math>m = N^{0.5}</math>)</i>								
(Intercept)	0.5223*** (0.0116)	0.5059*** (0.0093)	0.6125*** (0.0131)	0.8071*** (0.0248)	0.5037*** (0.0157)	0.5100*** (0.0099)	0.9182*** (0.0361)	1.0064*** (0.0455)
Inflation	-7.4465** (3.4964)						2.1738 (2.8094)	0.2664 (3.9656)
Unemployment		0.2155** (0.0813)					0.6103*** (0.1070)	0.6458*** (0.0882)
Tbill			-0.0374*** (0.0037)				0.0381*** (0.0095)	0.0737*** (0.0119)
Gov.Bonds				-0.0566*** (0.0045)			-0.1001*** (0.0114)	-0.1342*** (0.0139)
GDP					-1.8885** (0.6830)			-0.0508 (0.6461)
Recession						-0.0211 (0.0297)	0.0273 (0.0237)	-0.0044 (0.0321)
adj. $R^2$	0.0115	0.0194	0.2501	0.3435	0.0617	-0.0016	0.4017	0.6342
<i>Panel D: LW estimator (<math>m = N^{0.5}</math>)</i>								
(Intercept)	0.3837*** (0.0120)	0.3567*** (0.0094)	0.4945*** (0.0123)	0.7241*** (0.0203)	0.3528*** (0.0164)	0.3526*** (0.0100)	0.7975*** (0.0299)	0.8827*** (0.0404)
Inflation	-12.9398*** (3.5222)						2.3408 (2.5542)	1.0516 (3.7541)
Unemployment		0.2536** (0.0821)					0.3982*** (0.0925)	0.6321*** (0.0825)
Tbill			-0.0428*** (0.0030)				0.0303*** (0.0082)	0.0610*** (0.0107)
Gov.Bonds				-0.0655*** (0.0034)			-0.0976*** (0.0094)	-0.1289*** (0.0123)
GDP					-1.5143** (0.7193)			1.3986** (0.6124)
Recession						0.0321 (0.0315)	0.0494** (0.0217)	-0.0035 (0.0305)
adj. $R^2$	0.0360	0.0248	0.3775	0.5215	0.0300	0.0001	0.5449	0.7117



Table 11: Long Memory in Developed and Emerging Countries – Alternative Estimates

This table presents the cross-sectional regressions of the memory estimates on the dummy variables. The investigated countries are the eighty-two following [Pukthuanthong & Roll \(2015\)](#) over the period from 1964 until 2015. Long memory is estimated with the GPH estimator and a bandwidth choice of  $m = N^{0.6}$  and  $m = N^{0.7}$  in Panel A and B, respectively. The GPH estimator is applied to absolute returns and a bandwidth of  $m = N^{0.5}$  in Panel C and Panel D shows results relying on the LW estimator and  $m = N^{0.5}$ . Stars indicate significance of the mean differences: \* significant at  $p < 0.10$ ; \*\* $p < 0.05$ ; \*\*\* $p < 0.01$

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
<i>Panel A: GPH estimator (<math>m = N^{0.6}</math>)</i>						
(Intercept)	0.2515*** (0.0183)	0.3573*** (0.0275)	0.2616*** (0.0177)	0.2814*** (0.0188)	0.3345*** (0.0182)	0.2209*** (0.0428)
OECD (TRTH)	0.1242*** (0.0308)					
Emerging (TRTH)		-0.0907** (0.0332)				
Developed (MSCI)			0.1206*** (0.0334)			0.1612** (0.0505)
Emerging (MSCI)				0.0500 (0.0355)		0.1104** (0.0505)
Frontier (MSCI)					-0.1189*** (0.0317)	-0.0053 (0.0495)
adj. $R^2$	0.1583	0.0738	0.1297	0.0119	0.1388	0.2189
<i>Panel B: GPH estimator (<math>m = N^{0.7}</math>)</i>						
(Intercept)	0.2083*** (0.0167)	0.3166*** (0.0262)	0.2225*** (0.0165)	0.2462*** (0.0180)	0.3022*** (0.0168)	0.2112*** (0.0397)
OECD (TRTH)	0.1415*** (0.0281)					
Emerging (TRTH)		-0.0853** (0.0317)				
Developed (MSCI)			0.1278*** (0.0312)			0.1391** (0.0468)
Emerging (MSCI)				0.0434 (0.0339)		0.0785* (0.0468)
Frontier (MSCI)					-0.1330*** (0.0292)	-0.0420 (0.0458)
adj. $R^2$	0.2318	0.0715	0.1632	0.0078	0.1959	0.2630

Long Memory in Developed and Emerging Countries – Alternative Estimates Continued

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
<i>Panel C: LW estimator (<math>m = N^{0.5}</math>)</i>						
(Intercept)	0.2546*** (0.0147)	0.3028*** (0.0212)	0.2551*** (0.0137)	0.2587*** (0.0139)	0.3054*** (0.0135)	0.2449*** (0.0329)
OECD (TRTH)	0.0544** (0.0246)					
Emerging (TRTH)		-0.0423 (0.0256)				
Developed (MSCI)			0.0667** (0.0260)			0.0769* (0.0388)
Emerging (MSCI)				0.0540** (0.0263)		0.0678* (0.0388)
Frontier (MSCI)					-0.0958*** (0.0235)	-0.0353 (0.0380)
adj. $R^2$	0.0456	0.0209	0.0646	0.0380	0.1618	0.1838
<i>Panel D: GPH estimator (absolute returns; <math>m = N^{0.5}</math>)</i>						
(Intercept)	0.3938*** (0.0140)	0.4584*** (0.0195)	0.3932*** (0.0131)	0.4154*** (0.0136)	0.4228*** (0.0139)	0.3842*** (0.0338)
OECD (TRTH)	0.0502** (0.0235)					
Emerging (TRTH)		-0.0685** (0.0236)				
Developed (MSCI)			0.0657** (0.0247)			0.0747* (0.0399)
Emerging (MSCI)				-0.0136 (0.0257)		0.0177 (0.0399)
Frontier (MSCI)					-0.0340 (0.0243)	0.0046 (0.0390)
adj. $R^2$	0.0422	0.0839	0.0701	-0.0090	0.0117	0.0498

Table 12: International Portfolio Sorts – Alternative Long Memory Estimates

This table presents the average macroeconomic variables of the tertile portfolios sorted by the memory parameter. The investigated countries are the eighty-two following [Pukthuanthong & Roll \(2015\)](#) over the period from 1964 until 2015. Long memory is estimated with the GPH estimator and a bandwidth choice of  $m = N^{0.6}$  and  $m = N^{0.7}$  in Panel A and B, respectively. The GPH estimator is applied to absolute returns and a bandwidth of  $m = N^{0.5}$  in Panel C and Panel D shows results relying on the LW estimator and  $m = N^{0.5}$ . The column *LMS* reports the difference of the third and first portfolio with t-statistics in squared brackets.

	T1	T2	T3	T3- T1 (LMS)	
<i>Panel A: GPH estimator (<math>m = N^{0.6}</math>)</i>					
Inflation	0.0038	0.0035	0.0033	-0.0005	[-0.6361]
Unemployment	7.7592	7.5606	6.8455	-0.9137	[-2.4542]
Tbill	11.9322	8.4317	11.3281	-0.6040	[-0.2547]
Gov.Bonds	10.2931	8.0374	8.0045	-2.2886	[-3.4877]
GDP	0.0018	0.0059	0.0038	0.0020	[1.7396]
BNS	-3.7743	-0.2091	-0.1562	3.6182	[2.0425]
BNS-I	0.0955	0.0148	0.0095	-0.0860	[-3.9556]
<i>Panel B: GPH estimator (<math>m = N^{0.7}</math>)</i>					
Inflation	0.0037	0.0031	0.0034	-0.0003	[-0.4056]
Unemployment	7.5144	7.4730	6.8688	-0.6456	[-1.3959]
Tbill	13.4881	9.9356	8.6620	-4.8262	[-1.4858]
Gov.Bonds	10.1239	8.4567	7.3953	-2.7287	[-6.3381]
GDP	0.0037	0.0033	0.0083	0.0046	[4.0613]
BNS	-3.6394	-0.2806	-0.1811	3.4583	[2.0498]
BNS-I	0.0904	0.0197	0.0113	-0.0791	[-3.5078]
<i>Panel C: GPH estimator (absolute returns; <math>m = N^{0.5}</math>)</i>					
Inflation	0.0037	0.0031	0.0033	-0.0004	[-0.5899]
Unemployment	7.7897	7.5074	6.7241	-1.0656	[-3.0034]
Tbill	13.6766	9.4347	8.6176	-5.0591	[-1.4161]
Gov.Bonds	9.5664	8.9168	7.8552	-1.7113	[-3.1334]
GDP	0.0044	0.0041	0.0066	0.0022	[1.7534]
BNS	-2.5185	-1.5721	-0.4765	2.0419	[2.8122]
BNS-I	0.0698	0.0382	0.0242	-0.0456	[-4.2736]
<i>Panel D: LW estimator (<math>m = N^{0.5}</math>)</i>					
Inflation	0.0042	0.0041	0.0047	0.0005	[0.8343]
Unemployment	7.3763	7.1598	6.6214	-0.7549	[-3.2149]
Tbill	13.0206	10.3177	9.8895	-3.1312	[-1.2597]
Gov.Bonds	9.9875	8.6120	7.9389	-2.0485	[-3.7036]
GDP	-0.0011	0.0056	0.0069	0.0079	[2.5104]
BNS	-4.0822	-0.9068	-0.4097	3.6724	[2.3223]
BNS-I	0.1148	0.0323	0.0203	-0.0945	[-4.4816]

Table 13: Long Memory of the U.S. – Predictive Regressions

This table presents the coefficients from the regressions of the memory parameter on the macroeconomic variables for the U.S. for the period from 1964 until 2015. The regressors are the log consumer price index, the log unemployment, treasury bill and the government bond rates and GDP growth lagged by  $h$  quarters. Recession is the indicator function that represents periods of expansion and recession defined by the NBER. All macroeconomic variables are monthly except for GDP, hence Model 5 and Model 8 are on a quarterly basis. Long memory is estimated with the GPH estimator and a bandwidth parameter of  $m = N^{0.5}$ . Stars indicate significance of the mean differences: \* significant at  $p < 0.10$ ; \*\*  $p < 0.05$ ; \*\*\*  $p < 0.01$

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8
<i>Panel A: h = 1</i>								
(Intercept)	0.4141*** (0.0110)	-0.2506*** (0.0748)	0.5431*** (0.0159)	0.7936*** (0.0298)	0.4041*** (0.0197)	0.4106*** (0.0121)	-0.0709 (0.0817)	-0.5865** (0.2577)
Inflation	-7.0080*** (1.3868)						-5.9748*** (1.0603)	-2.9321** (1.3907)
Unemployment		0.3704*** (0.0416)					0.5207*** (0.0471)	0.8053*** (0.1399)
Tbill			-4.6635*** (0.4380)				7.1164*** (0.8769)	8.0271*** (1.7104)
Gov.Bonds				-7.1906*** (0.5306)			-12.2448*** (0.9129)	-12.7036*** (1.7059)
GDP					-1.5167* (0.8577)			2.4658** (1.0392)
Recession						-0.0174 (0.0363)	0.0706** (0.0261)	0.0094 (0.0477)
adj. $R^2$	0.0747	0.2044	0.2699	0.3754	0.0206	-0.0025	0.5744	0.5654
<i>Panel B: h = 2</i>								
(Intercept)	0.4130*** (0.0113)	-0.2238** (0.0756)	0.5758*** (0.0149)	0.8045*** (0.0309)	0.4051*** (0.0198)	0.3989*** (0.0120)	0.3890*** (0.0924)	0.0728 (0.2718)
Inflation	-4.7089** (1.4380)						-4.3664*** (1.1830)	0.5189 (1.5008)
Unemployment		0.3554*** (0.0421)					0.1884*** (0.0522)	0.3474** (0.1456)
Tbill			-5.3730*** (0.3853)				-0.2507 (0.9404)	-1.1136 (1.7749)
Gov.Bonds				-7.1496*** (0.5328)			-5.7493*** (1.0173)	-4.6440** (1.8860)
GDP					-1.2340 (0.8512)			2.2171* (1.1180)
Recession						0.0876** (0.0360)	0.1299*** (0.0291)	0.0509 (0.0527)
adj. $R^2$	0.0310	0.1878	0.3889	0.3707	0.0108	0.0160	0.4704	0.4701
<i>Panel C: h = 4</i>								
(Intercept)	0.4134*** (0.0112)	-0.2527*** (0.0749)	0.5527*** (0.0157)	0.7942*** (0.0300)	0.4045*** (0.0198)	0.4070*** (0.0121)	0.0494 (0.0887)	-0.4678* (0.2687)
Inflation	-5.6349*** (1.4095)						-4.4262*** (1.1396)	-1.8303 (1.4541)
Unemployment		0.3715*** (0.0417)					0.4305*** (0.0508)	0.7147*** (0.1455)
Tbill			-4.8666*** (0.4214)				4.9223*** (0.9381)	5.7397** (1.7702)
Gov.Bonds				-7.1153*** (0.5280)			-10.2998*** (0.9809)	-10.6401*** (1.7865)
GDP					-1.3775 (0.8563)			2.6813** (1.0822)
Recession						0.0153 (0.0363)	0.0795** (0.0280)	0.0347 (0.0499)
adj. $R^2$	0.0470	0.2053	0.3033	0.3727	0.0155	-0.0027	0.5086	0.5244