The Bias of Realized Volatility

Janis Becker, Christian Leschinski[1]

Leibniz University Hannover

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Abstract

Realized volatility underestimates the variance of daily stock index returns by an average of 14 percent. This is documented for a wide range of international stock indices, using the fact that the average of realized volatility and that of squared returns should be the same over longer time horizons. It is shown that the magnitude of this bias cannot be explained by market microstructure noise. Instead, it can be attributed to correlation between the continuous components of intraday returns and correlation between jumps and previous/subsequent continuous price movements.

JEL-Numbers: G11, G12, G17

Keywords: Return Volatility · Realized Volatility · Squared Returns

[1] Corresponding author:
Phone: +49-511-762-5383
Fax: +49-511-762-3923
E-Mail: leschinski@statistik.uni-hannover.de

1 Introduction and Main Finding

Volatility is at the heart of everything from risk management to derivative pricing and asset management. While estimates of the unobserved volatility process were originally obtained using GARCH and stochastic volatility models, today high frequency data has become widely available and realized volatility (RV) has been adopted as the standard measure. Due to its nature as a non-parametric estimate that is consistent for the integrated variance in price processes that behave as semimartingales, realized volatility is often even treated as a direct observation of the underlying volatility process. This drastic improvement in the quality of volatility estimates has also led to major advances in volatility forecasting and risk management.

Previous contributions on the shortcomings of realized volatility have mostly focused on the effect of market microstructure noise and violations of the assumption that the price process can be observed frictionless at arbitrarily small time intervals.

Here, we focus on the semimartingale assumption and show that realized volatility is a biased estimator for the variance of stock index returns on daily and longer horizons. The bias is negative, so that the stock market risk is systematically underestimated. This effect is demonstrated for a wide range of international stock market indices and the average magnitude of the bias is 14 percent. The RV of the S&P 500, for example, underestimates the mean level of the variance by 15 percent.

We show that the reason for this bias lies in the presence of dependence between aggregates of returns within a trading day, as recently documented by Gao et al. (2018).

It is customary to model the log-price process $p(\tau)$ as a jump diffusion so that

$$dp(\tau) = \mu(\tau)d\tau + \sigma(\tau)dB(\tau) + \xi(\tau)dq(\tau), \tag{1}$$

where $\mu(\tau)$ is of finite variation, while $\sigma(\tau)$ is the instantaneous or spot volatility, strictly positive, stationary, (almost) surely square integrable and stochastically independent of the standard Brownian motion $B(\tau)$. Furthermore, $q(\tau)$ is a Poisson process uncorrelated with $B(\tau)$ and governed by the jump intensity $\lambda(\tau)$ so that $P(dq(\tau) = 1) = \lambda(\tau)d\tau$, which implies a finite number of jumps in the price path per time period. The scaling factor $\xi(\tau)$ denotes the magnitude of the jump in the return process if a jump occurs at time τ. Jumps are often associated with big unexpected events or macroeconomic announcements that make the price process discontinuous, while the semimartingale component $\sigma(\tau)dB(\tau)$ models the continuous part of its evolution.

For the sake of the argument presented here, we will assume that $\mu(\tau) = 0$, for all τ. That means that the equity premium is zero, which is a reasonable assumption on short time horizons such as days because it is so small. Nevertheless, this is purely for expositional

purposes and the arguments could easily be extended to allow for $\mu(\tau) \neq 0$.
Denote the continuously compounded return at day t by $r_t = p(t) - p(t-1)$, for $t = 1, ..., T$. Since $\mu(\tau) = 0$, we have

$$r_t = \int_{t-1}^{t} dp(\tau)d\tau = \int_{t-1}^{t} \sigma(\tau)dB(\tau) + \sum_{t-1 \leq \tau \leq t} J(\tau).$$

$$\text{Therefore,} \quad E[r_t^2] = Var[r_t] = IV_t + E\left[\sum_{t-1 \leq \tau \leq t} J^2(\tau)\right], \tag{2}$$

$$\text{where} \quad IV_t = \int_{t-1}^{t} \sigma^2(\tau)d\tau,$$

and $J(\tau) = \xi(\tau)dq(\tau)$ is non-zero only if there is a jump at time τ. This is due to the assumed independence between the continuous components and jump components, the independence of the increments of the Brownian motion, and the independence between successive jumps.
Equation (2) shows that squared returns are an unbiased estimator for the daily variance. It is, however, well known that squared returns are extremely noisy and inconsistent, since there is only a single daily return per trading day (cf. Andersen and Bollerslev (1998a)).
Realized volatility, on the other hand, makes use of the availability of high frequency data. If M intraday returns are observed, then the realized volatility is given by

$$RV_t = \sum_{i=1}^{M} r_{it}^2,$$

where r_{it} is the ith intraday return. Under the assumption that the log-price process is a semimartingale, Barndorff-Nielsen and Shephard (2001) derive the consistency and asymptotic normality of realized volatility as an estimate for the quadratic variation QV_t of the price process.
The consistency of RV_t for QV_t also extends to jump diffusions (cf. Andersen and Benzoni (2009)), but the properties of the quadratic variation depend on the properties of the underlying price process.
For semimartingales the quadratic variation equals the integrated variance so that $QV_t = IV_t$. If $p(t)$ follows a jump-diffusion such as (1), on the other hand,

$$QV_t = IV_t + \sum_{t-1 \leq \tau \leq t} J^2(\tau). \tag{3}$$

That means the quadratic variation equals the integrated variance plus the sum of squared jumps. For a review of these concepts cf. Andersen and Benzoni (2009).

Equations (2) and (3) therefore imply that

$$E[r_t^2] = Var[r_t] = E[RV_t]. \tag{4}$$

This equality is the basis for the arguments made in this paper. It implies the convergence of long term averages of r_t^2 and RV_t, so that for $\overline{r_t^2} = T^{-1}\sum_{t=1}^{T} r_t^2$ and $\overline{RV_t} = T^{-1}\sum_{t=1}^{T} RV_t$, we have

$$\overline{r_t^2} \xrightarrow{p} \overline{RV_t},$$

as $T \to \infty$. Furthermore, we have

$$\sqrt{T}\Delta\sigma^2 = \sqrt{T}\left(\overline{r_t^2} - \overline{RV_t}\right) \xrightarrow{d} N(0, V), \tag{5}$$

where the long run variance V of the differential $r_t^2 - RV_t$ can be estimated with HAC estimators, so that we can test the hypothesis that (4) is true using (5).

Note that the first equality in (2) and (4) holds generally as long as $\mu(\tau) = 0$. The squared returns are an unbiased estimate of the daily variance. The equality $E[RV_t] = Var[r_t]$, on the other hand, only holds under the assumption that the log-price process $p(t)$ follows a jump-diffusion or a semimartingale, since this implies the independence within and between continuous changes and jumps. A rejection of (4) is therefore indicative of a bias in RV_t and not in r_t^2.

To summarize, both realized volatility and squared returns approximate the same underlying volatility process with the difference being that squared returns are noisier. Therefore, deviations between the two volatility measures should be random and cancel each other out over time. Consequently, average squared return and average realized volatility should be equal given a long enough horizon.

Figure 1 shows that this is not the case for a wide cross section of stock indices. Here, and in the following, we use 15-minute data for the years 1996 until 2017 from the *Thomson Reuters Tick History* data base. The annualized average RV is plotted against the annualized average squared return for 22 commonly considered indices, such as the S&P 500, the DAX, and the SSEC. It can be seen that for all indices, except for the DAX, the average squared return is larger than the average realized volatility in the same time

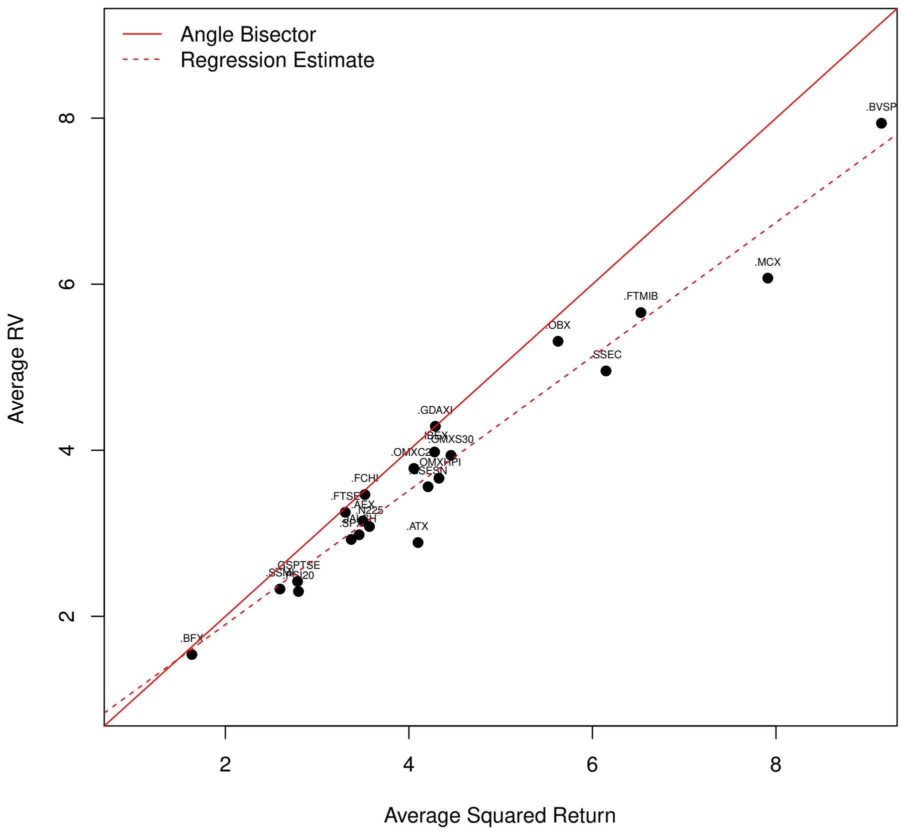

Figure 1: Average annualized volatility estimate for 22 indices using squared returns respectively RVs. The RV estimates are calculated from 15-minute data and the squared returns are adjusted for overnight returns such that both estimates are based on the same time horizon.

period.[1]

Figure 2 sheds light on the relation of average RV and average squared return over time. In the left plot, the average volatility estimate of the two estimators in a rolling window of 750 observations is displayed for the S&P 500. Again, it can be observed that the average RV is systematically smaller than the average squared return. This is not only true for isolated periods but holds all the time. However, the difference between the two time series seems to be larger in times of high volatility, such as the subprime mortgage crisis, indicating that average RV and average squared return differ by a factor rather

[1] It should be noted that the squared returns are calculated from open-to-close returns so that overnight returns are excluded and the time horizon is the same as that for the realized volatility. As a robustness check, Figure 7 in the appendix shows the results of the same analysis using realized volatilities for 31 stock indices from the 'Oxford-Man Institute's realised library' compiled by Heber et al. (2009) and considered by Shephard and Sheppard (2010) and Han and Kristensen (2014), among others. These RVs are calculated from 10-minute data. It can be seen that the analysis yields very similar results which underlines the robustness of our finding.

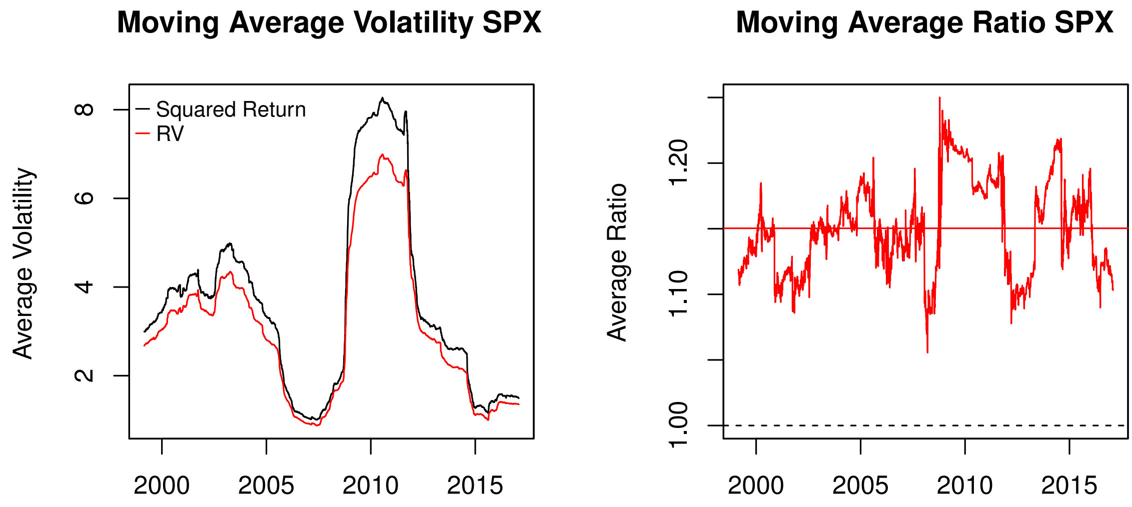

Figure 2: Left: average S&P 500 volatility estimate of the last 750 observations using squared returns respectively RVs extracted from 15-minute returns. Right: ratio between rolling averages of the squared return and RV.

than a constant. This factor is plotted over time on the right hand side of Figure 2. It seems relatively stable over time with an average of 1.15, which means that the RV underestimates the variance of the S&P 500 by 15 percent.[2]

Table 1 presents more detailed results for all 22 indices. It is apparent that the effect is especially pronounced for ATX, MCX, and SSEC with a factor between average squared return and average RV of 1.42, 1.30, and 1.24. This results in standard deviations that are larger by 3.25, 3.5, respectively 2.53 percentage points per annum than indicated by the RV estimator. For DAX, FCHI, and FTSE with factors of 1.00, 1.01, and 1.02 on the other hand, the effect is negligible. Averaged over all indices, the mean RV is 14 percent smaller than the mean squared return. This amounts to an annualized underestimation of the standard deviation by 1.34 percentage points.

The table further reports the test statistics $t_{HAC} = \sqrt{T}(\overline{r_t^2} - \overline{RV_t})/\sqrt{V_{HAC}}$ for robust t-tests of the null hypothesis that $E[RV_t] = E[r_t^2]$. Here, V_{HAC} is the long run variance of $r_t^2 - RV_t$ which is estimated using the method of Andrews (1991). As a robustification we also report the t_{MAC} statistic of Robinson (2005) and Abadir, Distaso, and Giraitis (2009), which accounts for the possibility of long memory in $r_t^2 - RV_t$. This might be present since both, r_t^2 and RV_t, are commonly found to be highly persistent (cf. Kruse, Leschinski, and Will (2018)). To account for the fact that the unconditional fourth moment of

[2]Here and hereafter, we focus our analysis mostly on the S&P 500. Plots for other indices show that investigating any of the indices for which the effect is present would have yielded similar results. These are available from the authors upon request.

RIC	Country	$\overline{r^2}$	$\overline{RV}$	$\overline{r^2}/\overline{RV}$	$\sqrt{\overline{r^2}} - \sqrt{\overline{RV}}$	t_{HAC}	t_{MAC}	t_{MOM}	T
.AEX	Netherlands	3.50	3.15	1.11	0.96	2.91***	2.48**	3.59***	4,567
.ATX	Austria	4.10	2.89	1.42	3.25	8.56***	4.31***	6.91***	4,179
.BFX	Belgium	1.63	1.54	1.06	0.38	1.91*	1.41	1.37	5,275
.BSESN	India	4.21	3.56	1.18	1.65	6.63***	3.86***	8.03***	4,948
.BVSP	Brazil	9.15	7.94	1.15	2.07	4.08***	2.58***	5.13***	4,728
.GDAXI	Germany	4.29	4.29	1.00	-0.00	-0.01	-0.01	0.91	5,264
.FCHI	France	3.52	3.47	1.02	0.14	0.58	0.49	1.24	5,272
.FTMIB	Italy	6.53	5.66	1.15	1.76	3.23***	3.43***	3.77***	1,942
.FTSE	Great Britain	3.31	3.25	1.02	0.15	0.53	0.87	2.77***	5,218
.GSPTSE	Canada	2.79	2.42	1.15	1.14	2.77***	3.73***	7.35***	3,654
.IBEX	Spain	4.28	3.98	1.08	0.74	2.74***	2.31**	3.40***	5,188
.JALSH	South Africa	3.46	2.98	1.16	1.32	4.72***	4.72***	6.27***	3,216
.MCX	Russia	7.91	6.07	1.30	3.48	6.19***	2.36**	7.78***	3,892
.N225	Japan	3.57	3.08	1.16	1.34	3.83***	0.76	4.59***	5,080
.OBX	Norway	5.62	5.31	1.06	0.67	1.44	4.87***	4.40***	2,679
.OMXC20	Denmark	4.05	3.78	1.07	0.69	1.78*	4.42***	4.48***	2,821
.OMXHPI	Finland	4.33	3.66	1.18	1.66	4.12***	4.04***	6.59***	2,835
.OMXS30	Sweden	4.46	3.94	1.13	1.27	3.13***	2.88***	5.39***	3,005
.PSI20	Portugal	2.80	2.30	1.22	1.56	4.89***	4.24***	5.08***	4,866
.SPX	United States	3.37	2.93	1.15	1.26	4.96***	3.47***	5.77***	5,183
.SSEC	China	6.15	4.96	1.24	2.53	7.24***	5.37***	9.10***	5,019
.SSMI	Switzerland	2.59	2.33	1.11	0.85	3.39***	1.90*	3.90***	4,770

Table 1: The table reports the average squared return $\overline{r^2}$ per annum in percent, the average realized volatility $\overline{RV}$ per annum in percent, and the ratio between the two volatility measures for all of the 22 considered indices. For better assessing the degree of the bias $\sqrt{\overline{r^2}} - \sqrt{\overline{RV}}$ is stated, which gives the average percentage points that the standard deviation implied by the two measures deviates per annum. Moreover, the table reports the results of robust t-tests t_{HAC} for the null hypothesis that the two volatility estimates are equal. As it is commonly found in the literature that squared returns and RV are highly persistent, t_{MAC} (cf. Robinson (2005) and Abadir, Distaso, and Giraitis (2009)), which accounts for this degree of persistence, is also stated. Moreover, we report t_{MOM}, which yields valid inference, if the return distribution does not exhibit unconditional finite fourth moments. For all tests ***, (**), and [*] indicate that the null hypothesis $E[r_t^2] = E[RV_t]$ is rejected at the 1%, (5%), or [10%] level. Positive test statistics thereby imply that squared returns are significantly larger on average. Finally, T gives the number of days considered for estimation.

the return distribution might not exist, we conduct an additional test t_{MOM}, for which the difference between RV and squared return is standardized by an estimate of the conditional standard deviation of the series. The test results suggest that for 16 (t_{HAC}), 13 (t_{MAC}), respectively 19 (t_{MOM}) indices the squared returns are significantly larger than the realized variances at the one percent level.

To summarize, average squared returns and average RVs are not identical in expectations. Instead, mean squared returns are larger by a factor of 1.14. This observation is time consistent and can be found for all of the 22 considered indices except DAX, FCHI, and FTSE.

Recalling Equation (4) and the considerations stated thereafter, this implies that realized volatility is a biased estimator for the variance of daily index returns. The next section provides a detailed investigation of possible explanations for this bias. Here, we provide evidence that the deviation between squared returns and RV is caused by dependencies in intraday returns that violate the assumptions required for consistency of RVs as an estimator for the daily volatility. Section 3 then concludes.

2 Explaining the Bias of Realized Volatility

To determine the source of the difference between average squared returns and average RVs, it is useful to decompose the observed continuously compounded return r_{it} into its components.

It is now broadly accepted that stock prices can be represented by a jump-diffusion model such as (1) (cf. Aït-Sahalia (2004); Barndorff-Nielsen and Shephard (2007); Corsi, Pirino, and Reno (2010)). Consequently, we can decompose the continuously compounded stock return r_{it} into jump component J_{it}, continuous component C_{it}, and equity premium. As mentioned before, we assume that the equity premium is zero to ease the presentation.[3]

Another important component of the observed return at high frequencies is market microstructure noise due to price discreteness, bid-ask spreads, trades taking place at different markets and networks, gradual response of prices to a block trade, difference in information contained in orders of different sizes, strategic order flows, and recording errors. Starting with Zhou (1996), numerous contributions find these effects to significantly influence the observed intraday return at high frequencies such as 1-second data. At low frequencies, however, the effect is often found to be negligible. To capture this characteristic, market microstructure noise is denoted by $\eta_{it,M}$ in the following, with M defined as the number of intraday observations and $E[\eta_{it,M}] = 0$. We then assume that market microstructure effects are not present on a daily frequency, i.e. $Var(\eta_{it,1}) = 0$, and that $Var(\eta_{it,M})$ is monotonically increasing with the sampling frequency M.

The observed continuously compounded return can then be written as

$$r_{it} = C_{it} + J_{it} + \eta_{it,M},$$

[3] Our results would not be altered by a nonzero equity premium unless it would exhibit sizable intraday variation (which is theoretically implausible).

so that for the two volatility estimators it holds that

$$RV_t = \sum_{i=1}^{M} (C_{it} + J_{it} + \eta_{it,M})^2 \tag{6}$$

$$\text{and} \quad r_t^2 = \left(\sum_{i=1}^{M} C_{it} + \sum_{i=1}^{M} J_{it} \right)^2 .$$

Calculating the difference between the two estimators yields

$$\begin{aligned} r_t^2 - RV_t = & \sum_{i,j=1, i \neq j}^{M} C_{it} C_{jt} + \sum_{i,j=1, i \neq j}^{M} J_{it} J_{jt} + 2 \sum_{i,j=1, i \neq j}^{M} C_{it} J_{jt} \\ & - \sum_{i=1}^{M} \eta_{it,M} \eta_{it,M} - 2 \sum_{i=1}^{M} C_{it} \eta_{it,M} - 2 \sum_{i=1}^{M} J_{it} \eta_{it,M} . \end{aligned}$$

To simplify the notation, let $AB_t = \sum_{i,j=1, i \neq j}^{M} A_{it} B_{jt}$ and $AB_t^* = \sum_{i=1}^{M} A_{it} B_{it}$, such that

$$r_t^2 - RV_t = CC_t + JJ_t + 2CJ_t - \eta\eta_{t,M}^* - 2C\eta_{t,M}^* - 2J\eta_{t,M}^* . \tag{7}$$

The first two terms capture the intraday dependencies in continuous and jump component. If for example $E[CC_t]$ is positive, then positive and negative intraday continuous returns would tend to occur in clusters. It would therefore be more likely that C_{it} is a large positive return if $(C_{1t}, ..., C_{i-1t}, C_{i+1t}, ... C_{Mt})$ are large positive returns. The third term captures the dependencies between the leads and lags of jump and continuous component. If $E[CJ_t]$ is positive, then it would be more likely to observe positive continuous returns at days where a positive jump occurs. The fourth term captures the variance of the market microstructure noise and the last two terms capture intraday dependencies between the noise component and the continuous and jump components. If for example $E[C\eta_{t,M}^*]$ is positive, then it is more likely to observe positive microstructure noise $\eta_{it,M}$ if C_{it} is large and positive.

As mentioned before, when calculating the RV estimator it is commonly assumed that the log-price process follows a jump diffusion such as (1) and that markets are frictionless. This implies that all of the terms in Equation (7) are zero in expectation and the expected values of squared returns and RVs are identical. However, Figures 1 and 2 together with Table 1 show that the average squared return is systematically larger than the average RV for a wide cross section of stock indices. Hence, at least one of the terms in Equation (7) has to be significantly larger than zero to explain the negative bias of the RV estimator as an estimator for the daily volatility. In the following, we therefore analyze each of the terms in Equation (7) separately, to determine the source of the bias.

2.1 Market Microstructure Noise

It is well established that market microstructure effects cause biased RV estimates (cf. e.g. Hansen and Lunde (2006) or Bandi and Russell (2006)). Therefore, it is tempting to conjecture that the difference between the average squared return and the average realized volatility can be attributed to the presence of microstructure noise. Since microstructure noise is not observable and cannot be estimated without access to tick data, this conjecture can only be refuted on the basis of plausibility arguments. These, however, are quite compelling.

First, the results presented here are obtained using 15-minute data to mitigate the impact of microstructure noise right from the start. Microstructure effects are commonly found to be relevant on high frequencies such as 1-second data. For sampling frequency lower than five minutes, as considered here, Bandi and Russell (2008) argue that the effect of market microstructure noise is negligible. This is also confirmed by Figures 8 and 9 in the appendix, which show that repeating our analysis for 5, and 30-minute data yields qualitatively similar results.

Second, market microstructure noise only generates a negative bias in the realized volatility if $\eta\eta^*_{t,M} < -2C\eta^*_{t,M} - 2J\eta^*_{t,M}$. This would imply negative correlation between noise and continuous component respectively between noise and jump component which outweighs the variance of the market microstructure noise. This is typically not the case as can be seen for example in the volatility signature plots of Hansen and Lunde (2006), Bandi and Russell (2006), and Aït-Sahalia, Mykland, and Zhang (2011). Here, market microstructure effects generate a positive bias in the realized volatility. In contrast to that, the bias observed here is negative.

Third, as a final robustness check, Figure 10 in the appendix repeats the analysis of Figure 1 for realized volatilities from the Oxford Man Realized Library estimated using the realised kernel estimator of Barndorff-Nielsen et al. (2008) that is constructed to be robust to market microstructure noise. Again, it can be observed that the average squared returns are significantly larger.

We therefore conclude, that market microstructure effects cannot explain the difference between average RVs and average squared returns documented in Section 1. Since we use 15-minute data, it seems reasonable to assume that the magnitude of η_{it} is negligible and Equation (7) can be further simplified such that

$$r_t^2 - RV_t \approx CC_t + JJ_t + 2CJ_t. \tag{8}$$

2.2 Jumps and Continuous Returns

To determine the relative magnitude of the remaining terms in Equation (8), we need to decompose the intraday returns r_{it} into their continuous and jump components. While numerous model-free estimators and tests have been proposed to disentangle the contribution of jumps and continuous components to the daily RV (cf. e.g. Barndorff-Nielsen and Shephard (2004), Aït-Sahalia and Jacod (2009), and Corsi, Pirino, and Reno (2010)), only the methodology of Lee and Mykland (2007) is able to determine jump and continuous components for every intraday return. The idea of Lee and Mykland (2007) is to compare each 15-minute return to an estimate of the volatility using the previous K observations. If the 15-minute return is large in comparison to the volatility of the previous observations, then it is concluded that a jump occured. Since $Var(C_{it}) = O(M^{-1})$, the jump asymptotically dominates the continuous component as $M \to \infty$ and r_{it} is a suitable estimator for J_{it}. Consequently, if the test rejects the null of no jump at time i on day t, we conclude that a jump of size r_{it} has occurred.

Lee and Mykland (2007) suggest to estimate the volatility of the previous K observations using the bipower variation introduced by Barndorff-Nielsen and Shephard (2004). However, Corsi, Pirino, and Reno (2010) show that this estimator is substantially biased in finite sample leading to a large underestimation of the jump component. To circumvent this problem, we consider their threshold bipower variation estimator. This is less affected by small sample bias and has the same limit as bipower variation in probability.[4]

The test statistic then evolves as

$$\mathcal{L}_{it} = \frac{r_{it}}{\sqrt{\hat{\sigma}^2_{it}}}, \qquad \text{with} \qquad \hat{\sigma}^2_{it} = \frac{\pi}{2}\frac{1}{K-2}\sum_{j=i-M+2}^{i-1} |r_j||r_{j-1}| I(r_j^2 \leq \theta) I(r_{j-1}^2 \leq \theta).$$

Here, $I(\cdot)$ is the indicator function and θ is a threshold, which is estimated using the approach suggested by Corsi, Pirino, and Reno (2010) that ensures that jumps do not influence the estimation of $\hat{\sigma}^2_{it}$.

In the absence of jumps, a single test is standard normally distributed. For multiple testing as it is performed here, critical values are derived based on Gaussian extreme value theory.

Since the variance estimator $\hat{\sigma}^2_{it}$ is the threshold bipower variation by Corsi, Pirino, and Reno (2010), the test is still consistent if a jump has already occurred in one of the previous K observations. As suggested by Lee and Mykland (2007) we set $K = 156$ and employ a significance level of one percent to decrease the likelihood of spuriously detecting jumps.

[4]The results using the bipower variation estimator are qualitatively similar and available upon request.

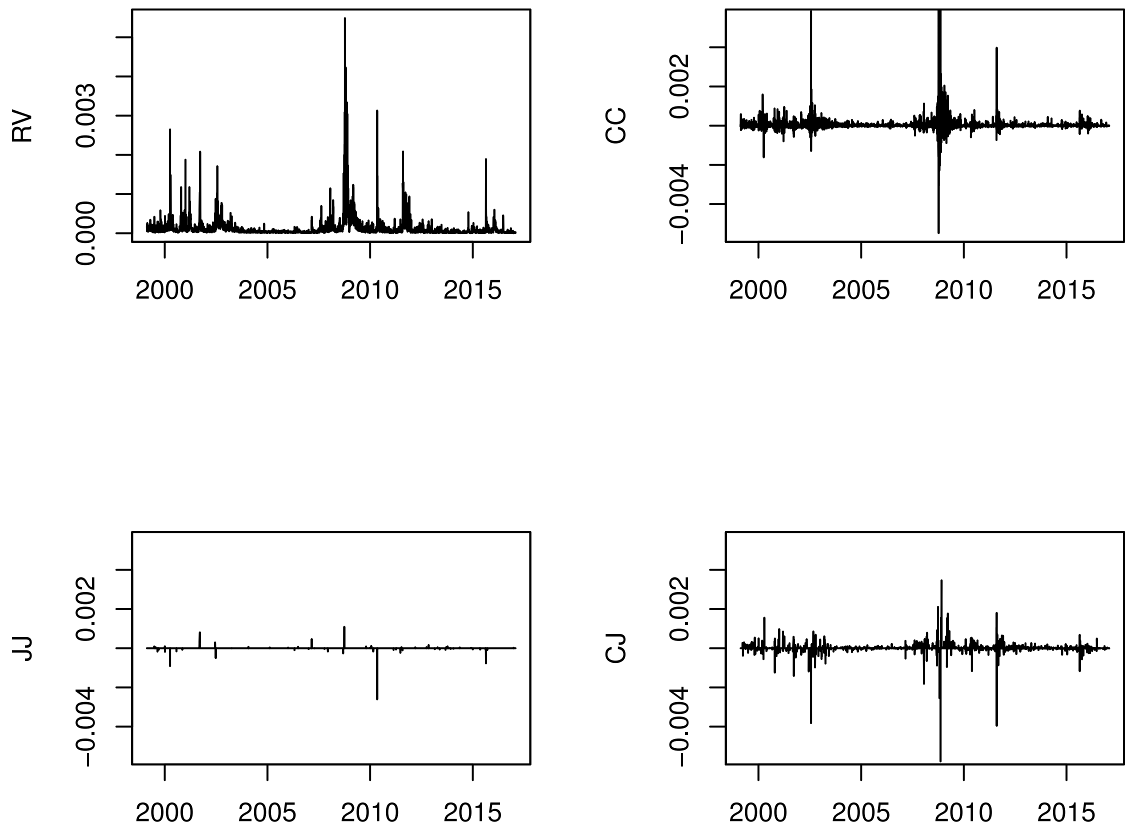

Figure 3: Daily values of RV_t, CC_t, JJ_t, and CJ_t based on 15-minute data of the S&P 500. CC_t, JJ_t, and CJ_t are determined using a slight modification of the methodology by Lee and Mykland (2007). This means that we employ the threshold bipower variation to estimate the instantaneous volatility instead of the bipower variation as this is biased in small samples.

The daily values of the three components of Equation (8) estimated using this methodology are plotted in Figure 3 for the S&P 500. The procedure indicates that of the 5,184 days in the sample, a jump occurred on 922 and there were only 77 days on which more than one jump occurred. Consequently, for the S&P 500 the component JJ_t, which is only nonzero if two jumps occur at the same day, is negligible.

Panel A of Table 2 provides t-statistics robust to autocorrelation and heteroscedasticity for the null hypothesis that $E[CC_t] = 0$, respectively $E[CJ_t] = 0$ or $E[JJ_t] = 0$.[5] For all indices with a significant bias, it can be seen that the component CC_t is positive and

[5] In analogy, Tables 5 and 6 in the appendix state t_{MAC} and t_{MOM}, i.e. the t-statistics when accounting for persistence respectively infinite unconditional fourth moments of the return distribution. It can be seen that the results are qualitatively similar.

RIC	Country	A 15-minute data CC	CJ	JJ	B seasonally adjusted CC	CJ	JJ	C 5-minute data CC	CJ	JJ
.AEX	Netherlands	3.21***	0.64	0.38	3.18***	0.17	0.43	2.23**	1.81**	0.38
.ATX	Austria	10.47***	0.64	-0.51	10.09***	-0.21	-0.57	11.26***	1.57	2.67***
.BFX	Belgium	1.78*	0.94	-0.54	1.66*	1.20	-0.74	1.10	0.80	1.04
.BSESN	India	7.17***	-0.46	-0.59	7.10***	-1.51	1.42	6.79***	3.49***	-0.72
.BVSP	Brazil	6.10***	1.62	-1.76*	4.60***	0.51	-1.55	6.82***	4.12***	-0.71
.GDAXI	Germany	-0.02	0.38	-0.66	0.12	-0.31	0.07	-0.68	1.51	0.82
.FCHI	France	0.97	-0.13	-0.69	0.51	-0.05	0.55	2.08**	2.10**	0.77
.FTMIB	Italy	3.99***	1.89*	-0.40	3.46***	1.94*	-0.53	3.28***	1.56	0.47
.FTSE	Great Britain	2.04**	0.18	-2.07**	2.16**	-0.98	-1.92*	-3.53***	1.06	1.90*
.GSPTSE	Canada	5.05***	0.18	-0.48	5.21***	-0.87	-1.65*	6.60***	-0.24	-1.78*
.IBEX	Spain	3.53***	0.60	-0.35	3.07***	0.73	0.41	2.95***	1.67*	0.27
.JALSH	South Africa	5.79***	1.91*	-1.07	5.31***	1.35	-0.17	4.97***	3.81***	-0.66
.MCX	Russia	7.31***	1.81*	-0.69	6.10***	3.03***	-0.73	7.62***	1.84*	0.73
.N225	Japan	3.34***	2.17**	1.45	3.71***	1.49	1.69*	3.58***	4.98***	2.76***
.OBX	Norway	2.01**	0.43	-1.26	1.91*	-0.26	0.53	2.87***	-1.00	-2.20**
.OMXC20	Denmark	4.11***	-0.77	-1.71*	2.00**	1.44	-2.22**	4.92***	0.55	-2.01**
.OMXHPI	Finland	5.18***	1.65*	-1.13	4.43***	0.50	0.88	6.65***	1.65*	0.41
.OMXS30	Sweden	2.93***	1.86*	-1.11	2.52**	2.11**	-1.19	2.95***	1.03	-0.77
.PSI20	Portugal	6.31***	-0.42	0.99	5.61***	0.47	1.04	4.22***	0.47	0.95
.SPX	United States	4.83***	1.12	-1.98*	4.79***	0.68	-2.33**	5.39***	3.27**	-0.74
.SSEC	China	7.68***	2.43**	1.03	7.78***	2.59***	0.99	8.49***	3.91***	1.38
.SSMI	Switzerland	2.79***	0.78	1.49	2.37**	3.17***	1.48	1.54	2.36**	1.46

Table 2: The table reports t-statistics for the null hypothesis that $E[CC_t] = 0$, respectively $E[CJ_t] = 0$ or $E[JJ_t] = 0$. ***, (**), and [*] then indicate that the null hypothesis is rejected at the 1%, (5%), or [10%] level. In Panel A CC_t, CJ_t, and JJ_t are determined using 15-minute data and the methodology by Lee and Mykland (2007) with the threshold bipower variation instead of the bipower variation to estimate the instantaneous volatility. In analogy, Panel B shows the test results when the instantaneous volatility estimates are adjusted for intraday seasonality as found by Andersen and Bollerslev (1997) and Andersen and Bollerslev (1998b) and in Panel C the three components are estimated using 5-minute data.

significantly different from zero at the one percent level. For the S&P 500, for example, the value of the test statistic is 4.83. The components CJ_t and JJ_t, on the other hand, are not indicated to be significantly different from zero at the one percent level for any of the indices. It can further be seen that for the DAX and the FCHI, for which Table 1 reports no significant bias of the RV estimator, CC_t is not significantly different from zero. For the FTSE, Table 2 reports significant dependence at the five percent level in the continuous component, although Table 1 states that there is no significant bias. The reason for this observation is that for the FTSE there is significant negative dependence in the jump component JJ_t at the five percent level which compensates for the bias caused by CC_t.

To show the robustness of the results, Panel B of Table 2 repeats the analysis adjusting the volatility for intraday seasonality as documented by Andersen and Bollerslev (1997)

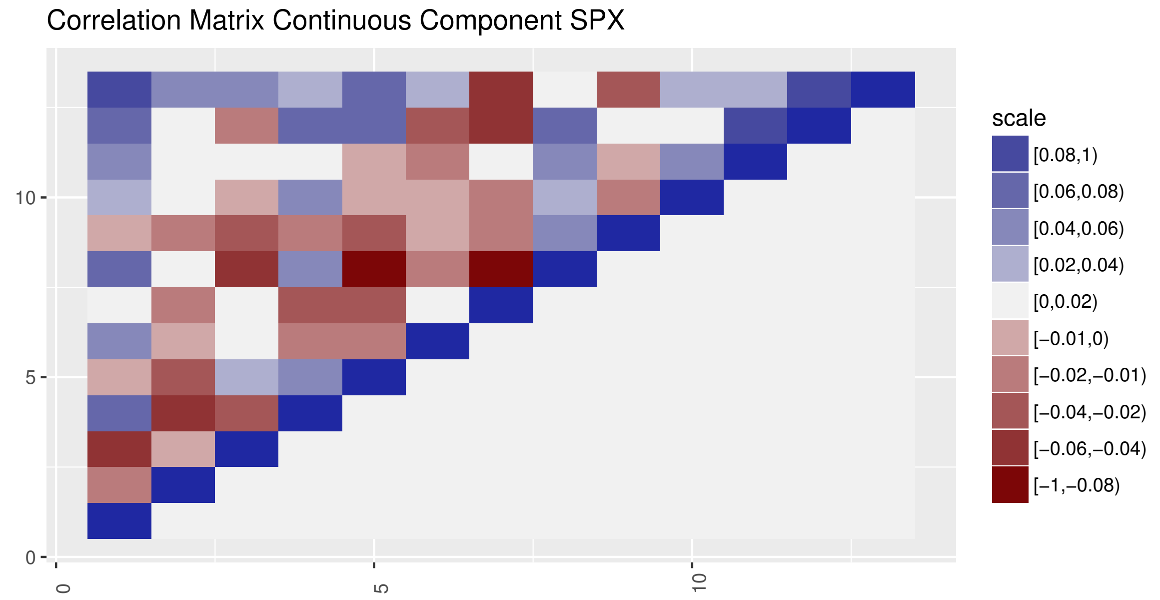

Figure 4: Correlation matrix for CC_t for the S&P 500 using 30-minute Returns.

and Andersen and Bollerslev (1998b) among others, and Panel C reports the test statistics using 5-minute instead of 15-minute data. The results are qualitatively similar with CC_t being positive and significantly different from zero for most indices and CJ_t and JJ_t being insignificant for most indices.

The term CC_t captures the dependence structure in C_{it}. Let $c_t = (C_{1t},...,C_{Mt})'$ denote the vector of continuous returns on day t and let ι be an $M \times 1$ vector of ones. Then

$$CC_t = \iota'(c_t c_t')\iota - c_t' c_t.$$

It is well established, that the autocorrelation function of r_{it} is essentially zero at all leads and lags. This justifies the semimartingale assumptions imposed in (1) that implies that all off-diagonal elements of $E[c_t c_t']$ are zero so that $E[CC_t] = 0$.

An unfortunate property of the univariate autocorrelation function, however, is the fact that it masks dependencies that do not depend on the lag, but on the location of the returns within a trading day. In a recent paper Gao et al. (2018) find significant correlation between 30-minute intraday returns of the S&P 500. This concerns in particular the last two returns in a trading day and the first and last returns of the day. They argue that these correlation patterns stem from investors infrequent re-balancing of their portfolios and late-informed investors who trade early morning information in the last hour, where liquidity is larger.

To shed further light into the dependence structure of the continuous components of the intraday returns of the S&P 500, Figure 4 shows their average correlation matrix. In line

Return locations within a trading day		Correlation	p-Value	Critical p-Value
$\rho_{5,8}$	*(11:30 - 12:00, 13:00 - 13:30)*	-0.0888	0.0004	0.0006
$\rho_{1,13}$	*(09:30 - 10:00, 15:30 - 16:00)*	0.1314	0.0010	0.0013
$\rho_{1,4}$	*(09:30 - 10:00, 11:00 - 11:30)*	0.0740	0.0010	0.0019
$\rho_{12,13}$	*(15:00 - 15:30, 15:30 - 16:00)*	0.1379	0.0016	0.0026
$\rho_{11,12}$	*(14:30 - 15:00, 15:00 - 15:30)*	0.0815	0.0025	0.0032

Table 3: The table shows all correlations between half hour returns of the S&P 500 that are significantly different from zero after accounting for the multiple testing problem. The last column states the corresponding critical p-values for an alpha of five percent when applying Simes correction (cf. Simes (1986)).

with Gao et al. (2018), the correlation matrix shows a momentum effect between first and last half hour of the trading day and in the last hour of the trading day. However, the plot also reveals positive as well as negative correlation between the other half hour returns of the S&P 500. When testing for the joint significance of all pairwise correlation coefficients we obtain a chi-square statistic of 168, which vastly exceeds the critical value of 110 at the 1 percent level.

When testing for the significance of individual correlations, there is a multiple testing problem. We account for this by applying Simes correction (cf. Simes (1986)), which consists of ordering all p-values in ascending order and then comparing them with $\frac{\alpha}{N}, \frac{2\alpha}{N}, ..., \frac{N\alpha}{N}$, with $N = 78$ being the number of performed tests. If any of the ordered p-values exceeds its respective threshold, the null hypothesis that the corresponding correlation between the two returns equals zero is rejected. This approach is designed to be less conservative and thus more powerful than the more primitive Bonferroni correction. Table 3 reports all combinations of returns for which this is the case at an alpha level of five percent. The table states that there is significant positive correlation between first and fourth, first and thirteenth (last), fourth and twelfth, eleventh and twelfth and the last two half hour returns. In contrast, negative correlation that is significantly different from zero only exists between the fifth and the eighth half hour return. With regard to the strength of the dependency, the correlation between first and last and second last and last half hour return are found to be the largest.

To summarize, there is significant positive correlation in continuous index returns. This does not only hold when considering the correlations separately but also when taking them all together. This violates the semimartingale assumption that is necessary for the consistency of the RV as an estimator for the variance of daily stock returns, and causes

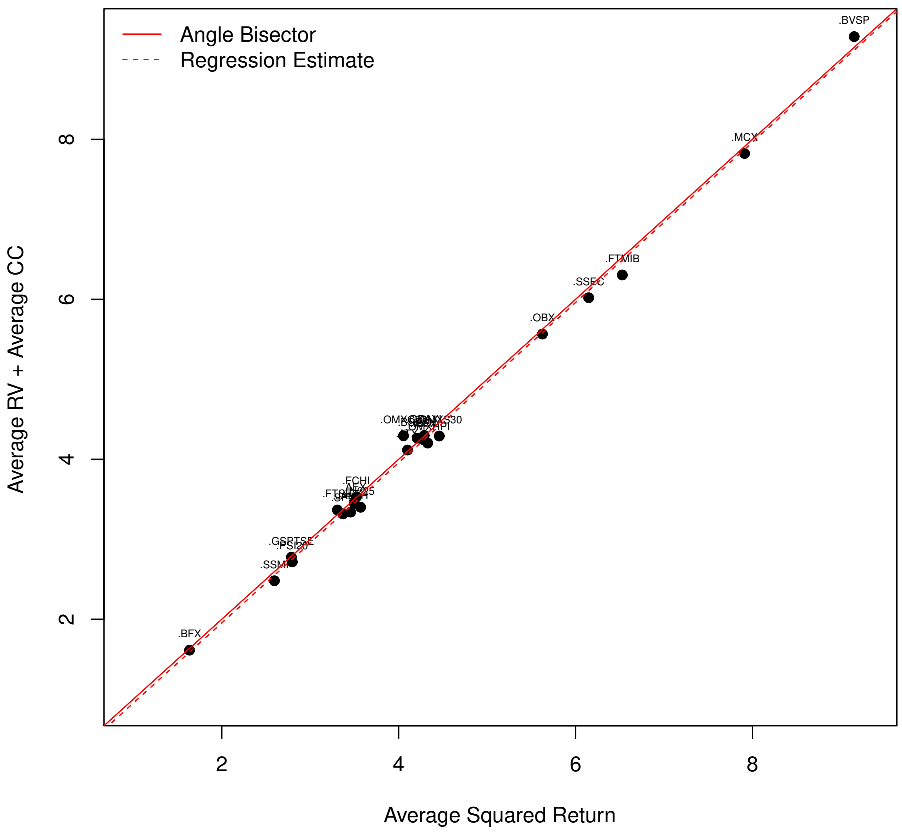

Figure 5: In analogy to Figure 1 with the difference that the RV estimates are corrected by CC_t. Again, the RV estimates are calculated from 15-minute data and the squared returns are adjusted for overnight returns such that both estimates are based on the same time horizon.

a significant negative bias as observed in Figure 1.[6]

To further illustrate this finding, Figure 5 repeats the analysis of Figure 1 and plots the average squared return against the sample average of the corrected realized volatility measure $\widetilde{RV}_t = RV_t + CC_t$. From Equation (8), adding CJ_t and JJ_t to $\widetilde{RV}_t$ would give exactly r_t^2. It can be seen that accounting for the CC_t component almost completely eliminates the difference observed in Figure 1.

This is also confirmed by the results in Table 4 that repeats the analysis from Table 1, but for the corrected realized volatility $\widetilde{RV}_t$. Now, the ratio of the two volatility estimators

[6] There is a growing body of literature which provides evidence that jumps are often erroneously identified when estimating them from 5, or 15-minute data. When considering tick data, where estimation precision is higher, the jump component is found to account for only a small fraction of the total price variation making it almost negligible (cf. Christensen, Oomen, and Podolskij (2014) and Bajgrowicz, Scaillet, and Treccani (2015)). It should be noted that performing our analysis without differentiating between jump and continuous component yields qualitatively the same results, i.e. there is correlation in index returns which cause biased RV estimates.

RIC	Country	$\overline{r^2}$	$\overline{\widetilde{RV}}$	$\overline{r^2}/\overline{\widetilde{RV}}$	$\sqrt{\overline{r^2}} - \sqrt{\overline{\widetilde{RV}}}$	t_{HAC}	t_{MAC}	t_{MOM}	T
.AEX	Netherlands	3.50	3.45	1.02	0.14	0.95	0.82	0.79	4,567
.ATX	Austria	4.10	4.12	1.00	-0.04	-0.15	-0.13	0.70	4,179
.BFX	Belgium	1.63	1.61	1.01	0.09	0.74	0.86	0.61	5,275
.BSESN	India	4.21	4.26	0.99	-0.13	-0.97	-0.87	-0.81	4,948
.BVSP	Brazil	9.15	9.28	0.99	-0.22	-0.65	-0.96	0.13	4,728
.GDAXI	Germany	4.29	4.30	1.00	-0.02	-0.12	-0.11	-0.23	5,264
.FCHI	France	3.52	3.53	1.00	-0.02	-0.13	-0.15	-0.15	5,272
.FTMIB	Italy	6.53	6.30	1.04	0.44	1.00	0.69	1.84*	1,942
.FTSE	Great Britain	3.31	3.37	0.98	-0.16	-0.71	-0.91	-0.49	5,218
.GSPTSE	Canada	2.79	2.78	1.00	0.03	0.09	0.05	0.73	3,654
.IBEX	Spain	4.28	4.24	1.01	0.09	0.39	0.41	-0.14	5,188
.JALSH	South Africa	3.46	3.34	1.04	0.32	1.43	2.12**	1.41	3,216
.MCX	Russia	7.91	7.82	1.01	0.16	0.56	0.56	1.66*	3,892
.N225	Japan	3.57	3.40	1.05	0.46	2.09**	1.87*	2.65***	5,080
.OBX	Norway	5.62	5.57	1.01	0.13	0.37	1.40	0.87	2,679
.OMXC20	Denmark	4.05	4.29	0.94	-0.58	-1.88*	-1.83*	-0.81	2,821
.OMXHPI	Finland	4.33	4.20	1.03	0.30	1.08	1.58	2.10**	2,835
.OMXS30	Sweden	4.46	4.29	1.04	0.40	1.53	5.17***	2.69***	3,005
.PSI20	Portugal	2.80	2.72	1.03	0.24	1.01	1.10	1.21	4,866
.SPX	United States	3.37	3.32	1.02	0.15	0.86	1.33	0.28	5,183
.SSEC	China	6.15	6.02	1.02	0.26	1.76*	2.30**	2.67***	5,019
.SSMI	Switzerland	2.59	2.48	1.05	0.36	2.57**	3.41***	1.97**	4,770

Table 4: In analogy to Table 1 with $\widetilde{RV}_t = RV_t + CC_t$. Again, average squared return, average adjusted realized volatility, and average deviation between the two are stated per annum in percent.

ranges from 0.94 to 1.05 resulting in an annualized difference in standard deviation ranging from only -0.58 to 0.46 percentage points. Accordingly, the null hypothesis that $E[r_t^2 - \widetilde{RV}_t] = 0$ is rejected in only a few instances.

The relative magnitude of the effect of CC_t can be seen in Figure 6 that repeats the analysis from Figure 2, but including the rolling average of $\widetilde{RV}_t$. In line with the results in Table 2, it can be seen that CC_t captures most of the difference between squared returns and RV for the S&P 500. This can also be seen in the right plot, which shows that the ratio between average squared return and $\widetilde{RV}$ is close to one for the whole time period.

To emphasize that these results do not only hold for the S&P 500 but for all considered indices, Figures 11 to 14 in the appendix show pre and post correction plots for the SSEC, the BSESN, the BVSP, and the DAX. For the SSEC, the BSESN, and the BVSP it can again be observed that correcting RV_t with CC_t eliminates the bias. As mentioned above, for the DAX the bias is less pronounced for the sample average but still, Figure 14 shows

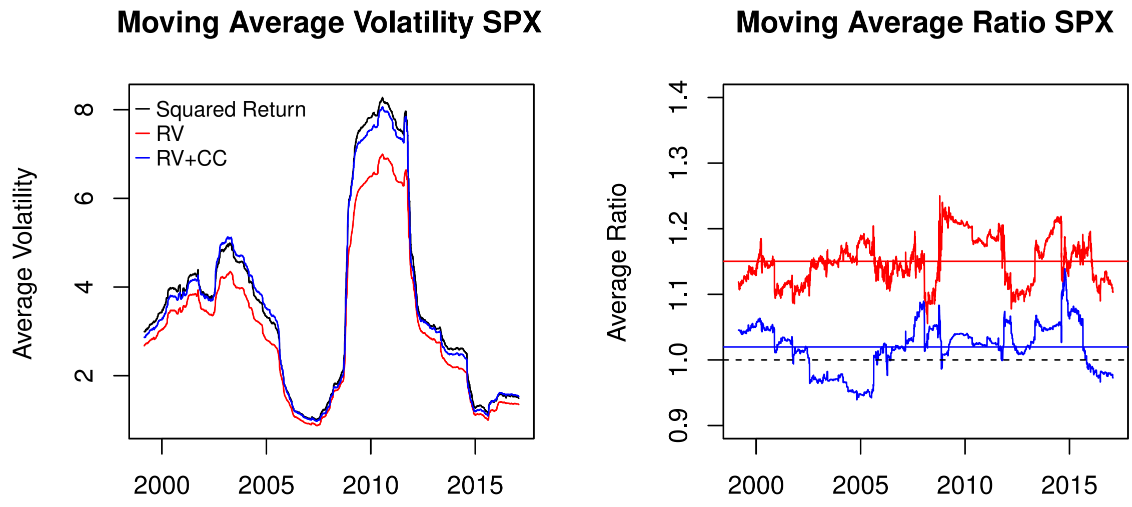

Figure 6: In analogy to Figure 2 with $\widetilde{RV} = RV_t + CC_t$. Again, both plots depict the moving average of the previous 750 observations.

that adjusting with CC_t is meaningful. Plots for the other indices yield qualitatively similar results. Since CJ_t and JJ_t are insignificant in most cases in Table 2, we therefore conclude that the dependence in CC_t explains the bias of RV.

The most intuitive idea to eliminate this bias would be to add CC_t to the daily RV estimate, i.e. use $\widetilde{RV}_t$ as the volatility estimate. While this solves the bias problem, it brings back the noise problem, since $\widetilde{RV}_t$ is almost as noisy as r_t^2 itself.

Figure 4 indicated that for the S&P 500 there is primarily correlation in the first and last hour of the trading day. As a final check whether this correlation is the reason for the observed bias, we repeated our analysis excluding the first and last hour of the trading day for the calculation of RV and squared return. As expected, average squared return and average RV now have a ratio of 0.97 and are not indicated to be significantly different from another at any level. For the other indices, however, it is not necessarily correlation in the first and last hour of the trading day that causes the negative bias reported in Table 1. If we repeat our analysis for these indices excluding the first and last hour of the trading day, then there is still a significant negative bias of the RV estimator for 12 (t_{HAC}), 8 (t_{HAC}), respectively 11 (t_{MOM}) indices at the one percent level. More detailed results can be found in Table 7 in the appendix that repeats the analysis of Table 1 when excluding the first and last hour of the trading day.

3 Conclusion

As an ex post measure of the quadratic variation of the price process, realized volatility has become the standard measure for volatility estimation. While RV is often used to estimate the variance of daily stock returns, this is only a valid approach if the log-price process is a semimartingale or a jump-diffusion.

As shown here, there are significant correlations between intraday returns that are in contradiction to the semimartingale assumption and cause a considerable bias if RV is used as an estimate for the variance of daily or weekly index returns.

While previous research on market microstructure effects has focused on frictions in event time, these results indicate that structural effects in calendar time should be investigated further to illuminate the source of these intraday dependencies.

Another important task for further research is the development of bias-corrected RV estimates that combine the unbiasedness of squared returns with the low variance of RV estimates. A simple way to do this would be to assume that the sum of the correlations between the intraday returns is constant, so that $E[CC_t/\sigma_t^2] = \rho$. In this case $E[RV_t]$ and $E[r_t^2]$ differ by the constant $1+\rho$, that can be estimated by $\overline{r_t^2}/\overline{RV_t}$.

Acknowledgements

We want to thank Neil Shephard for his helpful comments.

Appendix

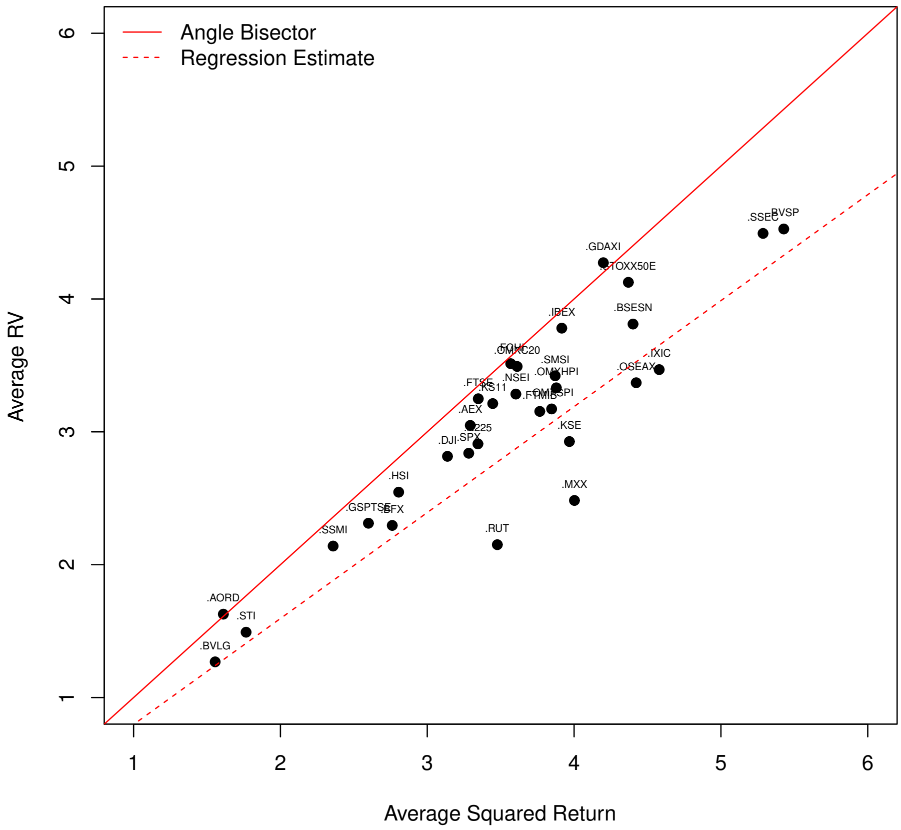

Figure 7: In analogy to Figure 1 using the data of the realized library for the 31 indices. The RV estimates are calculated from 10-minute data and the squared returns are adjusted for overnight returns such that both estimates are based on the same time horizon.

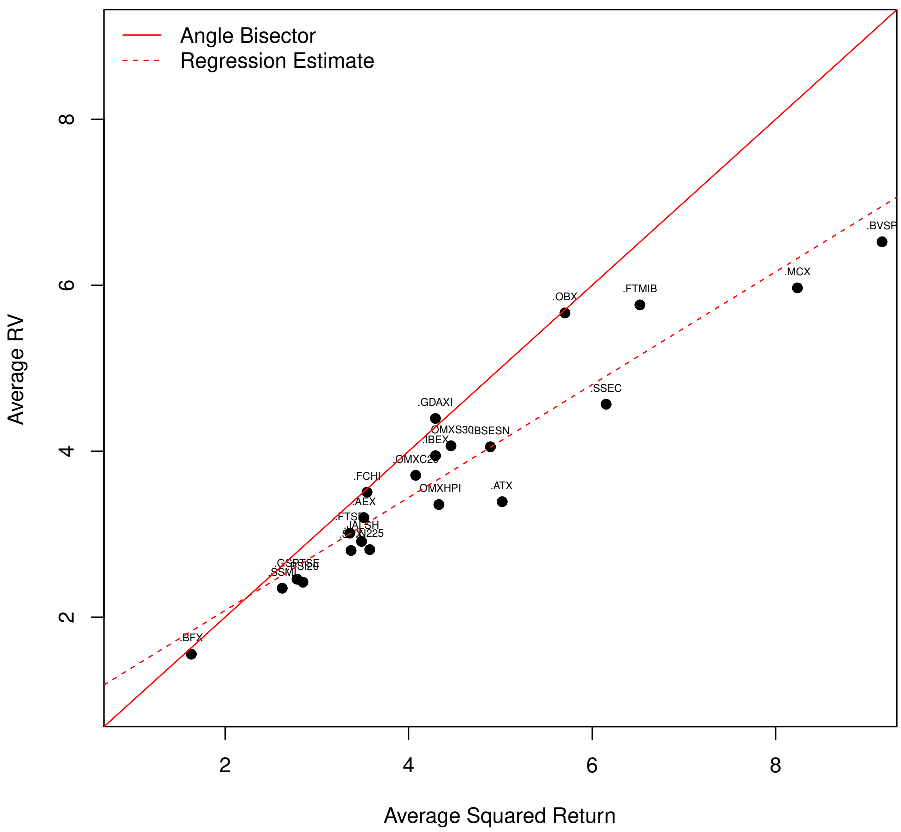

Figure 8: In analogy to Figure 1 with the RV estimates now calculated from 5-minute data. As before, squared returns are adjusted for overnight returns such that both estimates are based on the same time horizon.

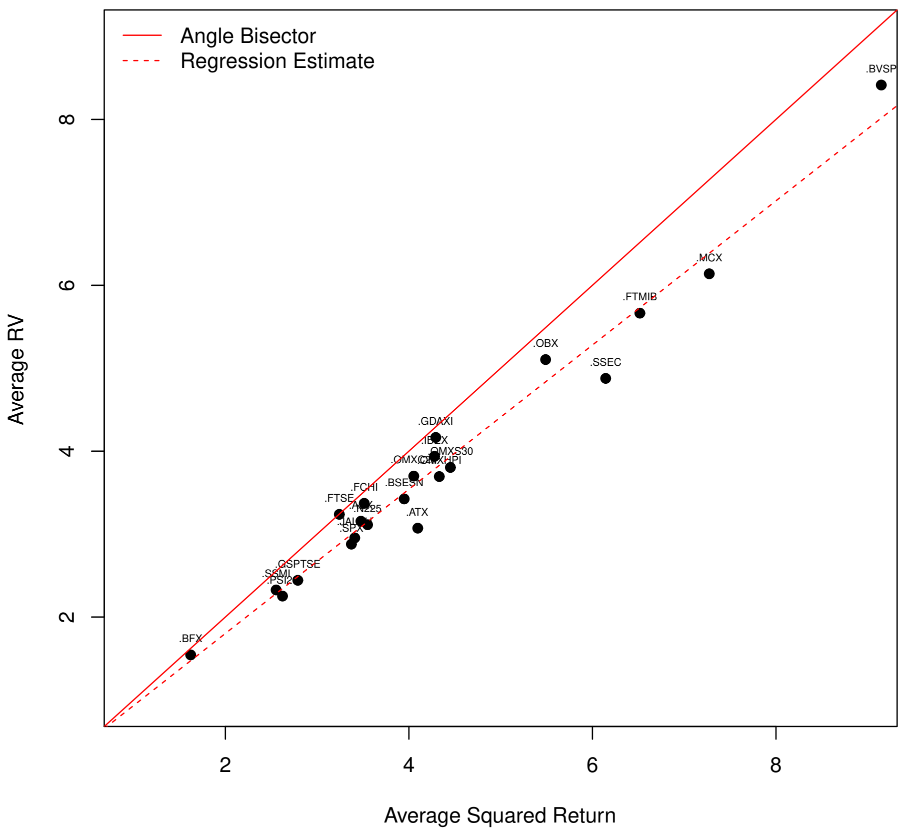

Figure 9: In analogy to Figure 1 with the RV estimates now calculated from 30-minute data. As before, squared returns are adjusted for overnight returns such that both estimates are based on the same time horizon.

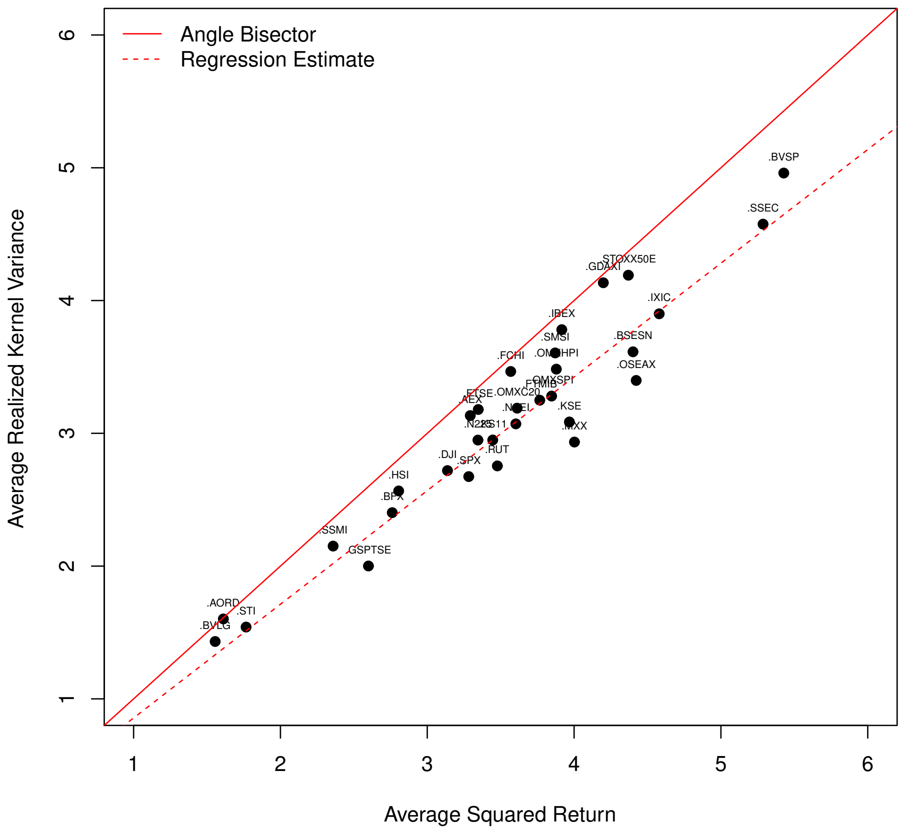

Figure 10: In analogy to Figure 7 again using the data of the realized library for the 31 indices. Now, however, the realized kernel variance, which is robust to market microstructure noise, is depicted on the y-axis.

RIC	Country	A 15-minute data CC	CJ	JJ	B seasonally adjusted CC	CJ	JJ	C 5-minute data CC	CJ	JJ
.AEX	Netherlands	3.09***	0.56	0.29	2.32**	0.23	0.42	1.50	1.21	0.67
.ATX	Austria	2.81***	0.50	-0.47	2.49**	-0.18	-0.54	3.07***	1.18	0.72
.BFX	Belgium	0.74	1.63	-0.42	0.74	1.16	-0.94	0.60	0.66	0.89
.BSESN	India	2.21**	-0.40	-0.58	2.42**	-1.08	1.29	2.23**	3.51***	-0.71
.BVSP	Brazil	1.74*	1.40	-1.35	2.32**	0.46	-1.31	1.58	1.91*	-0.69
.GDAXI	Germany	-0.04	0.35	-0.66	0.14	-0.33	0.06	-0.53	1.60	1.29
.FCHI	France	1.17	-0.15	-0.69	0.64	-0.05	0.64	1.68*	1.81*	0.72
.FTMIB	Italy	4.16***	1.19	-0.40	4.15***	1.75*	-0.54	2.19**	2.90***	0.99
.FTSE	Great Britain	1.46	0.24	-1.83	2.60***	-0.60	-1.85*	-2.70***	1.67*	3.32***
.GSPTSE	Canada	2.53**	0.05	-0.61	1.92*	-0.24	-0.87	2.68***	-0.34	-1.58
.IBEX	Spain	3.73***	0.57	-0.34	3.01***	0.76	0.41	2.80***	2.31**	0.23
.JALSH	South Africa	8.44***	2.57**	-0.96	7.96***	2.00**	-0.17	6.98***	2.55**	-1.01
.MCX	Russia	2.42**	1.88*	-0.62	1.71*	2.35**	-0.53	1.68*	1.98**	0.46
.N225	Japan	2.71***	1.16	1.13	1.33	0.65	1.71*	0.26	3.37***	2.92***
.OBX	Norway	5.51***	1.56	-0.33	7.41***	-1.08	0.36	5.36***	-2.81**	-1.69*
.OMXC20	Denmark	4.31***	-1.11	-0.56	3.77***	2.56**	-0.94	4.19***	1.89*	-1.30
.OMXHPI	Finland	5.85***	2.83***	-0.95	11.05***	1.06	0.95	4.14***	3.94***	0.46
.OMXS30	Sweden	2.86***	3.77***	-1.11	2.66***	2.49**	-1.16	2.25**	1.92*	-0.89
.PSI20	Portugal	3.24***	-0.49	1.03	4.19***	0.41	1.08	0.84	0.53	0.87
.SPX	United States	1.39	1.68*	-2.52**	1.47	0.67	-2.43**	2.26**	3.73***	-0.62
.SSEC	China	4.97***	3.10***	0.79	5.54***	3.86***	0.63	4.12***	3.21***	1.22
.SSMI	Switzerland	2.33**	0.87	1.56	1.66*	3.88***	1.54	0.96	2.38**	1.50

Table 5: In analogy to Table 2 with MAC instead of HAC test statistics. Again Panel A is based on 15-minute data, Panel B adjusts for intraday seasonality and Panel C is based on 5-minute data.

		A			B			C		
		15-minute data			seasonally adjusted			5-minute data		
RIC	Country	CC	CJ	JJ	CC	CJ	JJ	CC	CJ	JJ
.AEX	Netherlands	2.84***	0.32	0.81	2.85***	1.81*	0.24	0.88	2.22***	1.09
.ATX	Austria	12.13***	1.17	-0.35	12.46***	0.48	-0.61	13.05***	0.77	2.67***
.BFX	Belgium	1.42	1.00	-0.52	1.80*	1.61	-0.79	0.70	-0.75	1.52
.BSESN	India	8.25***	-0.24	-0.96	8.75***	-0.46	0.79	6.67***	4.31***	-2.12**
.BVSP	Brazil	7.37***	0.22	-1.74*	6.22***	0.60	-1.54	11.63***	2.97***	-0.51
.GDAXI	Germany	0.85	1.02	0.04	0.87	-0.46	0.56	-2.41**	2.70***	2.04**
.FCHI	France	0.98	0.79	-0.67	0.49	0.55	0.39	-0.98	2.93***	1.46
.FTMIB	Italy	3.90***	2.64***	-0.44	4.38***	1.95*	-0.68	2.15**	3.02***	0.60
.FTSE	Great Britain	3.99***	1.44	-1.48	5.01***	-0.01	-1.86*	6.34***	1.99**	1.27
.GSPTSE	Canada	8.71***	1.34	-0.13	8.00***	-1.39	-1.33	12.14***	0.60	-1.13
.IBEX	Spain	4.94***	0.98	-0.28	4.15***	1.58	0.22	4.03***	1.71*	0.20
.JALSH	South Africa	6.97***	1.36	-0.84	6.36***	2.46**	-0.52	6.88***	3.65***	0.10
.MCX	Russia	12.04***	1.88*	-0.68	10.79***	3.57***	-0.68	13.38***	4.10***	1.33
.N225	Japan	3.87***	1.62	2.05**	2.38**	1.63	1.70*	3.91***	5.09***	3.32***
.OBX	Norway	4.19***	1.89*	-1.19	4.64***	0.35	0.08	5.21***	-0.02	-1.58
.OMXC20	Denmark	6.22***	0.43	-1.87*	4.98***	1.45	-2.11*	6.40***	1.52	-1.71*
.OMXHPI	Finland	8.12***	2.53**	-0.61	7.82***	1.29	0.91	9.56***	2.67***	3.02***
.OMXS30	Sweden	4.86***	1.74*	-1.06	5.03***	2.35**	-1.12	3.82***	1.23	-0.86
.PSI20	Portugal	6.85***	-0.34	1.00	6.64***	0.82	1.03	5.45***	1.59	0.99
.SPX	United States	5.66***	2.83***	-1.81*	5.87***	0.95	-2.35**	6.44***	3.74***	-0.10
.SSEC	China	9.43***	2.51**	0.63	9.30***	2.52**	0.83	13.29***	4.95***	1.76*
.SSMI	Switzerland	4.36***	1.16	1.46	4.36***	3.82***	1.48	2.63***	2.21**	1.47

Table 6: In analogy to Table 2 with MOM instead of HAC test statistics. Again Panel A is based on 15-minute data, Panel B adjusts for intraday seasonality and Panel C is based on 5-minute data.

RIC	Country	$\overline{r^2}$	$\overline{RV}$	$\overline{r^2}/\overline{RV}$	$\sqrt{\overline{r^2}} - \sqrt{\overline{RV}}$	t_{HAC}	t_{MAC}	t_{MOM}	T
.AEX	Netherlands	2.01	1.93	1.04	0.28	1.28	0.83	1.58	4,567
.ATX	Austria	1.92	1.28	1.49	2.52	8.44***	2.79***	9.04***	4,179
.BFX	Belgium	0.88	0.85	1.05	0.21	1.46	0.65	1.76*	5,274
.BSESN	India	1.82	1.78	1.02	0.15	0.74	1.10	1.76*	4,947
.BVSP	Brazil	3.83	3.25	1.18	1.52	3.15***	3.28***	5.96***	4,728
.GDAXI	Germany	2.43	2.55	0.95	-0.40	-1.97**	-1.69*	-2.05**	5,263
.FCHI	France	2.09	2.18	0.96	-0.32	-1.68*	-0.65	-0.65	5,272
.FTMIB	Italy	3.20	2.89	1.11	0.91	2.71***	1.80*	2.96***	1,942
.FTSE	Great Britain	1.42	1.40	1.01	0.07	0.46	0.29	1.58	5,218
.GSPTSE	Canada	0.86	0.73	1.17	0.71	3.26***	5.87***	9.21***	3,654
.IBEX	Spain	2.39	2.24	1.07	0.49	2.63***	2.44**	2.90***	5,188
.JALSH	South Africa	1.30	1.14	1.14	0.74	4.12***	3.49	5.20***	3,216
.MCX	Russia	4.05	3.30	1.23	1.95	5.74***	3.51***	7.97***	3,892
.N225	Japan	1.33	1.18	1.12	0.65	3.44***	3.39***	3.60***	5,080
.OBX	Norway	2.14	1.89	1.14	0.91	2.68***	4.51***	5.00***	2,678
.OMXC20	Denmark	1.64	1.42	1.15	0.89	3.51***	3.26***	3.93***	2,821
.OMXHPI	Finland	2.16	1.76	1.22	1.40	4.35***	2.83***	6.42***	2,835
.OMXS30	Sweden	1.97	1.86	1.06	0.39	1.52	0.71	1.19	3,005
.PSI20	Portugal	1.44	1.29	1.11	0.62	3.30***	1.66*	2.43**	4,866
.SPX	United States	1.18	1.22	0.97	-0.18	-1.16	-0.55	1.41	5,183
.SSEC	China	2.14	2.08	1.03	0.22	1.01	0.68	1.73*	5,019
.SSMI	Switzerland	1.46	1.37	1.07	0.39	1.51	1.67*	2.19**	4,770

Table 7: In analogy to Table 1 with the difference that average RV and average squared return are now calculated without including the first and last hour of the trading day. Again, average squared return, average adjusted realized volatility, and average deviation between the two are stated per annum in percent.

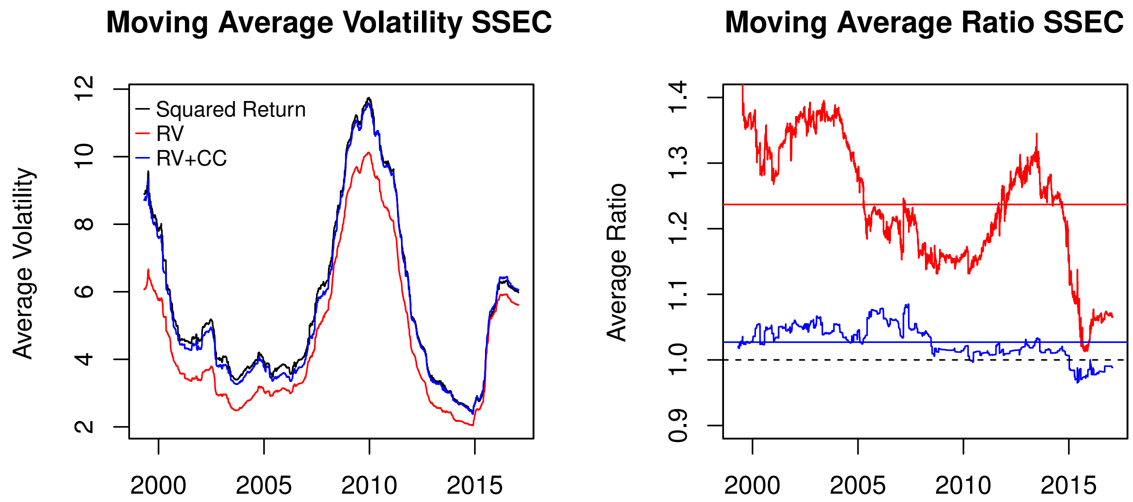

Figure 11: In analogy to Figure 2 with $\widetilde{RV} = RV_t + CC_t$ for the SSEC. Again, both plots depict the moving average of the previous 750 observations.

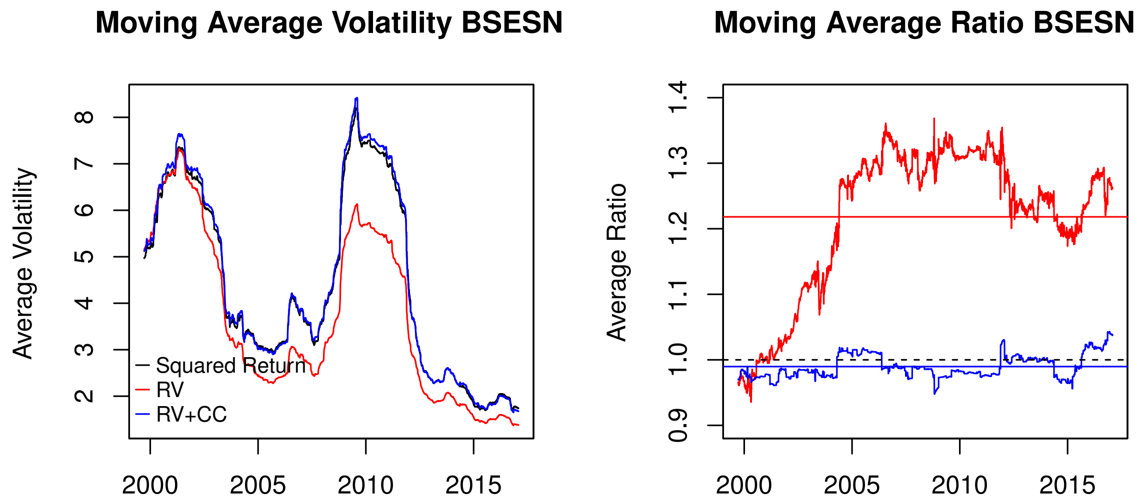

Figure 12: In analogy to Figure 2 with $\widetilde{RV} = RV_t + CC_t$ for the BSESN. Again, both plots depict the moving average of the previous 750 observations.

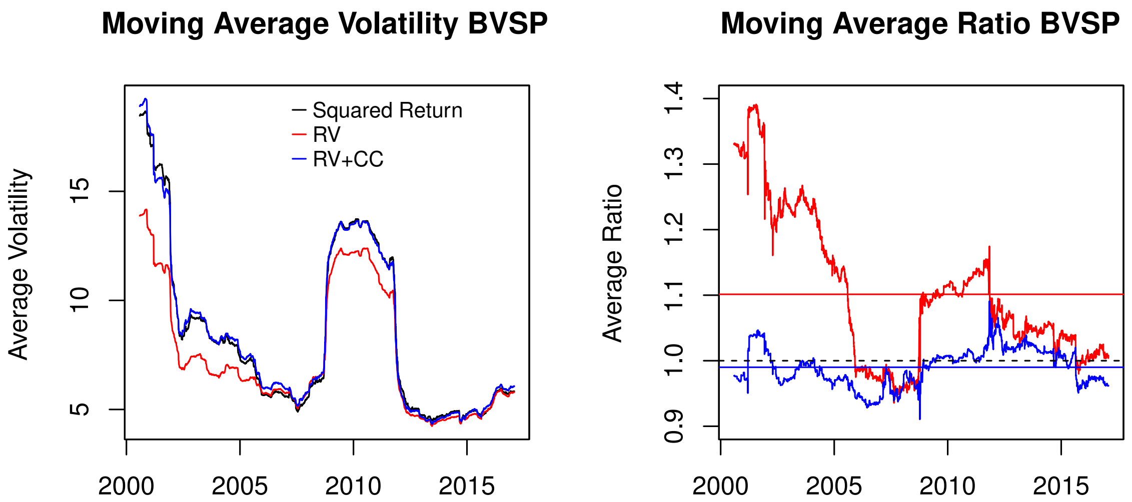

Figure 13: In analogy to Figure 2 with $\widetilde{RV} = RV_t + CC_t$ for the BVSP. Again, both plots depict the moving average of the previous 750 observations.

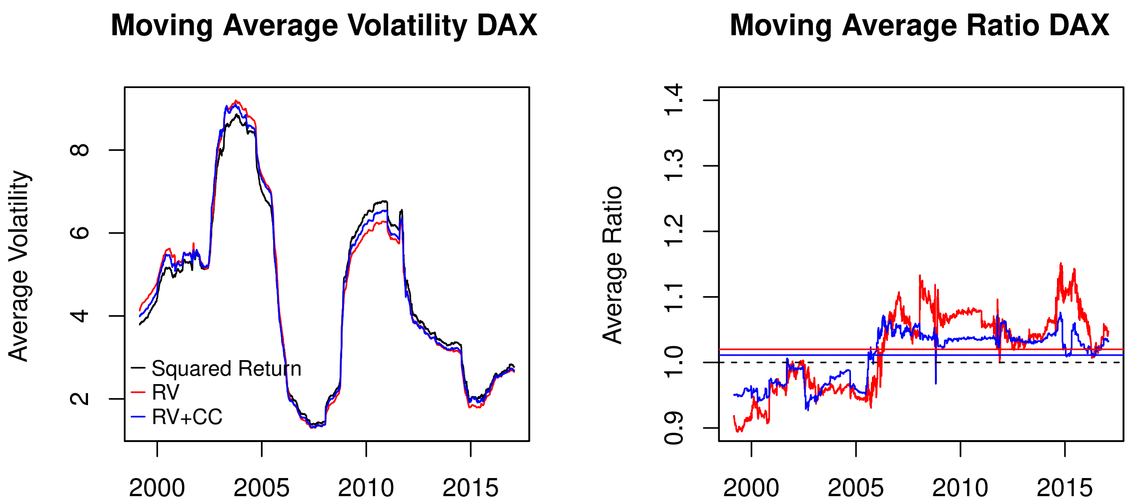

Figure 14: In analogy to Figure 2 with $\widetilde{RV} = RV_t + CC_t$ for the DAX. Again, both plots depict the moving average of the previous 750 observations.

References

Abadir, K., W. Distaso, and L. Giraitis (2009). “Two estimators of the long-run variance: beyond short memory”. In: *Journal of Econometrics* 150(1), pp. 56–70.

Aït-Sahalia, Y. (2004). “Disentangling diffusion from jumps”. In: *Journal of Financial Economics* 74(3), pp. 487–528.

Aït-Sahalia, Y., J. Jacod, et al. (2009). “Testing for jumps in a discretely observed process”. In: *The Annals of Statistics* 37(1), pp. 184–222.

Aït-Sahalia, Y., P. Mykland, and L. Zhang (2011). “Ultra high frequency volatility estimation with dependent microstructure noise”. In: *Journal of Econometrics* 160(1), pp. 160–175.

Andersen, T. and L. Benzoni (2009). “Realized volatility”. In: *Handbook of financial time series*. Springer, pp. 555–575.

Andersen, T. and T. Bollerslev (1997). “Intraday periodicity and volatility persistence in financial markets”. In: *Journal of Empirical Finance* 4(2-3), pp. 115–158.

Andersen, T. and T. Bollerslev (1998a). “Answering the skeptics: Yes, standard volatility models do provide accurate forecasts”. In: *International Economic Review*, pp. 885–905.

Andersen, T. and T. Bollerslev (1998b). “Deutsche mark–dollar volatility: intraday activity patterns, macroeconomic announcements, and longer run dependencies”. In: *The Journal of Finance* 53(1), pp. 219–265.

Andrews, D. (1991). “Heteroskedasticity and autocorrelation consistent covariance matrix estimation”. In: *Econometrica: Journal of the Econometric Society*, pp. 817–858.

Bajgrowicz, P., O. Scaillet, and A. Treccani (2015). “Jumps in high-frequency data: Spurious detections, dynamics, and news”. In: *Management Science* 62(8), pp. 2198–2217.

Bandi, F. and J. Russell (2006). “Separating microstructure noise from volatility”. In: *Journal of Financial Economics* 79(3), pp. 655–692.

Bandi, F. and J. Russell (2008). “Microstructure noise, realized variance, and optimal sampling”. In: *The Review of Economic Studies* 75(2), pp. 339–369.

Barndorff-Nielsen, O. and N. Shephard (2001). “Non-Gaussian Ornstein–Uhlenbeck-based models and some of their uses in financial economics”. In: *Journal of the Royal Statistical Society: Series B (Statistical Methodology)* 63(2), pp. 167–241.

Barndorff-Nielsen, O.. and N. Shephard (2004). “Power and Bipower Variation with Stochastic Volatility and Jumps”. In: *Journal of Financial Econometrics* 2(1), pp. 1–37.

Barndorff-Nielsen, O. and N. Shephard (2007). “Variation, jumps, market frictions and high frequency data in financial econometrics”. In: *Advances in Economics and Econometrics. Theory and Applications*. Cambridge University Press, pp. 328–372.

Barndorff-Nielsen, O., P. Hansen, A. Lunde, and N. Shephard (2008). “Designing realized kernels to measure the ex post variation of equity prices in the presence of noise”. In: *Econometrica* 76(6), pp. 1481–1536.

Christensen, K., R. Oomen, and M. Podolskij (2014). “Fact or friction: Jumps at ultra high frequency”. In: *Journal of Financial Economics* 114(3), pp. 576–599.

Corsi, F., D. Pirino, and R. Reno (2010). “Threshold bipower variation and the impact of jumps on volatility forecasting”. In: *Journal of Econometrics* 159(2), pp. 276–288.

Gao, L., Y. Han, S. Li, and G. Zhou (2018). “Market intraday momentum”. In: *Journal of Financial Economics*.

Han, H. and D. Kristensen (2014). “Asymptotic theory for the QMLE in GARCH-X models with stationary and nonstationary covariates”. In: *Journal of Business & Economic Statistics* 32(3), pp. 416–429.

Hansen, P. and A. Lunde (2006). “Realized variance and market microstructure noise”. In: *Journal of Business & Economic Statistics* 24(2), pp. 127–161.

Heber, G., A. Lunde, N. Shephard, and K. Sheppard (2009). “Oxford-Man Institute’s realized library, version 0.2”. In: *Oxford-Man Institute, University of Oxford*.

Kruse, R., C. Leschinski, and M. Will (2018). “Comparing Predictive Accuracy under Long Memory, With an Application to Volatility Forecasting”. In: *Journal of Financial Econometrics*.

Lee, S. and P. Mykland (2007). “Jumps in financial markets: A new nonparametric test and jump dynamics”. In: *The Review of Financial Studies* 21(6), pp. 2535–2563.

Robinson, P. (2005). “Robust covariance matrix estimation: HAC estimates with long memory/antipersistence correction”. In: *Econometric Theory* 21(1), pp. 171–180.

Shephard, N. and K. Sheppard (2010). “Realising the future: forecasting with high-frequency-based volatility (HEAVY) models”. In: *Journal of Applied Econometrics* 25(2), pp. 197–231.

Simes, J. (1986). “An improved Bonferroni procedure for multiple tests of significance”. In: *Biometrika* 73(3), pp. 751–754.

Zhou, B. (1996). “High-frequency data and volatility in foreign-exchange rates”. In: *Journal of Business & Economic Statistics* 14(1), pp. 45–52.