Testing for breaks in the cointegrating relationship: On the stability of government bond markets' equilibrium

Paulo M. M. Rodrigues[1], Philipp Sibbertsen[2] , and Michelle Voges[2]

This version: June 25, 2019

Abstract
In this paper, test procedures for no fractional cointegration against possible breaks in the persistence structure of a fractional cointegrating relationship are introduced. The tests proposed are based on the supremum of the Hassler and Breitung (2006) test statistic for no cointegration over possible breakpoints in the long-run equilibrium. We show that the new tests correctly standardized converge to the supremum of a chi-squared distribution, and that this convergence is uniform. An in-depth Monte Carlo analysis provides results on the finite sample performance of our tests. We then use the new procedures to investigate whether there was a dissolution of fractional cointegrating relationships between benchmark government bonds of ten EMU countries (Spain, Italy, Portugal, Ireland, Greece, Belgium, Austria, Finland, the Netherlands and France) and Germany with the beginning of the European debt crisis.

Keywords: Fractional cointegration · Persistence breaks · Hassler-Breitung test · Changing Long-run equilibrium
JEL classification: C12, C32

---

[1]Banco de Portugal and Nova School of Business and Economics
[2]Institute of Statistics, Faculty of Economics and Management, Leibniz University Hannover, D-30167 Hannover, Germany

1 Introduction

Since the seminal works of Engle and Granger (1987) and Johansen (1988) cointegration testing has become an important topic of research, both theoretically as well as empirically. The equilibrium relationship between economic and financial variables postulated by many economic theories is typically assumed to be constant over time, i.e., cointegrating relationships do not change. However, this assumption may be too restrictive.

A constant long-run equilibrium may be questionable in light of the growing empirical evidence that economic and financial time series may display persistence changes over time (see, inter alia, Kim (2000), Kim et al. (2002), Busetti and Taylor (2004), and Harvey et al. (2006), for tests when the order of integration is integer; and Giraitis and Leipus (1994), Beran and Terrin (1996), Beran and Terrin (1999), Sibbertsen and Kruse (2009), Hassler and Scheithauer (2011), Hassler and Meller (2014), and Martins and Rodrigues (2014), for tests when the order of integration is some real number). Hence, it is natural to expect that changes in the persistence of economic and financial time series may also originate changes in the long-run equilibrium. This has been substantiated in recent years by a vast literature documenting changes in the historical behaviour of economic and financial variables; see among others, McConnell and Perez-Quiros (2000), Herrera and Pesavento (2005), Cecchetti et al. (2006), Kang et al. (2009) and Halunga et al. (2009).

The impact of structural breaks in the deterministic kernels on cointegration has been widely analysed (see e.g. Hansen (1992), Quintos and Phillips (1993), Hao (1996), Andrews et al. (1996), Bai and Perron (1998), Kuo (1998), Inoue (1999), Johansen et al. (2000), and Lütkepohl et al. (2003), but less attention has been given to the impact of changes in the actual long-run equilibrium (see Martins and Rodrigues (2018)). The focus of this paper is to propose new tests capable of detecting changes in fractional cointegration relationships. We introduce procedures designed to detect changes in the long-run equilibrium between macroeconomic or financial variables based on rolling, recursive forward and recursive reverse estimation of the Hassler and Breitung (2006) test, in the spirit of the approaches proposed by e.g. Davidson and Monticini (2010). Asymptotic results are derived and the performance of the new tests evaluated in an in-depth Monte Carlo exercise. In particular, special attention is devoted to the case of unknown orders of integration of the variables involved due to its empirical relevance. Furthermore, we apply the new test statistics to the government bond market of the European Monetary Union (EMU) finding evidence of segmented fractional cointegration with breaks at the beginning of the European debt crisis.

This paper is organized as follows. Section 2 presents the model specification and assumptions; Section 3 introduces the tests for no cointegration under persistence breaks, a break point estimator, and corresponding asymptotic theory; Section 4 discusses the results of an in-depth Monte Carlo analysis on the finite sample properties of the new tests; Section 5 illustrates the application of the new procedures to the EMU government bond market; Section 6 concludes the paper and finally, an appendix collects all the proofs.

2 Model Specification and Assumptions

Consider an m-dimensional process $\mathbf{x_t}$ integrated of order d, $I(d)$, and let y_t be an one-dimensional $I(d)$ process as well. The processes $\mathbf{x_t}$ and y_t are said to be fractionally cointegrated if, considering the regression,

$$y_t = \mathbf{x}_t' \beta + u_t, \qquad t = 1, \ldots, T, \tag{1}$$

u_t is integrated of order $I(d-b)$ with $b > 0$.

In what follows the focus is on testing the null hypothesis of no fractional cointegration, $H_0 : b = 0$. The usual alternative in this setting is to have fractional cointegration over the whole range of observations, $H_1 : b > 0$. However, we are interested in testing for segmented fractional cointegration. This means that the fractional cointegration relationship may hold only in subsamples of the period under analysis. Therefore, our alternative hypothesis is $H_1 : b_t > 0$, for $t = \lfloor \lambda_1 T \rfloor + 1, \ldots, \lfloor \lambda_2 T \rfloor$ and $b_t = 0$ elsewhere, with $0 \le \lambda_1 < \lambda_2 \le 1$.

The test statistics that will be proposed are based on the approach of Hassler and Breitung (2006), who provide a regression-based test for the null of no fractional cointegration on the residuals, $\hat{u}_t$, of a model as in (1). Before presenting the relevant test statistics let us make the following assumptions:

Assumption 1: Let y_t and $\mathbf{x_t}$ be fractionally integrated of orders d_1 and d_2, respectively with $y_t = 0$ and $\mathbf{x}_t = 0$ for $t \le 0$.

Assumption 2: The vector $\mathbf{v}_t' := (v_{1,t}, \mathbf{v}_{2,t}') = (\Delta_+^{d_1} y_t,\ \Delta_+^{d_2} \mathbf{x}_t')$, is a stationary vector autoregressive process of order p of the form

$$\mathbf{v}_t = A_1 \mathbf{v}_{t-1} + \cdots + A_p \mathbf{v}_{t-p} + \varepsilon_t \tag{2}$$

where $\Delta_+^{d_1} y_t := (1-L)^{d_1} y_t I(t>0)$, $\Delta_+^{d_2} \mathbf{x}_t := (1-L)^{d_2} \mathbf{x}_t I(t>0)$, $I(\cdot)$ is the indicator function, L denotes the usual backshift or lag operator and the error process ε_t is assumed to be independent and identically distributed (iid) with mean zero and covariance matrix,

$$\boldsymbol{\Sigma} := \begin{pmatrix} \sigma_{11}^2 & \sigma_{21}' \\ \sigma_{21} & \boldsymbol{\Sigma}_{22} \end{pmatrix}.$$

3 Testing for no cointegration under persistence breaks

As in Hassler and Breitung (2006) the cointegrating vector β is not identified under the null hypothesis of no cointegration. Thus, considering that $d_1 = d_2 = d$, we define the following regression model,

$$\Delta_+^d y_t = \Delta_+^d \mathbf{x}_t' \beta + e_t, \qquad \beta := \boldsymbol{\Sigma}_{22}^{-1}\sigma_{21} \tag{3}$$

where $e_t := v_{1,t} - \mathbf{v}_{2,t}'\boldsymbol{\Sigma}_{22}^{-1}\sigma_{21}$.

The LM test for no cointegration is then applied to the OLS residuals, $\hat{e}_t$, obtained from (3), i.e.,

$$\Delta_+^d y_t = \Delta_+^d \mathbf{x}_t'\hat{\beta} + \hat{e}_t$$

where

$$\hat{e}_t := e_t - \sum_{t=1}^{T} \mathbf{v}_{2,t}' e_t \left(\sum_{t=1}^{T} \mathbf{v}_{2,t}\mathbf{v}_{2,t}' \right)^{-1} \mathbf{v}_{2,t}.$$

Specifically, to implement the tests proposed by Hassler and Breitung (2006) and Demetrescu et al. (2008), which is the approach followed in this paper, a regression framework is considered, viz.,

$$\hat{e}_t = \phi \hat{e}_{t-1}^* + \sum_{i=1}^{p} \gamma_i \hat{e}_{t-i} + a_t, \qquad t = 1, \ldots, T, \tag{4}$$

where $\hat{e}_{t-1}^* := \sum_{j=1}^{t-1} j^{-1}\hat{e}_{t-j}$ and a_t is a martingale difference sequence. Equation (4) is used to test the null $H_0 : \phi = 0$ $(b = 0)$ against the alternative $H_1 : \phi < 0$ $(b > 0)$.

Remark 3.1: Under local alternatives of the form $\mathrm{H}_1 : b = c/\sqrt{T}$ with a fixed $c > 0$, it can be shown that $\phi = -c/\sqrt{T} + O\left(T^{-1}\right)$ and that $\{a_t\}$ is a fractionally integrated noise component. As a result, the heterogenous behavior of ϕ and the different stochastic properties of a_t provide a sound statistical basis to identify the order of fractional integration of $\{\hat{e}_t\}$. Despite the apparent theoretical simplicity of this framework, the fact that $\hat{e}_{t-1}^*$ converges in mean square sense to $e_{t-1}^{**} := \sum_{j=1}^{\infty} j^{-1} e_{t-j,d}$ under the null hypothesis and Assumption 1, with $\left\{e_{t-1}^{**}\right\}$ being a stationary linear process with non-absolutely summable coefficients, is a source of major technical difficulties for the asymptotic analysis in this context; see e.g. Hassler et al. (2009). □

Remark 3.2: Demetrescu et al. (2008) and Hassler et al. (2009) derive the asymptotic theory of the fractional integration tests under least-squares (LS) estimation of the set of parameters $\kappa := \left(\phi, \gamma_1, ..., \gamma_p\right)'$ of a regression as in (4), and show that these are $\sqrt{T}$-consistency and asymptotic normal under fairly general conditions. As a result, in a conventional setting as in (4) $\mathrm{H}_0 : \phi = 0$ can be tested by means of a standard t-ratio, or some measurable transformation such as its squares. If our assumptions are strengthened such that $a_t \sim iid\mathcal{N}\left(0, \sigma^2\right)$, the specific harmonic weighting upon which $\left\{e^*_{t-1}\right\}$ is constructed in (4) also ensures efficient testing. □

In this paper we concentrate on the case of iid errors, e_t, ($p = 0$ in (4)) although it is also possible to allow for serial correlation in the innovations. Following Demetrescu et al. (2008) this can be accommodated through parametric augmentation as in (4) allowing for $p > 0$.

3.1 The Test Statistics

As we are interested in testing for no fractional cointegration against the alternative of segmental fractional cointegration, we apply the Hassler and Breitung (2006) test on a subinterval defined by the truncation points λ_1 and λ_2 with $0 \leq \lambda_1 < \lambda_2 \leq 1$. Thus, for λ_1 and λ_2 fixed we consider the statistic,

$$t(\hat{e}(\lambda_1, \lambda_2)) = \frac{\sqrt{\lfloor \lambda_2 T \rfloor - \lfloor \lambda_1 T \rfloor} \sum_{t=\lfloor \lambda_1 T \rfloor + 1}^{\lfloor \lambda_2 T \rfloor} \hat{e}_t(\lambda_1, \lambda_2) \hat{e}^*_{t-1}(\lambda_1, \lambda_2)}{\sqrt{\sum_{t=\lfloor \lambda_1 T \rfloor + 1}^{\lfloor \lambda_2 T \rfloor} \hat{e}^{*2}_{t-1}(\lambda_1, \lambda_2)} \sqrt{\frac{1}{T-1} \sum_{t=\lfloor \lambda_1 T \rfloor + 1}^{\lfloor \lambda_2 T \rfloor} \hat{e}^2_t(\lambda_1, \lambda_2)}} \tag{5}$$

where $\hat{e}_t(\lambda_1, \lambda_2)$ are the subsample based residuals and $\hat{e}^*_{t-1}(\lambda_1, \lambda_2)$ the corresponding harmonic weighted residuals as defined in (4).

However, since the breakpoints, λ_1 and λ_2, are usually unknown we adopt the split sample testing approach proposed by Davidson and Monticini (2010), and define the following sets on which the tests will be performed:

$$\Lambda_S = \left\{ \left\{ 0, \frac{1}{2} \right\}, \left\{ \frac{1}{2}, 1 \right\} \right\} \tag{6}$$

$$\Lambda_{0f} = \{\{0, s\} : s \in [\lambda_0, 1]\} \tag{7}$$

$$\Lambda_{0b} = \{\{s, 1\} : s \in [0, 1 - \lambda_0]\} \tag{8}$$

$$\Lambda_{0R} = \{\{s, s + \lambda_0\} : s \in [0, 1 - \lambda_0]\} \tag{9}$$

where Λ_S represents a simple split sample with just two elements; Λ_{0f} and Λ_{0b} denote forward- and backward-running incremental samples, respectively of minimum length $\lfloor \lambda_0 T \rfloor$ and maximum length T; Λ_{0R} defines a rolling sample of fixed length $\lfloor \lambda_0 T \rfloor$, and finally $\lambda_0 \in (0,1)$ is fixed and needs to be chosen by the practitioner. Davidson and Monticini (2010) consider two additional sets, namely $\Lambda_S^* = \Lambda_S \cup \{0,1\}$ and $\Lambda_{0R}^* = \Lambda_{0R} \cup \{0,1\}$.

Therefore, considering the sets in (6) to (9), our proposed test procedures against breaks in the fractional cointegration relation are the split sample tests,

$$\mathcal{T}_S := \max_{\{\lambda_1,\lambda_2\}\in\Lambda_S} t^2(\hat{e}(\lambda_1,\lambda_2)); \tag{10}$$

$$\mathcal{T}_S^* := \max_{\{\lambda_1,\lambda_2\}\in\Lambda_S^*} t^2(\hat{e}(\lambda_1,\lambda_2)); \tag{11}$$

the incremental (recursive) tests

$$\mathcal{T}_{I_f}(\lambda) := \max_{\lambda_0\le\lambda\le 1} t^2(\hat{e}(0,\lambda)); \tag{12}$$

$$\mathcal{T}_{I_b}(\lambda) := \max_{0\le\lambda\le 1-\lambda_0} t^2(\hat{e}(\lambda,1)); \tag{13}$$

the rolling sample test

$$\mathcal{T}_R(\lambda) := \max_{0\le\lambda\le 1-\lambda_0} t^2(\hat{e}(\lambda,\lambda+\lambda_0)); \tag{14}$$

$$\mathcal{T}_R^*(\lambda) := \max_{\{\lambda_1,\lambda_2\}\in\Lambda_{0R}^*} t^2(\hat{e}(\lambda_1,\lambda_2)). \tag{15}$$

We can state these statistics in general form as,

$$\mathcal{T}_K(\lambda_1,\lambda_2) := \max_{\lambda_1\in\Lambda_1,\lambda_2\in\Lambda_2} t^2(\hat{e}(\lambda_1,\lambda_2)),\ K = S, S^*, I_f, I_b, R, R^\star. \tag{16}$$

3.2 Asymptotic Results

To characterize the asymptotic behavior of the test statistics in (10) - (15), consider first Theorem 1 provided next, which states the asymptotic normality of the test statistic in (5) and which is the main building block of the test statistics $\mathcal{T}_K(\lambda_1,\lambda_2)$, $K = S, S^*, I_f, I_b, R, R^\star$.

Theorem 1. *Assuming that the data is generated from (1) and that Assumptions 1 and 2 hold, it follows under the null hypothesis of no fractional cointegration that, as $T \to \infty$,*

$$t(\hat{e}(\lambda_1,\lambda_2)) \Rightarrow N(0,1), \tag{17}$$

where $\Rightarrow$ denotes weak convergence.

Hence, based on the result of Theorem 1 we can now state the limit results for the test statistics introduced in (10) - (15).

Theorem 2. *Assuming that the data is generated from (1) and that Assumptions 1 and 2 hold, under the null hypothesis of no fractional cointegration it follows, as $T \to \infty$, that*

$$\mathcal{T}_K(\lambda_1, \lambda_2) \Rightarrow \sup_{\lambda_1 \in \Lambda_1, \lambda_2 \in \Lambda_2} \chi_1^2, \; K = S, S^*, I_f, I_b, R, R^*. \tag{18}$$

As a next step we provide an estimator of the break point τ under the alternative. The estimator basically consists of minimizing the sum of squared residuals of a regression as in (3). Thus, our break point estimator is

$$\hat{\tau} = \underset{\tau \in \Delta}{arg\ inf}\ [\tau T]^{-2\hat{d}} \sum_{t=1}^{[\tau T]} \hat{e}_t^2(\tau) \tag{19}$$

where, $\Delta := (\delta; (1-\delta))$ and $0 < \delta < 0.5$ is an interval eliminating the first and last observations to have enough observations at hand for the break point estimation. For this statistic, the following consistency result can be stated:

Theorem 3. *Assuming that the break is from the cointegrated subsample to the non-cointegrated subsample and that Assumptions 1 and 2 hold, as $T \to \infty$, than*

$$\hat{\tau} \to \tau_0. \tag{20}$$

where τ_0 denotes the true break fraction.

Remark 3.3: If the break is from the non-cointegrated to the cointegrated sample then the reversed sum of squared residuals, from T to $\lfloor \tau T \rfloor$, can be used to consistently estimate the break fraction τ_0. □

4 Monte Carlo Study

In this Section, we analyze the finite-sample properties of the residual-based tests for segmented fractional cointegration introduced above by means of Monte Carlo simulation. The data generation process (DGP) considered for the empirical size and power analysis is

$$y_t = x_t + e_t, \qquad t = 1, ..., T \tag{21}$$

$$x_t = x_{t-1} + v_t, \tag{22}$$

$$(1-L)^{(1-b_t)} e_t = a_t, \tag{23}$$

where

$$\begin{pmatrix} v_t \\ a_t \end{pmatrix} \sim iidN\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}\right).$$

For $\rho = 0$, x_t is strictly exogenous whereas for $\rho \neq 0$, x_t is correlated with e_t (i.e. endogenous).

For implementation of the tests we compute the OLS residuals,

$$\hat{e}_t = y_t - \hat{\alpha} - \hat{\beta}x_t, \tag{24}$$

run the test regression in (4) on these residuals $(\hat{e}_t)$ and compute the different test statistics introduced in the previous section, i.e., $\mathcal{T}_S^*$, $\mathcal{T}_{I_f}(\lambda_0)$, $\mathcal{T}_{I_b}(\lambda_0)$, and $\mathcal{T}_R(\lambda_0)$, as well as the full sample test proposed by Hassler and Breitung (2006), which we denote as $\mathcal{T}_{HB}$. All results reported are for a 5% significance level and are based on 5000 Monte Carlo replications. We present results for sample sizes $T = \{250, 500\}$.

For benchmarking purposes, we consider the test statistics computed either for iid innovations as in Breitung and Hassler (2002) or using Eicker-White's correction against heteroskedasticity as in Demetrescu et al. (2008).

To compute the critical values for the tests we generate data from

$$y_t = x_t + e_t, \qquad t = 1, ..., T \tag{25}$$

$$(1-L)^{d_1} x_t = v_t, \tag{26}$$

$$(1-L)^{d_1} e_t = a_t, \tag{27}$$

with $d_1 = \{0.5, 0.6, ..., 1\}$ and computed the critical values as the average of the critical values obtained for each d_1 considered at a specific significance level (see Table 1).

Table 1: Critical Values for Subsample Tests

	$\mathcal{T}_S^*$	$\mathcal{T}_{I_f}(\lambda_0)$	$\mathcal{T}_{I_b}(\lambda_0)$	$\mathcal{T}_R(\lambda_0)$
		$T = 250$		
1%	9.438	7.722	7.699	7.172
5%	5.960	4.458	4.471	4.112
10%	4.470	3.130	3.133	2.867
		$T = 500$		
1%	8.888	7.387	7.405	6.862
5%	5.737	4.293	4.296	3.955
10%	4.381	3.000	3.006	2.767

Note: For implementation of the tests we considered $\lambda_0 = 0.5$ and all results are based on 5000 Monte Carlo replications.

4.1 Empirical rejection frequencies

For the analysis of the finite sample rejection frequencies under the null and alternative hypothesis, we consider three experiments:

Experiment 1: Constant cointegration relation over the whole sample.

Experiment 2: Spurious regime in the first part of the sample and a fractional cointegrated regime in the second part, i.e.,

$$\begin{cases} b_t = 0 & for \quad t = 1, ..., \lfloor \lambda T \rfloor \\ b_t > 0 & for \quad t = \lfloor \lambda T \rfloor + 1, ..., T \end{cases} . \tag{28}$$

Experiment 3: Fractional cointegrated regime in the first part of the sample and a spurious regime in the second part of the sample, i.e.,

$$\begin{cases} b_t > 0 & for \quad t = 1, ..., \lfloor \lambda T \rfloor \\ b_t = 0 & for \quad t = \lfloor \lambda T \rfloor + 1, ..., T \end{cases} \tag{29}$$

with $\lambda \in \{0.3, 0.5, 0.7\}$ in both experiments 2 and 3.

In the case of Experiment 1, data is generated from (21) - (23), where y_t and x_t are both I(1) variables and $b_t = b = \{0, 0.05, 0.10, ..., 0.50\}$ which allows us to look at the empirical rejection frequencies under the null hypothesis (empirical size, $b = 0$) as well as under the alternative (finite sample power, $b_t > 0$). The first observation we can make from the upper panel of Table 2 is that for $T = 250$, with the exception of $\mathcal{T}_{HB}$ (which displays an empirical size of 8.4%), all other tests have acceptable finite sample size (ranging between 5.2% and 6.1%). As the sample size increases to $T = 500$ all tests improve in size (for $\mathcal{T}_{HB}$ the empirical rejection frequency under the null hypothesis reduces to 6.4% whereas for the other subsample tests it ranges between 4.5% and 4.9%). Also in terms of power an improvement is observed. In the lower panel with endogenous x_t, we observe lower empirical sizes for $T = 250$ compared to the exogenous case and slightly higher sizes for $T = 500$. The power is always better than with exogenous x_t. Overall, all tests are relatively robust to endogeneity. Note, that of the set of sequential tests proposed, the best performing in both cases are the recursive tests, $\mathcal{T}_{I_f}(\lambda_0)$ and $\mathcal{T}_{I_b}(\lambda_0)$, although, as expected, $\mathcal{T}_{HB}$ displays in the case of Experiment 1 the overall best performance.

Table 2: Rejection frequencies of tests - Experiment 1 ($\lambda_0 = 0.5$)

$\rho = 0$										
	$T = 250$					$T = 500$				
b	$\mathcal{T}_S^*$	$\mathcal{T}_{I_f}(\lambda_0)$	$\mathcal{T}_{I_b}(\lambda_0)$	$\mathcal{T}_R(\lambda_0)$	$\mathcal{T}_{HB}$	$\mathcal{T}_S^*$	$\mathcal{T}_{I_f}(\lambda_0)$	$\mathcal{T}_{I_b}(\lambda_0)$	$\mathcal{T}_R(\lambda_0)$	$\mathcal{T}_{HB}$
0.00	0.0612	0.0612	0.0580	0.0524	0.0842	0.0492	0.0460	0.0450	0.0480	0.0640
0.05	0.1538	0.1826	0.1812	0.1154	0.2256	0.2084	0.2518	0.2558	0.1580	0.3066
0.10	0.3844	0.4448	0.4482	0.2584	0.5144	0.6246	0.6954	0.6942	0.4314	0.7516
0.15	0.6908	0.7560	0.7576	0.4856	0.8110	0.9322	0.9590	0.9590	0.7582	0.9698
0.20	0.8990	0.9386	0.9390	0.6970	0.9608	0.9958	0.9982	0.9982	0.9436	0.9992
0.25	0.9878	0.9950	0.9952	0.8792	0.9968	0.9998	1	1	0.9950	1
0.30	0.9992	1	0.9998	0.9574	1	1	1	1	0.9992	1
0.35	0.9998	1	1	0.9920	1	1	1	1	1	1
0.40	1	1	1	0.9984	1	1	1	1	1	1
0.45	1	1	1	0.9998	1	1	1	1	1	1
0.50	1	1	1	1	1	1	1	1	1	1

$\rho = 0.8$										
	$T = 250$					$T = 500$				
b	$\mathcal{T}_S^*$	$\mathcal{T}_{I_f}(\lambda_0)$	$\mathcal{T}_{I_b}(\lambda_0)$	$\mathcal{T}_R(\lambda_0)$	$\mathcal{T}_{HB}$	$\mathcal{T}_S^*$	$\mathcal{T}_{I_f}(\lambda_0)$	$\mathcal{T}_{I_b}(\lambda_0)$	$\mathcal{T}_R(\lambda_0)$	$\mathcal{T}_{HB}$
0.00	0.0482	0.0538	0.0512	0.0394	0.0804	0.0516	0.0550	0.0558	0.0472	0.0664
0.05	0.1592	0.1994	0.2050	0.0948	0.2636	0.3144	0.3746	0.3652	0.1492	0.4034
0.10	0.4546	0.5468	0.5488	0.2172	0.6258	0.7966	0.8526	0.8502	0.4262	0.8622
0.15	0.7984	0.8718	0.8684	0.4186	0.9074	0.9834	0.9888	0.9896	0.7478	0.9898
0.20	0.9646	0.9806	0.9796	0.6300	0.9876	0.9996	0.9998	0.9998	0.9386	0.9998
0.25	0.9964	0.9992	0.9986	0.7892	0.9996	1	1	1	0.9882	1
0.30	0.9998	0.9998	1	0.9150	1	1	1	1	0.9986	1
0.35	1	1	1	0.9674	1	1	1	1	0.9996	1
0.40	1	1	1	0.9860	1	1	1	1	1	1
0.45	1	1	1	0.9942	1	1	1	1	1	1
0.50	1	1	1	0.9990	1	1	1	1	1	1

In the case of Experiment 2, the sample is divided into two sub-periods where in the first sub-period there is no cointegration ($b = 0$) and in the second the variables are cointegrated ($b > 0$). We allow the change into the cointegrated regime to be early in the sample ($\lambda = 0.3$), in the middle of the sample ($\lambda = 0.5$) and late in the sample ($\lambda = 0.7$). We consider a similar exercise in Experiment 3 except that the first sub-period corresponds to cointegration ($b > 0$) and the second to a spurious regression ($b = 0$). From Table 3 we observe first that the overall best performing test of the sequential tests introduced is $\mathcal{T}_S^*$ followed by $\mathcal{T}_{I_f}(\lambda_0)$. The overall test $\mathcal{T}_{HB}$, although slightly oversized, also displays interesting power performance. The good behavior of $\mathcal{T}_S^*$ is clearly observable in the larger sample ($T = 500$) where it stands out particularly for $\lambda = 0.5$ and $\lambda = 0.7$. For $\lambda = 0.3$ the difference of $\mathcal{T}_S^*$ with regards to $\mathcal{T}_{HB}$ is not as marked.

Table 4 reports results for the case where there is cointegration in the first sub-period and in the second sub-period the results are spurious. In this case the rolling approach $\mathcal{T}_R(\lambda_0)$ displays interesting behavior, particularly for $b_t > 0.15$ and $T = 250$ and for $b_t > 0.1$ when $T = 500$. The $\mathcal{T}_S^*$ statistic also displays good power performance.[1]

[1]We have also performed simulations with EW corrected statistics, however since the results are qualitatively similar to those reported in Tables 2 - 4 we have decided not to include them in the paper for the sake of space. These can however be obtained from the authors.

Table 3: Rejection frequencies of tests - Experiment 2 ($\lambda_0 = 0.5$)

	$T = 250$					$T = 500$				
b	$\mathcal{T}_S^*$	$\mathcal{T}_{I_f}(\lambda_0)$	$\mathcal{T}_{I_b}(\lambda_0)$	$\mathcal{T}_R(\lambda_0)$	$\mathcal{T}_{HB}$	$\mathcal{T}_S^*$	$\mathcal{T}_{I_f}(\lambda_0)$	$\mathcal{T}_{I_b}(\lambda_0)$	$\mathcal{T}_R(\lambda_0)$	$\mathcal{T}_{HB}$
					$\lambda = 0.3$					
0	0.055	0.058	0.061	0.054	0.076	0.058	0.055	0.056	0.056	0.065
0.05	0.079	0.077	0.079	0.051	0.104	0.082	0.083	0.083	0.057	0.096
0.10	0.101	0.100	0.103	0.050	0.128	0.133	0.125	0.136	0.052	0.141
0.15	0.134	0.129	0.144	0.051	0.166	0.189	0.173	0.182	0.053	0.191
0.20	0.161	0.151	0.167	0.054	0.189	0.254	0.222	0.238	0.051	0.243
0.25	0.202	0.178	0.194	0.053	0.221	0.311	0.265	0.272	0.052	0.293
0.30	0.237	0.210	0.230	0.050	0.257	0.375	0.325	0.339	0.050	0.351
0.35	0.281	0.247	0.262	0.051	0.298	0.453	0.393	0.410	0.052	0.420
0.40	0.310	0.275	0.293	0.056	0.324	0.499	0.424	0.437	0.052	0.454
0.45	0.353	0.307	0.313	0.049	0.359	0.537	0.467	0.473	0.058	0.493
0.50	0.397	0.341	0.353	0.055	0.393	0.594	0.514	0.527	0.052	0.543
					$\lambda = 0.5$					
0	0.051	0.060	0.060	0.048	0.078	0.054	0.059	0.057	0.054	0.069
0.05	0.092	0.094	0.099	0.063	0.126	0.114	0.115	0.118	0.090	0.132
0.10	0.159	0.147	0.155	0.091	0.182	0.279	0.224	0.239	0.164	0.248
0.15	0.269	0.207	0.227	0.142	0.260	0.474	0.331	0.345	0.228	0.361
0.20	0.411	0.285	0.298	0.204	0.336	0.658	0.426	0.436	0.283	0.454
0.25	0.530	0.345	0.358	0.240	0.400	0.775	0.532	0.531	0.344	0.560
0.30	0.640	0.409	0.418	0.267	0.463	0.832	0.593	0.594	0.373	0.619
0.35	0.727	0.462	0.473	0.297	0.518	0.871	0.657	0.654	0.404	0.676
0.40	0.770	0.515	0.518	0.325	0.565	0.894	0.707	0.700	0.425	0.727
0.45	0.811	0.565	0.566	0.328	0.618	0.906	0.739	0.732	0.432	0.757
0.50	0.832	0.611	0.612	0.348	0.653	0.924	0.766	0.766	0.452	0.783
					$\lambda = 0.7$					
0	0.060	0.062	0.058	0.053	0.085	0.056	0.056	0.058	0.054	0.066
0.05	0.114	0.128	0.133	0.072	0.166	0.154	0.167	0.178	0.080	0.188
0.10	0.207	0.216	0.232	0.090	0.266	0.342	0.360	0.364	0.119	0.386
0.15	0.346	0.344	0.347	0.105	0.398	0.583	0.546	0.538	0.137	0.572
0.20	0.509	0.465	0.467	0.125	0.518	0.739	0.657	0.646	0.181	0.679
0.25	0.625	0.534	0.542	0.145	0.592	0.847	0.741	0.724	0.210	0.757
0.30	0.726	0.619	0.613	0.159	0.660	0.887	0.780	0.766	0.239	0.796
0.35	0.798	0.669	0.664	0.177	0.714	0.912	0.818	0.810	0.279	0.831
0.40	0.837	0.710	0.700	0.197	0.738	0.927	0.837	0.825	0.311	0.850
0.45	0.871	0.742	0.735	0.227	0.774	0.933	0.843	0.832	0.336	0.854
0.50	0.884	0.761	0.751	0.237	0.789	0.946	0.868	0.854	0.369	0.878

Table 4: Rejection frequencies of tests - Experiment 3 ($\lambda_0 = 0.5$)

	$T = 250$					$T = 500$				
b	$\mathcal{T}_S^*$	$\mathcal{T}_{I_f}(\lambda_0)$	$\mathcal{T}_{I_b}(\lambda_0)$	$\mathcal{T}_R(\lambda_0)$	$\mathcal{T}_{HB}$	$\mathcal{T}_S^*$	$\mathcal{T}_{I_f}(\lambda_0)$	$\mathcal{T}_{I_b}(\lambda_0)$	$\mathcal{T}_R(\lambda_0)$	$\mathcal{T}_{HB}$
					$\lambda = 0.3$					
0	0.061	0.061	0.058	0.052	0.084	0.058	0.055	0.054	0.050	0.076
0.05	0.111	0.137	0.128	0.115	0.171	0.152	0.169	0.165	0.162	0.208
0.10	0.257	0.273	0.270	0.258	0.326	0.399	0.401	0.394	0.416	0.462
0.15	0.438	0.432	0.426	0.479	0.504	0.709	0.661	0.655	0.753	0.717
0.20	0.653	0.617	0.603	0.682	0.677	0.915	0.832	0.832	0.934	0.868
0.25	0.830	0.741	0.735	0.863	0.788	0.980	0.910	0.908	0.988	0.929
0.30	0.927	0.828	0.819	0.940	0.864	0.995	0.939	0.937	0.998	0.954
0.35	0.962	0.873	0.865	0.978	0.899	0.998	0.958	0.954	0.998	0.966
0.40	0.986	0.908	0.902	0.993	0.933	1.000	0.974	0.973	1.000	0.979
0.45	0.995	0.926	0.920	0.997	0.944	0.999	0.982	0.980	1.000	0.986
0.50	0.997	0.948	0.943	0.998	0.961	0.999	0.981	0.979	1.000	0.986
					$\lambda = 0.5$					
0	0.058	0.059	0.061	0.057	0.081	0.049	0.048	0.050	0.051	0.069
0.05	0.097	0.095	0.093	0.230	0.123	0.115	0.112	0.114	0.360	0.152
0.10	0.193	0.169	0.163	0.509	0.222	0.311	0.237	0.229	0.686	0.288
0.15	0.350	0.250	0.243	0.726	0.305	0.591	0.365	0.363	0.879	0.425
0.20	0.529	0.344	0.334	0.845	0.406	0.823	0.495	0.494	0.962	0.556
0.25	0.702	0.430	0.413	0.926	0.494	0.934	0.602	0.593	0.987	0.651
0.30	0.828	0.516	0.504	0.965	0.574	0.970	0.678	0.675	0.997	0.724
0.35	0.888	0.560	0.551	0.983	0.623	0.980	0.752	0.746	0.996	0.789
0.40	0.937	0.633	0.623	0.991	0.684	0.989	0.780	0.773	0.998	0.820
0.45	0.953	0.673	0.664	0.994	0.721	0.989	0.813	0.817	0.998	0.845
0.50	0.967	0.711	0.703	0.996	0.756	0.991	0.849	0.848	0.999	0.877
					$\lambda = 0.7$					
0	0.058	0.057	0.055	0.057	0.080	0.051	0.052	0.050	0.054	0.076
0.05	0.071	0.079	0.072	0.079	0.104	0.077	0.085	0.077	0.095	0.113
0.010	0.108	0.107	0.104	0.123	0.139	0.120	0.123	0.117	0.155	0.154
0.15	0.136	0.135	0.129	0.158	0.172	0.181	0.165	0.154	0.223	0.206
0.20	0.163	0.161	0.155	0.197	0.202	0.241	0.222	0.208	0.292	0.269
0.25	0.205	0.191	0.183	0.238	0.245	0.285	0.262	0.250	0.340	0.314
0.30	0.230	0.217	0.212	0.268	0.272	0.351	0.310	0.296	0.411	0.357
0.35	0.263	0.249	0.241	0.306	0.306	0.402	0.359	0.353	0.456	0.418
0.40	0.291	0.274	0.265	0.341	0.328	0.436	0.398	0.388	0.485	0.462
0.45	0.347	0.321	0.314	0.386	0.376	0.496	0.447	0.444	0.543	0.504
0.50	0.368	0.341	0.332	0.413	0.401	0.527	0.484	0.482	0.566	0.543

We also apply the break point estimator to data from Experiment 3 and residuals from a regression without constant in order to detect a break from cointegration to no cointegration. Table 5 shows the estimated break fraction for different choices of δ. This choice does not have any influence on the results. Therefore for practical purposes, a small δ is recommended in order to keep a large part of the data in the analysis. With small b, there is a tendency to locate the break in the middle of the sample, but the results improve as the cointegrating strength b increases and for the largest b the accuracy is good. Hence, with strong cointegrating relations, the break point estimator delivers reliable results. If there is permanent cointegration, the break is estimated at the end of the admissible window. If the data is generated from Experiment 2, the regression residuals are reversed before applying the break point estimator. The results remain the same and are available upon request.

Table 5: Break point estimates with $T = 1000$ and 5000 Monte Carlo replications.

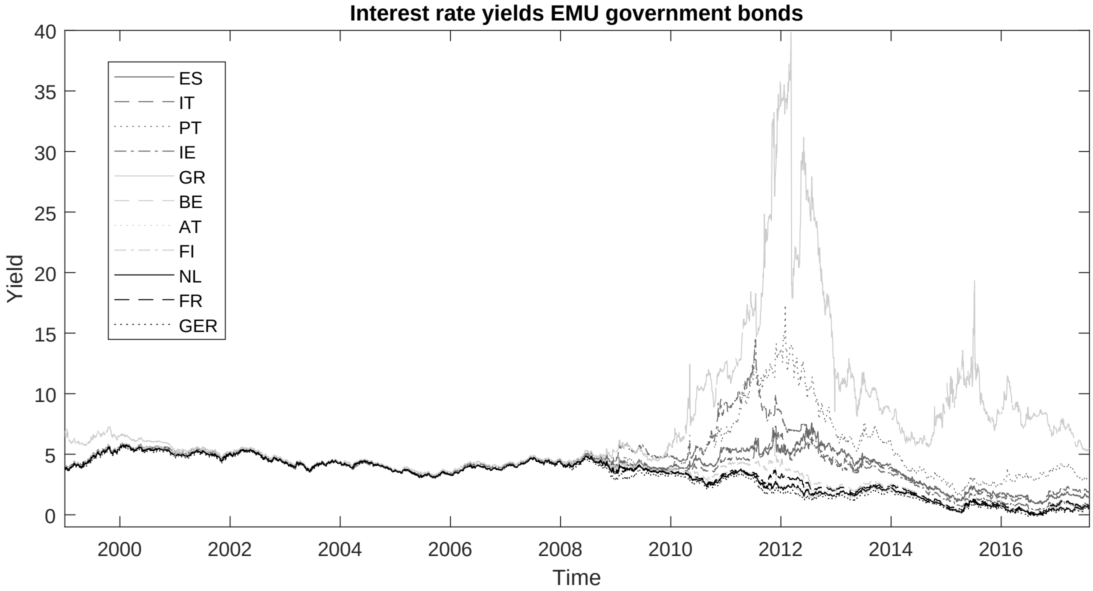
δ	0.05			0.1			0.15		
$b \backslash \lambda$	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7
0.10	0.564	0.604	0.688	0.559	0.598	0.676	0.550	0.589	0.659
0.15	0.503	0.558	0.667	0.509	0.560	0.666	0.514	0.559	0.665
0.20	0.461	0.526	0.661	0.458	0.526	0.658	0.472	0.524	0.660
0.25	0.424	0.499	0.655	0.437	0.501	0.657	0.436	0.503	0.658
0.30	0.410	0.483	0.654	0.412	0.488	0.656	0.414	0.494	0.659
0.35	0.389	0.470	0.653	0.397	0.473	0.656	0.404	0.478	0.656
0.40	0.373	0.458	0.655	0.381	0.461	0.655	0.392	0.470	0.656
0.45	0.365	0.446	0.648	0.374	0.457	0.651	0.387	0.463	0.653
0.50	0.358	0.448	0.647	0.375	0.453	0.648	0.380	0.458	0.653
no break		0.938			0.890			0.842	

5 Empirical Application

In this Section, we apply the tests introduced in Section 3 to benchmark government bonds of countries that are part of the European Monetary Union (EMU). The analysis is based on daily observations between 01.01.1999 and 08.08.2017 (about 4,800 observations per country) of 10-year-to-maturity benchmark government bonds of eleven EMU countries (Spain, Italy, Portugal, Ireland, Greece, Belgium, Austria, Finland, the Netherlands, France and Germany). The data ist obtained from Thomson Reuters Eikon.

According to Leschinski et al. (2018), market integration requires the existence of a (fractional) cointegrating relationship among the goods of the market under consideration. Regarding the European bond market, it is generally accepted that the market is integrated after the introduction of the Euro and prior to the EMU debt crisis or at

Figure 1: Yields of EMU government bonds.



least up to the subprime mortgage crisis (Baele et al. (2004), Ehrmann et al. (2011), Pozzi and Wolswijk (2012), Christiansen (2014), and Ehrmann and Fratzscher (2017), among others) so that we would expect fractional cointegration during this period. This conclusion is supported by Figure 1 that shows how the bond yields co-move in the beginning. When the crisis began in 2008-2010, they drift apart so that no market integration and no cointegration is assumed any longer. Therefore, it is likely that testing for no cointegration over the full sample does not allow us to reject the null hypothesis. However, with the new tests introduced in this paper we expect to be able to detect cointegration with breaks in the cointegrating relationship in the sense that under the alternative we have fractional cointegration in a certain subsample and no cointegration elsewhere.

Table 6: p-values of ADF- and KPSS-tests.

	ES	IT	PT	IE	GR	BE	AT	FI	NL	FR	GER
ADF	0.93	0.93	0.93	0.92	0.93	0.93	0.93	0.93	0.93	0.93	0.93
KPSS	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01

The order of integration of our data is unknown so that we apply unit root and stationarity tests (Table 6). The ADF-test, augmented based on the Schwert's rule and including a drift, cannot reject the unit root and the KPSS-test rejects stationarity for all countries leading to the conclusion that $d_i = 1$ for all countries' yields. This might be implausible from an economic perspective. However, the finite sample behavior suggests a unit root which is consistent with results available in the literature on fractional cointegration, confer for example Chen and Hurvich (2003) and Nielsen (2010). The cointegrating regressions are carried out in a bivariate setting where the yield of country i, y_{it}, is regressed on the German yield, $y_{GER,t}$:

$$y_{it} = \beta_0 + \beta_1 y_{GER,t} + e_t, \text{ for } i = 1, ..., 10. \tag{30}$$

The residuals obtained from the regressions in (30) are used for testing in the split, incremental and rolling sample versions of the test where λ_0 is set to 0.2 and 0.5, respectively. The Hassler-Breitung test is applied to the full sample. In order to account for autocorrelation, we augment the lagged regression (4) using the Schwert's rule as suggested in Demetrescu et al. (2008), and we use Eicker-White (EW) heteroscedasticity-robust standard errors as it is more suitable in our empirical setting. The results are given in Table 7 and bold numbers indicate rejection at the 5% significance level.

Table 7: Values of test statistic with $\lambda_0 = 0.2$ and $\lambda_0 = 0.5$ with EW heteroscedasticity-robust standard errors, and parametric augmentation to correct for autocorrelation (Schwert's rule).

	$\mathcal{T}_{HB}$	$\mathcal{T}_S^*$	$\mathcal{T}_{I_f}(0.2)$	$\mathcal{T}_{I_b}(0.2)$	$\mathcal{T}_R(0.2)$	$\mathcal{T}_{I_f}(0.5)$	$\mathcal{T}_{I_b}(0.5)$	$\mathcal{T}_R(0.5)$
ES	0.05	0.05	1.85	0.70	**8.02**	1.85	0.04	1.66
IT	0.31	0.31	2.51	1.88	**15.31**	2.51	0.79	1.87
PT	2.46	2.46	2.55	3.26	**14.87**	2.55	2.66	2.14
IE	0.04	0.04	**4.90**	4.09	**28.71**	**4.90**	0.09	2.65
GR	0.29	0.55	3.18	2.21	**5.65**	3.18	2.21	2.70
BE	0.45	1.67	**8.30**	2.20	**15.68**	**8.30**	0.66	**6.52**
AT	2.91	4.20	**11.45**	**8.77**	**37.06**	**4.57**	4.22	**6.38**
FI	3.43	**28.03**	**33.84**	**5.98**	**24.43**	**29.30**	**5.00**	**28.92**
NL	**11.42**	**11.42**	**19.34**	**11.15**	**23.19**	**11.60**	**11.15**	**11.36**
FR	2.91	2.91	**11.99**	**5.53**	**11.92**	**11.99**	**5.45**	**9.18**

The Hassler-Breitung test does not reject the null of no cointegration on the full sample for all countries except for the Dutch yield, and the split sample test finds cointegration between the German and Dutch and the German and Finnish yields. The incremental tests with $\lambda_0 = 0.2$ reject the null hypothesis for Austria, Finland, the Netherlands and France in the backward-rolling window and additionally for Ireland and Belgium in the forward-rolling window. Thus, segmented cointegration is found for countries that were less affected by the financial crisis and no cointegration for those more strongly affected. The rolling sample tests rejects the null of no cointegration for all regression pairs. Overall, the results meet the expectation that the European yields are not cointegrated over the whole period. With the new tests for segmented cointegration, we find that the European yields were cointegrated in at least part of the sample.

Davidson and Monticini (2010) recommend the use of $\lambda_0 = 0.5$ because a break must occur in either the first half of the sample or the second. Nonetheless, choosing $\lambda_0 = 0.2$ leads neither to disadvantages nor to advantages which was also confirmed in the Monte Carlo exercise. With $\lambda_0 = 0.5$, the results for the incremental tests are very similar to those with $\lambda_0 = 0.2$, but we get less rejections with the rolling sample test. This could imply that a shorter period than 50% of the sample is fractionally cointegrated or, at least, that the evidence for segmented fractional cointegration for the countries that were most affected by the financial crisis is ambiguous.

The finding of segmented cointegration for the Netherlands does not contradict the rejection of the Breitung-Hassler-test as it also has power, albeit less, in the presence of segmented cointegration. The other way round, the tests for segmented cointegration also have power if the cointegrating relation is permanent as they include the full sample as well.

Table 8: Break date estimates with $\delta = 0.05$.

ES	IT	PT	IE	GR
05.05.2010	24.05.2010	27.04.2010	28.04.2010 15.08.2014*	22.04.2010
BE	**AT**	**FI**	**NL**	**FR**
21.11.2008	14.12.2001	06.12.2002	21.10.2002	21.11.2008

In order to gain a deeper understanding of the dynamics, we estimate the break date with the break point estimator proposed in (19) based on the regression residuals (without constant). We set $\delta = 0.05$ and impose a minimum length of $\lfloor 0.1T \rfloor$ between the sequentially estimated breaks. The results are given in Table 8. The breaks for Spain, Italy, Portugal, Ireland and Greece are estimated in April and May of 2010, hence shortly after the start of the European debt crisis. For France and Belgium we obtain the exact same date in November 2008, i.e. two years earlier than for the previous countries. For Austria, Finland and the Netherlands the breaks are located at the end of 2001 and 2002. We also look at reversed residuals in order to identify potential breaks from no cointegration to cointegration that are indicated by an asterisk. There is one found for Ireland implying that the Irish yield is cointegrated with the German one until 2010, then the cointegrating relationship temporarily dissolves and reemerges in 2014.

If we consider the sample starting 1999 up to the first break, there is still evidence of unit roots in the data and we find the breaks given in Table 9. As they are also 'forward'-breaks implying the dissolution of cointegration, they contradict the first found break dates. In the sample between the break date estimates, we do not find 'backward'-breaks that would justify the first break, except for Italy. For Italy, it implies a short period of no cointegration between 2002 and 2004. For the other countries, the 'backward'-break might be too small to be detected or there is a smooth transition. Therefore, it is not clear for Spain, Portugal, Ireland and Greece at which point exactly the relationship with Germany dissolves. The test results in Table 7 suggest a short period of cointegration because the rolling test rejects with $\lambda_0 = 0.2$ but not with $\lambda_0 = 0.5$ for these countries.

Table 9: Break dates with $\delta = 0.05$ before the first break in Table 8.

ES	IT	PT	IE	GR
04.03.2002	16.10.2002	10.12.2001	30.09.2008	30.10.2008
BE	AT	FI	NL	FR
—	05.01.2001	08.05.2000*	11.02.2000*	13.12.2007

Strictly speaking, the direction of the estimated break dates for Finland and the Netherland in 2000 and 2002 imply no cointegration for most of the sample. This contradicts the findings of the tests in Table 7 that state rather strong evidence of cointegration, in particular for the Dutch yield. Therefore, we conclude that they are permanently cointegrated.

Considering the sample from the first break date until 2017, we estimate the break dates in Table 10. Those are 'backward'-breaks implying the emergence of a fractional cointegrating relation. They are located in 2012 and 2013 for most of the countries. For Austria, there is another 'forward'-break in 2008, but after that we also find a 'backward'-break on 05.09.2012.

Table 10: Break dates with $\delta = 0.05$ after the first break in Table 8.

ES	IT	PT	IE	GR
24.05.2013*	02.05.2013*	12.12.2012*	11.12.2014*	12.10.2012*
BE	AT	FI	NL	FR
11.12.2012*	30.10.2008	—	—	05.09.2012*

In Table B.1 in the appendix, all found break dates from sequential estimation are collected, and in all subsamples the data still exhibits unit roots. The table contains further break dates for some countries in 2000 and in 2016 that imply no cointegration at the edges of the sample. However, the dates are very close to the edges, and the Monte Carlo simulation showed estimates very close to the margins in the case of permanent cointegration. Therefore, the validity of the breaks in the small subsamples close to the edges is doubtful and we rather suspect continuous cointegration in the border-subsamples.

All in all, based on the co-movements in Figure 1 and the rejections in Table 7, we conclude that the yields of the countries were fractionally cointegrated with that of Germany after the introduction of the euro until the European debt crisis. The break point estimates point to the dissolution of fractional cointegrating relationships and market integration at the beginning of the European debt crisis in 2010 although the breaks might have occurred earlier for Spain, Italy, Portugal and Ireland. In 2012/2013 the cointegrating relationships are reestablished. For Finland and the Netherlands the results indicate permanent cointegration.

6 Conclusion

In this paper, we present tests for the null of no fractional cointegration against the alternative of segmented fractional cointegration. To do this we develop new tests based on the procedure of Hassler and Breitung (2006) combined with ideas from Davidson and Monticini (2010). We introduce split sample, forward- and backward-running incremental sample and rolling sample tests for segmented cointegration. We show that the limit distribution of all of these statistics converge to the supremum of a chi-squared distribution. Furthermore, a break point estimator based on minimizing the sum of squared residuals is also proposed.

An in-depth Monte Carlo analysis shows the satisfying size and power properties of our tests in various situations. However, it turns out that the split sample test performs best in terms of power when the break occurs from the spurious to the fractionally cointegrated regime wherever the breakpoint is. On the other hand, if the break is from the fractionally cointegrated regime to the spurious regime, the rolling window test has the best power properties for all possible breakpoints. Therefore, we recommend application of both the split sample and the rolling window tests.

As segmented fractional cointegration is a very likely empirical situation we investigate daily EMU government bonds between January 1999 and August 2017. We find constant fractional cointegration for the Dutch and Finish government bond yields with Germany. For the other countries, namely Spain, Italy, Portugal, Greece, Ireland, Belgium, and France we find segmented fractional cointegration with a period of no fractional cointegration during the European debt or financial crisis.

Acknowledgement:
Financial support of DFG under grant SI 745/9-2 as well as from the Portuguese Science Foundation (FCT) through project PTDC/EGE-ECO/28924/2017, and (UID/ECO/00124/2013 and Social Sciences DataLab, Project 22209), POR Lisboa (LISBOA-01-0145-FEDER-007722 and Social Sciences DataLab, Project 22209) and POR Norte (Social Sciences DataLab, Project 22209), is gratefully acknowledged. We are also very grateful to Uwe Hassler and Fabrizio Iacone as well as participants of the Long Memory Workshop 2018 in Hannover for inspiring discussions and helpful comments on the paper.

References

Andrews, D. W., Lee, I., and Ploberger, W. (1996). Optimal changepoint tests for normal linear regression. *Journal of Econometrics*, 70(1):9 – 38.

Baele, L., Ferrando, A., Hördahl, P., Krylova, E., and Monnet, C. (2004). Measuring financial integration in the euro area. Technical report, ECB Occasional Paper, No.14.

Bai, J. and Perron, P. (1998). Estimating and testing linear models with multiple structural changes. *Econometrica*, 66(1):47–78.

Beran, J. and Terrin, N. (1996). Testing for a change of the long-memory parameter. *Biometrika*, 83(3):627–638.

Beran, J. and Terrin, N. (1999). Amendments and corrections: Testing for a change of the long-memory parameter. *Biometrika*, 86(1):233–233.

Busetti, F. and Taylor, A. R. (2004). Tests of stationarity against a change in persistence. *Journal of Econometrics*, 123(1):33–66.

Cecchetti, S. G., Flores-Lagunes, A., and Krause, S. (2006). Has monetary policy become more efficient? A cross-country analysis. *The Economic Journal*, 116(511):408–433.

Chen, W. W. and Hurvich, C. M. (2003). Semiparametric estimation of multivariate fractional cointegration. *Journal of the American Statistical Association*, 98(463):629–642.

Christiansen, C. (2014). Integration of European bond markets. *Journal of Banking & Finance*, 42:191–198.

Davidson, J. and Monticini, A. (2010). Tests for cointegration with structural breaks based on subsamples. *Computational statistics & data analysis*, 54(11):2498–2511.

Demetrescu, M., Kuzin, V., and Hassler, U. (2008). Long memory testing in the time domain. *Econometric Theory*, 24(1):176–215.

Ehrmann, M. and Fratzscher, M. (2017). Euro area government bonds–fragmentation and contagion during the sovereign debt crisis. *Journal of International Money and Finance*, 70:26–44.

Ehrmann, M., Fratzscher, M., Gürkaynak, R. S., and Swanson, E. T. (2011). Convergence and anchoring of yield curves in the euro area. *The Review of Economics and Statistics*, 93(1):350–364.

Engle, R. F. and Granger, C. W. (1987). Co-integration and error correction: Representation, estimation, and testing. *Econometrica*, 55(2):251–276.

Giraitis, L. and Leipus, R. (1994). A posteriori and sequential methods of change-point detection in FARIMA-type time series. In *Probability Theory and Mathematical Statistics: Proceedings of the Sixth Vilnius Conference*, pages 485–496.

Halunga, A. G., Osborn, D. R., and Sensier, M. (2009). Changes in the order of integration of US and UK inflation. *Economics Letters*, 102(1):30–32.

Hao, K. (1996). Testing for structural change in cointegrated regression models: some comparisons and generalizations. *Econometric Reviews*, 15(4):401–429.

Harvey, D. I., Leybourne, S. J., and Taylor, A. R. (2006). Modified tests for a change in persistence. *Journal of Econometrics*, 134(2):441–469.

Hassler, U. and Breitung, J. (2006). A residual-based LM-type test against fractional cointegration. *Econometric Theory*, 22(6):1091–1111.

Hassler, U. and Meller, B. (2014). Detecting multiple breaks in long memory the case of us inflation. *Empirical Economics*, 46(2):653–680.

Hassler, U., Rodrigues, P. M., and Rubia, A. (2009). Testing for general fractional integration in the time domain. *Econometric Theory*, 25(6):1793–1828.

Hassler, U. and Scheithauer, J. (2011). Detecting changes from short to long memory. *Statistical Papers*, 52(4):847–870.

Herrera, A. M. and Pesavento, E. (2005). The decline in US output volatility: structural changes and inventory investment. *Journal of Business & Economic Statistics*, 23(4):462–472.

Inoue, A. (1999). Tests of cointegrating rank with a trend-break. *Journal of Econometrics*, 90(2):215 – 237.

Johansen, S. (1988). Statistical analysis of cointegration vectors. *Journal of Economic Dynamics and Control*, 12(2-3):231–254.

Johansen, S., Mosconi, R., and Nielsen, B. (2000). Cointegration analysis in the presence of structural breaks in the deterministic trend. *The Econometrics Journal*, 3(2):216–249.

Kang, K. H., Kim, C.-J., and Morley, J. (2009). Changes in US inflation persistence. *Studies in Nonlinear Dynamics & Econometrics*, 13(4).

Kim, J.-Y. (2000). Detection of change in persistence of a linear time series. *Journal of Econometrics*, 95(1):97–116.

Kim, J.-Y., Belaire-Franch, J., Amador, R. B., et al. (2002). Corrigendum to "detection of change in persistence of a linear time series" [JoE 95 (2000) 97-116]. *Journal of Econometrics*, 109(2):389–392.

Kuo, B.-S. (1998). Test for partial parameter instability in regressions with I(1) processes. *Journal of Econometrics*, 86(2):337–368.

Leschinski, C., Voges, M., and Sibbertsen, P. (2018). Integration and disintegration of EMU government bond markets. Technical report, Leibniz Universität Hannover, Wirtschaftswissenschaftliche Fakultät.

Lütkepohl, H., Saikkonen, P., and Trenkler, C. (2003). Comparison of tests for the cointegrating rank of a VAR process with a structural shift. *Journal of Econometrics*, 113(2):201 – 229.

Martins, L. F. and Rodrigues, P. M. (2014). Testing for persistence change in fractionally integrated models: An application to world inflation rates. *Computational Statistics & Data Analysis*, 76:502–522.

Martins, L. F. and Rodrigues, P. M. (2018). Testing for segmented cointegration. *mimeo*.

McConnell, M. M. and Perez-Quiros, G. (2000). Output fluctuations in the United States: What has changed since the early 1980's? *American Economic Review*, 90(5):1464–1476.

Nielsen, M. Ø. (2010). Nonparametric cointegration analysis of fractional systems with unknown integration orders. *Journal of Econometrics*, 155(2):170–187.

Pozzi, L. and Wolswijk, G. (2012). The time-varying integration of euro area government bond markets. *European Economic Review*, 56(1):36–53.

Quintos, C. E. and Phillips, P. C. (1993). Parameter constancy in cointegrating regressions. *Empirical Economics*, 18(4):675–706.

Sibbertsen, P. and Kruse, R. (2009). Testing for a break in persistence under long-range dependencies. *Journal of Time Series Analysis*, 30(3):263–285.

A Technical Appendix

Before we prove the Theorems define

$$\mathbf{e}'(\lambda_1,\lambda_2) := (e_{\lfloor\lambda_1 T\rfloor+2},\ldots,e_{\lfloor\lambda_2 T\rfloor})$$

and

$$\mathbf{e}^{*'}(\lambda_1,\lambda_2) := (e^*_{\lfloor\lambda_1 T\rfloor+1},\ldots,e^*_{\lfloor\lambda_2 T\rfloor}).$$

Proof of Theorem 1:

From Lemma A in Hassler and Breitung (2006) we have directly:

$$\begin{aligned}
\frac{1}{\lfloor\lambda_2 T\rfloor-\lfloor\lambda_1 T\rfloor}\mathbf{e}'(\lambda_1,\lambda_2)\mathbf{e}(\lambda_1,\lambda_2) &\overset{P}{\to} \sigma^2 \\
\frac{1}{(\lfloor\lambda_2 T\rfloor-\lfloor\lambda_1 T\rfloor)^{1/2}}\mathbf{e}'(\lambda_1,\lambda_2)\mathbf{e}^*(\lambda_1,\lambda_2) &\Rightarrow N\left(0;\sigma^4\frac{\pi^2}{6}\right) \\
\frac{1}{\lfloor\lambda_2 T\rfloor-\lfloor\lambda_1 T\rfloor}\mathbf{e}^{*'}(\lambda_1,\lambda_2)\mathbf{e}^*(\lambda_1,\lambda_2) &\overset{P}{\to} \sigma^2\frac{\pi^2}{6}.
\end{aligned} \tag{A.1}$$

The rest of the proof follows exactly the lines of the proof of proposition 3 in Hassler and Breitung (2006) with the only difference that we localize their arguments to the interval $t=\lfloor\lambda_1 T\rfloor+1,\ldots,\lfloor\lambda_2 T\rfloor$. For ease of readability we recall their arguments here. Defining $\hat{e}_t(\lambda_1,\lambda_2) = e_t(\lambda_1,\lambda_2) - \mathbf{e}'(\lambda_1,\lambda_2)\mathbf{V}_2(\lambda_1,\lambda_2)(\mathbf{V}_2'(\lambda_1,\lambda_2)\mathbf{V}_2(\lambda_1,\lambda_2))^{-1}v_{2,t}(\lambda_1,\lambda_2)$ and $\hat{e}^*_{t-1}(\lambda_1,\lambda_2) = e^*_{t-1}(\lambda_1,\lambda_2) - \mathbf{e}'(\lambda_1,\lambda_2)\mathbf{V}_2(\lambda_1,\lambda_2)(\mathbf{V}_2'(\lambda_1,\lambda_2)\mathbf{V}_2(\lambda_1,\lambda_2))^{-1}v^*_{2,t-1}(\lambda_1,\lambda_2)$ we have

$$\begin{aligned}
\hat{\mathbf{e}}'(\lambda_1,\lambda_2)\hat{\mathbf{e}}(\lambda_1,\lambda_2) &= \mathbf{e}'(\lambda_1,\lambda_2)\mathbf{e}(\lambda_1,\lambda_2) - r_T'\mathbf{V}_2'(\lambda_1,\lambda_2)\mathbf{e}(\lambda_1,\lambda_2), \\
\hat{\mathbf{e}}^{*\prime}(\lambda_1,\lambda_2)\hat{\mathbf{e}}^*(\lambda_1,\lambda_2) &= \mathbf{e}^{*\prime}(\lambda_1,\lambda_2)\mathbf{e}^*(\lambda_1,\lambda_2) - 2r_T'\mathbf{V}_2^{*\prime}(\lambda_1,\lambda_2)\mathbf{e}^*(\lambda_1,\lambda_2) \\
&\quad + r_T'\mathbf{V}_2^{*\prime}(\lambda_1,\lambda_2)\mathbf{V}_2^*(\lambda_1,\lambda_2)r_T, \\
\hat{\mathbf{e}}^{*\prime}(\lambda_1,\lambda_2)\hat{\mathbf{e}}(\lambda_1,\lambda_2) &= \mathbf{e}^{*\prime}(\lambda_1,\lambda_2)\mathbf{e}(\lambda_1,\lambda_2) - r_T'\mathbf{V}_2^{*\prime}(\lambda_1,\lambda_2)\mathbf{e}(\lambda_1,\lambda_2) \\
&\quad - r_T'\mathbf{V}_2'(\lambda_1,\lambda_2)\mathbf{e}^*(\lambda_1,\lambda_2) + r_T'\mathbf{V}_2^{*\prime}(\lambda_1,\lambda_2)\mathbf{V}_2(\lambda_1,\lambda_2)r_T,
\end{aligned}$$

with $r_T = (\mathbf{V}_2'(\lambda_1,\lambda_2)\mathbf{V}_2(\lambda_1,\lambda_2))^{-1}\mathbf{V}_2'(\lambda_1,\lambda_2)\mathbf{e}(\lambda_1,\lambda_2)$, $\mathbf{V}_2 = \left(\mathbf{V}_{2,2}',\ldots,\mathbf{V}_{2,T}'\right)$. By Assumption 2 and the iid assumption for v_t it holds

$$\begin{aligned}
\mathbf{V}_2'(\lambda_1,\lambda_2)\mathbf{e}(\lambda_1,\lambda_2) &= O_P(T^{1/2}), \\
r_T &= O_P(T^{-1/2}), \\
\mathbf{V}_2^{*\prime}\mathbf{e}^* &= O_P(T),
\end{aligned}$$

and

$$\frac{1}{\lfloor\lambda_2 T\rfloor-\lfloor\lambda_1 T\rfloor}\mathbf{V}_2^{*\prime}(\lambda_1,\lambda_2)\mathbf{e}(\lambda_1,\lambda_2) \;\to\; 0,$$
$$\frac{1}{\lfloor\lambda_2 T\rfloor-\lfloor\lambda_1 T\rfloor}\mathbf{V}_2^{\prime}(\lambda_1,\lambda_2)\mathbf{e}^{*}(\lambda_1,\lambda_2) \;\to\; 0.$$

From (A.1) we now have:

$$\frac{1}{\lfloor\lambda_2 T\rfloor-\lfloor\lambda_1 T\rfloor}\hat{\mathbf{e}}^{\prime}(\lambda_1,\lambda_2)\hat{\mathbf{e}}(\lambda_1,\lambda_2)$$
$$=\frac{1}{\lfloor\lambda_2 T\rfloor-\lfloor\lambda_1 T\rfloor}\mathbf{e}^{\prime}(\lambda_1,\lambda_2)\mathbf{e}(\lambda_1,\lambda_2)+o_P(1)\overset{P}{\to}\sigma^2$$
$$\frac{1}{(\lfloor\lambda_2 T\rfloor-\lfloor\lambda_1 T\rfloor)^{1/2}}\hat{\mathbf{e}}^{\prime}(\lambda_1,\lambda_2)\hat{\mathbf{e}}^{*}(\lambda_1,\lambda_2)$$
$$=\frac{1}{(\lfloor\lambda_2 T\rfloor-\lfloor\lambda_1 T\rfloor)^{1/2}}\mathbf{e}^{\prime}(\lambda_1,\lambda_2)\mathbf{e}^{*}(\lambda_1,\lambda_2)+o_P(1)\Rightarrow N\left(0;\sigma^4\frac{\pi^2}{6}\right)$$
$$\frac{1}{\lfloor\lambda_2 T\rfloor-\lfloor\lambda_1 T\rfloor}\hat{\mathbf{e}}^{*\prime}(\lambda_1,\lambda_2)\hat{\mathbf{e}}^{*}(\lambda_1,\lambda_2)$$
$$=\frac{1}{\lfloor\lambda_2 T\rfloor-\lfloor\lambda_1 T\rfloor}\mathbf{e}^{*\prime}(\lambda_1,\lambda_2)\mathbf{e}^{*}(\lambda_1,\lambda_2)+o_P(1)\overset{P}{\to}\sigma^2\frac{\pi^2}{6}$$

which proves the theorem. □

Proof of Theorem 2:

The proof follows directly from the results in Theorem 1 and the arguments in Davidson and Monticini (2010). □

Proof of Theorem 3:

Assume that the break is from cointegration to non-cointegration. This is before the break the residuals are of integration order $d-b$ whereas they are of order d after the break. Denote by $\hat{d}$ the estimated integration order based on the whole sample. Then we have $d-b\leq\hat{d}\leq d$.

We thus have

$$\lfloor\tau T\rfloor^{-2\hat{d}}\sum_{t=1}^{\lfloor\tau T\rfloor}\hat{e}_t^2(\tau)=O_P(T^{(d-b)-\hat{d}})1_{[\tau\leq\tau_0]}+\infty 1_{[\tau>\tau_0]}$$

which proves the theorem. □

B Supplementary Tables

Table B.1: All break dates in pairwise cointegrating regressions with the German yield. **Bold dates** indicate 'forward'-breaks and *italic dates* indicate 'backward'-breaks.

	ES	IT	PT	IE	GR	BE	AT	FI	NL	FR
1999										
2000	*23.03.2000*	*31.01.2000*	*09.05.2000*		*03.02.2000*		*25.01.2000*	*08.05.2000*	*11.02.2000*	
2001		*10.04.2001*	**10.12.2001**				**14.12.2001**	**19.04.2001**		
2002	**04.03.2002**	**16.10.2002**						**06.12.2002**	**21.10.2002**	
2003				**04.07.2003**						
2004		*30.04.2004*								
2005										
2006										
2007										**13.12.2007**
2008	**05.09.2008**	**05.09.2008**	**15.09.2008**	**30.09.2008**	**30.10.2008**	**21.11.2008**	**30.10.2008**			**21.11.2008**
2009										
2010	**05.05.2010**	**24.05.2010**	**27.04.2010**	**28.04.2010**	**22.04.2010**					
2011										
2012			*12.12.2012*	**02.08.2012**	*12.10.2012*	*11.12.2012*	*05.09.2012*			*05.09.2012*
2013	*24.05.2013*	*02.05.2013*								
2014				*15.08.2014*	**29.12.2014**					
2015							**24.06.2015**			
2016	**29.01.2016**	**02.02.2016**	**29.01.2016**	**08.02.2016**		**08.02.2016**				**06.01.2016**
2017										