Estimating the transition time from school to university using a stochastic mortality model

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Abstract

The sufficient provision of university places is of high sociopolitical importance as it facilitates the participation in higher education as well as the accumulation of human capital both being important factors for wealth and economic growth. The aim of this paper is to use the concept of mortality rates for forecasting the transition from school graduates into university in order to enrich and enhance the planning of the needed university places for first-year students. The transition rates differentiated in 16 Federal States and two types of university entrance qualifications were interpreted as mortality rates, fitted with the classical Lee-Carter approach and forecasted with automatically selected ARIMA processes. The results suggest that the constancy assumption used for the original planning provides indeed the best approach for many Federal States. Nevertheless, the application of the well-established Lee-Carter mortality model offers the opportunity to estimate an alternative number of first-year students and to open up a range of possible solutions.

JEL-Classification: C22, I21, I23, J11
Keywords: Mortality rates, mortality model, transition rates, university enrolment, first-year students

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1 Introduction

The provision of university places for first-year students facilitates the participation in education as well as the accumulation of human capital. The higher level of education achieved by university education supports participation opportunities in the labour market and income prospect so that a precarious living condition in old age becomes less likely. Additionally, a huge number of empirical studies give evidence that the accumulation of human capital has a positive impact on GDP.[1] Thus, the provision of a sufficient number of university places meeting the demand of school leavers who want to enrol in university is of high sociopolitical importance.

The economic model that was constructed to provide estimations about the future demand for higher education projected the demand for university places in each Federal State in Germany until 2030 and was supposed to offer more planning security.[2] One important model component is the transition from school to university. There is no time limit for the gap between the achievement of the university admittance qualification and the enrolment into university and many different aspects influence the transition process in Germany. Nevertheless, most school leavers start studying immediately or one year after graduating from school. Additionally, a tendency towards a shortening of the transition time can be observed in the past. In the current model version however it was assumed that the transition behaviour remains unchanged for the future.

The aim of this paper is to use the concept of mortality rates for forecasting the transition rates in order to enrich and enhance the planning of the needed university places for first-year students.

Based on the stochastic mortality model proposed by Lee and Carter (1992) the rather strict assumption of constant transition rates was relaxed and the impact on the model results assessed: The transition rates for 16 Federal States and two different types of university entrance qualification were interpreted as mortality rates and fitted according to the classical Lee-Carter approach (Lee and Carter, 1992). The fitted values were forecasted with ARIMA processes until 2030 and integrated in the model. The thus calculated projection results for first-year students were compared to the former projection with constant transition rates.

The results showed that the constancy assumption was the best approach for many transition rates. Where changes in the historical transition behaviour prevail, the trends estimated using the Lee-Carter model proved to be a good alternative.

[1] An overview and summary of the related literature is given in OECD (2001, Chapter 2).

[2] A documentation of the model is available upon request in German language.

However, some estimated changes might be too large and the Lee-Carter model too restrictive.

Overall, the methodology of mortality rates and of stochastic mortality models can be used to model the transition from school to studies. Adding this additional estimation routine to the existing model provides the opportunity to generate an alternative number of first-year students. Comparing both estimation results – the original and the alternative ones – opens up a possible range for the projection and helps to get an idea about the future related uncertainty in the number of first-year students.

The remainder of the paper is structured as follows. The section 2 provides an overview over the factors determining the number of first-year students in Germany and their transition from school to studies. It also includes a short description of the structure of the existing projection model. In section 3 the mortality model used for estimating the transition rates is specified and the fitting results are given. The section 4 summarises the forecasting results and compares the new approach with the original calculation results of the model. In section 5 the conclusion can be found.

2 The number of first-year students in Germany

In 2017, 441 thousand pupils graduated from school in Germany holding a qualification that entitled them for studies.[3] The majority of them (78.4 %) received a general university entrance qualification (*Allgemeine Hochschulreife*), only 21.6 % achieved an entrance qualification for universities of applied sciences (*Fachhochschulreife*).[4] Compared to 2000 the overall number of school graduates increased by 27 % or 1.4 % p. a. The number of school graduates holding a general university entrance qualification grew with a change of 34.2 % or 1.7 % p. a. faster than those with entrance qualification for universities of applied sciences (6.3 % or 0.4 % p. a.).

These school graduates have the choice between studies, vocational training, a combination of studies and vocational training or entering the job market. In the 2013 graduation cohort there was a clear preference for studies: 80.1 % started studying after at least four years. In average, the propensity to study is higher for school graduates holding a general university entrance qualification (84.6 %) than for

[3] The numbers all refer to StBA (2018).

[4] The entrance qualification for universities of applied sciences (*Fachhochschulreife*) is an upper secondary education qualification that entitles students to study at a university of applied sciences or to take up a Bachelor's degree programme at some universities.

graduates with an entrance qualification for universities of applied sciences (48 %).[5] Compared to the 2000 graduation cohort the decision in favour of studies gained importance with an increase in the propensity to study by 8.9 percentage points. Here the change was mainly driven by the graduates holding an entrance qualification for universities of applied sciences as their propensity to study grew by 14.9 percentage points compared to a 5.6 percentage points increase in case of the general university entrance qualification.

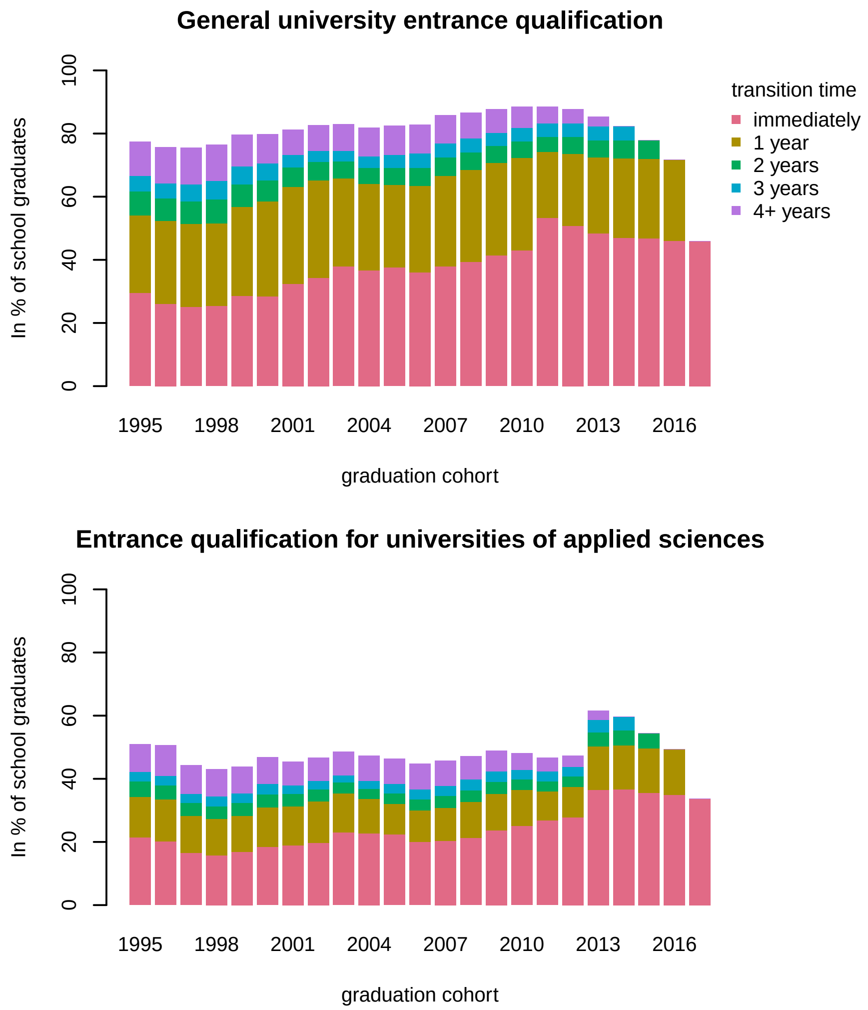
The decision in favour of studies does not yet determine the time of the enrolment in university.[6] The time gap between finishing school and starting studies can vary and depends on administrative regulations and individual plans. In order to study the school graduates need to select the subject and the university they want to apply for. Some subjects with extra restrictions require applications in July for enrolment in October. Additionally, universities of applied sciences often request the certificate of a subject related internship in advance. Some school graduates (due to the tight time schedule or out of other reasons) choose to wait for one year and fill the time gap with voluntary service, an internship, military service, jobbing, a year abroad or similar. If the enrolment in university follows after the completion of a vocational training the time gap between school and studies would be at least two years.

Figure 1 shows the distribution of the different transition times for each graduation cohort. In average over the cohorts 1995–2013 most of the school graduates decide to start their studies right after school: 36.4 % of the school graduates holding a general university entrance qualification and 22 % of those with an entrance qualification for universities of applied sciences. 27.1 % and 11.6 % respectively wait one year. Taken together, the transitions with no or a one year time lag represent the preferred transition time of school graduates. Only 6 % and 3.8 % have a time gap between school and studies of two years. 4.5 % and 3 % wait three years. A transition time of at least four years is in average used by 8.5 % and 7.3 % of the school graduates. This involves mostly very late transitions not occurring after four years but rather a lot later. As a consequence, the shares are lower the younger the graduation cohort as they could not be observed yet. The remaining 17.4 % and 52.4 % did not start studying at all.

[5] The average refers to the graduation cohorts 2000–2013.

[6] Here the term university is used to equally refer to universities as well as universities of applied sciences.

Figure 1: Transition time between school and studies for the graduation cohorts 1995–2017 (first-year students in percent of all school graduates of the respective graduation cohort)



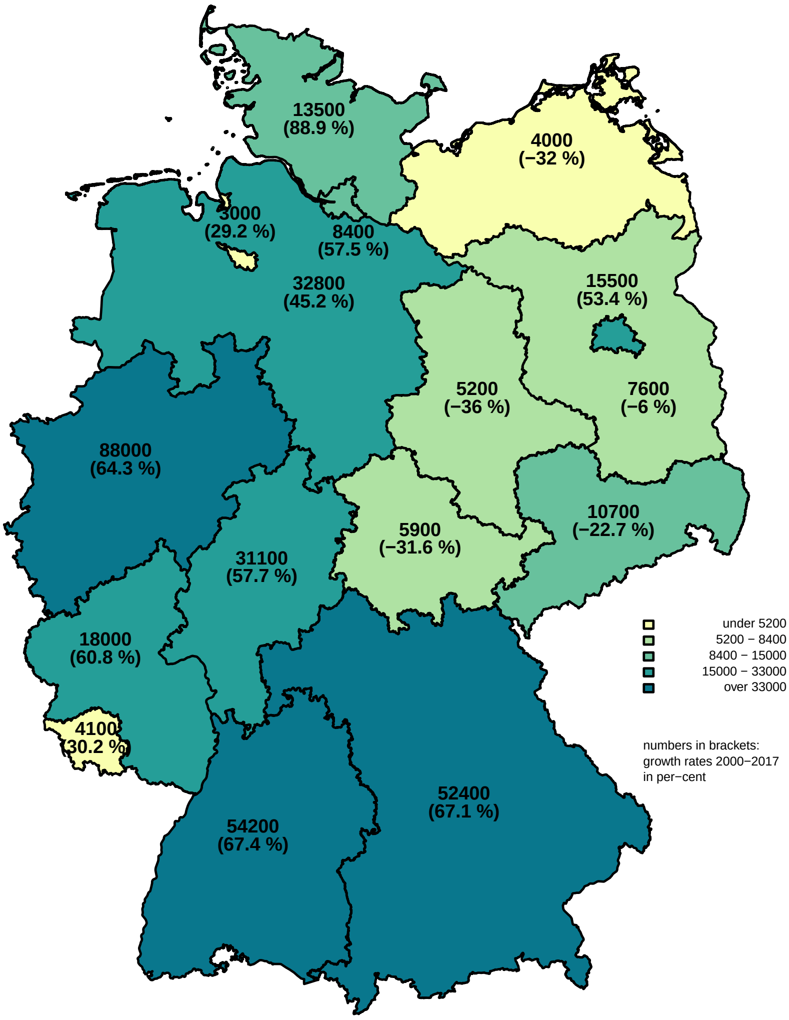
Source: StBA 2018, own calculation and figure.

Overall, 354 thousand first-year students with a German university entrance qualification[7] did enrol in university in 2017. Important determinants are – as described above – the number of school graduates that are allowed to study, their willingness to do so and the time they need to enrol in university. Compared to 2000 their number increased by 45.6 % or 2.2 % p. a. and hence faster than the number of school graduates with university entrance qualification.

[7]The term university entrance qualification equally comprise the general university entrance qualification as well as the entrance qualification for universities of applied sciences.

The number of first-year students arising in each Federal State are not equally distributed over Germany (see Figure 2).[8] The lowest number of first-year students come from *Bremen* (3 thousand) and the highest number from Nordrhein-Westfalen (88 thousand). The growth rates differ considerably as well: while between 2000 and 2017 the number of first year students is declining in Brandenburg, Mecklenburg-Vorpommern, Sachsen, Sachsen-Anhalt and Thüringen, extremely high positive changes can be realised in Schleswig-Holstein, Baden-Württemberg and Bayern.

Figure 2: Number of first-year students in 2017 and change compared to 2000 in per-cent

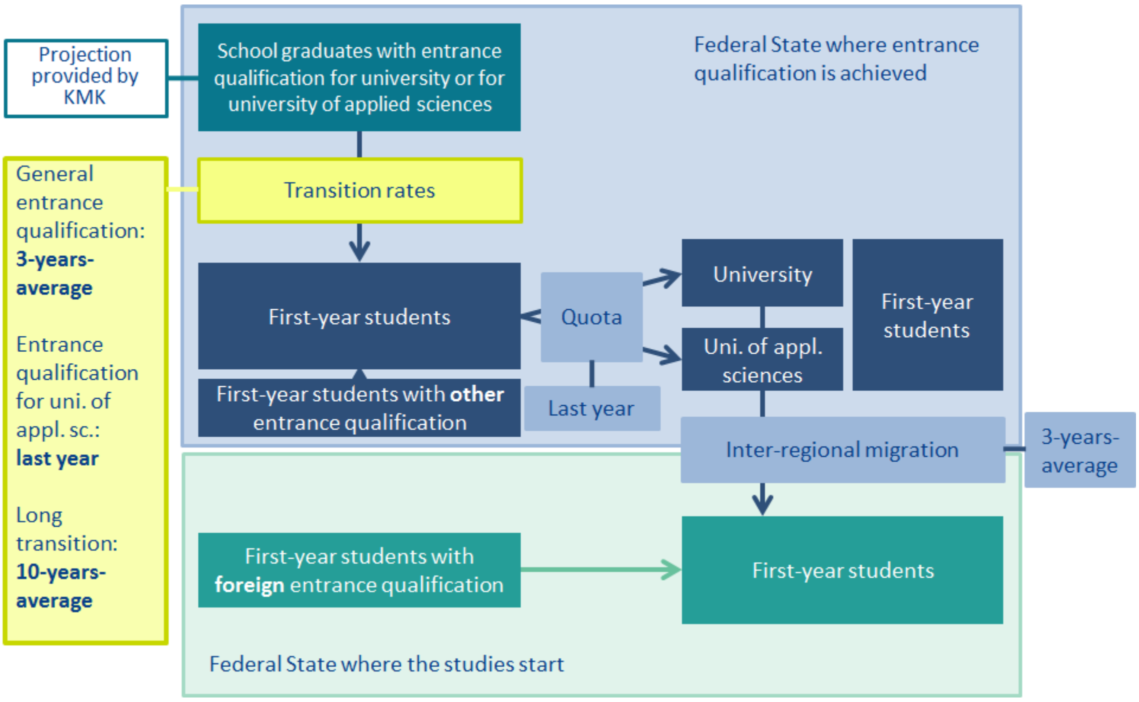


Source: StBA 2018, own calculations and figure. Map: GADM data, Version 2.8.

[8]In the data set at hand the first-year student is attributed to the Federal State where the university entrance qualification is received and not where the university is located.

These spatial differences reflect differences in the number of school graduates, the propensity to study as well as the transition behaviour. For the projection of the first-year students until 2030 these regional differences were considered modelling each Federal State separately. An overview over the model structure is given in Figure 3.

Figure 3: Model structure for the projection of first-year students until 2030



The basis of the model was an exogenous projection of the school leavers with university entrance qualification provided by the *Kultusministerkonferenz* (Standing Conference of the Ministers of Education and Cultural Affairs of the Federal States in Germany) (KMK 2018). The school leavers were differentiated by the type of university entrance qualification and the Federal State where they received it for the graduation years 2018–2030. Depending on the specific transition behaviour the overall number of first-year students in a given calendar year for each Federal State and subdivided into the two types of university entrance qualification could be calculated. It was assumed that the transition behaviour of school graduates with a general university entrance qualification for no time lag as well as for time lags of one to three years is constant during the projection time and follows a three years average. In case of the school graduates holding a entrance qualification for universities of applied sciences the last available value was held constant.[9] For the long transitions of four or more years an average value of ten years was taken for both types of university entrance qualifications. The estimated number of first-year students were distributed to universities and universities of applied sciences using a constant quota. Up to this calculation step the Federal State represents the place where the university entrance qualification was received. However, the number of first-year students according to the Federal State were the university is located is of much higher interest. Therefore, the migration behaviour of the first-year students is considered using a constant inter-regional migration matrix. Finally, first-year students with foreign university entrance qualification are added resulting in the total number of first year students differentiated by type of university and by the Federal State of the university location.

The constancy assumption of the transition rates is quite restrictive. Figure 4 shows the development of the transition rates over time for Germany. The transition rates represent the conditional probability of a school leaver for a given time gap between school and studies to enrol in university in a given calendar year.[10]

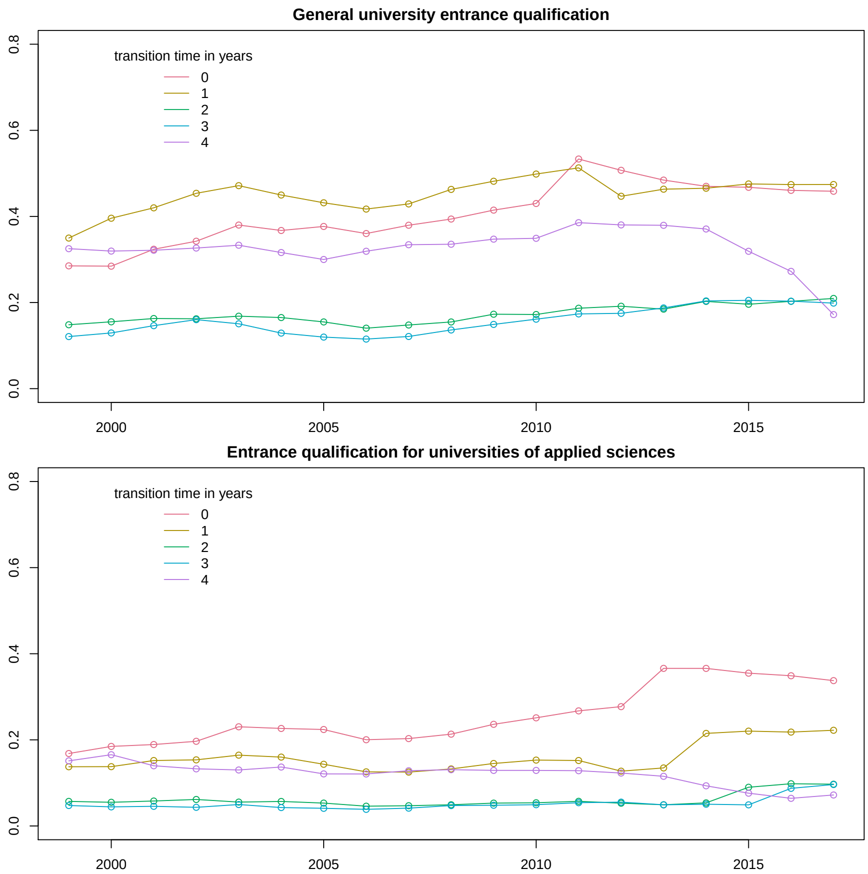
For the different transition times as well as for the different types of university entrance qualifications changes can be observed during 1999–2017. With regard to the general university entrance qualification the probability for an immediate transition from school to studies started at 28.5 % in 1999 and increased by 17.3 percentage points reaching 45.8 % in 2017. The probability for enrolment in university one year after school increased as well but on a slower pace: the transition rate grew by 12.4 percentage points. Due to this development the transition rates for school graduates with no time gap or a one year time gap are quite similar since 2014. The transition rates for time gaps of two and three years increased by 6.1 and 7.8 percentage points reaching values of 21 % and 19.9 %. Only the enrolment probability for school leavers finishing school at least four years ago seems to be declining (-15.3 percentage points between 1999 and 2017). However, the decline in the transition probability in 2015, 2016 and 2017 does not represent a real probability reduction but merely reflects that those transitions are indeed very late transitions taking place a lot later than four years after graduation and are therefore not yet detectable in the numbers.

[9] Due to recent changes in the data collection of the survey the three years average would give spurious results.

[10] For example, in 2017 the probability of a school graduate who received his general university entrance qualification three years ago to enrol in university is 19.9 %.

Consequently, for 1999–2014 this transition rate exhibited a positive change from 32.5 % to 37.1 % as well.

Figure 4: Transition rates for different time gaps between school and university enrolment for Germany 1999–2017



Note: The transition rates represent a conditional share, i. e. the number of school graduates who waited x years with the enrolment into university relative to all school leavers who hold an university entrance qualification since x years and did not yet enrol into university. The values encompass all German and foreign students that hold a German university entrance qualification. Persons with a foreign university entrance qualification are not considered.
Source: StBA 2018, own calculation and figure.

For the transition rates of school graduates holding an entrance qualification for universities of applied sciences a similar development can be observed (see second graph in Figure 4). The transition rates for an immediate transition and time gaps of one, two and three years increased by 16.9, 8.5, 4 and 4.9 percentage points. For the long transitions of at least four years the rate exhibits a declining tendency throughout the whole observation period: it decreases from 15.1 % in 1999 to 9.3 % in 2014 and continue to decrease thereafter.

Overall, these tendencies or observable trends seem to speak against the constancy assumption. Interpreting the transition rates as mortality rates and using the methodology of mortality models offers an opportunity to forecast the trends underlying in these transition rates.

3 Modelling the transition behaviour using a mortality model

The Lee-Carter model (Lee and Carter, 1992) is one of the most famous and widely used mortality models. It is often applied as benchmark model to mortality data (Novokreshchenova, 2016, p. 3). The estimation function is (Lee and Carter, 1992, p. 660):

$$tr(x,t) = a_x + b_x k_t + \epsilon_{xt} \tag{1}$$

Applied to the problem at hand, i. e. the estimation of the transition rates, tr_{xt} is the log of the transition rate for transition time $x = 0, \dots, 4+$ and calendar year $t = 1999, \dots, 2017$. a_x represents the time invariant mean value of the respective transition rate with transition time x and $b_x k_t$ the time varying component. More precisely, k_t is a common time trend for all transition rates. b_x is the specific reaction of transition time x on changes in the common time trend and stays constant over time. ϵ_{xt} is an error term with zero mean and variance σ^2. The model is identifiable subject to the parameter restrictions $\sum b_x = 1$ and $\sum k_t = 0$. In the classical Lee-Carter model the trend parameter k_t follows a random walk with drift:

$$k_t = \delta + k_{t-1} + \xi_t \tag{2}$$

Especially for the rather short time series 1999–2017 the Lee-Carter model offers some attractive features as only a_x, b_x and the time series k_t have to be estimated. For the forecasting the problem reduces to estimating a ARIMA process for k_t. Additionally, the Lee-Carter model is a well-known and accepted approach that proved to be useful in many applications. Thus, it can be seen as a good starting point in estimating the transition rates.

The model was fitted using the R package StMoMo provided by Villegas, Kaishev, and Millossovich (2018). The average transition rates a_x for the time period 1999–2017 are listed in Table 1. The transition time specific reactions to changes in the common trend are given in Table 2. The general time trend in the overall transition time is depicted in Figure 5.

For the general university entrance qualification the average transition probabilities for immediate enrolment and enrolment after one year were highest compared to the other transition times and took similar values (see left part of Table 1). The average transition rate for immediate enrolment ranged from 0.28 in Hamburg to 0.51 in Saarland. The mean value of the probability for a transition gap of one year varied from 0.3 in Brandenburg to 0.56 in Saarland. The average probabilities for enrolment after two and three years were less than half as high and exhibit values from 0.1 in Brandenburg to 0.22 in Hamburg and 0.1 in Brandenburg to 0.2 in Niedersachsen respectively. A late transition, i. e. after at least four years, became in average more probable again: The minimum probability lay at 0.24 in Sachsen whereas the maximum reached 0.42 in Hamburg.

Table 1: Average transition probability (a) for each transition time x (immediately or time gaps of 1, 2, 3 and at least 4 years)

	General UEQ					Uni. of Applied Sc. EQ				
	0	1	2	3	4+	0	1	2	3	4+
BW	0.35	0.51	0.22	0.14	0.28	0.25	0.20	0.08	0.06	0.13
BY	0.50	0.56	0.21	0.18	0.35	0.35	0.25	0.09	0.07	0.16
BE	0.35	0.39	0.20	0.14	0.37	0.31	0.27	0.13	0.09	0.26
BB	0.34	0.30	0.10	0.10	0.25	0.22	0.10	0.04	0.03	0.11
HB	0.36	0.46	0.17	0.16	0.36	0.30	0.21	0.10	0.10	0.27
HH	0.28	0.40	0.22	0.16	0.42	0.17	0.14	0.06	0.03	0.08
HE	0.44	0.51	0.20	0.19	0.37	0.31	0.19	0.07	0.07	0.16
MV	0.37	0.35	0.11	0.12	0.28	0.33	0.15	0.06	0.04	0.11
NI	0.37	0.44	0.17	0.20	0.38	0.23	0.13	0.05	0.04	0.10
NW	0.42	0.42	0.16	0.17	0.35	0.18	0.11	0.04	0.04	0.10
RP	0.49	0.50	0.18	0.19	0.37	0.23	0.16	0.06	0.05	0.10
SL	0.51	0.56	0.20	0.19	0.37	0.24	0.13	0.05	0.06	0.13
SN	0.37	0.39	0.13	0.10	0.24	0.32	0.18	0.07	0.05	0.13
ST	0.42	0.38	0.12	0.13	0.29	0.31	0.13	0.04	0.03	0.10
SH	0.34	0.43	0.17	0.16	0.34	0.21	0.14	0.06	0.04	0.09
TH	0.39	0.38	0.13	0.12	0.26	0.26	0.12	0.05	0.04	0.10

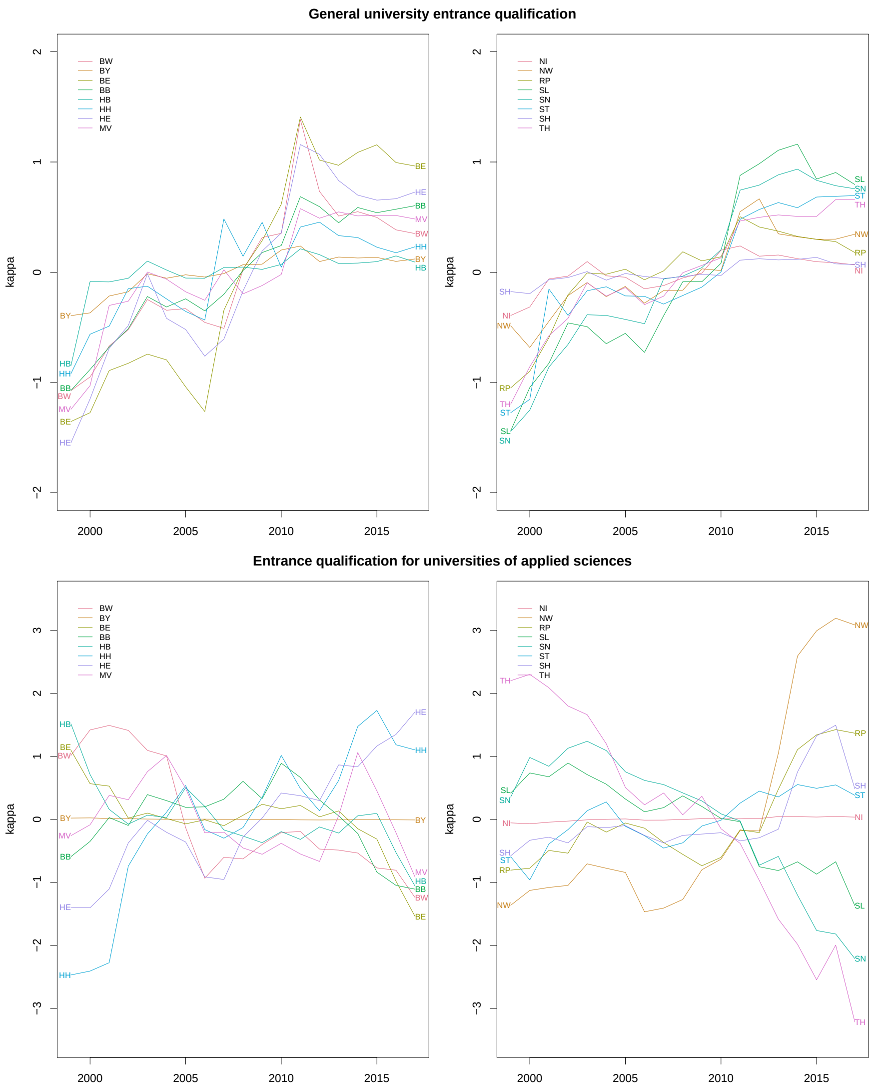
Note: General UEQ: General university entrance qualification, Uni. of Applied Sc. EQ: Entrance qualification for universities of applied sciences. The Federal States are: BW: Baden-Württemberg, BY: Bayern, BE: Berlin, BB: Brandenburg, HB: Bremen, HH: Hamburg, HE: Hessen, MV: Mecklenburg-Vorpommern, NI: Niedersachsen, NW: Nordrhein-Westfalen, RP: Rheinland-Pfalz, SL: Saarland, SN: Sachsen, ST: Sachsen-Anhalt, SH: Schleswig-Holstein, TH: Thüringen.

Quelle: StBA 2018, own calculations.

With regard to the average transition rates for entrance qualifications for universities of applied sciences (right part of Table 1) the mean values for all transition times were generally lower than those of the general university entrance qualification and the differences between the Federal States was higher. The mean probability for an immediate enrolment after school e. g. ranged from 0.17 in Hamburg to 0.35 in Bayern. Transitions with time gaps of two or three years reached in average only values below 0.13 and 0.1.

Huge differences between the types of the university entrance qualifications also appeared in the common trend component for the Federal States: For the general university entrance qualification a positive development for κ for all Federal States could be assessed (see upper part of Figure 5). The highest positive changes for the time period 1999–2017 were estimated for Berlin, Hessen, Saarland and Sachsen. The slowest growth showed for Schleswig-Holstein and Niedersachsen.

Figure 5: Fitted long-term common transition time k_t



Note: The Federal States are: BW: Baden-Württemberg, BY: Bayern, BE: Berlin, BB: Brandenburg, HB: Bremen, HH: Hamburg, HE: Hessen, MV: Mecklenburg-Vorpommern, NI: Niedersachsen, NW: Nordrhein-Westfalen, RP: Rheinland-Pfalz, SL: Saarland, SN: Sachsen, ST: Sachsen-Anhalt, SH: Schleswig-Holstein, TH: Thüringen.
Source: StBA 2018, own calculation and figure.

Contrary to that, the common transition probability for the entrance qualification for universities of applied sciences was characterised by a negative development for the majority of all Federal States. Only 7 out of 16 Federal States showed a positive change in the transition from school to university. The highest decreases were estimated for Thüringen, Berlin and Bremen; positive developments resulted for Nordrhein-Westfalen, Hamburg, Hessen, Rheinland-Pfalz, Schleswig-Holstein, Sachsen-Anhalt and Niedersachsen. Additionally, for Niedersachsen and Bayern the common trend component was almost stable not varying a lot from zero.

Finally, the reactions of the transition rates to changes in the common trend varied between the type of university entrance qualification and the Federal States as well (see Table 2).

Table 2: Reaction b of each transition time x=0,...,4+ to changes in the common trend k

	General UEQ					Uni. of Applied Sc. EQ				
	0	1	2	3	4+	0	1	2	3	4+
BW	0.29	0.15	0.21	0.28	0.08	0.04	0.18	0.25	0.13	0.40
BY	0.86	0.16	-0.05	0.29	-0.26	-19.40	1.65	0.69	-4.27	22.33
BE	0.26	0.20	0.19	0.23	0.13	-0.06	-0.04	0.05	0.09	0.96
BB	0.45	0.13	0.16	0.23	0.02	0.27	0.11	0.05	0.08	0.49
HB	1.12	-0.14	0.10	0.18	-0.26	-0.52	0.03	0.38	0.35	0.77
HH	0.37	0.27	0.20	0.21	-0.06	0.27	0.32	0.22	0.19	-0.01
HE	0.19	0.16	0.23	0.30	0.12	0.27	0.24	0.26	0.37	-0.14
MV	0.41	0.19	0.20	0.18	0.01	0.36	0.60	0.20	-0.07	-0.09
NI	0.76	0.10	-0.04	0.38	-0.20	5.33	-0.19	-0.32	2.96	-6.78
NW	0.51	0.02	0.16	0.37	-0.05	0.37	0.36	0.26	0.14	-0.13
RP	0.46	0.07	0.06	0.32	0.08	0.30	0.39	0.39	0.26	-0.34
SL	0.17	0.04	0.23	0.35	0.20	-0.14	0.41	0.24	0.10	0.39
SN	0.38	0.16	0.18	0.23	0.05	0.04	0.26	0.23	0.09	0.39
ST	0.40	0.14	0.16	0.28	0.02	0.54	0.21	0.17	0.32	-0.24
SH	1.62	-0.10	-0.50	0.39	-0.41	0.55	0.59	0.30	0.12	-0.57
TH	0.47	0.21	0.18	0.20	-0.05	0.05	0.22	0.20	0.19	0.34

Note: General UEQ: General university entrance qualification, Uni. of Applied Sc. EQ: Entrance qualification for universities of applied sciences. The Federal States are: BW: Baden-Württemberg, BY: Bayern, BE: Berlin, BB: Brandenburg, HB: Bremen, HH: Hamburg, HE: Hessen, MV: Mecklenburg-Vorpommern, NI: Niedersachsen, NW: Nordrhein-Westfalen, RP: Rheinland-Pfalz, SL: Saarland, SN: Sachsen, ST: Sachsen-Anhalt, SH: Schleswig-Holstein, TH: Thüringen.

Quelle: StBA 2018, own calculations.

Most of the transition times encompassed positive as well as negative reactions to changes in the common trend. However, negative reactions occured not very often. A negative β_x implies that the probability for a transition at transition time x becomes less likely if the overall probability for enrolment in university is increasing.

Exemptions to the mixed results were the immediate transition and the transition after three years for school graduates holding a general university entrance qualification. For these rates solely positive reactions to changes in the common trend were estimated. An increasing overall probability to start studying translated in a higher probability for immediate enrolment or for enrolment after three years for all Federal States.

Very high impacts of changes in the trend components on the transition times could be found for Niedersachsen and Bayern for the entrance qualification for universities of applied sciences. As described above both Federal States were characterised by extremely low changes in the trend component. This rather static development was compensated by strong reactions, i. e. the transition rates in Niedersachsen and Bayern reacted very sensitive to small changes in the long-term trend.

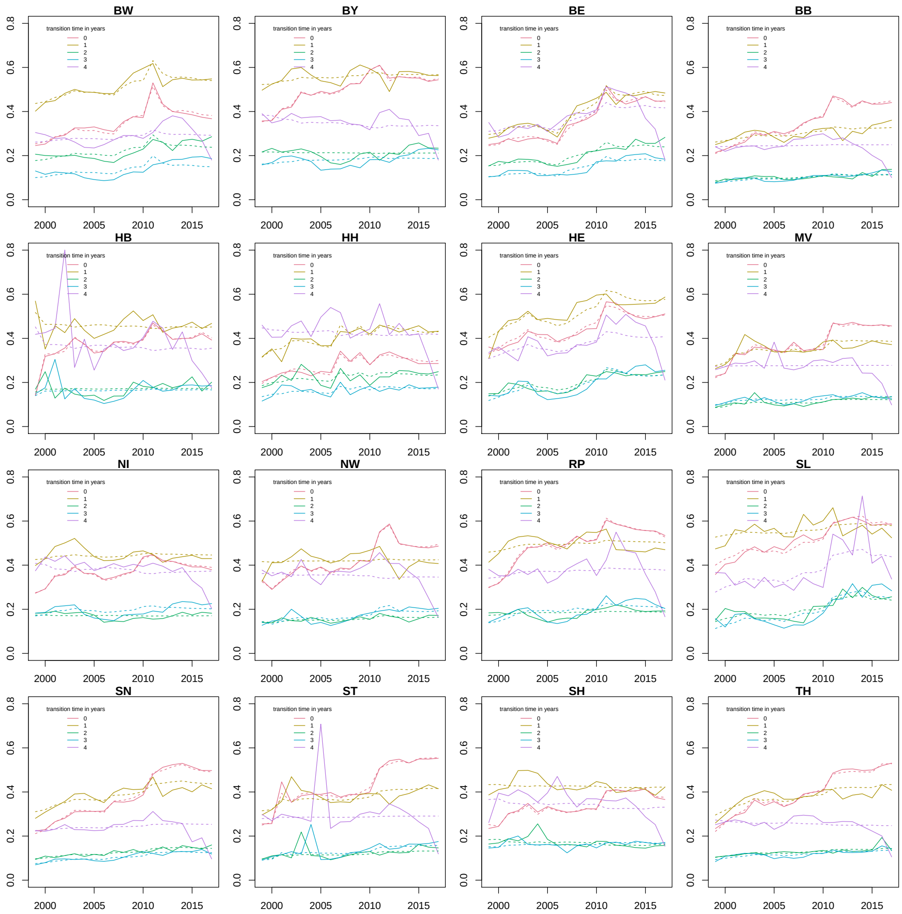
The overall estimation results for the transition rates composed of the elements described above are displayed in Figure 6 and Figure 11.[11] The fitted transition rates are represented by the dashed lines and compared to the respective actual historic transition rates represented by the solid lines. The fitted transition rates seem to meet the development of the actual transition rates fairly well so that they should provide a good basis for forecasting.

However, the residuals of the fitted models depicted in Figure 12 and Figure 13 (in the Appendix) suggest that there were some systematic pattern not kept by the estimation functions. The pattern were somewhat better identifiable for the transition model of school graduates holding a general university entrance qualification but were nevertheless also present for school graduates holding an entrance qualification for universities of applied sciences. The regular pattern for the general university entrance qualification could be explained by changes in education policies called “G8”. The schooling time in higher secondary education leading to a general university entrance qualification was reduced from nine to eight years. As education policies rest in the responsibility of each Federal State the year of the implementation varied and thus the overall implementation spanned over the whole observation period. The shortening of the schooling time lead to double cohorts and to special reactions of the graduates from double cohorts as well as from graduates of the cohort preceding a double cohort. As the implementation was completed in 2016 no further impact is expected for the graduation cohorts after 2017. It can hence be assumed that the omitted effects captured by the residuals are most likely caused by the policy measure “G8”. Not including the policy impacts in the estimation function but leaving it to the residuals should not distort the estimation results for the long-run transition rates. As for the fitted models for the entrance

[11]Figure 11 can be found in the Appendix.

qualification for universities of applied sciences the pattern in the residuals might absorb changes in method of collecting the data. Overall, the goodness-of-fit of the estimated transition rates seems to be reasonably high so that they can be used for a projection.

Figure 6: Fitted (dashed lines) and actual (solid lines) transition rates for different time gaps between school and university enrolment for school graduates holding the general university entrance qualification 1999–2017



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Source: StBA 2018, own calculation and figure.

As the components a_x and b_x are time invariant only k_t has to be forecasted. Lee and Carter (1992, p. 663) use an ARIMA process and find that a random walk with drift describes k_t best (see Equation 2). For the identification of the appropriate ARIMA process representing the common time trend of the transition rates the automated procedure of the forecast package for R (Hyndman and Khandakar, 2008) was used. The selected ARIMA models are summarised in Table 3.

Table 3: Selected ARIMA processes representing the common trend component k of the transition rates for each Federal State and each type of university entrance qualification

	General UEQ				Uni. of Applied Sc. EQ			
	ARIMA	drift	AR	MA	ARIMA	drift	AR	MA
BW	(0,1,0)	–	–	–	(0,1,0)	–	–	–
BY	(0,1,0)	0.0286	–	–	(0,1,0)	-0.0017	–	–
BE	(0,1,0)	–	–	–	(0,1,0)	0.1467	–	–
BB	(0,1,0)	0.0931	–	–	(1,0,0)	–	0.8907	–
HB	(0,1,0)	–	–	–	(0,1,0)	0.1432	–	–
HH	(0,1,0)	–	–	–	(0,1,0)	–	–	–
HE	(0,1,0)	0.1264	–	–	(0,1,0)	0.1722	–	–
MV	(0,1,0)	0.0959	–	–	(2,0,0)	–	0.9892, -0.5600	–
NI	(0,1,0)	–	–	–	(0,1,0)	0.0051	–	–
NW	(0,1,0)	–	–	–	(1,1,0)	–	0.5223	–
RP	(0,2,1)	–	–	-0.6754	(0,1,0)	0.1208	–	–
SL	(0,1,0)	0.1242	–	–	(0,1,0)	–	–	–
SN	(0,1,0)	0.1223	–	–	(0,2,2)	–	–	-0.9229, 0.6205
ST	(0,1,0)	0.1094	–	–	(0,1,0)	–	–	–
SH	(0,1,0)	–	–	–	(0,1,0)	–	–	–
TH	(0,1,0)	0.1033	–	–	(1,1,0)	-0.2910	-0.4961	–

Note: General UEQ: General university entrance qualification, Uni. of Applied Sc. EQ: Entrance qualification for universities of applied sciences. The Federal States are: BW: Baden-Württemberg, BY: Bayern, BE: Berlin, BB: Brandenburg, HB: Bremen, HH: Hamburg, HE: Hessen, MV: Mecklenburg-Vorpommern, NI: Niedersachsen, NW: Nordrhein-Westfalen, RP: Rheinland-Pfalz, SL: Saarland, SN: Sachsen, ST: Sachsen-Anhalt, SH: Schleswig-Holstein, TH: Thüringen.

For the projection of the common trend for the general university entrance qualification related transition rates a random walk with drift was chosen for half of the Federal States. Simultaneously, for seven out of 16 Federal states a random walk without drift is the best forecasting solution implying that the transition rates stay constant for the whole projection period. The original model assumption of constant transition rates therefore seems to be reasonable for many Federal States. For only one Federal State a completely different ARIMA process is selected: The k_t-value for Rheinland-Pfalz is best forecasted with linear exponential smoothing.

In case of the entrance qualification for universities of applied sciences the random walk with drift was the best solution for only six out of 16 Federal States. The outcome of constant transition rates (a random walk without drift) was selected for five Federal States. Consequently, for five Federal States other ARIMA models than random walk proved to be the best forecasting option. In Brandenburg and Nordrhein-Westfalen k_t followed a first-order autoregressive model with zero mean and in Mecklenburg-Vorpommern a second-order autoregressive model with zero mean. For Thüringen a first-order autoregressive model with constant was chosen. The common trend component in Sachsen is best represented by linear exponential smoothing.

For Bremen the random walk with drift for the common trend component related to the entrance qualification for universities of applied sciences yield inconsistent estimation results: The transition rate for immediate enrolment in university became larger than one. For the implementation into the model all transition rates for this case were therefore kept constant.

4 Results

The forecast results for the transition rates are shown in Figure 7 and Figure 8. The bold lines represent the past development from 1999–2017. The dashed lines depict the fitted transition rates in the period 1999–2017 and the forecasted transition rates in the period 2017–2030. The year 2017 is marked by the vertical line.

For some Federal States the predicted growth in the transition rates – especially for the immediate transition – seem to be very high. This is especially the case for Bayern, Mecklenburg-Vorpommern, Rheinland-Pfalz and Sachsen-Anhalt in case of school leavers holding a general university entrance qualification. Regarding the entrance qualification for universities of applied sciences the changes in Bayern, Berlin, Brandenburg, Mecklenburg-Vorpommern, Niedersachsen, Nordrhein-Westfalen, Rheinland-Pfalz, Sachsen and Thüringen are significantly higher than in the past.

Compared to the conservative approach of constant transition rates, the forecasted transition rates could hence be seen as representing a more progressive development. Implementing the projected transition rates into the model offers the opportunity for an alternative development of first-year students that could be interpreted as an upper bound of the c.p. development.[12]

The different model results for the total number of first-year students are shown in Figure 9. The green line represents the original model projection with constant transition rates. The red line shows the projection with the alternative transition rates based on the mortality model. The difference between the two projection is depicted by the bar chart.

The difference between both developments was small in the beginning (569 first-year students) and reached a maximum of 14771 persons in 2030. The drop in the difference between original and new projection in 2025 was due to a loss of a graduate cohort caused by changes in the education policies in Bayern: as especially the probability for the direct enrolment after school was increasing in the estimated

[12]In a later step transition rates projections resulting from other mortality models or growth assumptions could be added to the model as well so that a whole range of possible projection results could be offered and compared.

transition rates, the immediate effect of the lost cohort is higher than with the constant ones.

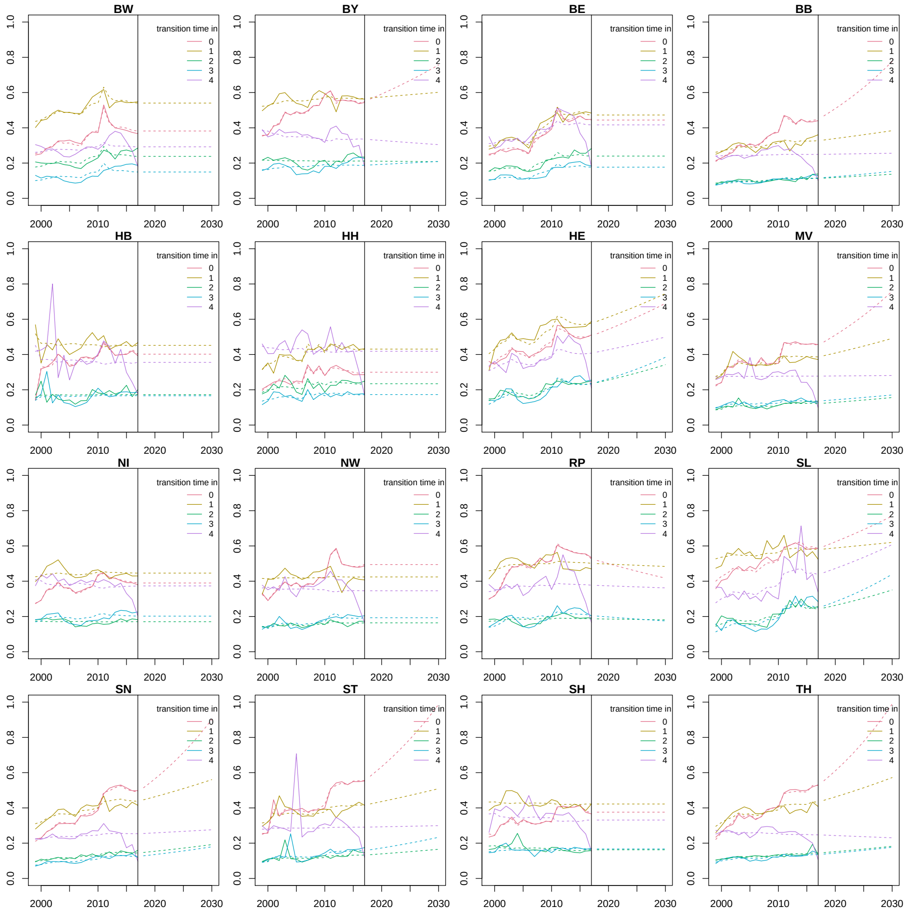
The generally small difference between both lines revealed the high impact of the demographic change: With an ageing population the number of school graduates was quite stable growing only by -0.9 % between 2017 and 2030. The difference in the number of first-year students could hence be explained by the increasing propensity to study.

The highest deviation between the new and the original modelling approach resulted for Bayern. The absolute difference amounted to 3643 in 2030. The already mentioned drop in 2025 was a single event with no long-term influence.

Also noticeable more university places for first-year students than originally planned might be necessary in Hessen, Sachsen and Berlin conditional on an ongoing increase in the transition rates. Almost no difference to the original projection resulted for Hamburg, Bremen, Saarland and Schleswig-Holstein.

Nordrhein-Westfalen was the only Federal State were less university places were needed when including the forecasted transition rates in the model. The number of first-year students is by -399 persons lower in 2030 than originally calculated with constant transition rates.

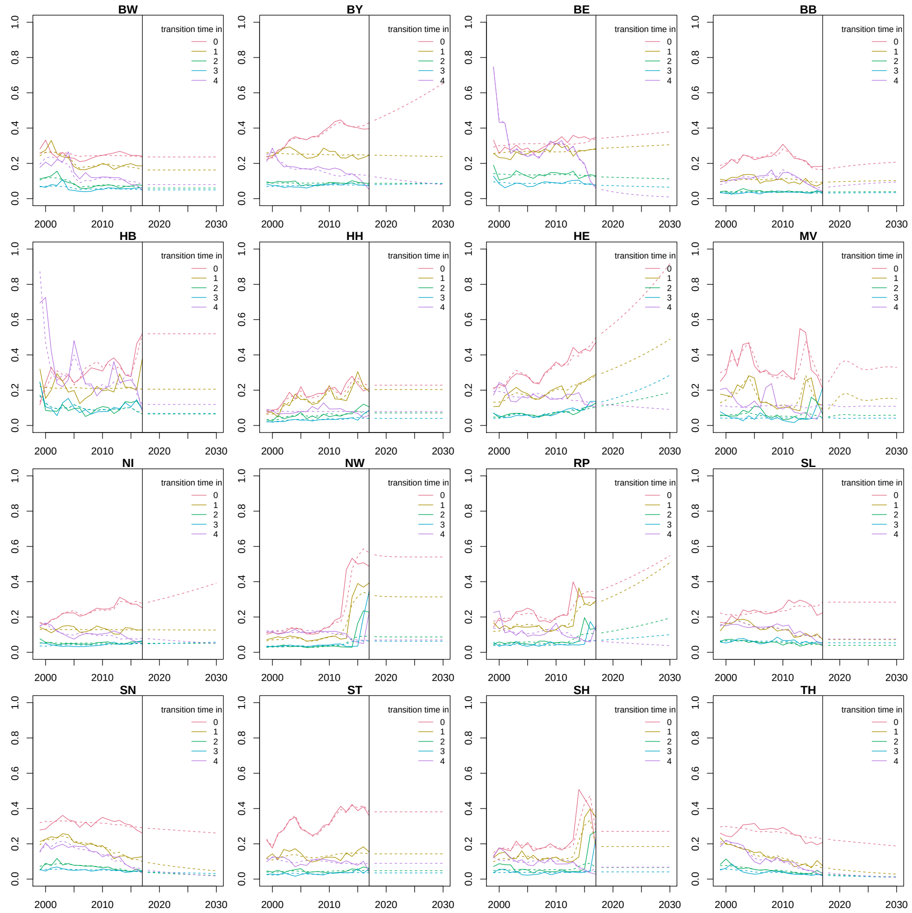
Figure 7: Transition rates for different time gaps between school and university enrolment for school graduates holding the general university entrance qualification 1999–2030



Note: The Federal States are: BW: Baden-Württemberg, BY: Bayern, BE: Berlin, BB: Brandenburg, HB: Bremen, HH: Hamburg, HE: Hessen, MV: Mecklenburg-Vorpommern, NI: Niedersachsen, NW: Nordrhein-Westfalen, RP: Rheinland-Pfalz, SL: Saarland, SN: Sachsen, ST: Sachsen-Anhalt, SH: Schleswig-Holstein, TH: Thüringen. The fitted and projected transition rates are depicted by the dashed lines, the actual transition rates by the bold lines. The vertical line marks the start of the projection period.

Source: StBA 2018, own calculation and figure.

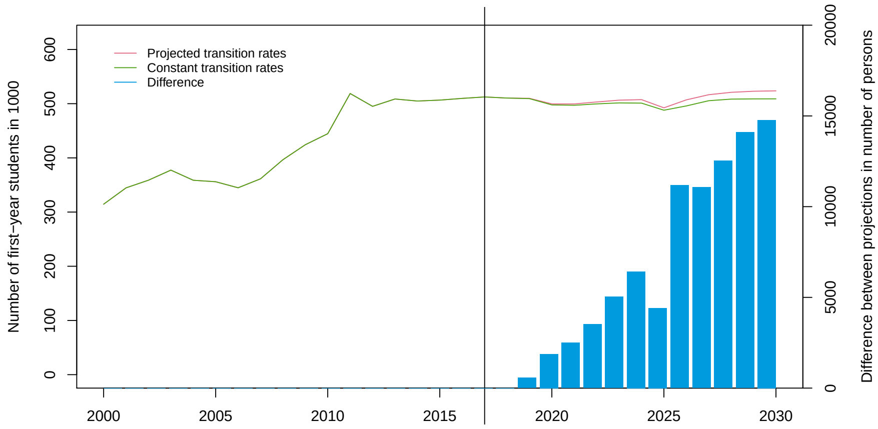
Figure 8: Transition rates for different time gaps between school and university enrolment for school graduates holding the entrance qualification for universities of applied sciences 1999–2030



Note: The Federal States are: BW: Baden-Württemberg, BY: Bayern, BE: Berlin, BB: Brandenburg, HB: Bremen, HH: Hamburg, HE: Hessen, MV: Mecklenburg-Vorpommern, NI: Niedersachsen, NW: Nordrhein-Westfalen, RP: Rheinland-Pfalz, SL: Saarland, SN: Sachsen, ST: Sachsen-Anhalt, SH: Schleswig-Holstein, TH: Thüringen. The fitted and projected transition rates are depicted by the dashed lines, the actual transition rates by the bold lines. The vertical line marks the start of the projection period.

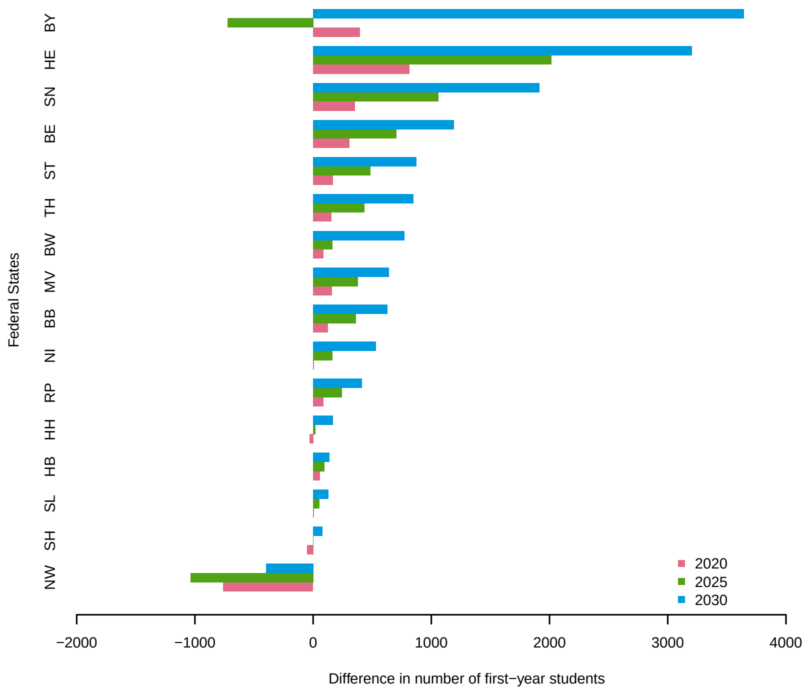
Source: StBA 2018, own calculation and figure.

Figure 9: Number of first-year students based on constant and forecasted transition rates (in 1000, left axis) and difference between the projections (right axis) 1999–2030



Source: StBA 2018, own calculation and figure.

Figure 10: Difference in the number of first-year students based on constant and forecasted transition rates for the Federal States in 2020, 2025 and 2030



Source: StBA 2018, own calculation and figure.

5 Conclusion

The results showed that the constancy assumption was the best approach for many transition rates. Where changes in the historical transition behaviour prevail, the trends estimated using the Lee-Carter model proved to be a good alternative. However, some estimated changes might be too large suggesting a restriction of the possible growth/decline in the transition rates. Additionally, the Lee-Carter model is quite restrictive: As b_x is constant over time t and k_t is constant over transition time x no transition time and calendar time interactions are possible. Cohort effects are not considered as well. The advantage is, however, that with the rather short time series of only 18 years just one parameter has to be forecasted. Furthermore, the Lee-Carter model is a well-known model and widely used and accepted as benchmark. For the problem at hand it also provides a trustworthy option for generating an alternative projection.

Overall, the methodology of mortality rates and of stochastic mortality models can be used to model the transition from school to studies. Adding this additional estimation routine to the existing model provides the opportunity to generate an alternative number of first-year students. Comparing both estimation results – the original and the alternative ones – opens up a possible range for the projection and helps to get an idea about the future related uncertainty in the number of first-year students.

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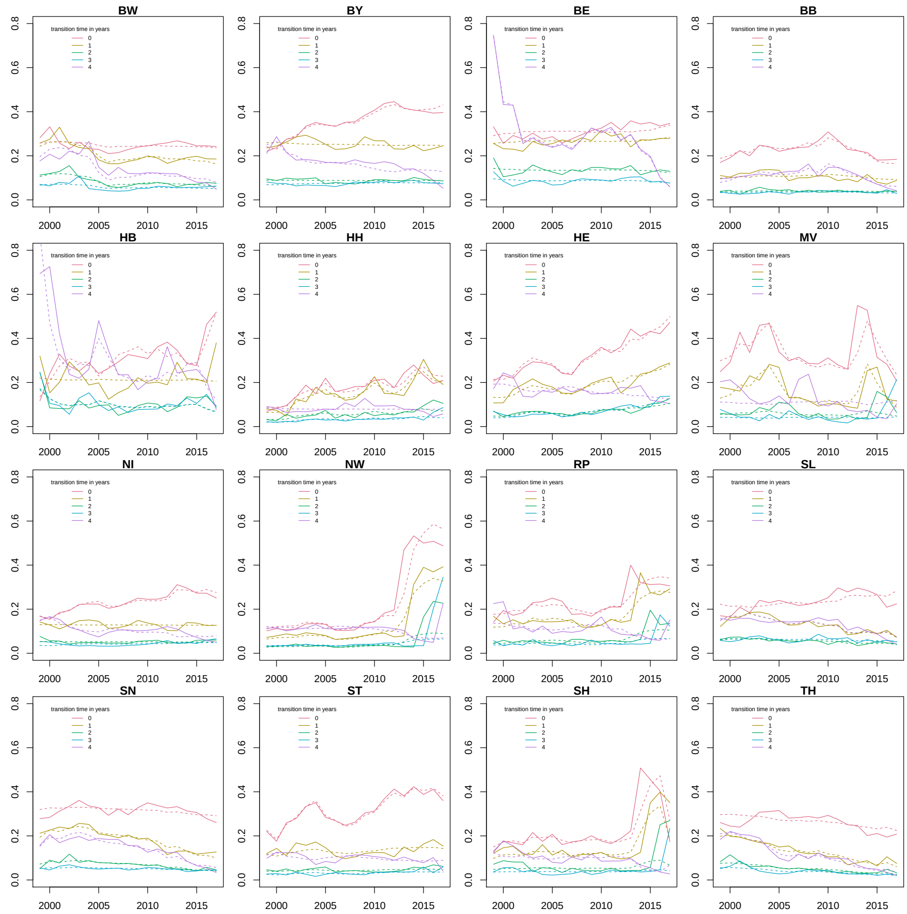
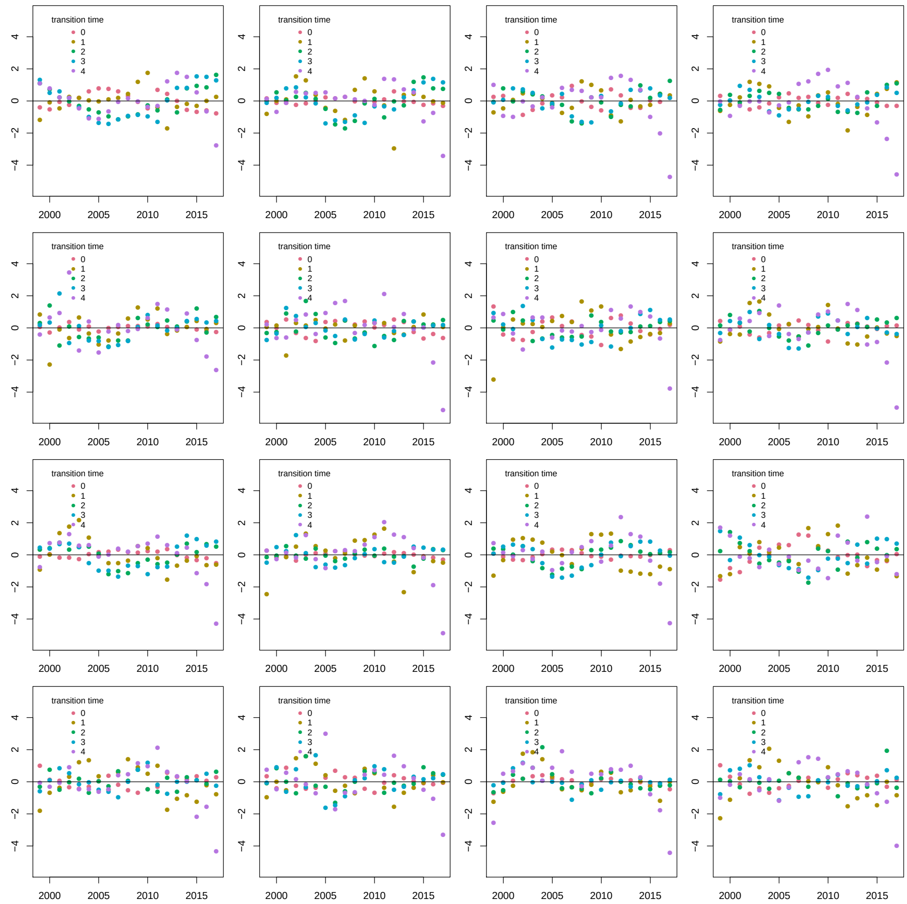
Appendix

Figure 11: Fitted (dashed lines) and actual (solid lines) transition rates for different time gaps between school and studies for school graduates holding the entrance qualification for universities of applied sciences 1999–2017



Note: The Federal States are: BW: Baden-Württemberg, BY: Bayern, BE: Berlin, BB: Brandenburg, HB: Bremen, HH: Hamburg, HE: Hessen, MV: Mecklenburg-Vorpommern, NI: Niedersachsen, NW: Nordrhein-Westfalen, RP: Rheinland-Pfalz, SL: Saarland, SN: Sachsen, ST: Sachsen-Anhalt, SH: Schleswig-Holstein, TH: Thüringen.
Source: StBA 2018, own calculation and figure.

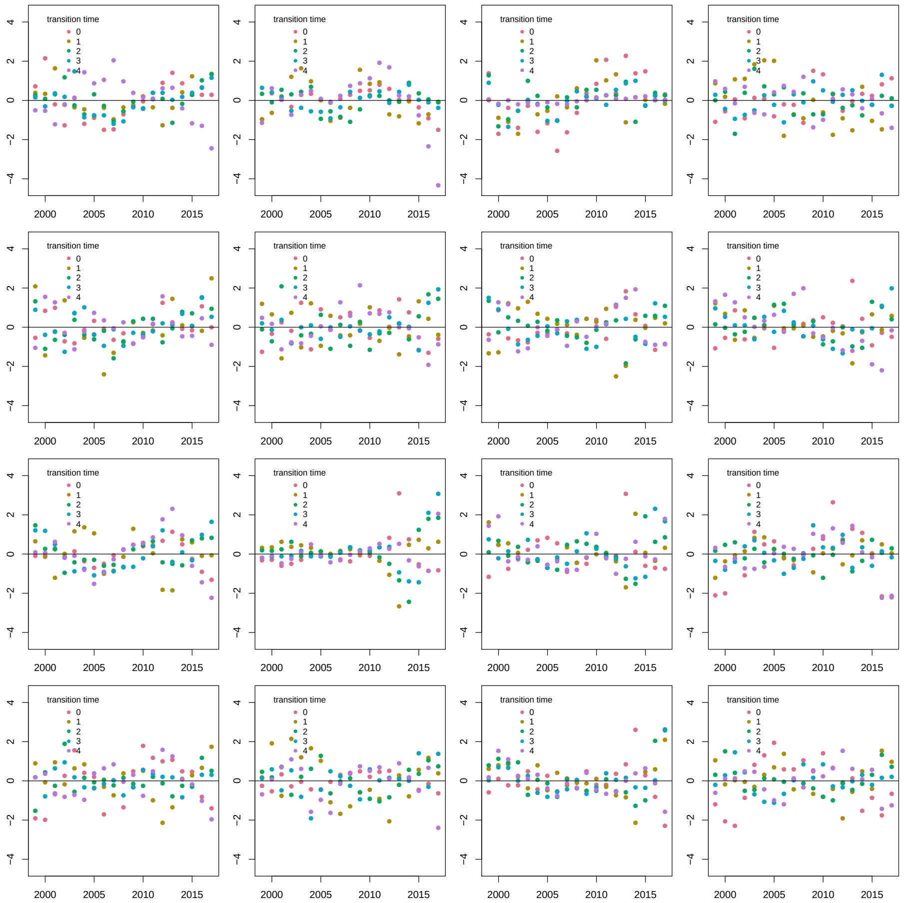
Figure 12: Scaled deviance residuals for the case of the general university entrance qualification



Note: The Federal States are: BW: Baden-Württemberg, BY: Bayern, BE: Berlin, BB: Brandenburg, HB: Bremen, HH: Hamburg, HE: Hessen, MV: Mecklenburg-Vorpommern, NI: Niedersachsen, NW: Nordrhein-Westfalen, RP: Rheinland-Pfalz, SL: Saarland, SN: Sachsen, ST: Sachsen-Anhalt, SH: Schleswig-Holstein, TH: Thüringen.

Source: StBA 2018, own calculation and figure.

Figure 13: Scaled deviance residuals for the case of the entrance qualification for universities of applied sciences



Note: The Federal States are: BW: Baden-Württemberg, BY: Bayern, BE: Berlin, BB: Brandenburg, HB: Bremen, HH: Hamburg, HE: Hessen, MV: Mecklenburg-Vorpommern, NI: Niedersachsen, NW: Nordrhein-Westfalen, RP: Rheinland-Pfalz, SL: Saarland, SN: Sachsen, ST: Sachsen-Anhalt, SH: Schleswig-Holstein, TH: Thüringen.
Source: StBA 2018, own calculation and figure.