Miles–Ezzell's WACC Approach Yields Arbitrage

Andreas Löffler
FB Wirtschaftswissenschaften, Universität Hannover,
Königsworther Platz 1,
30167 Hannover, Germany.

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A simple counterexample shows that the widely used WACC approach to value leverage firms developed by Miles and Ezzell can create an arbitrage opportunity. The only consequence to be drawn is that their WACC approach cannot be applied under the circumstances assumed by Miles and Ezzell.

We show how the WACC has to be modified in order to obtain proper results. We develop a theory in continuous as well as discrete time. In discrete time it turns out that with a further assumption on the cash flows of the firm formulas similar to Miles and Ezzell's results can be verified. This assumption requires that the increments of cash flows have to be uncorrelated. This is a much weaker assumption then independent increments which is used in models of random walk.

keywords: WACC, leverage ratio, tax shield

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1 Introduction

A firm does create value not only by its cash flows but also by the way it is financed. Debt interest payments are tax–deductible, and therefore debt financing will be somewhat cheaper than equity financing. The difference of the values of the levered and the unlevered firm is known as the tax shield. It was already shown by Modigliani & Miller (1963) how such a tax shield is to be evaluated if the cash flow of the firm form a perpetual rent. Myers (1974) extended this approach by assuming that the firm determines the future amount of debt today. This approach is known as APV (adjusted present value) theory. In a recent paper Clubb & Doran (1995) have extended Myers' APV–formula for a debt management policy involving a one–period lag in the revision of the firm's debt schedule.

In many cases the assumption of a deterministic future amount of debt is not satisfied, since in particular in practical applications the target of a financing decision is given by the leverage ratio of the firm instead of the amount of debt (see for example Brealey & Myers (1996, p. 513) or Ross, Westerfield & Jaffe (1996, p. 463)). In this case it cannot be assumed that the tax relief will be riskless since tax payments are uncertain:

> "Even though the firm might issue riskless debt, if financing policy is targeted to realized market values, the amount of debt outstanding in future periods is not known with certainty (unless the investment is riskless) ..."
> Miles & Ezzell (1980, p. 721).

In a discrete time setup[1] and under the sharp restriction of a constant leverage ratio Miles and Ezzell (see Miles & Ezzell (1980)) developed a theory how these tax payments should be evaluated. This approach is known as weighted average cost of capital theory (WACC, see for example Grinblatt & Titman (1998, chapter 12.3)). This approach is widely used today and can be considered as a standard technique for valuation of leverage firms.[2] In this introduction we present a simple model in which this WACC formula leads to an arbitrage opportunity. Hence, their formula cannot be applied without further assumptions.

This paper will develop a new WACC theory in continuous time as well as discrete time. Our apporach uses an arbitrage argument and will heavily rely on the use of an equivalent martingale measure. Furthermore, it turns out that we can dispense the assumption of a constant leverage ratio. This assumption is obviously very restrictive, since in many applications as for example LBO's the leverage ratio usually drops down very fast (see for example Newbould, Chatfield & Anderson (1992)). In discrete

[1] Taggart (1991, p. 12) has a continuous time approach. But his approach is heuristic and does not contain rigoros proofs. Also the later paper by Harris & Pringle (1985) is not based on a rational argument but on "pedagogic advantages" (p.241).

[2] "Discounted Cash flow is the dominant investment–evalutaion technique. WACC is the dominant discount rate used in DCF analyses" Bruner et. al. (1998).

time we have to make a further assumption regarding the probability distribution of cash flows. The cash flows of the firm (after being adjusted by a time varying growth rate) have to form a martingale under the subjective probability measure. This assumption corresponds to the stochastic differential equation used in the continuous time theory: there the conditional expectation of the increment of values of the firm is proportional to the current firm value. Using this assumption formulas similar to Miles & Ezzell (1980) can be verified.

We do not discuss the issue whether the exogenous leverage ratio is optimal. The tax system we use in our analysis implies that the higher the leverage ratio the higher will be the value of the firm's cash flows. Therefore, it is suboptimal to choose a financing policy without complete debt financing. The question what determines an optimal policy is not our concern.

To present our counterexample we consider a model with two periods. Cash flows after tax of the unlevered firm are as shown in figure 1. The (subjective) probabilities are choosen such that at $t = 1$ up– and down–movements occur with probability 0.5 and at $t = 0$ the movements (up, down or middle) occur with probability $\frac{1}{3}$.

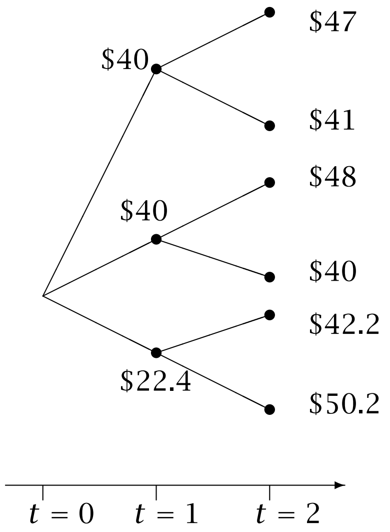

Figure 1: Cash flows of the unlevered firm (after tax) $\widetilde{\mathrm{CF}}^U$ creating an arbitrage strategy

We furthermore assume that the cost of capital are constant at $r^U = 10\%$ and let the riskless rate be $r_f = 5\%$. Since we can evaluate the expectation of the cash flows we get the following values of the unlevered firm

$$V_0^U = \$68, \quad V_1^U(u,m) = \$40, \qquad V_1^U(d) = \$42.$$

We will use a tax rate of 34%. We furthermore assume that the levered firm will maintain a leverage ratio of 58.09581%. From the Miles–Ezzell-formula (see Miles & Ezzell (1980, p. 726)) we get the weighted average cost of capital as

$$\text{WACC} = \left(1 - \frac{\tau r_f}{1 + r_f}\, l\right)(1 + r^U) - 1 = 8.965423\%$$

If we discount the exptected cash flows using this WACC we have the following value of the levered firm

$$V_0^L = \frac{\mathrm{E}[\widetilde{\mathrm{CF}}_1^U]}{1 + \text{WACC}} + \frac{\mathrm{E}[\widetilde{\mathrm{CF}}_2^U]}{(1 + \text{WACC})^2} = \$69. \tag{1}$$

Assume now that shares of the levered and the unlevered firm trade at a market. Consider furthermore a riskless asset traded at a value of one today. To construct an arbitrage opportunity an investor uses the following strategy. At time $t = 0$ she holds one share of the levered firm long and one share of the unlevered firm short. She furthermore sells for the amount of \$1 riskless assets at $t = 0$. At time $t = 1$ she still holds the risky assets and after paying retirement and interest from the bond she sells riskless assets for \$0.36859 again. The following table summarizes her strategy.

cash flows (in \$)	$t = 0$	$t = 1$	$t = 2$
from levered firm	-69	$+\widetilde{\mathrm{CF}}_1^L$	$+\widetilde{\mathrm{CF}}_2^L$
from unlevered firm	68	$-\widetilde{\mathrm{CF}}_1^U$	$-\widetilde{\mathrm{CF}}_2^U$
from selling riskless in $t = 0$	1	-1.05	
from selling riskless in $t = 1$		0.36859	-0.38702
sum	0	>0	>0

Table 1: The arbitrage strategy of the investor using the Miles-Ezzell–formula

We now evaluate the payments of the strategy. At time $t = 0$ no payment has to be made. At time $t = 1$ the difference of the cash flows of the unlevered and the levered firm are given by the tax advantages from debt

$$\widetilde{\mathrm{CF}}_1^L - \widetilde{\mathrm{CF}}_1^U = \tau r_f l V_0^L = 0.68146 = 1.05 - 0.36859$$

and hence the net cash flow of her strategy is zero. At $t = 1$ only one period is left. The value of the levered firm is therefore determined by the relation

$$V_1^L(u, m, d) = V_1^U(u, m, d) + \frac{\tau r_f l V_1^L(u, m, d)}{1 + r_f} \qquad \Longrightarrow \qquad V_1^L(u, m, d) \geq 40.37981.$$

Hence, at $t = 2$ regardless of the state the investor has tax advantages as high as

$$\tilde{\mathrm{CF}}_2^L - \tilde{\mathrm{CF}}_2^U = \tau r_f l V_1^L(u,m,d) \geq 0.3988$$

and the net cash flow is higher than zero. To summarize: the investor receives money at $t = 1, 2$ without facing the risk of earlier payments. This is an arbitrage opportunity.

The paper is organized as follows. We start with the WACC theory in discrete time and establish results similar to Miles & Ezzell (1980). The next chapter develops the model in discrete time. The last section closes the paper.

2 The WACC theory in discrete time

There are T periods of time $t = 0, 1, \dots, T$. With time t information evolves about the true state of the world. This can be formalized using a probability space and a filtration $\mathcal{F}_t$ (for details see Duffie (1988, p. 130)). There is a firm income tax τ, interest payments reduce taxes. An investor values a firm with lifetime T, at $t = T$ we assume transversality (value of the levered and unlevered firm are zero).

We assume that the cost of capital are constant, hence the value at time t satisfies the equation

$$\tilde{\mathrm{V}}_t^u = \frac{\mathrm{E}[\tilde{\mathrm{V}}_{t+1}^U + \tilde{\mathrm{CF}}_{t+1} | \mathcal{F}_t]}{1 + r^U}, \tag{2}$$

where r^U are the cost of capital of the unlevered firm. Our assumption that the cost of capital are constant is made for simplicity, the proofs reveal that we can easily suspend it. If the market is free of arbitrage there is an equivalent martingale measure Q such that the value of the unlevered firm V_t^U for all $t \leq T - 1$ satisfies the following recursive equation, for details see Harrison & Kreps (1979)

$$\tilde{\mathrm{V}}_t^U = \frac{\mathrm{E}_Q[\tilde{\mathrm{V}}_{t+1}^U + \tilde{\mathrm{CF}}_{t+1} | \mathcal{F}_t]}{1 + r_f}. \tag{3}$$

At time t the levered firm has debt $l_t V_t^L$ and therefore a tax relief $\tau r_f l_t V_t^L$ compared to the unlevered firm. This tax relief implies a difference of the levered firm value V_t^L to the unlevered firm value V_t^U which will be denoted as tax shield T_t. The value of a levered cash flow stream at time t equals the market value of the unlevered cash flow stream plus the tax shield T_t at time t:

$$\tilde{\mathrm{V}}_t^L = \tilde{\mathrm{V}}_t^U + \tilde{\mathrm{T}}_t. \tag{4}$$

At time t the amount of debt B_t will induce a riskless tax shield at time $t+1$, consequently we have

$$\widetilde{\mathrm{T}}_t = \frac{\mathrm{E}_Q[\widetilde{\mathrm{T}}_{t+1}|\mathcal{F}_T] + \tau r_f B_t}{1+r_f}. \tag{5}$$

To derive our valuation formula we have to make a further assumption regarding the probability distribution of cash flows. This will enable us to prove a formula that in the case of constant leverage ratios was already provided by Miles & Ezzell (1980) and Miles & Ezzell (1985). In continuous time theory the assumption of a Brownian motion implies that the conditional expectation of the increment of values of the firm is proportional to the current firm value. In discrete time we use this condition for the cash flows instead of the firm values.

Assumption 1 (probability distribution of cash flows) *The cash flows satisfy*

$$\mathrm{E}[\widetilde{\mathrm{CF}}_{t+1} - \widetilde{\mathrm{CF}}_t|\mathcal{F}_t] = g_t \cdot \widetilde{\mathrm{CF}}_t \tag{6}$$

where $g_t > 0$ is deterministic.

Notice that it would not make sense to assume growing cash flows $\widetilde{\mathrm{CF}}_t = (1 + g_t) \cdot \widetilde{\mathrm{CF}}_1$: since cash flows at time t are adapted the random variable $\widetilde{\mathrm{CF}}_t$ had to be $\mathcal{F}_1$–measurable. But this is to say that there would not have been any uncertainty about the future dividends today.

Our assumption is for example satisfied if the cash flows are a product of independent random variables. To this end let the cash flows be

$$\widetilde{\mathrm{CF}}_t = (1+g_1+Y_1)\cdots(1+g_t+Y_t)$$

where the Y_t are independent with expectation zero and $\mathcal{F}_t$ be the filtration generated by $Y_1, \ldots, Y_t$. It is straightforward to show that these cash flows satisfy equation (6).

The amount of debt is subject to the financing policy of the firm. Therefore, this amount might be a random variable itself. We now make the following assumption concerning the debt schedule of the firm.

Assumption 2 (debt schedule) *The leverage ratio $l_t = \frac{B_t}{V_t^L}$ ($t \geq 0$) is deterministic.*

This implies, that today ($t = 0$) the investor knows what the leverage ratio at time $t > 0$ will be. It is not required that the leverage ratio will be the same at each date $t > 0$. The investor does not need to know the absolute amount of equity and debt at time $t > 0$.

We are now able to prove the following result which is due to Miles & Ezzell (1980).

Proposition 1 (WACC formula in discrete time) *If the leverage ratios satisfy assumption 2 and the cash flows satisfy assumption 1 the value of the levered company is given by*

$$V_0^L = \sum_{t=1}^{T} \frac{\mathrm{E}[\widetilde{\mathrm{CF}}_1]}{\prod_{k=1}^{t}\left\{(1 - \frac{\tau r_f}{1+r_f} l_{k-1})(1+r^U)\right\}}. \tag{7}$$

If the leverage ratio is constant this is the Miles & Ezzell (1980) formula. As in their paper the denominator can easily be shown as the weighted average of the cost of capital of debt and equity. If furthermore lifetime is infinite we arrive at the Miles & Ezzell (1985) result.

Proof. We start with some preliminary results. From (6) and (2) we get

$$\tilde{V}_t^U = \sum_{i=t+1}^{T} \frac{\mathrm{E}[\widetilde{\mathrm{CF}}_i|\mathcal{F}_t]}{(1+r^U)^{i-t}} = \sum_{i=t+1}^{T} \frac{\prod_{k=t}^{i-1}(1+g_k)}{(1+r^U)^{i-t}} \widetilde{\mathrm{CF}}_t =: A_t \cdot \widetilde{\mathrm{CF}}_t.$$

Notice that A_t is the inverse of the dividend ratio of the firm. For A_{t-1} the following holds

$$\begin{aligned} A_{t-1} &= \sum_{i=t}^{T} \frac{\prod_{k=t-1}^{i-1}(1+g_k)}{(1+r^U)^{i-(t-1)}} \\ &= \frac{1+g_{t-1}}{1+r^U} + \frac{1+g_{t-1}}{1+r^U} \sum_{i=t+1}^{T} \frac{\prod_{k=t}^{i-1}(1+g_k)}{(1+r^U)^{i-t}} \\ &= \frac{1+g_{t-1}}{1+r^U}(1+A_t). \end{aligned}$$

This and (3) imply

$$\begin{aligned} & \frac{\mathrm{E}_Q[A_t \cdot \widetilde{\mathrm{CF}}_t + \widetilde{\mathrm{CF}}_t|\mathcal{F}_{t-1}]}{1+r_f} = \tilde{V}_{t-1}^U = A_{t-1} \cdot \widetilde{\mathrm{CF}}_{t-1} \\ \Longrightarrow \quad & \frac{1+A_t}{1+r_f} \mathrm{E}_Q[\widetilde{\mathrm{CF}}_t|\mathcal{F}_{t-1}] = A_{t-1} \widetilde{\mathrm{CF}}_{t-1} \\ \Longrightarrow \quad & \frac{\mathrm{E}_Q[\widetilde{\mathrm{CF}}_t|\mathcal{F}_{t-1}]}{1+r_f} = \frac{1+g_{t-1}}{1+r^U} \widetilde{\mathrm{CF}}_{t-1}. \end{aligned} \tag{8}$$

Taking the expectation $\mathrm{E}_Q[\cdot|\mathcal{F}_{t-2}]$ and applying the law of iterated expectation gives us

$$\frac{\mathrm{E}_Q[\widetilde{\mathrm{CF}}_t|\mathcal{F}_{t-2}]}{1+r_f} = \frac{1+g_{t-1}}{1+r^U} \mathrm{E}_Q[\widetilde{\mathrm{CF}}_{t-1}|\mathcal{F}_{t-2}]$$

Applying the above result (8) for $t-1$ we get using (6)

$$\frac{\mathrm{E}_Q[\widetilde{\mathrm{CF}}_t|\mathcal{F}_{t-2}]}{1+r_f} = \widetilde{\mathrm{CF}}_{t-2}(1+r_f)\frac{(1+g_{t-1})\cdot(1+g_{t-2})}{(1+r^U)^2} = \mathrm{E}[\widetilde{\mathrm{CF}}_t|\mathcal{F}_{t-2}]\frac{1+r_f}{(1+r^U)^2}.$$

Continuing our calculations we arrive at

$$\frac{\mathrm{E}_Q[\widetilde{\mathrm{CF}}_t|\mathcal{F}_k]}{(1+r_f)^{t-k}} = \frac{\mathrm{E}[\widetilde{\mathrm{CF}}_t|\mathcal{F}_k]}{(1+r^U)^{t-k}} \tag{9}$$

for any $t > k \geq 1$.

At time $t = T-1$ the values of the levered and the unlevered firm satisfy

$$\left(1 - \frac{\tau r_f}{1+r_f} l_{T-1}\right) \tilde{\mathrm{V}}^L_{T-1} = \tilde{\mathrm{V}}^U_{T-1},$$

Using (3) the firm's value at $T-1$ can be written:

$$\tilde{\mathrm{V}}^L_{T-1} = \frac{\mathrm{E}_Q[\widetilde{\mathrm{CF}}_T|\mathcal{F}_{T-1}]}{(1 - \frac{\tau r_f}{1+r_f} l_{T-1})(1+r_f)}. \tag{10}$$

From (5) we get the tax shield using the last equation (remember $T_T = 0$)

$$\begin{aligned}\tilde{\mathrm{T}}_{T-1} &= \frac{\tau r_f}{1+r_f} l_{T-1} \tilde{\mathrm{V}}^L_{T-1} \\ &= \frac{\tau r_f}{1+r_f} l_{T-1} \frac{\mathrm{E}_Q[\widetilde{\mathrm{CF}}_T|\mathcal{F}_{T-1}]}{(1 - \frac{\tau r_f}{1+r_f} l_{T-1})(1+r_f)}.\end{aligned}$$

(5) implies now (we also need the law of iterated expectation, see for example Williams (1991, p. 88))

$$\tilde{\mathrm{T}}_{T-2} = \frac{\tau r_f}{1+r_f} l_{T-1} \frac{\mathrm{E}_Q[\widetilde{\mathrm{CF}}_T|\mathcal{F}_{T-2}]}{(1 - \frac{\tau r_f}{1+r_f} l_{T-1})(1+r_f)^2} + \frac{\tau r_f}{1+r_f} l_{T-2} \tilde{\mathrm{V}}^L_{T-2}. \tag{11}$$

With (4) and some laborious calculations we get an equation of the firm's levered cash flow stream at time $T-2$:

$$\tilde{\mathrm{V}}^L_{T-2} = \frac{\mathrm{E}_Q[\widetilde{\mathrm{CF}}_T|\mathcal{F}_{T-2}]}{(1 - \frac{\tau r_f}{1+r_f} l_{T-2})(1 - \frac{\tau r_f}{1+r_f} l_{T-1})(1+r_f)^2} + \frac{\mathrm{E}_Q[\widetilde{\mathrm{CF}}_{T-1}|\mathcal{F}_{T-2}]}{(1 - \frac{\tau r_f}{1+r_f} l_{T-2})(1+r_f)}. \tag{12}$$

By induction we get the following value of the levered cash flow stream at time 1 analogously to (10) and (12):

$$V^L_0 = \sum_{t=1}^{T} \frac{\mathrm{E}_Q[\widetilde{\mathrm{CF}}_t|\mathcal{F}_0]}{\prod_{k=1}^{t} \left\{(1 - \frac{\tau r_f}{1+r_f} l_{k-1})(1+r_f)\right\}}.$$

Using (9) this is

$$V^L_0 = \sum_{t=1}^{T} \frac{\mathrm{E}[\widetilde{\mathrm{CF}}_t|\mathcal{F}_0]}{\prod_{k=1}^{t} \left\{(1 - \frac{\tau r_f}{1+r_f} l_{k-1})(1+r^U)\right\}}$$

Since the conditional expectation with respect to $\mathcal{F}_0$ is just the expectation under the subjective probability this is the desired result. ■

3 The WACC theory in continuous time

The future $t > 0$ is uncertain, time horizon is the interval $[0, T]$. First we consider the value of the levered firm V_t^L. The firm has a deterministic payout–ratio δ_t, where the after–tax cash flow at time t is given by $\delta_t V_t^L$.

Assumption 3 (probability distribution of cash flows) *Under the subjective probability measure the firm has drift r_t^L and volatility σ_t^L (all variables are subject to several conditions that ensure the equation has indeed a solution, for details see Duffie (1988, p. 228))*

$$dV_t^L = (r_t^L - \delta_t)V_t^L dt + \sigma_t^L V_t^L dW_t, \tag{13}$$

where W_t is the standard Brownian Motion.

The firm is financed by the amount S_t of stocks (equity) and the amount B_t of bonds (debt). Interest payments from debt are certain, the instantaneous riskless rate is given by r_t and may vary with time t. As in the theory of discrete time the leverage ratio is deterministic.

Assumption 4 (debt schedule) *The leverage ratio of the firm $l_t = \frac{B_t}{V_t^L}$ is deterministic and differentiable.*

We can prove the following result.

Proposition 2 (WACC formula in continuous time) *The value of the tax shield is given by*

$$T_t = V_t^L \int_t^T \tau r_s l_s \, e^{\int_t^s -\delta_u du} ds =: V_t^L \cdot L_t.$$

Furthermore, the volatility of the leverage and unleveraged firm coincide $\sigma_t^L = \sigma_t^U$ and the drift of the leveraged and the unleveraged firm satisfy the following relation

$$r_t^L = r_t^U + \frac{\dot{L}_t}{1 - L_t}. \tag{14}$$

Proof. If the market is free of arbitrage there is a risk neutral probability measure Q such that the discounted value of the firm is a martingale. Q is given by

$$\frac{dW^Q}{dW} = \exp\left(- \int_0^T \frac{r_t^L - r_t}{\sigma_t^L} dW_t - \int_0^T \frac{1}{2} \left(\frac{r_t^L - r_t}{\sigma_t^L} \right)^2 dt \right).$$

If we change the measure using Girsanov's formula we get

$$dV_t^L + \delta_t V_t^L dt = r_t V_t^L dt + \sigma_t^L V_t^L dW_t^Q. \tag{15}$$

Now let us consider the tax shield. We can evaluate this tax shield using the standard technique by calculating the deflated expectation under the risk neutral probability measure

$$T_t = \mathrm{E}^Q\left[\int_t^T e^{-\int_t^s r_u du} \tau r_s l_s V_s^L ds \mid \mathcal{F}_t\right].$$

Since the leverage ratio is deterministic everything except V_s is nonstochastic. Using Fubini the value of the tax shield simplifies to

$$T_t = \int_t^T \tau r_s l_s e^{-\int_t^s r_u du} \mathrm{E}^Q\left[V_s^L \mid \mathcal{F}_t\right] ds.$$

V_s^L is a solution of the stochastic differential equation (15). Hence, its expectation can be evaluated and we get

$$\begin{aligned} T_t &= \int_t^T \tau r_s l_s\, e^{-\int_t^s r_u du} V_t^L\, e^{\int_t^s (r_u - \delta_u) du} ds \\ &= V_t^L \int_t^T \tau r_s l_s\, e^{\int_t^s -\delta_u du} ds. \end{aligned}$$

This proves the first part of the proposition.

The value of the unlevered firm is the sum of the tax shield and the value of the levered firm

$$V_t^L = V_t^u + T_t.$$

Collecting terms we get

$$V_t^U = V_t^L \cdot (1 - L_t).$$

If we apply Itô's lemma we get using (13)

$$\begin{aligned} dV_t^U &= \sigma_t^L V_t^U dW_t + \left(r_t^L V_t^U - \delta_t V_t^U - \dot{L}_t V_t^L\right) dt \\ &= \sigma_t^L V_t^U dW_t + \left(r_t^L - \frac{\dot{L}_t}{1 - L_t} - \delta_t\right) V_t^U dt \end{aligned}$$

and this was to be shown. ■

The value of the tax shield is determined by the value of the levered firm and a factor L_t that depends on the leverage ratio, the riskless rate of return and the payout ratio. If volatility, payout ratio and leverage ratio are constant, under an infinite time horizon L_t simplifies to

$$L_t = \frac{\tau r}{\delta} l.$$

4 Conclusion

In this paper we have shown that using the WACC approach of Miles & Ezzell (1980) can yield an arbitrage strategy. The reason for the existence of the arbitrage strategy is that the formula may give the wrong value of a levered firm even if the leverage ratio is held constant.

Nevertheless, if in discrete time the increment of the cash flows is proportional to the current cash flow we have shown that we can surmount the problems concerning the WACC approach. This condition on the cash flows can be interpreted as a discrete time analog of a Brownian motion. We were even able to generalize the results to the case where the leverage ratio of the firm is not constant but only deterministic. This WACC approach could also be provided in continuous time.

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