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Stochastic Environmental Policy, Risk–Taking, and Growth

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Abstract

This paper analyzes stochastic productive pollution within a model of endogenous growth. The extent to which the agents perceive their individual influence on aggregate pollution is parameterized. Recursive preferences allow for the separation between intertemporal substitutability and risk aversion.
Two different environmental policy regimes are distinguished: A stochastic pollution tax, which is uncorrelated with the technological uncertainty of pollution, is compared with a pure deterministic tax regime. The impact of the stochastic pollution tax on abatement effort and growth is ambiguous and depends on the parameter setting. Nevertheless, it is shown, that for a sufficiently high volatility of the pollution tax, the induced rise in uncertainty associated with pollution leads to an increase in abatement activities and therefore supports the corresponding deterministic environmental policy.

Zusammenfassung

Der vorliegende Beitrag analysiert die Auswirkungen stochastischer Umweltsteuern auf das dynamische Gleichgewicht einer Volkswirtschaft. Es liegt eine Produktionstechnologie zugrunde, in der die Umweltverschmutzung ein essentieller Produktionsfaktor ist und das Ausmaß der mit der Produktion verbundenen Umweltzerstörung einem stochastischen Prozeß unterliegt. Der individuelle Einfluss auf die Umweltverschmutzung wird von den Haushalten nur teilweise wahrgenommen. Es wird gezeigt, dass die internalisierende Wirkung deterministischer Umweltabgaben von einer stochastischen Umweltsteuer unterstützt wird, falls die Volatilität dieser Steuer hinreichend hoch ist. Die Auswirkungen einer stochastischen Umweltabgabe sind jedoch nicht eindeutig: insbesondere bei geringer Volatilität kann sie auch zu einer Senkung der Umweltschutzausgaben führen.

1 Introduction

Uncertainty is an essential feature of environmental aspects, since future consequences of present actions are unknown. Hence, in an economy populated by risk averse individuals any environmental policy has to take the underlying uncertainty into account. Moreover, it can be shown that stochastic environmental taxation may reverse the usual impact of fiscal policy on individual behavior. Environmental policy will be analyzed for a setting where pollution is stochastic, essential for production, and can be reduced by abatement activity. In order to disentangle the effects of intertemporal substitution and risk–taking, recursive preferences are assumed, which allow for a separation of the intertemporal elasticity of substitution and the degree of relative risk aversion.

Different contributions study the impact of environmental degradation on endogenous growth, as e. g. Gradus and Smulders (1993), Ligthart and van der Ploeg (1994), Bovenberg and Smulders (1995), Jones and Manuelli (1995), Byrne (1997) or Stokey (1998). The authors derive conditions for the feasibility of long–term growth and analyze environmental policy to internalize market failures. The growth impact of environmental aspects depends on different assumptions about the production structure. As long as deterioration of nature is inescapably linked to the production of the consumption good, optimal growth is limited by the trade–off between consumption and environment, see e. g. Stokey (1998). If instead the engine of growth is non–pollutant, as e. g. the accumulation of human capital in Gradus and Smulders (1993) as well as Byrne (1997), optimal growth can even be independent of environmental concerns. Smulders and Gradus (1996) show, that the incorporation of abatement activities leads to an ambiguous impact on growth: On the one hand, optimal growth tends to decline with an increase in environmental preferences. On the other hand, optimal growth tends to rise: With higher growth, it becomes easier to achieve a certain level of abatement effort.

Although uncertainty is an important determinant of both environmental degradation and the growth process, there are only few papers which address the impact of risk on feasible growth paths, as e. g. Clarke and Reed (1994), Baranzini and Bourguignon (1995), Beltratti et al. (1998) as well as Chichilnisky and Heal (1998). Uncertainty influences the dynamic equilibrium through different channels: First, capital accumulation as well as abatement expenditures are adjusted to the underlying uncertainty. The impact is ambiguous and depends crucially on the degree of relative risk aversion, which determines the motive for precautionary savings (see e. g. Soretz, 2003a,b). Second, any governmental activity changes not only expected values of net economic variables, but also their volatility. Hence, there are counter acting effects on the endogenous growth process, which were analyzed first in the seminal work of Eaton (1981) and more recently taken up e. g. by Turnovsky (1993, 1995b, 2000), Smith (1996), Clemens and Soretz (1997), or Corsetti (1997).

The individual influence on pollution is underestimated by the agents. Only part of the impact of individual capital accumulation as well as individual abatement expenditure on pollution is taken into account within intertemporal utility maximization. The extent, to which pollution is perceived to be exogenous to individual decisions is parameterized according to the formulation of congestion effects in the public goods literature (see e. g. Edwards, 1990; Glomm and Ravikumar, 1994; Turnovsky and Fisher, 1998). Partial perception of the individual influence on pollution can alternatively be interpreted as incomplete sense of responsibility for environmental concerns.

As long as pollution is perceived to be at least in part exogenous to individual decisions, market equilibrium deviates from social optimum. Therefore, various fiscal policy instruments are included. Government acts through the taxation of income, a subsidy on abatement expenditures and a stochastic tax on pollution. With respect to the pollution tax two settings are compared: First, it is assumed to be an additional exogenous pollution tax shock which increases the uncertainty associated to pollution within the individual intertemporal optimization. This assumption can be interpreted as discretionary environmental policy, which the agents are not able to anticipate. Second, a deterministic tax regime is analyzed, where all tax rates are known by the households. The impact of the stochastic environmental policy on equilibrium growth and pollution is compared with the corresponding deterministic policy. It is shown that the stochastic policy regime supports the reduction in pollution if the volatility of the pollution tax is sufficiently high. Nevertheless, the impact on equilibrium growth is ambiguous.

In order to disentangle the effects of risk–taking and intertemporal substitutability, recursive preferences are assumed. With this class of preferences the intertemporal elasticity of substitution and the degree of relative risk aversion are both constant (hence constant growth is feasible), but may be set separately. The setting draws back on Epstein and Zin (1989), Weil (1990) or Obstfeld (1994a,b). The expected utility case where the intertemporal elasticity of substitution equals the reciprocal of the degree of relative risk aversion is included as a special case.

The assumptions of the model are presented in section 2. Part 3 derives the intertemporal market equilibrium. Section 4 determines the impact of stochastic environmental policy on macroeconomic equilibrium, particularly on expected growth and abatement effort. The effects are shown to be ambiguous and to depend on the volatility of the stochastic pollution tax as well as on intertemporal substitutability. Section 5 gives a short conclusion.

2 The model

Three characteristics of the environment are considered in this paper: First, pollution causes disutility and leads to a trade–off between consumption and environmental care. Second, pollution is essential for the production of the homogenous consumption good. Third, the productive capacity of pollution is uncertain.

The economy is populated by a continuum of homogenous, infinitely long living individuals. The individuals can be interpreted as long–lived dynasties, because intergenerational altruism is particularly plausible with respect to environmental aspects. Intertemporal utility is defined by recursion in order to disentangle the effects of risk–taking and intertemporal substitution

$$G((1-\rho)U(t)) = \frac{1-\rho}{1-1/\varepsilon}(c(t)P(t)^{-\gamma})^{1-1/\varepsilon}h + e^{-\beta h}G((1-\rho)\mathrm{E}_t[U(t+h)]) \qquad (1)$$

where $c(t)$ and $P(t)$ denote individual consumption and aggregate pollution. $\beta > 0$ is the constant rate of time preference and $\gamma > 0$ indicates environmental anemities. With an increase in γ environmental preferences gain importance. The function G is given by

$$G(x) = \frac{1-\rho}{1-1/\varepsilon}x^{\frac{1-1/\varepsilon}{1-\rho}} \quad . \qquad (2)$$

The recursive specification of utility draws back on Epstein and Zin (1989) or Weil (1990) and was extended to continuous time by Svensson (1989) and Duffie and Epstein (1992). It allows for a separate setting of the constant intertemporal elasticity of substitution, $\varepsilon > 0$, and the degree of relative risk aversion, $\rho > 0$. The special case where $1/\varepsilon = \rho$ is included and the recursive preferences (1) then are equivalent to the usual expected utility form.[1]

Pollution is essential for production. In particular, the intratemporal elasticity of substitution between the input factors, pollution and capital, is assumed to be unity. Furthermore, the productive capacity of pollution, dp, is stochastic and evolves according to

$$dp = P(dt + \sigma_y dz_y(t)) \tag{3}$$

where $dz_y(t)$ is a Wiener process with $dz_y \sim N(0, dt)$. Alternatively, this technological risk of pollution can be interpreted as an uncertain level of environmental degradation associated with a given level of production. The individual production function results in

$$f(k(t)) = Ak(t)dp \quad . \tag{4}$$

$A > 0$ denotes the productivity parameter and $k(t)$ is a broad measure of physical capital available to the individual. The deterministic counterpart to this technology relies on Smulders and Gradus (1996) who analyze a general production function with the inputs pollution and capital and develop conditions for a positive sustainable rate of growth.

Pollution is an inevitable by–product of capital accumulation and can be reduced through abatement activities. Additionally, pollution is assumed to be a flow variable. Hence, I refer predominantly on pollutants which dissolve rather quickly. Since there is no rivalry in the disutility out of pollution, it depends on the relation between aggregate abatement effort, $E(t)$, and aggregate capital stock, $K(t)$,

$$P(t) = \left(\frac{E(t)}{K(t)}\right)^{-\alpha} \qquad \alpha > 0 \quad . \tag{5}$$

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[1] Of course, the given recursion only applies for $\varepsilon \neq 1$ and $\rho \neq 1$.

Moreover, the perception of the individual influence on pollution is parameterized. The ratio between abatement and capital is perceived to depend in part $(1-\delta)$ on individual behavior and in part (δ) on equilibrium values of the entire economy, which are exogenous to individual decisions. Hence, the perceived relation between abatement activity and capital stock, $\eta_p = (E/K)_p$, is given by

$$\eta_p = \left(\frac{E(t)}{K(t)}\right)^{\delta} \left(\frac{e(t)}{k(t)}\right)^{1-\delta} \qquad \delta \in [0,1) \tag{6}$$

where $e(t)$ denotes individual abatement effort. δ defines the extent to which the agents perceive pollution to be exogenous to their individual decisions. The setting of perception in equation (6) relies on the formulation of congestion effects in the public goods literature (see e. g. Edwards, 1990; Glomm and Ravikumar, 1994). In these lines, the parameter δ could also denote the joint degree of rivalry of capital and abatement in the "production" of pollution (see Turnovsky, 1995a, p. 405).

Since all agents are identical and population size is normalized to unity, individual and aggregate values are equal in equilibrium. Nevertheless, within individual optimization aggregate capital as well as aggregate abatement are given exogenously. Hence, the perception parameter δ is a measure for the consciousness of individual influence on pollution.

As long as the perception parameter is above zero, the agents underestimate their individual influence on pollution. Hence, there is a negative externality of capital accumulation and the equilibrium value of abatement effort is suboptimally low. Therefore, government intervenes through different channels: In order to internalize the negative external effect of capital, government levies an income tax at constant rate τ_y. The subsidy at rate τ_e fosters abatement activity.

Additionally, pollution is taxed stochastically at rate $\tau_P \sigma_P \, dz_P$ where dz_P is a Wiener process. With this concern, two settings will be distinguished: First,the stochastic pollution tax is assumed to be an additional exogenous disturbance which is uncorrelated to the pollution shock. This regime corresponds to the case of discretionary environmental policy, which cannot be anticipated by the households. In the second setting environmental policy is assumed to be completely deterministic. In this case, all tax rates are known by the households.

3 Market equilibrium

The agents choose consumption and abatement effort together with capital accumulation in order to maximize lifetime utility, $U(t)$, as defined by the recursion (1) and (2). Since partial perception of the individual influence on pollution is incorporated to the model, the agents feel at least in part responsible for environmental decay. Hence, there is an individual decision about the optimal level of abatement effort which leads to an interior solution. Nevertheless, as long as environmental responsibility is incomplete, that is $\delta > 0$, equilibrium abatement is suboptimal and gives rise to environmental policy.

Since differentiation with respect to time of the stochastic accumulation process of capital is impossible, Itô's Lemma is used to derive the stochastic Bellman equation. Additionally, let the value function, $J(k)$, denote maximum lifetime utility. The individual optimization problem then results in

$$\begin{aligned}\mathcal{B} &= \frac{1-\rho}{1-1/\varepsilon}(cP^{-\gamma})^{1-1/\varepsilon} - \beta G((1-\rho)J(k)) \\ &\quad +(1-\rho)G'((1-\rho)J(k))\left(J'(k)\frac{\mathrm{E}[dk]}{dt} + \frac{1}{2}J''(k)\,\sigma_k^2\right) \end{aligned} \tag{7}$$

with given initial values of physical capital, k_0, and of the stochastic disturbance, z_0. Regarding environmental policy as described above and taking the fiscal parameters as given, capital evolves according to

$$dk = [(1-\tau_y)AkP - c - (1-\tau_e)e]dt + (1-\tau_y)AkP\sigma_y dz_y - \tau_P P\sigma_P dz_P \tag{8}$$

with the variance of capital determined by the technological disturbance as well as the stochastic pollution tax. Within individual optimization, the entire fiscal policy is exogenously given. In particular, the stochastic process of pollution taxation which will be determined in equilibrium, is perceived to be exogenous. Hence, the variance of capital is determined out of (8) and is given by

$$\sigma_k^2 = (1-\tau_y)^2A^2k^2P^2\sigma_y^2 - 2(1-\tau_y)\tau_P AkP^2\sigma_{yP} + \tau_P^2P^2\sigma_P^2 \tag{9}$$

where the covariance between the technological disturbance and the pollution tax shock, σ_{yP}, as well as the variance of the pollution tax, σ_P^2, depend on the further assumptions about the stochastic process of pollution taxation which will be introduced in section 4.

The individual maximizes intertemporal utility subject to the capital accumulation equation (8) and with respect to consumption and abatement expenditures. Within optimization, pollution is perceived to depend on individual decisions according to assumption (6). Maximization of the stochastic Bellman equation (7) with respect to consumption and abatement effort together with capital accumulation leads to the following first order conditions

$$c^{-1/\varepsilon}P^{-\gamma(1-1/\varepsilon)} - G'J' \overset{!}{=} 0 \tag{10}$$

$$\begin{aligned} &-\alpha\gamma(1-\delta)c^{1-1/\varepsilon}P^{-\gamma(1-1/\varepsilon)}e^{-1} + \\ &\quad +G'\left(J'(\alpha(1-\delta)(1-\tau_y)AP\eta^{-1} + (1-\tau_e)) - \frac{1}{2}J''\frac{\partial\sigma_k^2}{\partial e}\right) \overset{!}{=} 0 \end{aligned} \tag{11}$$

$$\begin{aligned} &-\alpha\gamma(1-\delta)c^{1-1/\varepsilon}P^{-\gamma(1-1/\varepsilon)}k^{-1} + G''J'\left(J'\frac{\mathrm{E}[dk]}{dt} + \frac{1}{2}J''\sigma_k^2\right) + \\ &\quad +G'\Big(J'((1-\tau_y)AP(1+\alpha(1-\delta)) - \beta) \\ &\qquad +J''\left(\frac{\mathrm{E}[dk]}{dt} + \frac{1}{2}\frac{\partial\sigma_k^2}{\partial k}\right) + \frac{1}{2}J'''\sigma_k^2\Big) \overset{!}{=} 0 \end{aligned} \tag{12}$$

and the transversality condition

$$\lim_{t\to\infty} \mathrm{E}\left[e^{-\beta t}G((1-\rho)J(k))\right] = 0 \tag{13}$$

which must be satisfied in order to assure feasible consumption paths. In the pareto–optimal steady–state, the production function (4) reduces to the linear technology and the transversality condition is met for all positive consumption paths. Nevertheless, if responsibility for environmental quality is incomplete ($\delta > 0$), equilibrium growth deviates from optimal growth. Therefore, only parameter settings are considered, which additionally satisfy the transversality condition.

The necessary condition (10) assures equality of marginal utility out of consumption across time. The expenditures for consumption and abatement are adjusted through equation (11). This condition balances marginal utility out of consumption and marginal utility out of environmental care, as perceived by the individuals. Equation (12) together with (10) determine optimal capital accumulation.

The solution of the resulting system of stochastic differential equations usually is by conjecture. Due to constant relative risk aversion as well as constant intertemporal elasticity of substitution, the conjecture consists of constant ratios between consumption, capital and abatement. Equivalently, consumption, capital and abatement activities are presumed to grow at the same rate in the steady state. With the consumption ratio, $\mu \equiv c/k$, and the abatement ratio, η, equation (10) leads to the following CRRA guess for the value function

$$J(k) = \left(\mu \eta^{\alpha\gamma(1-\varepsilon)}\right)^{\frac{1-\rho}{1-\varepsilon}} \frac{k^{1-\rho}}{1-\rho} \tag{14}$$

and enables the determination of equilibrium consumption and abatement out of equations (11) and (12) according to

$$\begin{aligned} \alpha\gamma(1-\delta)\mu &= (1-\tau_y)\alpha(1-\delta)A\eta^{-\alpha} + (1-\tau_e)\eta + \frac{\rho}{2}\eta\frac{\partial\sigma_k^2}{\partial e}k^{-1} && (15) \\ (1-\varepsilon)(1-\tau_e)\eta &= \beta\varepsilon + (1-\varepsilon)(1-\tau_y)A\eta^{-\alpha} - \mu \\ &\quad + \frac{\rho}{2k}\left(\varepsilon\eta\frac{\partial\sigma_k^2}{\partial e} + \varepsilon\frac{\partial\sigma_k^2}{\partial k} - (\varepsilon+1)\sigma_k^2 k^{-1}\right) \quad . && (16) \end{aligned}$$

The conjecture of constant ratios μ and η will have to be confirmed subsequently, when the assumptions about the stochastic pollution tax allow for a determination of the equilibrium volatility of capital.

Expected growth of physical capital, $\varphi \equiv \mathrm{E}[dk]/(k\,dt)$, can be evaluated from equation (8) together with the equilibrium solutions for consumption as well as abatement, (15) and (16). It results in

$$\varphi = \varepsilon\left((1-\tau_y)A\eta^{-\alpha} - (1-\tau_e)\eta - \beta\right) - \frac{\varepsilon\rho}{2k}\left(\eta\frac{\partial\sigma_k^2}{\partial e} + \frac{\partial\sigma_k^2}{\partial k} - \frac{\varepsilon+1}{\varepsilon}\sigma_k^2 k^{-1}\right). \tag{17}$$

Equilibrium growth can be divided in two parts. The first term of the expected growth rate (17) corresponds to the Keynes–Ramsey–Rule of the according deterministic model. The second term describes the response of the risk averse individual to uncertainty and will be described subsequently in more detail for different assumptions concerning the stochastic taxation of pollution.

Due to the double impact of pollution, on the one hand on utility and on the other hand on production, the equilibrium cannot be determined explicitely. Nevertheless, the next section will show for regular parameter values that equations (15) and (16) describe a unique solution to consumption and abatement. Once the individually optimal choice about consumption and abatement is made, expected growth is determined by equation (17).

4 Environmental Policy

Environmental Policy consists of income taxation, a subsidy on abatement expenditures and the stochastic pollution tax. With respect to the stochastic disturbance of pollution taxation, dz_P, two different assumptions are distinguished: The first setting considers an exogenous tax shock, which is a Wiener process and is uncorrelated with the pollution shock. Hence, the covariance of the two shocks, σ_{yP}, is zero and the variance of capital sums up both disturbances. This setting describes a stochastic tax on pollution additionally to the deterministic subsidy, τ_e on abatement activity. The level of the stochastic pollution tax cannot be anticipated by the households (the expected value is zero). Therefore, it can be interpreted as the discretionary part of environmental policy. It will be shown that this stochastic environmental policy has an impact on macroeconomic equilibrium and can promote the corresponding deterministic policy.

In the second setting, which will be interpreted as a reference setting, the pollution disturbance is the only exogenous shock. All fiscal parameters are deterministic and known to the individuals. The revenues out of the taxation of stochastic income components are balanced with the stochastic tax on pollution, $dz_P = -(\tau_y \sigma_y)/(\tau_P \sigma_P) Akdz$. Since the variance of capital in the case of a balanced governmental budget can be attributed exclusively to the exogenous productivity shock, fiscal policy has no impact on equilibrium capital volatility in this setting.

The volatility of capital in equilibrium, depending on the policy setting, is then given by

$$\sigma_{k,u}^2 = k^2 \eta^{-2\alpha}((1-\tau_y)^2 A^2 \sigma_y^2 + \tau_P^2 \sigma_P^2) \tag{18}$$

$$\sigma_{k,d}^2 = A^2 k^2 \eta^{-2\alpha} \sigma_y^2 \quad . \tag{19}$$

Subsequently, the setting of two uncorrelated shocks is denoted by the subscript *u*, whereas the subscript *d* indicates the deterministic policy setting.

Macroeconomic equilibrium depends twofold on the policy setting: First, within the savings decision of risk averse individuals, the source of uncertainty matters. Hence, a difference in the uncertainty of future income streams induces a change in capital accumulation and therefore in equilibrium growth, φ. Second, the equilibrium abatement ratio, η, is influenced not only by the deterministic abatement subsidy, but also by the stochastic pollution tax. Therefore, the resulting level of pollution depends on the specific policy setting.

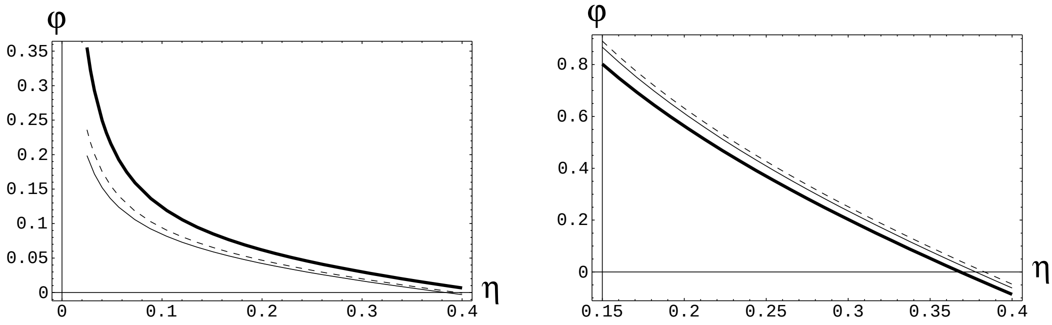
Growth effects: In the first step, the growth effects of the stochastic pollution tax will be analyzed based on arbitrary chosen abatement levels. The optimal level of environmental expenditures will be determined subsequently. With the assumptions about the stochastic process of pollution taxation as given above, equilibrium expected growth can be determined according to equation (17) and results in

$$\begin{aligned} \varphi_u &= \varepsilon\left((1-\tau_y)A\eta^{-\alpha}-(1-\tau_e)\eta-\beta\right) \\ &\quad +\frac{\rho\eta^{-2\alpha}}{2}(1-\varepsilon)\left((1-\tau_y)^2A^2\sigma_y^2+\tau_P^2\sigma_P^2\right) \end{aligned} \tag{20}$$

$$\varphi_d = \varepsilon\left((1-\tau_y)A\eta^{-\alpha}-(1-\tau_e)\eta-\beta\right)+\frac{\rho\eta^{-2\alpha}}{2}(1-\varepsilon+2\varepsilon\tau_y)A^2\sigma_y^2 \quad . \tag{21}$$

The considered policy settings differ with respect to the stochastic pollution tax. Hence, the second parts of the growth rates (20) and (21) reflect the different individual response towards the uncertainty in fiscal policy. With stochastic environmental policy, the uncorrelated additional pollution tax shock increases uncertainty. This leads to a positive income and a negative substitution effect on savings: On the one hand, an increase in risk reduces expected utility out of future income flows. Savings are increased in order to compensate for this impact and to equalize expected marginal utility across time (positive income effect). On the other hand, capital accumulation gets less attractive for risk averse individuals if uncertainty increases. There is an incentive to decrease savings (negative substitution effect).

Whether the income effect or the substitution effect dominates, depends on individual preferences as well as on the specific structure of risk. Precautionary savings, as defined by Leland (1968) and Sandmo (1970), offer a kind of insurance against future income risk. With the recursive specification of utility considered here, the intertemporal elasticity of substitution, ε, decides whether the income or the substitution effect dominates. The degree of relative risk aversion, ρ, determines the magnitude of additional savings in response to the uncertain pollution tax.



(a) *Precautionary savings ($\varepsilon < 1$).* (b) *No precautionary savings ($\varepsilon > 1$).*

Figure 1: *Growth effect of stochastic pollution tax.*

Figure 1 illustrates the difference in the equilibrium growth rates numerically.[2] Independently from the chosen policy setting, the negative slope indicates the trade–off between environmental expenditures (increasing η), and investment in physical capital (increasing φ).

The dashed lines show the growth rate for the deterministic policy, φ_d. Figure 1(a) shows the case of precautionary savings ($\varepsilon < 1$). Both solid lines represent stochastic pollution taxation, but with different levels of volatility of the pollution tax shock. With sufficiently high volatility (high $\tau_P\sigma_P$), the thick solid line applies and equilibrium growth exceeds the growth rate for the balanced budget setting

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[2] Parameters are set as follows: preferences $\beta = 0.03$, $\rho = 2$, $\varepsilon = 0.2$ for figure 1(a), $\varepsilon = 3$ for figure 1(b), $\gamma = 3$, production $A = 0.4$, $\alpha = 0.5$, $\sigma_y = 0.04$, perception $\delta = 0.9$, fiscal policy $\tau_y = 0.6$, $\tau_e = 0.4$, $\tau_P\sigma_P = 0.035$ for high volatility, $\tau_P\sigma_P = 0$ for low volatility.

due to precautionary savings. Note, that this means an increase in equilibrium growth for all possible environmental expenditure rates, η, and pollution levels, respectively. Vice versa, a sufficiently high stochastic pollution tax leads to a steady state, which is characterized by a less polluting growth process. For sufficiently low volatility (low $\tau_P\sigma_P$), the thin solid line applies and equilibrium growth falls below the growth rate of the deterministic policy regime. Such a policy is related to a decrease in uncertainty and, hence, lower precautionary savings.

Figure 1(b) illustrates the reversed argument:[3] In this plot, the intertemporal elasticity of substitution is set to $\varepsilon = 3$. Hence, individuals have no motive for precautionary savings and an increase in uncertainty due to a stochastic pollution tax instead reduces optimal capital accumulation. Again, the thick line represents high volatility and the thin line low volatility of the environmental policy.

Pollution effects: The stochastic pollution tax not only has a direct impact on growth, but furthermore influences the individual choice of abatement effort. Substitution of the assumptions about the policy settings into equations (15) and (16) leads to

$$\begin{aligned}
\alpha\gamma(1-\delta)\mu_u &= (1-\tau_y)\alpha(1-\delta)A\eta_u^{-\alpha}+(1-\tau_e)\eta_u \\
&\quad -\rho\alpha(1-\delta)\eta_u^{-2\alpha}((1-\tau_y)^2A^2\sigma_y^2+\tau_P^2\sigma_P^2) \qquad (22)\\
\mu_u &= \beta\varepsilon+(1-\varepsilon)((1-\tau_y)A\eta_u^{-\alpha}-(1-\tau_e)\eta_u) \\
&\quad -\frac{\rho(1-\varepsilon)}{2}\eta_u^{-2\alpha}((1-\tau_y)^2A^2\sigma_y^2+\tau_P^2\sigma_P^2) \qquad (23)
\end{aligned}$$

for the stochastic pollution tax and

$$\begin{aligned}
\alpha\gamma(1-\delta)\mu_d &= (1-\tau_y)\alpha(1-\delta)A\eta_d^{-\alpha}+(1-\tau_e)\eta_d \\
&\quad -\rho\alpha(1-\delta)\eta_d^{-2\alpha}A^2\sigma_y^2 \qquad (24)\\
\mu_d &= \beta\varepsilon+(1-\varepsilon)((1-\tau_y)A\eta_d^{-\alpha}-(1-\tau_e)\eta_d) \\
&\quad -\frac{\rho}{2}\eta_d^{-2\alpha}A^2\sigma_y^2(1-\varepsilon+2\varepsilon\tau_y) \qquad (25)
\end{aligned}$$

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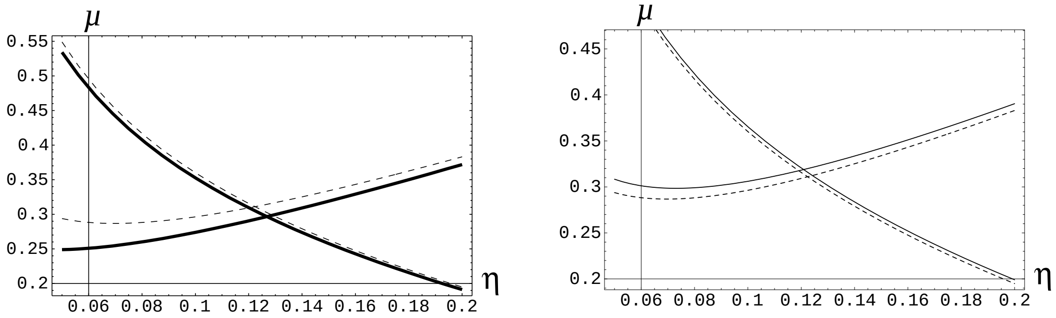
[3] Here, only values $\eta > 0.15$ are considered in order to exclude infeasible solutions.

for the deterministic policy setting, respectively. Equations (22) and (24) represent the optimal choice of abatement activity, equations (23) and (25) are derived from optimal capital accumulation.

Figure 2 illustrates the implications of a stochastic pollution tax on equilibrium abatement and consumption choice. The upward sloping curves describe the optimal composition of abatement and consumption according to equations (22) and (24). Since abatement and consumption are complementary goods, they are positively related within optimal abatement choice. The downward sloping curves display optimal capital accumulation (23) and (25). The negative relation between consumption and abatement results out of the individual budget constraint: if capital accumulation is set constant at the optimal level, consumption can only be increased at the cost of abatement.



(a) *High volatility.*

(b) *Low volatility.*

Figure 2: *Equilibrium abatement activity.*

Again, the situations with high (thick curves) and low (thin curves) volatility of the stochastic pollution tax are compared with the deterministic setting (dashed curves). The stochastic pollution tax induces a shift of both functions. Hence, the impact on environmental expenditures is not clearcut. Figure 2(a) shows that for a high volatility, the rise in environmental expenditures dominates. The stochastic pollution tax increases the uncertainty associated to pollution. Risk averse individuals will reinforce abatement activities in order to reduce pollution. Therefore, equilibrium pollution is diminished through stochastic pollution taxation with high volatility.

Figure 2(b) demonstrates, that the argument again reverses for low volatility tax rates. If the volatility of the environmental policy is sufficiently low, there is less capital risk than in the deterministic policy setting. Individuals react with decreased abatement activities and therefore higher equilibrium pollution.

To sum up, the impact of the stochastic pollution tax on individually optimal abatement and growth is ambiguous and depends on the volatility of the stochastic pollution tax. Sufficiently high volatility will increase environmental expenditures and therefore reduce equilibrium pollution. With this respect, a high volatility stochastic pollution tax will reduce pollution more efficiently than a deterministic policy regime.

5 Conclusion

This paper analyzes the impact of stochastic pollution taxation on equilibrium growth. On the one hand, pollution is essential for production and on the other hand it causes disutility. The individuals take only part of their influence on aggregate pollution into account. Hence, equilibrium capital accumulation as well as equilibrium pollution are suboptimally high and give rise to environmental policy.

The implications of a stochastic pollution tax are analyzed and compared with a deterministic tax regime. An exogenously given, additional pollution tax shock increases the uncertainty associated with pollution. Risk averse individuals take this additional risk into account within their intertemporal optimization. Hence, optimal abatement activity as well as optimal capital accumulation are adjusted simultaneously.

The impact of the stochastic pollution tax on macroeconomic equilibrium is analyzed numerically, since a closed form solution cannot be determined. The effect on equilibrium pollution is shown to be ambiguous and to depend on the volatility of the pollution tax. A sufficiently high volatility leads to a rise in equilibrium abatement activities in order to compensate for the increased risk due to taxation. Hence, this kind of stochastic environmental policy can support the corresponding deterministic policy.

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