The Simplest Unified Growth Theory

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Abstract. This paper provides a unified growth theory, i.e. a model that explains the very long-run economic and demographic development path of industrialized economies, stretching from the pre-industrial era to the present-day and beyond. Making strict use of Malthus' (1798) so-called preventive check hypothesis—that fertility rates vary inversely with the price of food—the current study offers a new and straightforward explanation for the demographic transition and the break with the Malthusian era. Employing a two-sector framework with agriculture and industry, the paper shows that agricultural productivity growth makes food goods, and therefore children, relatively less expensive. Industrial productivity growth, on the other hand, makes food goods, and therefore children, relatively more expensive. Fertility decline, according to the model, thus results from a period of relatively fast productivity growth in industry compared to agriculture. In part, this is caused by structural transformation in the form of labor leaving agriculture in favor of industry. The present framework lends support to existing unified growth theories and is well in tune with historical evidence about structural transformation.

Keywords: Economic Growth, Population Growth, Structural Change, Industrial Revolution.

JEL: O11, O14, J10, J13.

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1. Introduction

Over the past two centuries, a so-called Malthusian era, represented by stagnant standards of living and a positive relationship between income and fertility, has gradually been replaced by a so-called Modern Growth era, which in turn is marked by sustained economic growth and declining rates of fertility (a demographic transition). Building on seminal work by Galor and Weil (2000), several attempts have been made to try to merge the two eras, and to identify the underlying causes of the demographic transition. An incomplete list of studies includes Boucekkine et al. (2002), Doepke (2004), Galor and Moav (2002), Hansen and Prescott (2002), Jones (2001), Kögel and Prskawetz (2001), Lucas (2002), Strulik (2003) and Tamura (2002).

Common to these so-called unified growth theories is a complex apparatus, offered to motivate the profound shift in human fertility behavior associated with the demographic transition. The story of the fertility drop differs with the specific theory, but the gradual rise in the demand for human capital 'has led researchers to argue that the increasing role of human capital in the production process induced households to increase investment in the human capital of their offspring, ultimately leading to the onset of the demographic transition' (Galor, 2005).

The current paper offers a new and straightforward explanation for the demographic transition and the break with the Malthusian era – a theory which nicely complements ideas proposed in existing unified growth theories. More specifically, we provide an endogenous growth model which is consistent with the (stylized) development path of industrialized countries but does not rely on human capital accumulation as the driving force behind the demographic transition.

In the existing literature, a trade-off between child quantity and quality ultimately serves to generate a drop in fertility along with rising incomes. Fertility behavior in the present context, by contrast, relies entirely on Malthus' so-called *preventive check hypothesis* – that the tendency to matrimony, and thus implicitly the desire to give birth to children, is negatively correlated with the price of food (Malthus, 1798). The absence of a quality-quantity trade-off in the current paper eliminates the need of complex theoretical mechanisms hitherto required to explain the demographic transition, and is why we claim to provide the simplest unified growth theory so far proposed.

The current theory draws inspiration from a number of theoretical elements found in the existing literature. The micro foundation is borrowed from Weisdorf (2008), who shows that the Malthusian model conforms nicely to industrial development when the income-effect on the demand for children is removed (i.e. is set to zero). Learning-by-doing mechanisms, invoked in the current framework to trigger growth in productivity, are based on Matsuyama (1992) and Strulik (1997). Yet, the analogy to the R&D-driven growth literature based on Romer (1990) and Jones (1995) is also clearly visible.[1]

Employing a two-sector framework (with agricultural and industry), economic growth in the context of the present model derives from two sources. One is structural transformation in the form of labor being transferred from agriculture to industry. More specifically, agrarian productivity growth in conjunction with *Engel's law* (which states that the proportion of income spent on food falls as income rises) increases the share of labor allocated to industry and therefore raise the industrial sector's output without affecting the performance of the agricultural sector. The other source of growth derives from learning-by-doing in the production processes.

In brief, the development path proposed by the current theory can be described as follows. During early stages of development, the population size is relatively small; the share of labor employed in agriculture is relatively high; and standards of living are relatively close to subsistence. Because of a limited population size, learning-by-doing effects, to begin with, are relatively small. Hence, the agricultural sector's productivity growth is slow, yet faster than that of industry in which labor resources, and thus learning-by-doing effects, are even smaller.

Productivity growth in agriculture has two effects on development. On the one hand, higher productivity growth in agriculture *relative to industry* makes food, and therefore children, relatively less expensive; this raises fertility and gradually speeds up the rate of population growth. On the other hand, agricultural productivity growth, in combination with Engel's law, increases the share of labor allocated to industrial activities.

With learning-by-doing effects at work in both agriculture and industry, the transfer of labor out of agriculture gradually shifts the ratio of productivity growth in favor of industry. Sooner or later, therefore, the rate of the industrial sector's productivity growth surpasses that of agriculture. Subsequently, industrial goods are becoming affordable at relatively lower prices, implying that food, and therefore children, become relatively more expensive. Henceforth, fertility declines, and the rate of growth of population slows down, until population growth eventually (endogenously) comes to a halt.

[1]The current model can also be understood as a two-sector extension of Kremer's (1993) model on long-run human economic development, now capturing structural change and micro-founded population growth.

The current theory emphasizes two important features connected to the release of agricultural labor as a source of long-term economic growth. On the one hand, as the analysis below will clarify, a transfer of labor out of agriculture can persist for many (i.e. hundreds of) years before ultimately leading to substantial economic growth, a point often stressed by economic historians. As such, the industrial revolution in the present context is not a sudden break with the past, but a gradual outcome of long-standing processes taking place throughout the Malthusian era. On the other hand, as the analysis below will also establish, a slowdown of economic growth will eventually occur, as the beneficial effects of structural transformation gradually become exhausted.

The paper continues as follows. Section 2 describes the basic model, Section 3 explores dynamics and stability conditions, and Section 4 performs a quantitative analysis, highlighting the long-run development path predicted by the model. Section 5 looks at some empirical evidence related to the model's main predictions, and Section 6 discusses extensions and generalizations of the theoretical framework. Finally, Section 7 concludes.

2. The Model

2.1. **Introduction.** In the following, we describe the simplest unified growth theory, and we explain its main mechanisms and predictions. One of the main advantages of the theory's simplicity is that it provides a closed-form solution for fertility. However, simplicity often entails some shortcomings. In a later section, therefore, we discuss the shortcomings of the simple model, and we demonstrate, by relaxing some of the initial assumptions, that the simple model can be generalized (although at a cost to the closed-form solution for fertility), so as to account for these shortcomings.

To begin with, consider a closed economy with two sectors: an agricultural sector producing food, and an industrial (i.e. non-agricultural) sector producing manufactured goods. Economic activities extent over infinite (discrete) time denoted by t. We examine a two-period overlapping generations economy with childhood and adulthood. Productive and reproductive activities take place only during adulthood. For simplicity, reproduction is asexual, meaning that each individual is born to a single parent. Individuals are identical from every aspect, and each adult individual is endowed with one unit of labor, which is supplied inelastically to work.

Change in the size of the labor force (i.e. the adult population) between any two periods is given by

$$L_{t+1} = n_t L_t, \tag{1}$$

where n_t is the gross rate of growth of population. As we abstract from mortality, n_t also measures the rate of fertility in period t.

2.2. **Preferences.** The cause of the demographic transition in the current framework is to be found in the interaction between differential productivity growth and parental preferences. Individuals in the model maximize utility derived from the amount of offspring that they have, n_t, and from the number of manufactured goods that they consume, measured by m_t.

Suppose that individuals face a 'hierarchy of needs', so that having children (and feeding them) is a first-priority activity, whereas enjoying manufactured goods comes in second. In times of economic crises, this means that parents will adjust fertility relatively little compared to the consumption of manufactured goods. Such a behavior implies that the elasticity of the marginal utility is higher for children than for manufactured goods.

A convenient way of formalizing this is by making use of a so-called *quasi-linear* utility function, which can be written as

$$u_t = m_t + \gamma \ln n_t, \tag{2}$$

with γ being the weight put on children.

Intuitively, the quasi-linear description provides the strongest form of a hierarchy of needs, namely when the income-elasticity on the demand for children is zero. This implies that fertility is affected not by income changes but by the relative costs of raising children (which, for most of history, meant the relative price of food). Although the income-effect on fertility is absent, hierarchic needs in the current form involve an *indirect* income-effect on the demand for children. That is, combined with the labor market equilibrium condition below, productivity (and therefore income) growth has an ambiguous effect on fertility: agricultural productivity growth, by reducing the relative price of food, will increase fertility. Manufacturing productivity growth, on the other hand, by increasing the relative price of food, will reduce fertility. As will be demonstrated in Section 6, however, the quasi-linear 'zero-income-effect' preferences are not essential for the qualitative nature of the results obtained below.

To obtain the budget constraint, suppose that an individual, over the course of a lifetime, consumes a fixed quantity of food (i.e. calories), measured by a. For simplicity, food is demanded only during childhood, and some of it then stored for adulthood.[2] The price of manufactured goods is set to one, so p_t denotes the price of food, measured in terms of manufactured goods (i.e. the relative price of food). By setting $a \equiv 1$, it follows that each child consumes one unit of food.[3] This means that the total costs of raising n_t children, measured in terms of manufactured goods, is $p_t n_t$. The individual budget constraint thus reads

$$w_t = p_t n_t + m_t, \tag{3}$$

where w_t is the income of a representative individual, also measured in terms of manufactured goods.

The solution to the optimization problem implies that the demand for children is given by

$$n_t = \frac{\gamma}{p_t}. \tag{4}$$

Consistent with Malthus' (1798) *preventive check hypothesis*, the price-effect on the demand for children is negative. However, although there is no direct income-effect on fertility, an indirect income-effect will ultimately enter through the relative price of food, as will become apparent below.

In the existing literature, fertility responds directly to a change in income. In the present study, fertility responds to a change in the relative price of food. In turn, the price of food can be affected by sectoral shifts, such as labor leaving agriculture in favor of industry. This means that such structural transformations may account for changes in fertility. In accordance with stylized historical facts proposed by Crafts (1996) and Voth (2003), therefore, the present work is well in tune with the idea that persistent (and rapidly shifting rates of) population growth went on during early phases of industrialization, where income per capita was largely stagnant but when substantial structural transformation took place.

2.3. **Production.** In line with existing unified growth theories, agricultural production is subject to constant returns to labor and land, the latter being measured by X. Land is in fixed

[2]It will not affect the qualitative nature of the results, if, instead, the individual's food demand where to be divided over two periods.
[3]In Section 6, we relax this assumption by letting child (and thus food) expenditures increase with rising income.

supply, and the total amount is normalized to one (i.e. $X \equiv 1$). Industrial production is subject to constant returns to labor, implying, as is also common in the related literature, that land is not an important factor in manufacturing.[4]

Inspired by Matsuyama (1992), new technology in each of the two sectors appears as a result of learning-by-doing. More specifically, output and new technology in the respective sectors are produced according to the following production functions:

$$Y_t^A = \mu A_t^{\varepsilon} (L_t^A)^{\alpha} = A_{t+1} - A_t, \quad 0 < \alpha, \varepsilon < 1 \tag{5}$$

$$Y_t^M = \delta M_t^{\phi} L_t^M = M_{t+1} - M_t, \quad 0 < \phi < 1. \tag{6}$$

The variable A_t is total factor productivity in agriculture (superscript A for agricultural goods), and the variable M_t measures total factor productivity in industry (superscript M for manufactured goods). With $0 < \varepsilon, \phi < 1$, there are diminishing returns to learning in both sectors. As is not uncommon in the related literature, we abstract throughout from the use of capital in production in both sectors.

Define $g_t^Z \equiv (Z_{t+1} - Z_t)/Z_t$ to be the net rate of growth of a variable Z between any two periods. Then it follows that the net rate of productivity growth in the agricultural sector is $g_t^A \equiv \mu A_t^{\varepsilon-1} \left(L_t^A\right)^{\alpha}$. Similarly, the net rate of productivity growth in industry is $g_t^M = \delta M_t^{\phi-1} L_t^M$.

2.4. **Equilibrium.** Together, the variables L_t^A and L_t^M measure total labor input in agriculture and industry, respectively. They therefore fulfil the condition that

$$L_t^A + L_t^M = L_t. \tag{7}$$

The share of total labor devoted to agriculture is determined by the food market equilibrium condition. This condition says that total food supply, Y_t^A, equals total food demand, which (when each person consumes one unit of food) is $n_t L_t$. Using (5), the food market equilibrium condition implies that the share of farmers to the entire labor force is given by

$$\theta_t \equiv \frac{L_t^A}{L_t} = \left(\frac{n_t L_t^{1-\alpha}}{\mu A_t^{\varepsilon}}\right)^{\frac{1}{\alpha}}. \tag{8}$$

[4]In Section 6, we demonstrate that assuming diminishing returns-to-labour in manufacturing does not affect the qualitative results presented below.

Note that productivity growth in agriculture releases labor from the agricultural sector, whereas population growth has the opposite effect.

Consistent with existing unified growth theories, we assume that there are no property rights over land, meaning that the land rent is zero. This implies that labor in both sectors receives the sector's average product. The labor market equilibrium condition implies that the relative price of food adjusts, so that farmers and manufacturers earn the same income, i.e. $w_t = p_t Y_t^A / L_t^A = Y_t^M / L_t^M$. By the use of (4)-(8), this means that the relative price of food is given by

$$p_t = \frac{\left(\delta M_t^{\phi}\right)^{\alpha} (\gamma L_t)^{1-\alpha}}{\mu A_t^{\varepsilon}}. \tag{9}$$

Note that the relative price of food increases with productivity growth in industry and with the size of the population. Productivity growth in agriculture, on the other hand, has the opposite effect.

Finally, a second use of (4) in (9) provides the rate of fertility in a general equilibrium, and is

$$n_t = \mu \left(\frac{\gamma}{\delta}\right)^{\alpha} \frac{A_t^{\varepsilon}}{M_t^{\alpha\phi} L_t^{1-\alpha}}. \tag{10}$$

It follows that productivity growth in agriculture increases fertility, whereas productivity growth in industry and population growth have the opposite effect.

The simplest unified growth model is complete with equations (1) to (10).

3. Balanced and Unbalanced Growth in the Long-Run

In the following, we explore the balanced and unbalanced growth paths of the economy. Along a balanced growth path, all variables grow at constant rates (possibly zero). According to (5), the gross rate of agricultural productivity growth is given by $A_{t+1}/A_t = \mu (L^A{}_t)^{\alpha} / A_t^{1-\epsilon}$. Along a balanced growth path, the left-hand-side is constant by definition, so the right-hand-side must be constant as well. At the same time, the share of labor in agriculture must be constant, implying that L_A grows at the same rate as L. A constant growth of productivity in agriculture thus implies that

$$1 + g^A = \left(1 + g^L\right)^{\alpha/(1-\epsilon)}. \tag{11}$$

From similar reasoning, it follows from (6) that a constant rate of growth of productivity in manufacturing implies that

$$1 + g^M = \left(1 + g^L\right)^{1/(1-\phi)}. \tag{12}$$

According to (10), the gross rate of growth of population can be written as

$$\frac{n_{t+1}}{n_t} = \frac{\left(1 + g_t^A\right)^\epsilon}{\left(1 + g_t^M\right)^{\alpha\phi} n_t^{1-\alpha}} \quad \Rightarrow \quad 1 + g_{t+1}^L = \frac{\left(1 + g_t^A\right)^\epsilon \left(1 + g_t^L\right)^\alpha}{\left(1 + g_t^M\right)^{\alpha\phi}}. \tag{13}$$

Inserting (11) and (12) into (13), we obtain the equilibrium law of motion for population growth, which is given by

$$1 + g_{t+1}^L = \left(1 + g_t^L\right)^\eta, \qquad \eta \equiv \alpha + \frac{\alpha\epsilon}{1-\epsilon} - \frac{\phi\alpha}{1-\phi}. \tag{14}$$

Along a balanced growth path, the population grows at constant rate, i.e. $g_{t+1}^L = g_t^L = g^L$. This leaves two possibilities: either (i) there is no population growth (i.e. $g^L = 0$), meaning that the population level is constant, or (ii) there is positive or negative population growth (i.e. $g^L \neq 0$), meaning that the population level is constantly growing or shrinking.

The latter possibility applies only when the knife-edge condition $\eta = 1$ is fulfilled. Meanwhile, stability of the balanced growth path requires that $|\eta| < 1$. Hence, not only does growth on the knife-edge demand a very specific parameter constellation (cf. the definition of η). It also requires that the economy starts off *on* the balanced growth path, and then remains there forever. Since it is the transitional dynamics of the model that will be of interest, we will abstract from the knife-edge case throughout.

As for the former possibility—that in which there is balanced growth but no population growth—we can conclude, using (11) and (12), that balanced growth combined with no population growth also means no productivity growth. In other words, if $|\eta| < 1$, then the economy will ultimately reach a unique, stable balanced growth path, along which there is no (exponential) economic growth.

This is a noteworthy result. Since positive population growth cannot persist forever (because of a limited space), other long-run growth theories, such as Jones (2001), have to impose the condition that population growth is zero over the long run (which ultimately also leads to a zero rate of productivity growth as well). In the present context, by contrast, a stationary population level over the long-run occurs endogenously. Specifically, population growth in the current model 'digs its own grave' through the feedback-effect of productivity. The reason is that productivity growth in manufacturing increases the relative costs of having children. This slows down the growth of population, thus decelerating productivity growth for subsequent generations. Eventually, therefore, the economy endogenously converges to a state of zero growth in population and productivity. Meanwhile, as our numerical analysis will show below, the transitional dynamics may indeed involve centuries of positive exponential growth before a zero exponential growth path is eventually reached.[5]

While a *balanced* growth path involves a zero growth of population and income, an *unbalanced* growth path, characterized by imploding or exploding growth, may in principle exist. In the following, we explore the two cases of unbalanced growth, starting with the case of imploding growth. Imploding growth implies perpetual negative population growth. That is, n_t is smaller than one, and L_t, therefore, is decreasing. It is easy to see that imploding growth is not an option. This follows from the fact that g_t^A and g_t^M are bound to be non-negative, as 'forgetting-by-doing' does not take place. With $\lim_{L\to 0} g^A = 0$ and $\lim_{L\to 0} g^M = 0$, we get from (10) that $\lim_{L\to 0} n = const./L^{1-\alpha}$. As L_t converges to zero, n_t goes to infinity—a contradiction to the initial assumption of n_t being smaller than one. Hence, there will be no imploding growth. From an intuitive viewpoint, it is diminishing marginal returns-to-labor in agriculture (i.e. $\alpha < 1$) that prevents implosion. As the population decreases, agricultural productivity goes up. Food prices, therefore, go down, causing fertility, and thus next period's population level, to increase.

Explosive growth, on the other hand, cannot be ruled out if $\eta > 1$. In this case, the relative price of food ultimately goes to zero, and fertility, therefore, to infinity. With increasing rates of population growth, productivity growth in both sectors will grow hyper-exponentially, until fertility reaches infinity in finite time. A sufficient condition for stability of the balanced growth path, therefore, is that

$$\epsilon < \epsilon_{crit} \equiv \frac{1-\phi-\alpha+2\alpha\phi}{1-\phi+\alpha\phi}. \tag{15}$$

This condition says that the learning elasticity in agriculture cannot be too large. If (15) is violated, then the pace of growth of agricultural productivity, fueled by population growth, will lead to ever falling food prices, speeding up further the growth of population.

[5]Of course, the model is capable of generation persistent economic growth if the central mechanism of prices on fertility is disabled, for example through introduction of a minimum rate of fertility. This case is discussed in the extensions part of the paper.

Inspection of (15) reveals that the critical level of ϵ, i.e. ϵ_{crit}, is decreasing in α and increasing in ϕ. Intuitively, this means that, the larger are the counterbalancing forces of fixed land in agriculture and learning-effects in manufacturing, the larger the learning-effects in agriculture can be without leading to explosive growth.

The following proposition summarizes the considerations made above about balanced and unbalanced growth.[6]

Proposition 1. *There exists a unique balanced growth path for the simplest unified growth model. It implies zero population growth and zero (exponential) economic growth. If the learning elasticity in agriculture ϵ is sufficiently small, i.e. if (15) is not violated, then there is no unbalanced growth in the long-run.*

Note that Proposition 1 includes the possibility of a so-called 'Malthusian' trap. That is, there are adjustment paths towards the balanced growth path, along which an 'industrial revolution' will not emerge. If the learning elasticity in agriculture is sufficiently small, then population growth will 'swallowed up' agricultural productivity growth to such an extent that agricultural progress cannot free up enough labor for learning, and thus productivity growth, in manufacturing to gain momentum.[7]

If, on the other hand, agricultural learning-effects can free up adequate labor resources for manufacturing, then, eventually, the relative price of manufactured goods starts to decline, and the economy will go through (i) a demographic transition (a fertility transition, strictly speaking); (ii) a structural transformation whereby labor shifts from agriculture to industry; and (iii) a temporary take-off of economic growth (an 'industrial revolution'). Note, as is the aim of unified growth theory, that these events will occur endogenously, i.e. in the absence of exogenous shocks.

[6] It is worth emphasizing that $\epsilon < \epsilon_{crit}$ is a *sufficient* condition for stability. For its derivation, we have assumed that both g^A and g^M evolve in neighborhood of their steady-state path, (11) and (12), respectively. The condition could be relaxed, if the slowdown of population growth before the convergence to balanced growth appears sufficiently fast. Our numerical example below will verify this claim. For intuition, re-inspect (13). Imagine that adjustment dynamics imply that an 'agricultural revolution' occurs before an 'industrial revolution'. That is, consider a path along which g_t^A, because of diminishing returns-to-labor, is already declining, whereas g_t^M is still on the rise. The resulting fact, that g_t^A is converging to zero faster than g_t^M, relaxes the stability condition obtained from analytical considerations, which implicitly assumes that *all* growth rates are in the neighborhood of a balanced growth path. To see this, consider the limited case of $g^A \to 0$ in (13). Stability would then 'only' require that $\alpha\,(1-\phi) < 1$, which is always fulfilled. To summarize, therefore, condition (15) is a sufficient, but not a necessary condition to rule out explosive growth.

[7] To see this, consider the case of $\epsilon \to 0$ and start with $\theta_0 \to 1$.

The latter line of events (and the absence of a Malthusian trap) seems to have characterized Western-world countries in their transition from the pre-industrial era to the present day. In the following, therefore, we explore in more detail the transitional dynamics of the model.

4. Adjustment Dynamics: Growth in the Middle Ages, Industrialization, and the Productivity Slowdown

As a starting point, consider a pre-industrial, agricultural economy. That is, an economy in which the population level is relatively small; the share of labor employed in agriculture is relatively high; and the level of income per capita is relatively close to subsistence. Starting with a relatively small level of population, learning-by-doing effects, to begin with, are relatively small. Hence, the agricultural sector's productivity growth is slow, yet faster than that of industry, where labor input, and thus learning-by-doing effects, are even smaller.

Productivity growth in agriculture has two effects on development. On the one hand, higher productivity growth in agriculture *relative to industry* makes food, and therefore children, relatively less expensive; this raises fertility and gradually speeds up rates of population growth. On the other hand, agricultural productivity growth, in combination with Engel's law, increases the share of labor allocated to industrial activities.

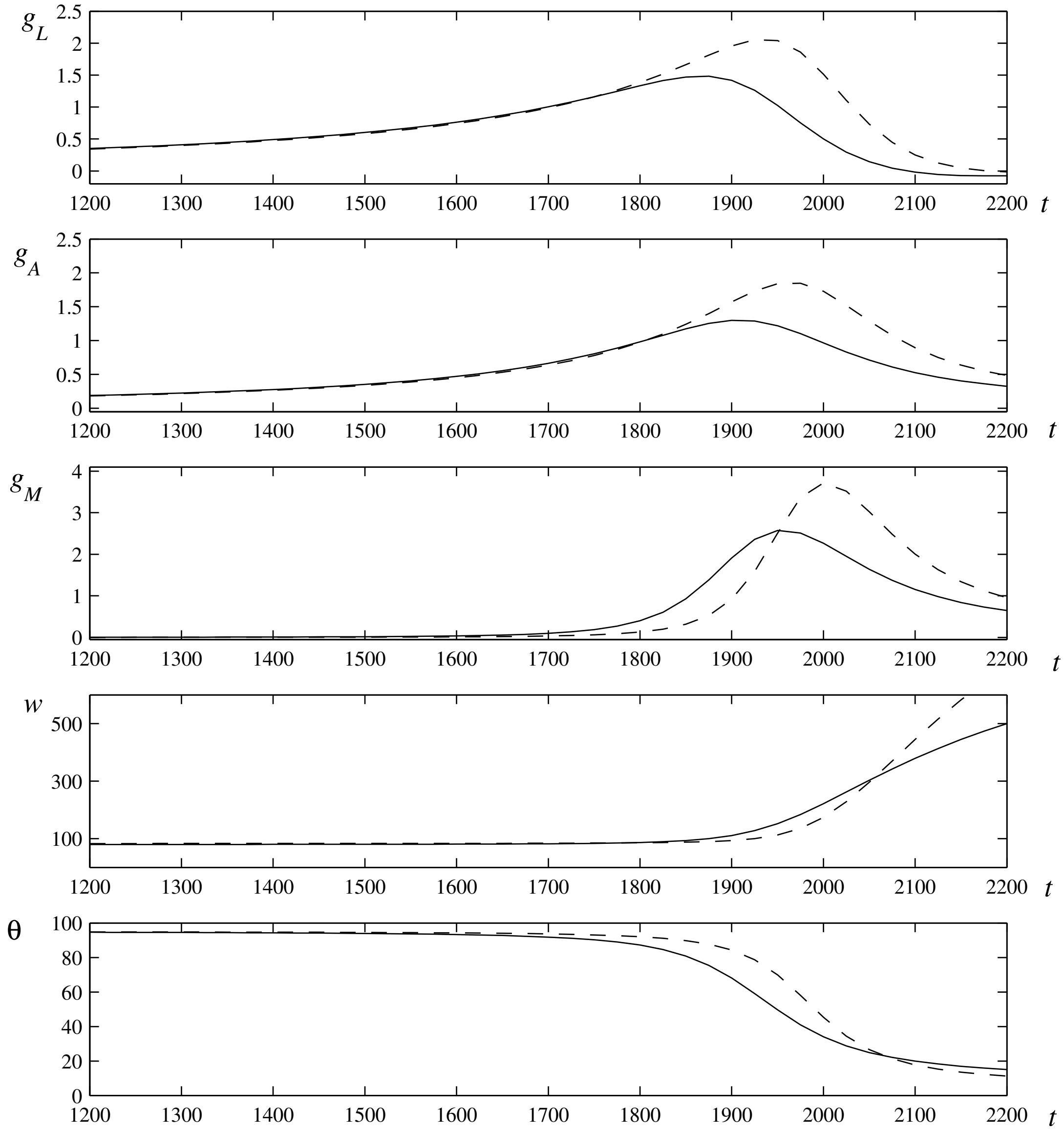
With learning-effects at work in both agriculture and industry, the transfer of labor out of agriculture gradually shifts the ratio of productivity growth in favor of industry. Sooner or later, therefore, the rate of the industrial sector's productivity growth surpasses that of agriculture. Subsequently, food, and therefore children, become relatively more expensive; fertility declines, and population growth slows down, until it eventually comes to a halt.

Observe, using (9), how the size of elasticities (i.e. ϵ and ϕ) shapes the path of fertility. The higher is ϵ, the stronger will be the economic impact on fertility increase during a take-off phase of agriculture (an 'agricultural revolution'). On the other hand, the higher is ϕ, the more pronounced will be the fertility decline (the demographic transition) during a take-off phase of industry (an 'industrial revolution').

In order to get a more exact picture of the adjustment dynamics, a calibration of the model is done below.[8] We calibrate the model such that it approximates the peak of the demographic

[8] Since we are simulating growth trajectories for a very simple model, our results will likely be inferior to those of large-scale CGE models. Such studies, using multi-factor, multi-sector models, and taking many important determinants into account (like capital accumulation, education, international trade, and energy production), are bound to produce more precise results than our's. In particular, it can be expected that our model makes large

Figure 1: Long-Run Dynamics According to the Simplest Unified Growth Model



Solid lines: benchmark case, dashed lines: higher preference for children: $\gamma = 4.5$ (instead of 3.4), otherwise benchmark parameters. From top to bottom the diagrams show population growth, productivity growth in agriculture, productivity growth in manufacturing, income per capita, and the labor share in agriculture (our measure of structural change).

transition in England and the subsequent slowdown of productivity growth. We set parameter values, so that population growth reaches its peak of 1.5 percent annually in the year 1875, and

approximation-errors at the end of the transition paths, when learning-by-doing effects level off, and because we have neither capital accumulation, nor education, to foster growth. More humbly, our experiment is to explore how much of economic and demographic history we can explain using possibly the simplest conceivable growth model that exists. See Harley and Crafts (2000) and Stokey (2001) for large scale CGE modelling of the industrial revolution.

so that total factor productivity (TFP) begins to slowdown in the year 1975, when it grows at a rate of 1.5 percent annually.

For the benchmark case, we set the following parameters: $\alpha = 0.8$, $\epsilon = 0.45$, $\phi = 0.3$, $\mu = 0.5$, $\delta = 1.5$, and $\gamma = 3.4$. Start values are $\theta_0 = L_0^A/L_0 = 0.95$, $L = 0.014$, $A_0 = 0.8$. The start value for M is obtained endogenously from $M_0 = (\gamma/(\delta\theta_0))^{1/\phi}$. For better readability of the results, it is assumed that a generation takes 25 years (approximately the length of the fecundity period), and we then translate generational growth rates into annual ones.[9]

Figure 1 demonstrates the adjustment path, running from the year 1200 on to the year 2200. The solid lines show the path of the benchmark economy. The dashed lines concern an alternative economy to be discussed further below.

To begin with, nearly the entire labor force is allocated to agriculture. In line with numbers provided by Galor (2005), industrial productivity growth, initially, is almost completely absent, while productivity growth in agriculture is around 0.15 percent annually. In the beginning, agricultural progress, manifesting itself in a slowly decreasing relative price for food, is almost entirely converted into population growth. Due to diminishing returns-to-labor in agriculture, however, income per capita is hardly affected by the growth of productivity.

Due to learning-by-doing, population growth (in a Boserupian manner) furthers productivity growth in agriculture.[10] Over the subsequent years, agricultural productivity growth (because the learning elasticity in agriculture in this case is sufficiently high) slowly builds up to reach 0.5 percent by the 18th century. By then, agriculture progress makes possible a substantial transfer of labor into manufacturing, causing an upsurge in industrial productivity growth.[11] By contrast to agriculture, productivity growth in the industrial sector derives from two sources: population growth and a transfer of labor from agriculture. From the turn of the 19th century onwards, therefore, economic growth gains momentum.

Although productivity growth in industry is now gradually on the rise, its rate of growth during the 19th century is still lower than that in agriculture. Consequently, the relative price of food is still falling, causing further increase in fertility and population growth. Strikingly,

[9] For individual experiments a spreadsheet containing our basic specification of the model can be downloaded from `http://kaldor.vwl.uni-hannover.de/holger/research/papers.php`.

[10] See Boserup (1981) for a detailed exploration on how population growth drives agricultural progress.

[11] This outcome of the model is in line with the observation that the direct labor input to produce a ton of grain – while staying almost constant for a long time in history – declined by 70 percent in the 19th century (Johnson, 2002).

although substantial structural and demographic changes take place in the 19th century, major improvements in standards of living, according to the model, do not appear before the 20th century. This phase of adjustment dynamics is well in tune with the demo-economic observations made in relation to England's industrial revolution. As has been emphasized by economic historians such as Crafts (1996) and Voth (2003), a high rate of structural change and considerable rates of population growth are accompanied by relatively low rates of economic growth.

In line with the empirical stylized facts, the fourth panel of Figure 1 shows that there is relatively slow income-growth in the centuries preceding the industrial revolution, and only a slight rise in income per capita during the initial phases of industrialization in the first half of the 19th century (we have normalized $y_{1875} = 100$). Meanwhile, the Figure also reveals a shortcoming of the model, and of its story of industrialization. From 1875 to 1990, income per capita in the present model grows at a factor 2.5. In England, by contrast, income per capita grew at a factor 5 (Maddison, 2001). This shortcoming can be explained by the neglect in the present framework of human and physical capital accumulation, which are estimated to have accounted for about half of the increase of income per capita in industrialized countries.

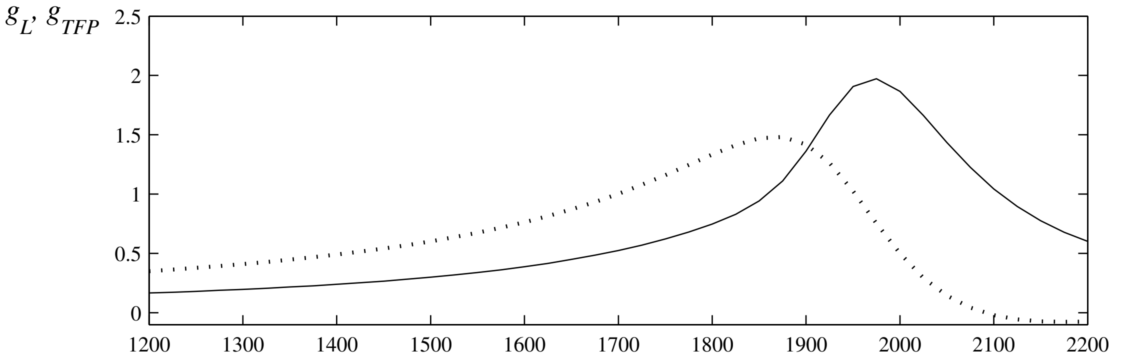
In order to keep track of the relationship between productivity growth on the one hand, and demographic growth on the other, implied total TFP growth is calculated using Domar weights. The implied TFP growth thus writes

$$g_t^{TFP} \equiv p_t \frac{Y_t^A}{Y_t} g_t^A + \frac{Y_t^M}{Y_t} g_t^M = p_t \theta \frac{y_t^A}{y_t} g_t^A + (1-\theta) \frac{y_t^M}{y_t} g_t^M,$$

where $Y = pY^A + Y^M$ measures GDP. Figure 2 combines the adjustment dynamics for implied TFP growth (the solid line) and population growth (the dotted line). As is evident from the illustration, the growth rate of population reaches its peak by the end of the 19th century, whereas implied TFP growth peaks about one century later.

This result nicely fits ideas proposed by Galor and Weil (2000). According to their theory, the drop in fertility (the demographic transition) does not emerge before the rate of TFP growth reaches substantial levels. In fact, the current model predicts a structural break for the correlation between TFP growth and population growth. As is evident from Figure 3, the correlation is positive before the year 1875, i.e. during the Malthusian phase, when TFP growth is relatively slow, and the rate of population growth is on the rise. By 1875, the rate of growth of population reaches its peak, and then starts to decline. However, TFP growth, at least for a

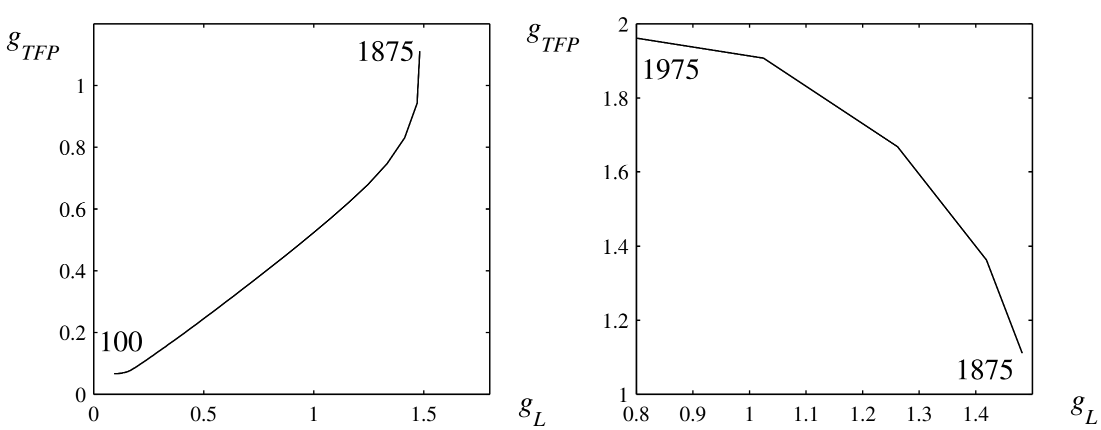
Figure 2: Long-Run Dynamics: Implied TFP Growth and Population Growth



Solid line: TFP growth, dotted line: population growth

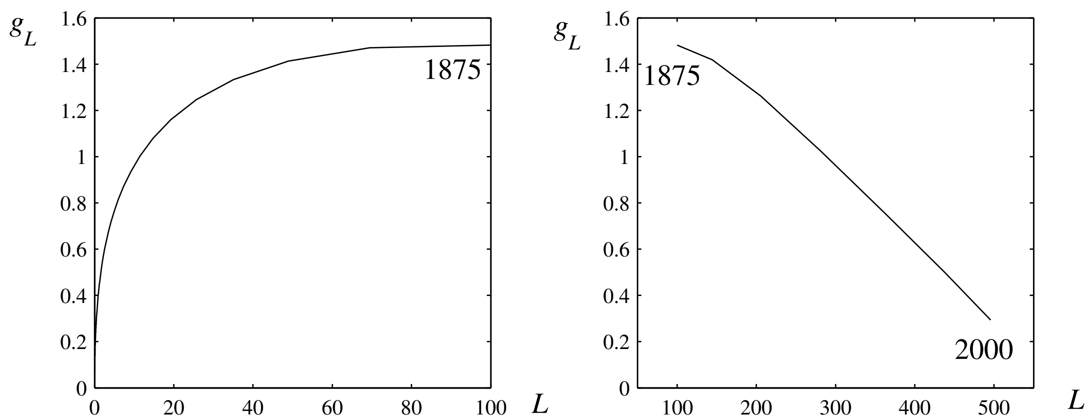
while, continues to rise, indicating a negative correlation between TFP and population growth after 1875. Empirically, this is not an unfamiliar result. For the past century, Bernanke and Guerkaynak (2002) provide supporting evidence for the negative correlation between TFP and population growth. For the pre-industrial period, by contrast, a positive relationship between TFP and population growth is well-documented by Clark (2007).[12]

Figure 3: Population-TFP-Nexus:
From Year 100 to 1875 (left) and from 1875 to 1975 (right)



[12]After 1975, a third phase, in which both population growth and TFP growth decline, can be observed, once again suggesting a (modest) positive correlation.

Figure 4: Population-Size and Population Growth:
From One Million B.C. to Year 1875 (left) and from 1875 to 2000 (right)



The model's prediction concerning the relationship between population size and population growth is also noteworthy. Using a one-sector variant of the current model, but with an exogenously imposed rate of population growth, Kremer (1993) establishes a remarkable positive correlation between the size and growth of population, a phenomenon unseen for any other species than humans. His result is supported empirically, though only weakly for the last data points concerning the 20th century (cf. his Figure 1).

Repeating Kremer's exercise using the present model, a structural break is generated in the year 1875. From one million BC to 1875, the correlation is positive, as suggested by Kremer. The slope is non-linear and the best fit of the correlation is obtained for the function $g_L = const{\cdot}L^{0.15}$. This suggests that the positive correlation was particularly strong during early times, and then became flatter as the population growth rate reaches its peak. After 1875, for industrialized and fully-developed countries, the correlation turns negative, and, indeed, becomes almost linear.

Finally, an analysis of variation in the taste for children provides insight with respect to understanding the differences in the timings of the industrial revolution and the demographic transition across countries. Compared to the benchmark case (solid lines), the dashed lines in Figure 1 capture the adjustment dynamics of an alternative economy. In the alternative economy, everything is identical to the benchmark case, except for γ which is set to 4.5 instead of 3.4. All other things being equal, parents in the alternative economy hence display a stronger taste for children than parents of the benchmark economy. As has been demonstrated by Strulik (2008), population growth, on average, peaks at higher rates in countries of lower latitude. Following

Strulik, therefore, the dashed lines of Figure 1 would capture a country situated closer to the equator than England.

Having a relatively strong taste for children involves parents having a relatively large demand for food. In the alternative economy, therefore, agriculture will dominate for more years than in the benchmark economy. Hence, when the industrial revolution sets off in the benchmark case, industrial productivity growth in the alternative economy is still relatively slow. In terms of Figure 1, the 'industrial revolution' (i.e. the transfer of labor from agriculture to industry) and the fertility drop in the alternative economy both start about 75 years (or three generations) later than in the benchmark case. However, once structural changes begin to emerge in the alternative economy, industrial productivity growth is faster than in the benchmark case. The reason, of course, is that the alternative economy's population level, and thus its learning-by-doing effects, are larger because of a stronger taste for children. In terms used by Bloom et al. (2001), the alternative economy ultimately earns its 'demographic dividend', and it eventually draws nearer to the benchmark economy, because of its higher growth rate of productivity and its faster rate of structural change. Effectively, a catching up (and eventual overtaking) is, therefore, at play.

5. Empirical Evidence

In this section, we resort to historical data to look for empirical evidence in support of the simplest unified growth theory. Producing such evidence would ideally involve two steps. First, one would explore the relationship between the relative rates of productivity growth in agriculture and industry on the one hand, and the relative price of food on the other. Next, one would study the correlation between relative prices and rates of fertility, with specific attention paid to the occurrence of a demographic transition.

The problem with the first step, however, is that contemporary industrialized countries, in the process of becoming industrialized, become increasingly dependent on foreign trade. Since our theory considers a closed economy, it makes no sense exploring empirically the link between sectoral productivity and relative prices, as captured by the model's equation (9). The only meaningful evidence to search for in the context of our theory, therefore, is a negative correlation between the relative price of food and the rate of fertility.

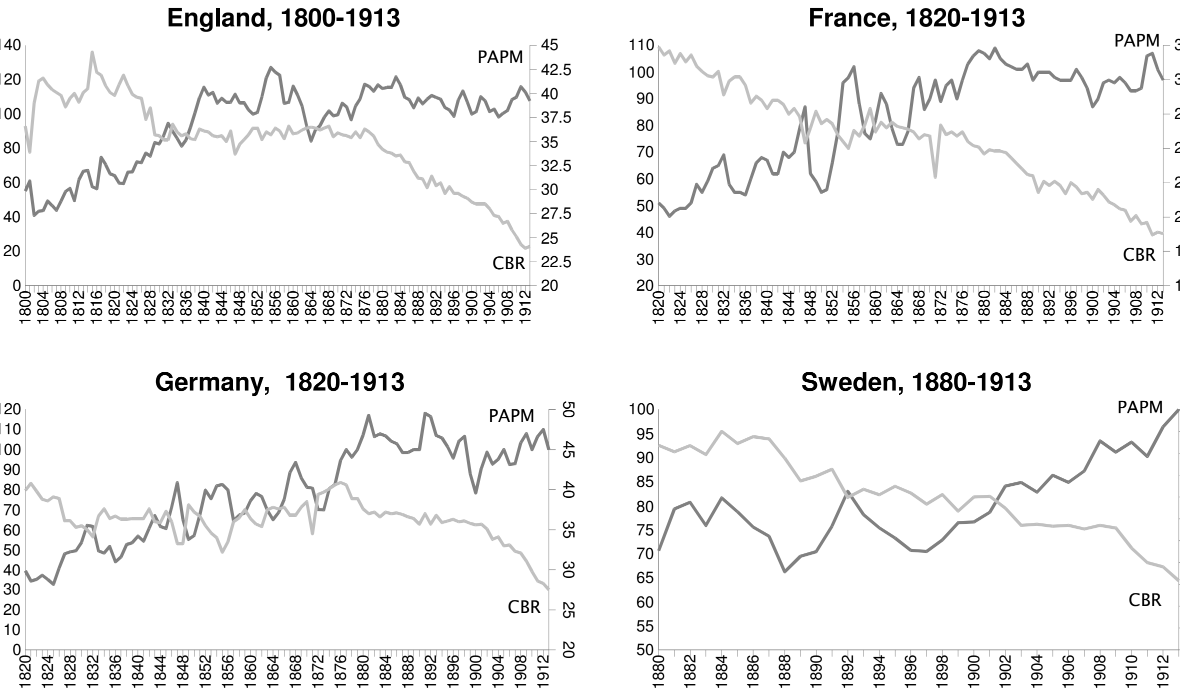
According to the model's equation (4), a comparatively high relative price of food implies a comparatively low rate of fertility, and vice versa. It is a well-documented fact in the fields of demography and economic history that food prices and fertility are negative correlated. Indeed, since variations in the price of food (or, more precisely, grain) are often used as a proxy for variations in the standard of living, the former has been used as a tool for testing the Malthusian population hypothesis. One such test is performed by Galloway (1988), who offers a detailed analysis of the effects on demographic variables of changes in European price levels before and under the industrial revolution. Following Galloway's findings, there is clear evidence of an inverse response to crude birth rates of food price variation in early modern Europe and beyond.

Meanwhile, whereas Galloway's and similar studies analyze the impact of *nominal* food price variation on fertility, our predictions rely on the argument that it is changes in *relative* food prices that matters to variation in fertility. Unfortunately, though, our need for relative rather than nominal prices severely limits the amount of evidence available. That is, although historical fertility data for contemporary industrialized countries is fairly easy to obtain, historical data on the relative price of food (explicitly defined as the price of agricultural goods divided by the price of industrial goods) is hard to come by. In fact, such prices for most countries do not exist before after the middle of the twentieth century, i.e. long after the historical fertility decline has ended. Meanwhile, four countries that are usually subject to very long-run economic and demographic analyses—England, France, Germany and Sweden—do indeed have these data available for appropriate periods.

When possible, the price data presented in the following is collected, so that it runs from shortly before the demographic transitions begin. In England, the onset of the demographic transition is to be found soon after the turn to the nineteenth century. Hence, for England, our data runs from 1800 on. In France, the fertility decline begins somewhat earlier. However, French price data does not exist before 1820, which makes 1820 the initial year. Compared to France and England, the demographic transitions in Germany and Sweden began relatively late, around 1875 in both cases. For Germany, price data dates back to 1820, whereas, in the Swedish case, such data is not available before 1880. Hence, 1820 and 1880 are the initial years for Germany and Sweden, respectively. For all countries, the data runs up until 1913.

Figure 6 plots a time-series for each of the four countries, showing the annual relative price of food (PAPM) together with the crude birth rate (CBR). For all four cases, there appear

Figure 5: Evidence: Relative Food Prices and Crude Birth Rates



Source: Crude birth rates for all four countries: Murphy and Dyson (1986). Price data: England: Agricultural prices, 1800-1805: Deane and Cole (1962). Agricultural prices, 1805-1913: Mitchell and Deane (1962). Industrial prices: Mitchell and Deane (1962). France: Agricultural and industrial prices: Bourguignon and Levy-Leboyer (1985). Germany: Agricultural and industrial prices: Wagemann (1935). Sweden: Agricultural prices: Schön (1995). Industrial prices: Larsson (2006). England: PAPM 1900=100; France: PAPM 1890=100; Germany and Sweden: PAPM 1913=100.

to be a downward trend in fertility, as well as an upward trend in the relative price of food, leaving a (stylized) impression of an overall negative correlation between the two variables.[13] In the case of France, the inverse relationship seems to be particularly clear-cut, at least up until the end of the 19th century. This is a noteworthy result, especially considering the fact that France—because of a relatively early onset of its demographic transition—does not fit well into the explanations of other unified growth theories.

6. Extensions and Generalizations

In this section, we check the robustness of the results of the basic framework proposed in Section 2. More specifically, we extent and generalize the simplest unified growth theory in four

[13]A simple regression analysis shows that negative correlation between the relative price of food and the birth rate is associated with an R^2-value of 40 percent for England, 67 percent for France, 12 percent for Germany, and 54 percent for Sweden. However, more substantial time-series analyses need to be carried out in order to appraise these conclusions. Such analyses, however, lie outside the scope of the present paper.

different directions. These concern (i) returns-to-labor in manufacturing; (ii) Engel's Law and the demand for food; (iii) the income-effect on the demand for children; and (iv) the long-run predictions of the basic model.

The generalizations imply that the simplest unified growth theory looses some of its simplicity. In particular, the extended model can no longer be solved by hand, or using a spreadsheet, meaning that numerical tools are needed in order to obtain a solution. Since this is true for each of the four generalizations, they will be introduced simultaneously (although their effects will be discussed separately).

The first generalization concerns returns-to-labor in the production of manufactured good. We want to demonstrate that the constant returns-to-labor assumption of the basic framework can be replaced by diminishing returns without affecting the model's qualitative conclusions. Hence, we replace (6) with a generalized production function for manufactured goods, which is given by

$$Y_t^M = \delta M_t^{\phi} \left(L_t^M\right)^{\lambda} = M_{t+1} - M_t, \quad 0 < \phi < 1, 0 < \lambda \leq 1. \tag{16}$$

It follows that the size of the parameter λ determines whether there is constant ($\lambda = 1$) or diminishing ($0 < \lambda < 1$) returns-to-labor in manufacturing.

As for the second extension, we want to introduce a less restrictive demand for food. In the basic model, children required a fixed amount of food for survival. This assumption complies with one aspect of Engel's Law, namely that the share of food expenditures to the total income falls with rising income. However, Engel's Law also permits that *total* food expenditure (and thus the total costs of children) increases with rising income, as long as the *share* of income spent on foodstuff goes down. In the interest of keeping most of model's original structure and tractability, we thus reformulate the household budget constraint, formerly given by (3), so that it now reads

$$w_t = p_t n_t (a + b w_t) + m_t, \quad a > 0, b \geq 0. \tag{17}$$

Note that the parameter a can be thought of as a 'subsistence' food consumption level, i.e. the number calories required to ensure the survival of a child.[14]

The third extension concerns the claim that quasi-linear preferences are used for tractability only, and that such preferences are not essential for the qualitative nature of the results to

[14]Alternative ways in which to include Engel's Law are provided by Strulik (2008) and Dalgaard and Strulik (2008).

hold. As was indicated in Section 4, a main feature of the preference function required for the basic results to hold is that the elasticity of the marginal utility is higher for children than for manufactured goods. A parameterization of a more general utility function that fulfils this condition (and is still in keeping with the ideas of the basic framework) would be given by

$$u_t = \gamma \ln n_t + \frac{m_t^{1-\sigma}}{1-\sigma}. \tag{18}$$

Having kept the log-form for fertility, we need the restriction that $\sigma < 1$ to establish our so-called hierarchy of needs.[15] By contrast to the basic framework, and in addition to the negative price-effect, using (18) there will be a direct (and positive) effect of income growth on fertility.

As will become apparent below, the first three model extensions described above will influence on the shape of the adjustment dynamics, but will have no effect on the qualitative nature of the model's results. The final extension, however, is deliberately made, so as to fundamentally change the results of the basic framework.

In the basic framework, productivity growth was directly linked to growth in the size of the population. Meanwhile, we want to show that the basic model can be extended, so that it matches the results of the existing endogenous growth literature, in which persistent growth of income per capita is a long-run feature. To this end, suppose that parents have a minimum number of children, symbolically denoted $\bar{n}$, that they would like to bring up. Rewriting (18) in order to include this, the extended utility function now reads

$$u_t = \gamma \ln(n_t - \bar{n}) + \frac{m_t^{1-\sigma}}{1-\sigma}. \tag{19}$$

which replaces (2) from the basic framework.

The new first-order condition for maximizing (19) with respect to (17) can be condensed into $\gamma = p_t(n_t - \bar{n})m_t^{\sigma}(a + bw_t)$. Together with (16), this provides the demand for children, as well as for manufactured goods. However, combining this with the equilibrium conditions for the goods market and the factor market, equations (1) and (5), we can no longer obtain a closed-form solution for fertility. Rather, the rate of fertility, the relative price of food, and the share of

[15]The restriction on σ could be relaxed by introducing a general iso-elastic form for utility for children, with elasticity of, say, σ_n, and then requiring $\sigma < \sigma_n$.

workers employed in agriculture are now jointly determined in the following set of equations:

$$\frac{p_t n_t a}{\theta_t(1 - b p_t n_t)} = \delta M_t^{\phi}(1-\theta)^{\lambda-1} L_t^{\lambda-1} \tag{20a}$$

$$p_t(n_t - \bar{n}) \left(a + b\frac{p_t n_t a}{\theta_t(1 - b p_t n_t)}\right) = \gamma \left(\delta M_t^{\phi}(1-\theta_t)^{\lambda} L_t^{\lambda-1}\right)^{\sigma} \tag{20b}$$

$$\frac{n_t L_t^{1-\alpha}}{\mu A_t^{\epsilon}} \left(a + b\frac{p_t n_t a}{\theta_t(1 - b p_t n_t)}\right) = \theta_t^{\alpha}. \tag{20c}$$

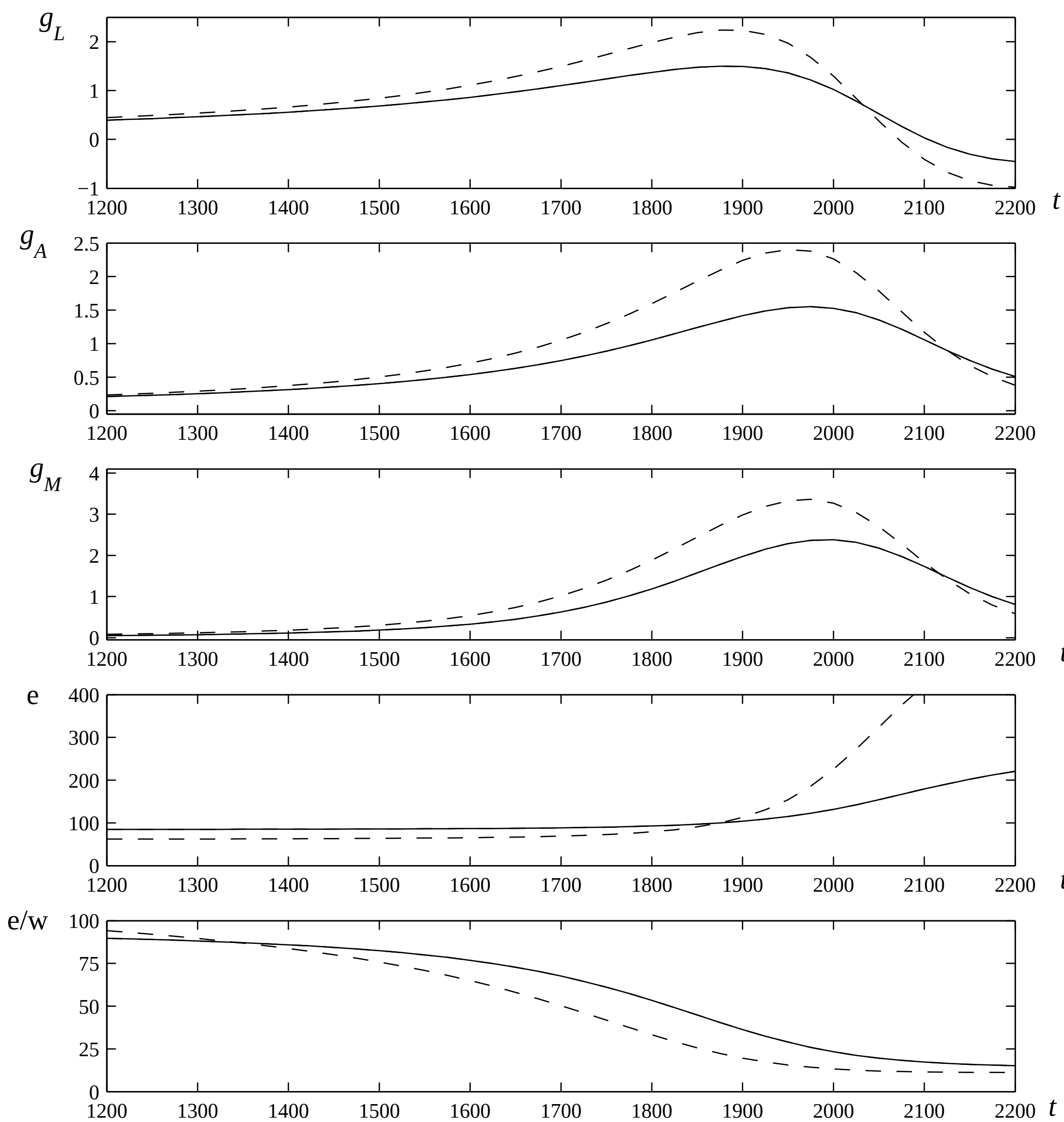
In the basic framework, it was implicitly assumed that $a = \lambda = 1$ and that $b = \sigma = \bar{n} = 0$. Under these assumptions, the above system of equations transforms into equations (4), (8), and (10) of the basic model. As for the general case, however, this can be solved numerically for the three unknowns, p_t, n_t and θ_t, for any given values of state variables L_t, A_t and M_t. The state variables themselves evolve according to (1), (5), and (16). Thus, we can solve numerically for the evolution of the economy for any given start values of L_0, A_0 and M_0.

What is to be expected from the extensions and generalizations introduced above? According to (20b), the introduction of $\sigma \in (0,1)$ allows for changes in income to impact on fertility. The direct income-effect on fertility, which was absent in the basic framework, is small at early stages of development, but is increasing with rising levels of income and with industrialization. Furthermore, the higher is σ, the more sluggish is the population growth slowdown during the second phase of demographic transition. In other words, the slowdown of TFP growth during the second phase of the demographic transition thus happens more quickly the smaller is σ.

Quite the opposite is the case when food (and thus child) expenditures become income-dependent. Having $b > 0$ instead of $b = 0$ makes children relatively more expensive, when income per capita is high. This means that adults living in relatively rich (i.e. industrialized) countries face a relatively high cost per child. It follows that an increase in b has a relatively modest effect on fertility at low income levels (i.e. before industrialization) when the costs of subsistence food consumption, a, have a dominant impact on fertility.

The two modifications discussed thus far (i.e. the introduction of σ and b different from zero) do not affect the qualitative (and hardly even the quantitative) results of the model. This is confirmed by Figure 6, where the solid lines show a case where the new parameters are given by $\sigma = 0.5$, $a = 0.9$, $\gamma = 3$ and $b = 0.1/\gamma = 0.033$. Note that the value of γ is slightly different from the original calibration; it is merely adjusted, so that the population growth rate peaks at 1.5 percent, which was also the case in the previous calibration. The remaining parameter values are kept intact from the benchmark specification. Finally, the parameter b is adjusted, so that the income-share of food expenditures converges to 10 percent from above, when income goes to infinity.

Figure 6: Long-Run Dynamics: Generalizations



Solid lines: $\sigma = 0.5$, $a = 0.9$, $b = 0.033$, $\gamma = 3$, all other parameters as for the benchmark case. Dashed lines: additionally $\lambda = 0.95$ From top to bottom the diagrams show population growth, productivity growth in agriculture, productivity growth in manufacturing, food expenditure, and the income share of food expenditure.

Comparing Figure 1 to Figure 6, the new transitional path is largely similar to that of the basic model. The most striking difference is the 'undershooting' behavior of the population growth rate: by the end of demographic transition, the population growth rate (i.e. g^L) approaches zero from below, resulting in a negative rate of growth of population by the 22nd century. On a whole, the adjustment dynamics for the population growth rate looks more like a 'turned J' than an 'inverted U', comprising a long and slow increase in fertility during the first phase of the transition, and then a comparably short and fast decrease in fertility during the second phase.

The fourth and fifth panel of Figure 6 illustrate the development of food expenditures, both in absolute terms, measured by e (with normalization $e_{1875} = 100$), and as a share of income, measured by e/w. In line with the empirical evidence on Engel's Law (see Grigg, 1994), we observe that food expenditures as a share of income start to fall long before the 'industrial revolution' sets in, and that it is down to just 50 percent by the mid-19th century. Eventually, food expenditures' share of income converges to the pre-fixed 10 percent.

Note also that there is relatively little movement in food expenditures, when measured in absolute terms, before the 19th century. After 'industrialization' sets in, food expenditures increase substantially. Yet, the model underpredicts the empirical increase in food expenditures per capita, which seems to have been much higher (*ibid.*). As already explained, this shortcoming can be attributed to the neglect of factor accumulation, the lack of which causes slower income growth, and, therefore, slower growth of food expenditures.

Next, we analyze the results of introducing diminishing instead of constant returns-to-labor in manufacturing. Hence, rather than $\lambda = 1$, we now set $\lambda = 0.95$. All other things being equal, the reduction of labor productivity in manufacturing, resulting from the introduction of diminishing returns-to-labor, downscales the performance of this sector relative to that of agriculture. This means that the relative price of food is comparatively lower, and fertility therefore higher, when diminishing returns-to-labor apply to manufacturing.

Indeed, diminishing returns in manufacturing involve two opposite effects on development. On the on hand, a higher rate of population growth tends to slow down the process of development, which postpones the 'industrial revolution' (a 'Malthusian' effect). On the other hand, once industrialization gains momentum, rates of productivity growth are higher than in the case of constant-returns; this arises from the 'demographic dividend' result discussed in Section 4.

For better readability of the results, the timing of events is normalized, so that the peak of the demographic transition is reached in 1870 with both constant and the diminishing returns-to-labor. The results are illustrated in Figure 6, where the solid lines represent constant returns-to-labor, and the dashed lines diminishing returns.

While the time-normalization tends to blur the slowdown of development (the 'Malthusian' effect), the Figure clearly demonstrates the scale-effect on productivity growth of having higher population levels (the 'demographic dividend' effect). However, during the second phase of demographic transition, the industrial sector expands more rapidly, leading ultimately to a faster reduction in rates of population growth, and therefore also to a stronger 'undershooting' behavior (i.e. negative population growth) in the first centuries of the new millennium.

The final element is the introduction of a minimum number of children, $\bar{n} > 1$. How does this modification affect the results of the model? For expositional purposes, we return in the following to the basic specification of the utility function, where $b = \sigma = 0$. This means that the optimal rate of fertility now reads $n_t = \bar{n} + \gamma/(ap_t)$. Thus, it follows that we disable one of the main mechanisms of the basic model, as population growth (instead of going to zero) now converges to $g^L = \bar{n} - 1$ for $p_t \to \infty$.

When considering the 'double' knife-edge condition, $\bar{n} = 1 = \phi$, the long-run results of the model compares with endogenous growth models of the first generation (based on Romer, 1990). The population level is stagnant, and persistent economic growth is made possible by constant returns to existing knowledge in the production of new knowledge.

For the more general case, where $\bar{n} > 1$ and $0 < \phi < 1$, the long-run results become identical to endogenous growth models of the second generation (based on Jones, 1995). With diminishing returns-to-knowledge (i.e. with $0 < \phi < 1$), persistent growth of productivity and income appears only in the case of persistent population growth. In the following, we explore this case in more detail.[16]

Balanced growth, in combination with positive population growth, over the long-run implies that the parameter values must be set, so that the industrial sector expands at a faster rate than

[16] The condition for balanced growth could be further relaxed, if we were to allow for knowledge-spillovers between the two sectors, as is the case in endogenous growth models of the third generation (see, for example, Strulik, 2005). However, since the sophistication of endogenous growth is not the focus of the present paper, this case is not pursued further.

that of agriculture. In the opposite case, food prices would be driven to zero, leading ultimately (as was discussed in Section 4) to exploding growth.

Along the balanced growth path, we now have that $1 + g^M = \left(1 + g^L\right)^{\lambda/(1-\phi)}$ and that $1 + g^A = \left(1 + g^L\right)^{\alpha/(1-\epsilon)}$. Balanced growth thus requires that

$$\frac{\lambda}{1-\phi} > \frac{\alpha}{1-\epsilon}. \tag{21}$$

In the standard-type endogenous growth models (e.g. Jones, 1995), there is no endogenous population growth. Furthermore, there is no agricultural sector, which means that (21) collapses into $\lambda/(1-\phi) > 0$. Jones (2001) does a calibration of his 1995-model, where he allows for endogenous population growth. Although he uses a different mechanism to explain fertility behavior, and even though he includes no agricultural sector (and, therefore, has no structural transformations), his calibration compares nicely with that of our extended model.

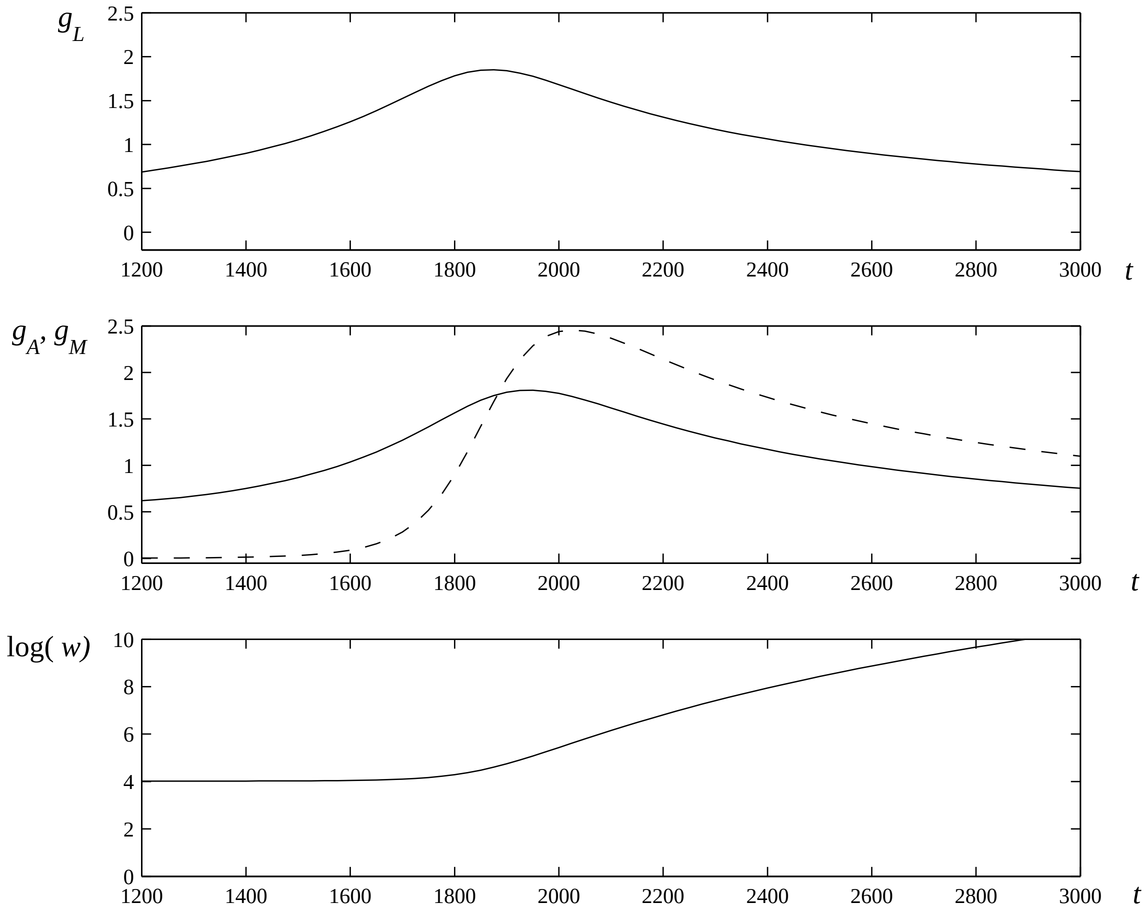
If food prices are increasing persistently, then the growth rate of population converges to $g^L = \bar{n} - 1$, while productivity growth in manufacturing converges to $\bar{n}^{\lambda/(1-\phi)}$. The rate of growth of income per capita is positive, yet lower than that of the industrial sector. The reason for this is that the agricultural sector is subject to diminishing returns, and that it needs to constantly expand its production in order to feed an ever increasing population.

Figure 7 demonstrate the trajectories for the case where $\bar{n} = 1.04$. The remaining parameter values are identical to those of the basic calibration in Section 4. The third panel illustrates the development of the log of income. Industrialization is synonymous to an endogenous structural break, after which income (which is largely constant before the 19th century) begins to grow (almost at a constant rate). The growth rate of productivity for manufacturing now peaks in the first half of the 21th century (or three generation later than in the basic model). However, even by the turn of the third millennium, its growth rate still exceeds one percent per year. The growth rate then slowly converges to a steady-state level of 0.22 percent annually.

These relatively high rates of productivity growth are made possible only because of a slowly converging rate of population growth. For the year 2100, the parameterization of the model predicts a population level five times that of the year 2000. By the year 2500, the population level has increased 500 fold compared to its year-2000 level.

Meanwhile, the Malthusian idea—that positive population growth (no matter how small) cannot persist forever—might still be valid, making the above scenario subject to suspicion.

Figure 7: Growth in the Very Long-Run: Year 1200 to 3000



As basic model except $\bar{n} = 1.04$. Second panel: solid lines: g_A, dashed lines: g_M.

If so, then we need not, as has been done in previous growth models, impose a constant rate of growth of population over the long run, or assume a particular set of parameter values that prevent an ever growing population. Instead, we can simply remove the assumption of minimum fertility. This will bring us back to the simplest unified growth theory (our basic model presented in Section 2). This model predicts that, as agricultural products get scarcer relative to manufactured goods, and when the population level, and thus the aggregate food demand, goes up, a price mechanism automatically ensures convergence towards a stable population.

7. Conclusion

This paper provides a unified growth theory, i.e. a model consistent with the very long-run economic and demographic development path of industrialized economies, stretching from the

Malthusian era to the present-day and beyond. Making strict use of Malthus' so-called *preventive check* hypothesis—that fertility rates vary inversely with the price of food—the current study offers a new and straightforward explanation for the demographic transition and the break with the Malthusian era.

Existing unified growth theories focus on human capital accumulation and a trade-off of child-quantity for child-quality as the driving force behind the demographic transition. The present study offers an alternative explanation, which nicely complements the ideas proposed in the existing literature, especially Galor and Weil (2000) and Kremer (1993).

Employing a two-sector framework with agriculture and industry, we demonstrate how fertility responds differently to productivity and income growth, depending on whether it emerges in agriculture or industry: agricultural productivity and income growth makes food goods, and therefore children, relatively less expensive. Industrial productivity and income growth, on the other hand, makes food goods, and therefore children, relatively more expensive.

In the basic model, an agricultural revolution is succeded by an industrial revolution, causing initial increase in fertility, followed by substantial decrease (a demographic transition). These events occur endogenously, after which the economy converges to a balanced growth path, along which there is zero population growth and zero (exponential) economic growth. We show some evidence in support of the main predition of the simplest unified growth theory: that fertility vary inversely with the relative price of food, and we demonstrate, by extending the model, that it is capable of matching the results, not only of existing unified growth theories, but also the endogenous growth literature in general.

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