How Child Costs and Survival Shaped the Industrial Revolution and the Demographic Transition*

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Abstract. This study provides a unified growth theory to correctly predict the initially negative and subsequently positive relationship between child mortality and net reproduction observed in industrialized countries over the course of their demographic transitions. The model captures the intricate interplay between technological progress, mortality, fertility and economic growth in the transition from Malthusian stagnation to modern growth. Not only does it provide an explanation for the demographic observation that fertility rates response with a delay to lower child mortality. It also identifies a number of turning points over the course of development, suggesting a high degree of complexity regarding the relationships between various economic and demographic variables.

Keywords: Economic Growth, Mortality, Fertility, Structural Change, Industrial Revolution.

JEL: O11, O14, J10, J13.

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1. Introduction

In recent years, several *unified growth theories* have been forwarded to try to motivate the historical shift from economic stagnation to modern growth. Following the seminal work by Galor and Weil (2000), the contributions include Boucekkine et al. (2002), Doepke (2004), Galor and Moav (2002), Galor and Mountford (2008), Jones (2001), Kögel and Prskawetz (2001), Lucas (2002), Strulik and Weisdorf (2008), Cervellati and Sunde (2005, 2007) and Tamura (2002). A part of this exercise consists of providing a micro foundation to motivate the fundamental relationships between economic and demographic variables, from pre-industrial times to the present day. One particular issue that scholars have been struggling with for a long time (although not only in the context of unified growth theory) is the impact of lower child mortality on fertility and net reproduction.[1] Most macroeconomic models are able to replicate the fact that a lower death risk of children leads to fewer births. However, since falling child mortality reduces the cost (price) of obtaining surviving offspring, the net rate of reproduction in these models ends up *increasing* in response to lower child mortality. That feature, however, is in contrast to the experience of most industrialized countries in the later part of their demographic transitions where the net rate of reproduction usually falls (Doepke 2005).

This study provides a model where the relationship between child mortality and net reproduction is positive during the early stages of development, but then turns negative during the later, more advanced, stages. This notion, which builds on the fact that fertility, and thus net reproduction, is affected not only by child mortality itself, but also by the cost of raising children, arises from combining two existing contributions: Strulik (2008) and Weisdorf (2008). First, inspired by Strulik (2008) we assume that parents care about surviving offspring as well as their nutritional status. For a given level of nutritional input, a drop in the child mortality rate brings parents to reduce their fertility, as more children now survive. At the same time, a higher survival probability causes parents to nourish their children better (a quantity-quality substitution effect). That, in turn, further improves the offsprings' survival probability, which again lowers fertility, and so on and so forth.[2] Second, inspired by Weisdorf (2008) we employ the fact that, in a closed economy, the cost of food goods (and thus of children) relative to other consumption goods, such as manufactured goods, is

[1]Net reproduction measures the number of offspring (normally women) living through to the end of their fertile age.
[2]This is consistent with empirical evidence which suggests that nutrition played a key role in Britain's mortality transition (Harris 2004), and that malnutrition has severe effects on child mortality (see Rice et al., 2000, Pelletier et al., 2003, and Caulfield et al., 2004).

affected by the growth of productivity in agriculture relative to that of industry. The effect from industry works though the labor market equilibrium condition. That is, when productivity in the industrial sector grows, industrial workers receive a higher wage rate. In a closed economy with free labor mobility, this means that agricultural terms of trade—i.e., the cost of food goods relative to manufactured goods—will have to adjust to insure that workers remain in agriculture. Therefore, if parents derive utility from children as well as manufactured goods, then a shift in the relative price of the two types of commodities (i.e. in the agricultural terms of trade), caused by a change in the productivity of industry relative to that of agriculture, will affect the parental decision regarding how many children to give birth to.

The key to understanding why the relationship between mortality and net reproduction changes over time, therefore, comes in the fact that fertility is influenced *both* by changes in mortality *and* by technological advancements in industry relative to agriculture. Below, we use these features to construct a unified growth theory that correctly predicts the relationship between child mortality and net reproduction over the course of a demographic transition. The theory accounts for the intricate interplay between technology, mortality, fertility and income per capita in the process from stagnation to growth. The theory is also in line with Engel's law, replicating the stylized fact that, as income rises, total food expenditures increase but the share of food expenditures to income falls. We show that the model is robust to different specifications regarding the type of preferences assigned to parents; to various assumptions concerning the sign of partial derivatives; and to the introduction of a cost of rearing children. When calibrated, the model points to several non-linearities in the relationship between variables, such as fertility and population growth, as well as population growth and TFP growth. This may explain why scholars studying the empirical relationships between economic and demographic variables in the long term have a hard time identifying their correlation.

The paper continues as follows. Section 2 provides a brief introduction to the stylized facts for Western Europe (particularly England) regarding the evolution of mortality, fertility and net reproduction over the long run. It also offers a summary of the theoretical literature related to the present work. Section 3 details the theoretical framework, and Section 4 explores its balanced growth dynamics. Section 5 calibrates the model and analyzes adjustment dynamics. Section 6 discusses extensions of the simple model towards a generalized utility function and the consideration of time costs of children. Section 7 concludes.

2. The empirical evidence and the related literature

In most Western European countries, the demographic transition occurred in the later half of the 19th century.[3] After a peak in the 19th or early 20th century, the birth rate in most countries dropped by roughly one-third over the subsequent 50 years. In England, the total fertility rate declined by close to 50 percent, from nearly five children per women in 1820 to 2.4 children by 1920. The crude birth rate followed a similar pattern, declining by 44 percent, from 36 per thousand inhabitants in 1820 to 20 in 1920 (see Figure 1).

With the exception of France and the US, a substantial mortality decline preceded the fall in fertility. In England, the mortality rate began to fall roughly one and a half century prior to the decline in fertility. During the early phases of England's so-called *mortality revolution*, the fall in mortality was primarily driven by lower child mortality. Since the fall in the child mortality rate occurred prior to the fall in fertility, falling mortality initially caused an increase in the net rate of reproduction. After the onset of the fertility decline, however, the nature of the relationship between mortality and net reproduction gradually changed. Falling child mortality was eventually outpaced by the decline in fertility, after which falling mortality was accompanied by falling rates of net reproduction.

Sources: Wrigley and Schofield (1984), Wrigley (1969) and Reher (2004).

Figure 1: Crude Birth Rates, Crude Death rates, and Net Reproduction Rates for England, 1721-1931

A unified growth theory that wants to explain the long-run evolution of mortality, fertility and net reproduction, therefore, has to account not only for the increase of net reproduction and the spike in birth rates observed prior to the fertility decline. It also needs to be able to motivate the subsequent decline in rates of fertility and net reproduction after the fertility transition sets in.

Economic studies on the relationship between child mortality and fertility go back at least to Becker and Barro (1988, 1989). In the Barro-Becker model, parents derive utility from surviving offspring. Their fertility decision is affected by the costs of producing surviving offspring. Lower child mortality reduces the costs of surviving offspring, so when the child mortality rate falls it induces parents to give birth to more children. In combination, the falling rate of child mortality and the rising rate of fertility lead to a higher rate of net reproduction. But since the net rate of reproduction was falling in the later stages of the demographic transition, the Barro-Becker type models are forced to come up with other explanations for the fertility decline in addition to falling child mortality in order to motivate falling net reproduction (Doepke, 2005).

[3] For a more detailed description of the Western Europe's economic and demographic patterns, see Galor (2005).

Among the recent studies, in which mortality plays a role in the process of development, a falling net rate of reproduction is motivated by a shift from child quality to child quantity, generated through parental investments in their children's education (Azarnert 2006; Ehrlich and Kim 2005; Kalemli-Ozcan, Ryder, and Weil 2000; Kalemli-Ozcan 2008, Soares 2005). Later refinements invoke a shift from exogenous to endogenous mortality in order to capture the long-run trends in economic and demographic variables. (Doepke 2005; Jones 2001; Hazan and Zoabi, 2006; Kalemli-Ozcan 2002, 2008; Lagerlöf 2003; Weisdorf, 2004). Jones (2001), Lagerlöf (2003) and Weisdorf (2004) compare directly to our work in that they analyze the effect of child mortality on fertility in the context of unified growth theory. Remarkably, they all share the feature that falling death rates have no impact on the parental decision regarding the number of births. In Jones (2001), the preferences of parents imply that the elasticity of substitution between consumption goods and children is always greater than one, an assumption that ultimately generates a drop in fertility as income grows. In Lagerlöf (2003) and Weisdorf (2004), the decline in fertility is a result of human capital accumulation, i.e. a parental trade-off effect from child quantity and quality.

Here, we set out to explore the direct influence of child mortality on the parental fertility decision. We do so by extending the unified growth theory proposed by Strulik and Weisdorf (2008). While our previous work neglects the role of death in development, the mortality rate in the current set-up is made endogenous—that is, the child mortalty rate is influenced partly by the level of nutrition that parents decide to provide for their offspring, and partly by general health factors (such as the availability of medicine, the provision of sewage etc), all of which are assumed to be exogenous to parents. When the general health factors improve, captured in the model by technological progress taking place in the manufacturing sector, parents find it advantageous to allocate more resources to nurish their children. This induced child quantity-quality trade-off provides a link from economic to demographic factors. This makes our work comparable to a study by Cervellati and Sunde (2007), who also investigate the linkage between mortality and fertility over the very long run. They focus on the interaction between education and adult longevity as a driving force behind economic growth, emphasizing the skill premium as a key factor in the transition from stagnation to growth. Their work is particularly comparable to ours in the sense that they, too, highlight the role of relative prices in development.

3. The Model

3.1. **Fertility, mortality, and net reproduction.** We consider a two-period overlapping generations economy with children and adults. Let L_t denote the number of adults in period t, and n_t the number of births per adult.[4] The birth rate (referred to also as the fertility rate) is determined endogenously below. The variable $\pi_t \in [0, 1]$ measures the survival probability of offspring. By the application of the law of large numbers, the variable π_t also measures the fraction of children who are born in period t and are still alive in period $t+1$. The net rate of reproduction, i.e. the number of offspring reaching the fertile age, is thus $\pi_t n_t$. The changes in the size of the labor force (identical to the size of the adult population) between any two periods can thus be expressed as

$$L_{t+1} = \pi_t n_t L_t. \tag{1}$$

We consider two types of child survival probabilities: an *extrinsic* and an *intrinsic* survival rate. The extrinsic rate, denoted $\bar{\pi}_t \in [0, 1]$, is exogenous to parents. Potentially, it could be affected by the geographic location of the family (its disease environment), the medical knowledge available, and the degree of urbanization. By contrast, the intrinsic survival rate can be influenced by parents through the amount of nutrition that they decide to invest in their offspring. Let h_t denote food expenditure per child. If we ignore the modern-day problem of obesity, which seems relevant in the present long-term context, then it seems reasonable to assume that food expenditures have a positive effect on the survival rate of offspring, but with diminishing returns.

Whether better nutrition is more effective when extrinsic survival is high or low, however, is an unsettled issue in the literature. Standard evolutionary arguments based, for example, on antagonist pleiotropy or disposable soma theory suggest that somatic investment (nutrition) is less effective in increasing survival when extrinsic survival rates are low (see e.g. Williams, 1957, Stearns 1992). That would suggest a positive cross derivative, i.e. $\partial^2 \pi_t / (\partial h \partial \bar{\pi}_t) > 0$. On the other hand, recent empirical evidence in biology (Williams and Day 2003) indicates that somatic investment does indeed become *more* effective as the extrinsic survival rates decrease, which would suggest a negative cross derivative, i.e. $\partial^2 \pi_t / (\partial h \partial \bar{\pi}_t) < 0$. The latter assumption has been introduced into the nutrition-based, long-run growth theory by Strulik (2008), and into evolutionary growth theory by Galor and Moav (2005). In light of these conflicting views, we leave the sign of the cross derivative generally undetermined and discuss its impact on comparative dynamics later on. A functional form, which

[4] To keep the model tractable, we assume that n_t is continuous, and that reproduction is asexual.

is simple enough to derive straightforward explicit solutions, is given by (2).

$$\pi_t = g(h_t, \bar{\pi}_t) = \frac{\lambda h_t \bar{\pi}_t}{h_t + \bar{\pi}_t + \nu}. \tag{2}$$

The symbols λ and ν are parameters measuring the influence of nutrition on survival probability, as well as the curvature of the survival function. We assume that $\lambda > 0$ and that $\nu > -\bar{\pi}_t$ for all t. The important derivatives are:

$$g_h \equiv \frac{\partial \pi_t}{\partial h_t} = \frac{\lambda \bar{\pi}_t(\nu + \bar{\pi}_t)}{(h_t + \bar{\pi}_t + \nu)^2} > 0, \quad g_{hh} \equiv \frac{\partial^2 \pi_t}{\partial \bar{\pi}_t^2} = -\frac{2\lambda \bar{\pi}_t(\nu + \bar{\pi}_t)}{(h_t + \bar{\pi}_t + \nu)^3} < 0,$$
$$g_{h\bar{\pi}} \equiv \frac{\partial^2 \pi_t}{\partial h_t \partial \bar{\pi}_t} = \frac{\lambda \left[2\bar{\pi}_t h_t + \nu(h_t + \bar{\pi}_t + \nu)\right]}{(h_t + \bar{\pi}_t + \nu)^3}.$$

It follows that nutritional input has a positive effect on the survival probability of children, but with diminishing returns. Since $\bar{\pi}_t$ enters $g(h_t, \bar{\pi}_t)$ in a symmetric way, we also conclude positive and diminishing returns of improving extrinsic survival. The sign of the cross-derivative, however, is undetermined. In particular, we have $g_{h\bar{\pi}} < 0$ for $-\bar{\pi} < \nu < -2\bar{\pi}\beta/\gamma$ and $g_{h\bar{\pi}} > 0$ for $\nu > -2\bar{\pi}\beta/\gamma$. In the calibration section this will allow us to investigate the impact of different cross-derivatives on the adjustment dynamics predicted by the model concerning the demographic transition and the process of industrialization.

3.2. **Preferences and optimization.** Adult individuals maximize utility, which is derived from three sources: surviving offspring, $\pi_t n_t$, the nutritional status of their offspring, h_t, and number of manufactured goods consumed, m_t. We assume that preferences are described by a utility function, in which the elasticity of marginal utility is higher for h_t and n_t than for m_t. This serves to capture the fact that parents, in times of crisis, will try to smoothen out their fertility, as well as the nutritional status of their offsprings, *more* than their consumption of other less vital (i.e. manufactured) goods (Livi-Bacci, 2006). The simplest utility function that captures such a 'hierarchy of needs' is of quasi-linear form as given by (3).

$$u_t = m_t + \beta \log(h_t) + \gamma \log(\pi_t n_t), \quad \gamma > \beta > 0. \tag{3}$$

A quasi-linear function allows us to obtain an explicit and analytically simple solution to the optimization problem of parents. In Section 6, we will generalize the utility function with the purpose of investigating numerically the robustness of the model in this regard. Furthermore, similar to Andreoni (1989) and Becker (1960) the parameter β measures the extent to which parents care about the nutritional status of their offspring. We will make the assumption that $\gamma > \beta$, which implies that parents without any children do not allocate income to child nutrition. In order to make the model tractable, we assume that nutritional goods are demanded only during childhood, and some of it is then stored for adulthood.[5] The price of manufactured goods is set to one, so that p_t denotes the price of one unit of nutrition (food), measured in terms of manufactured goods. We will let children who suffer child mortality perish at the end of the childhood period. This means that every child born consumes h_t units of nutrition, and that the total costs of raising n_t children, measured in terms of manufactured goods, is $p_t h_t n_t$. The budget constraint of an individual adult thus reads

$$w_t = p_t h_t n_t + m_t, \tag{4}$$

where w_t is parental income (also measured in terms of manufactured goods). In Section 6, we expand the model to also include a cost of child rearing and then investigate numerically how such a construction affects the results.

Inserting (2) and (4) into (3) leads to the following optimization problem:

$$\max_{n_t, h_t} u_t = w_t - p_t n_t h_t - \beta \log h_t + \gamma \log n_t + \gamma \log g(\pi_t, h_t).$$

The first-order conditions for a maximum are:

$$\frac{\partial u_t}{\partial n_t} = -p_t h_t + \frac{\gamma}{n_t} = 0 \tag{5}$$

$$\frac{\partial u_t}{\partial h_t} = -p_t n_t + \frac{\beta}{h_t} + \gamma \frac{g_h}{g} = 0. \tag{6}$$

In Appendix A, we show, using the second-order conditions, that the solution of (5) and (6) is in fact a maximum. Multiplying (6) by h_t and subtracting it from (5) multiplied by n_t gives us (7).

$$\gamma \frac{g_h}{g} \cdot h_t = \gamma - \beta \qquad \Rightarrow \qquad h_t = \frac{(\gamma - \beta)}{\gamma} \frac{g}{g_h}. \tag{7}$$

It thus follows that, regardless of the specification of the survival function, the optimal amount of food expenditure per child, h_t, is independent of the relative price of food, p_t. This result builds on the fact that prices multiplied by quantities make up total food expenditure, $p_t h_t n_t$, and that the quantities n_t and h_t enter expenditures symmetrically. The price of food, however, has a negative effect on the number of children demanded. This can be seen from inserting (7) into (5), which

5 It will not affect the qualitative nature of the results if, instead, the individual's nutritional demand were to be divided over two periods.

gives us that $n_t = \gamma/(p_t h_t)$. That captures the well-known child quantity-quality trade-off; a high child quality (or, in the Beckerian sense, a higher expenditure $p_t h_t$ per child) implies a lower child quantity.

By inserting (2) into (5) and (6) and solving for fertility and food expenditure, it follows that:

$$n_t = \frac{\gamma(\gamma - \beta)}{p_t \beta(\bar{\pi}_t + \nu)} \tag{8}$$

$$h_t = \frac{\beta(\bar{\pi}_t + \nu)}{\gamma - \beta}. \tag{9}$$

Note that the optimal expenditure per child, h_t, is a positive function of the extrinsic child survival rate $\bar{\pi}_t$ regardless of the sign of the cross-derivative. The key feature generating this result is that the survival elasticity of food expenditure is a positive function of the extrinsic survival rate, and a negative function of the food expenditures. This, in turn, follows from the facts that (i) the survival rate π_t is a positive function of the extrinsic survival rate $\bar{\pi}$; and that (ii) there are diminishing returns of food expenditure. To see this more clearly, note from (7) that parents prefer a constant survival elasticity of food expenditure, $\epsilon_h \equiv g_h \cdot h/g$, that is $\epsilon_h = (\gamma - \beta)/\gamma$. By inserting (2), we obtain that $\epsilon_h = (\bar{\pi} + \nu)/(\bar{\pi} + h + \nu)$ with $\partial\epsilon_h/\partial\pi > 0$ and $\partial\epsilon_h/\partial h < 0$. Thus, the optimal response to an improving extrinsic survival rate is to increase food expenditure. It should be noted that the parental preference for a constant survival elasticity is not driven by a particular specification of the survival function. Rather, it stems from the fact that child quality and quantity enter the utility function in a symmetric way (with the weights of β and γ).

From the demand function for offspring (8) we conclude that children are normal goods: the demand for children falls when the price of children rises. Due to the specific preference function, there is no direct income-effect on the demand for children. We will introduce a direct income-effect later on (in Section 7) by including time costs of child rearing. Note that an improvement in the extrinsic survival rate triggers a higher food expenditure, which improves the nutritional status of offsprings. This follows from the child quality-quantity trade-off effect which implies that an improvement in the extrinsic survival rate ($\bar{\pi}_t$) reduces the fertility rate (n_t) and increases the survival rate (π_t). It implies that there is a positive correlation between child mortality and fertility, a conclusion consistent with the empirical evidence (Galloway et al., 1998, Eckstein et al., 1999, Herzer et al., 2011).

The impact of the extrinsic survival rate on the *net* rate of fertility, i.e. on population growth, however, is a priori undetermined. By combining (8) and (9) with (2), we have that the net fertility rate is given by (10).

$$\pi_t n_t = \frac{(\gamma - \beta)\bar{\pi}\lambda}{p_t(\bar{\pi} + \nu)}, \qquad \frac{\partial(\pi_t n_t)}{\partial \bar{\pi}_t} = \frac{(\gamma - \beta)\lambda\nu}{p_t(\bar{\pi}_t + \nu)^2}. \tag{10}$$

It follows that the net rate of fertility is an increasing function of the influence of nutrition in reducing mortality, captured by the parameter λ.

The curvature parameter ν governs how the extrinsic survival rate affects the net fertility rate. Generally, higher $\bar{\pi}$ leads to higher net fertility directly and, indirectly, through the induced higher food expenditure and improved intrinsic survival (the survival effect) and it leads to lower net fertility through the induced reduction of births (the fertility effect). For the special case in which $\nu = 0$ the survival effect and the fertility effect balance each other and the model predicts that the net fertility rate is independent from the extrinsic survival rate. This is a standard result in Malthusian models with exogenous child mortality (see Galor, 2011, Ch. 4).

For $\nu > 0$ the model predicts – in line with recent evidence (Herzer et al. 2011) – that a lower child mortality rate leads to a lower birth rate and a higher net fertility rate. The survival effect dominates the fertility effect. While this prediction is shared with many other economic models of endogenous fertility (see Doepke, 2005), it appears to contradict the common view of demographers (see e.g. Cleland, 2001). Demographers normally emphasize that the demographic transition is triggered by falling mortality. Along the transition, the net rate of fertility eventually falls below its initial level (with a delay). The view of demographers suggests that the fertility effect dominates the survival effect. In the present model, we can capture this view by setting $\nu < 0$.

3.3. **Production.** We consider a dual-sector economy with agriculture and industry. In both sectors, new technology arises from learning-by-doing. More specifically, output, as well as new knowledge, occurs according to the following production functions:

$$Y_t^A = \mu A_t^{\varepsilon}(L_t^A)^{\alpha} = A_{t+1} - A_t, \quad 0 < \alpha, \varepsilon < 1 \tag{11}$$

$$Y_t^M = \delta M_t^{\phi} L_t^M = M_{t+1} - M_t, \quad 0 < \phi < 1. \tag{12}$$

The variable A_t measures TFP in agriculture, whereas M_t measures TFP in industry (manufacturing). The evolution of A and M arises from learning-by-doing, following Arrow (1962), Romer (1986), and – in a setup with endogenous fertility – Kremer (1993).

The learning-by-doing approach seems appropriate when investigating the technological and economic development throughout most of human history, simply because the bulk of technological improvements in history were not generated by scientists, at least not until after the mid 19th century (Mokyr, 2002). Since then, however, knowledge production has increasingly become a market activity, rendering the Arrow-Romer approach less appropriate. For this reason, we expect that the model loses some of it predictive power from the 20th century on.

Following Arrow (1962), we assume that there are diminishing returns to new knowledge in both sectors by demanding that $0 < \varepsilon, \phi < 1$. Agricultural production is subject to constant returns to labor and land. Land is assumed to be in fixed supply, and the total amount is normalized to one. With $0 < \alpha < 1$, there is thus diminishing returns to labor in agriculture. Industrial production, by contrast, is subject to constant returns to labor, implying that land is not an important factor in industrial production. As is standard in the related literature, we abstract throughout from the use of physical capital in production.

3.4. **Equilibrium.** The variables L_t^A and L_t^M measure the total amounts of labor input into agriculture and industry, respectively. Together, they make up the entire labor force, i.e.

$$L_t^A + L_t^M = L_t. \tag{13}$$

The share of workers devoted to agriculture, L_t^A/L_t, is determined by the market equilibrium condition for nutritional (i.e. food or agricultural) goods. The equilibrium condition says that the total supply of nutrition, Y_t^A, must be equal to the total demand, which—given that each child demands h_t units of food—is $h_t n_t L_t$. Using (11), the market equilibrium condition for nutrition thus implies that the fraction of workers necessary in agriculture to satisfy demand is given by (14).

$$\theta_t \equiv \frac{L_t^A}{L_t} = \left(\frac{h_t n_t L_t^{1-\alpha}}{\mu A_t^{\varepsilon}} \right)^{\frac{1}{\alpha}}. \tag{14}$$

Note that agricultural TFP growth is able to release labor from agriculture, while population growth and a higher level of nutrition per child have the opposite effect.

Suppose that there are no property rights over land, meaning that the land rent is zero, and thus that a representative adult individual receives the average product of the sector in which it is employed, $w_t^A = pY_t^A/L_t^A$, $w_t^M = Y_t^M/L_t^M$. The labor market equilibrium condition then implies that the real price of nutrition adjusts, so that agricultural and industrial workers are able to earn the same income, implying wages $w_t^A = w_t^M = w_t = p_t Y_t^A / L_t^A = Y_t^M / L_t^M$. By the use of (8)-(9) and (10)-(14) this means that the price of one unit of nutrition, measured in terms of manufactured goods, is

$$p_t = \frac{\left(\delta M_t^{\phi}\right)^{\alpha} (\gamma L_t)^{1-\alpha}}{\mu A_t^{\varepsilon}}. \tag{15}$$

It follows that the price of nutrition increases with TFP growth in industry, as well as with the size of the population, whereas TFP growth in agriculture has the opposite effect.

Inserting (15) into (10) gives us the net rate of reproduction in a general equilibrium, which is

$$\pi_t n_t = \frac{\mu A_t^{\varepsilon}}{\left(\delta M_t^{\phi}\right)^{\alpha} (\gamma L_t)^{1-\alpha}} \cdot \frac{(\gamma - \beta)\bar{\pi}_t \lambda}{\bar{\pi}_t + \nu}. \tag{16}$$

The two main forces affecting the net rate of reproduction are clearly visible in (16). The first term captures the negative effect from a higher price of nutrition. The second term contains the effect of a lower extrinsic child mortality rate, which may be positive or negative depending on the sign of ν.

Finally, it seems instructive to have a look at the determinants of the extrinsic mortality rate. Since nutritional expenditures influence only the intrinsic mortality rate, the determinants of $\bar{\pi}_t$ will include geography (i.e. the disease environment) as well as technical knowledge. As regards the influence of technical knowledge on mortality, we are inspired by Cutler et al. (2006, p. 116), who in their survey on the determinants of mortality conclude that "[k]nowledge, science and technology are the keys to any coherent explanation. Mortality in England began to decline in the wake of the Enlightenment, directly through application to health of new ideas about personal health and public administration, and indirectly through increased productivity". We capture this fact by assuming that advances in industrial knowledge, measured by M_t, are a good indicator of health-improving ideas including, for example, sewerage, water toilets, central heating, clinical devices, vaccines, pharmaceuticals, and medical knowledge in general. The role of geography works best if we treat it as a constant.

The process of industrialization may have affected child survival probabilities also through increasing rates of urbanization. For England and Wales, for example, Wrigley et al. (1997) document that falling child mortality occurred primarily in *rural* areas around the time of the industrial revolution. By contrast, in cities such as London child mortality initially increased as a result of the crowded living conditions that urban life entailed such that urban areas experienced a mortality revolution later than it was the case in the English countryside. We capture this effect by taking the share of manufacturing, i.e. the share of workers not occupied with agriculture, measured by $1-\theta_t$, as a proxy for the rate of urbanization, and include a negative impact of $1-\theta$ on the extrinsic child survival rate. Specifically, we assume that $\bar{\pi}_t$ evolves according to (17):

$$\bar{\pi}_t = a \cdot \left[1 - \mathrm{e}^{-bM_t + d(\theta_0 - \theta_t)}\right]. \tag{17}$$

Here, the parameter a reflects the impact of geography (and other relevant constants) on the survival rate; b captures the power of knowledge in reducing number of deaths; and d captures the negative impact of urbanization $(1-\theta_t)$, relative to the initial degree of urbanization (at time 0), i.e. $(1-\theta_t)-(1-\theta_0)$. Note that, by construction, the share $1-\theta_t$ is limited from above, whereas knowledge grows forever. Hence, we expect to have only a *temporary* effect on mortality of increasing rates of urbanization and that this effect eventually becomes dominated by the positive impact on mortality of an increasing level of technical knowledge. In the limit, therefore, it follows that $\lim_{M_t\to\infty} \bar{\pi}_t = a < 1$.

4. Balanced and Unbalanced Growth in the Long Run

In the following, we explore the balanced-growth dynamics of the model. Along a balanced growth path, all the variables are constant or grow at constant rates. Let a balanced growth rate of a variable x be denoted by g^x (to be identified by a missing time index). According to (11), the gross rate of growth of TFP in agriculture is given by $g_t^A = (A_{t+1} - A_t)/A_t = \mu (L^A{}_t)^{\alpha} / A_t^{1-\epsilon}$. Along a balanced growth path, the left hand side is constant by definition, so that the right hand side must be constant as well. Furthermore, the share of labor in agriculture must be constant, implying that L^A grows at the same rate as L. Thus, a constant rate of growth of TFP in agriculture requires that

$$1 + g^A = (1 + g^L)^{\alpha/(1-\epsilon)}. \tag{18}$$

Similarly, we get from (12) that a constant rate of growth of TFP in industry requires that

$$1 + g^M = (1 + g^L)^{1/(1-\phi)}. \tag{19}$$

Taking first differences of (16) the rate of population growth can be written as

$$1 + g_{t+1}^L = \frac{\bar{\pi}_{t+1}(\bar{\pi}_t + \nu)}{\bar{\pi}_t(\bar{\pi}_{t+1} + \nu)} \cdot \frac{(1 + g_t^A)^{\epsilon}(1 + g_t^L)^{\alpha}}{(1 + g_t^M)^{\alpha\phi}}. \tag{20}$$

Along a balanced growth path, the level of TFP in industry is either constant or is growing at a constant rate. In either case, $\bar{\pi}$ will eventually assume a constant value, meaning that, along a balanced growth path, the first term on the right-hand side will be equal to one. Using this information, and inserting (18) and (19) into (20), we find that the equilibrium law of motion for the growth of population is given by

$$1 + g_{t+1}^L = (1 + g_t^L)^\eta, \qquad \eta \equiv \alpha + \frac{\alpha\epsilon}{1-\epsilon} - \frac{\phi\alpha}{1-\phi}. \tag{21}$$

Along a balanced growth path, the population grows at constant rate, meaning that $g_{t+1}^L = g_t^L = g^L$. This leaves two possibilities for balanced growth. Either there is no population growth ($g^L = 0$) or – assuming the knife-edge condition that $\eta = 1$ – the population is growing or shrinking at a constant rate. However, it follows from (18) that $|\eta| < 1$ is required for stability reasons. Therefore, the growth-on-the-knife-edge case not only demands a very specific parameter constellation; it also requires that the economy starts off *on* a balanced growth path (with suitable initial values) and remains on this path forever (which is impossible in the event of shocks). This essentially eliminates the possibility of having balanced growth in combination with population growth. The implication—that there is no population growth on a balanced growth path—means that there is also no TFP growth in the steady state (as can be verified by looking at (18) and (19)). This conclusion is summarized in the following proposition.

Proposition 1. *There exists a unique balanced growth path with zero population growth and zero (exponential) economic growth. A sufficiently small knowledge-elasticity in agriculture, i.e.*

$$\epsilon < \frac{1-\phi-\alpha+2\alpha\phi}{1-\phi+\alpha\phi}, \tag{22}$$

prevents the case of unbalanced growth in the long-run.

The proof is found in the Appendix.[6]

5. Long-Run Adjustment Dynamics

5.1. **Intuition.** The aim of this section is to test if the model is capable of replicating the stylized development pattern of an industrialized economy, from its pre-industrial era up to the present-day

[6] Note that the term ‘unbalanced growth’ refers to an equilibrium growth path along which growth rates are exploding or imploding (and should not be confused with ‘unstable’, i.e. off-equilibrium growth.

and beyond.[7] Before turning to explore the model's adjustment dynamics towards the balanced-growth path, we will briefly go over the intuition behind the development path leading to the steady state.

Suppose we start off with an economy in which the population level is relatively small; the share of labor employed in agriculture is relatively high; and the level of income per capita is relatively close to subsistence. Furthermore, suppose that birth rates as well as child mortality, are both relatively high, meaning that the net rate of reproduction is close to that of replacement. Roughly speaking, these are the characteristics of a pre-industrial, agricultural society.

As explained in the model section, there are economies-of-scale to population size. Since the initial population level is low, learning-by-doing effects, to begin with, are relatively modest. Hence, TFP growth in agriculture is rather slow, yet faster than in industry where labor resources, and thus learning-by-doing effects, are even smaller.

Growing TFP in agriculture has two effects on development. On the one hand, because it releases labor from agriculture, it increases the share of labor allocated to industrial activities. On the other hand, higher TFP growth in agriculture *relative to industry* makes nutrition, and therefore children, relatively less expensive. According to (8) this raises fertility, which tends to increase the net rate of reproduction.

At the same time, with economies-of-scale at work, both in agriculture and industry, the transfer of labor out of agriculture gradually speeds up TFP growth in industry. As the expansion of industrial knowledge gains momentum, the *extrinsic* child survival probability begins to increase. This leads parents to allocate more resources to nutrition per child, which further improves the survival probability of the offspring through a reduction in *intrinsic* child mortality.

It is clear that falling mortality increases the net rate of reproduction through the survival effect. Yet, at the same time falling mortality reduces fertility, which, in isolation, diminishes the net rate of reproduction, as follows from (16). However, as long as advances in industrial knowledge and thus in child mortality reduction are relatively slow, the 'cheaper nutrition' effect on the net rate of reproduction dominates the 'mortality decline' effect. Hence, during early stages of development, the net rate of reproduction rate will go up. In this period, therefore, declining child mortality is accompanied by rising rates of birth and net reproduction.

[7] For a detailed description of the development course of industrialized countries, see Galor (2005).

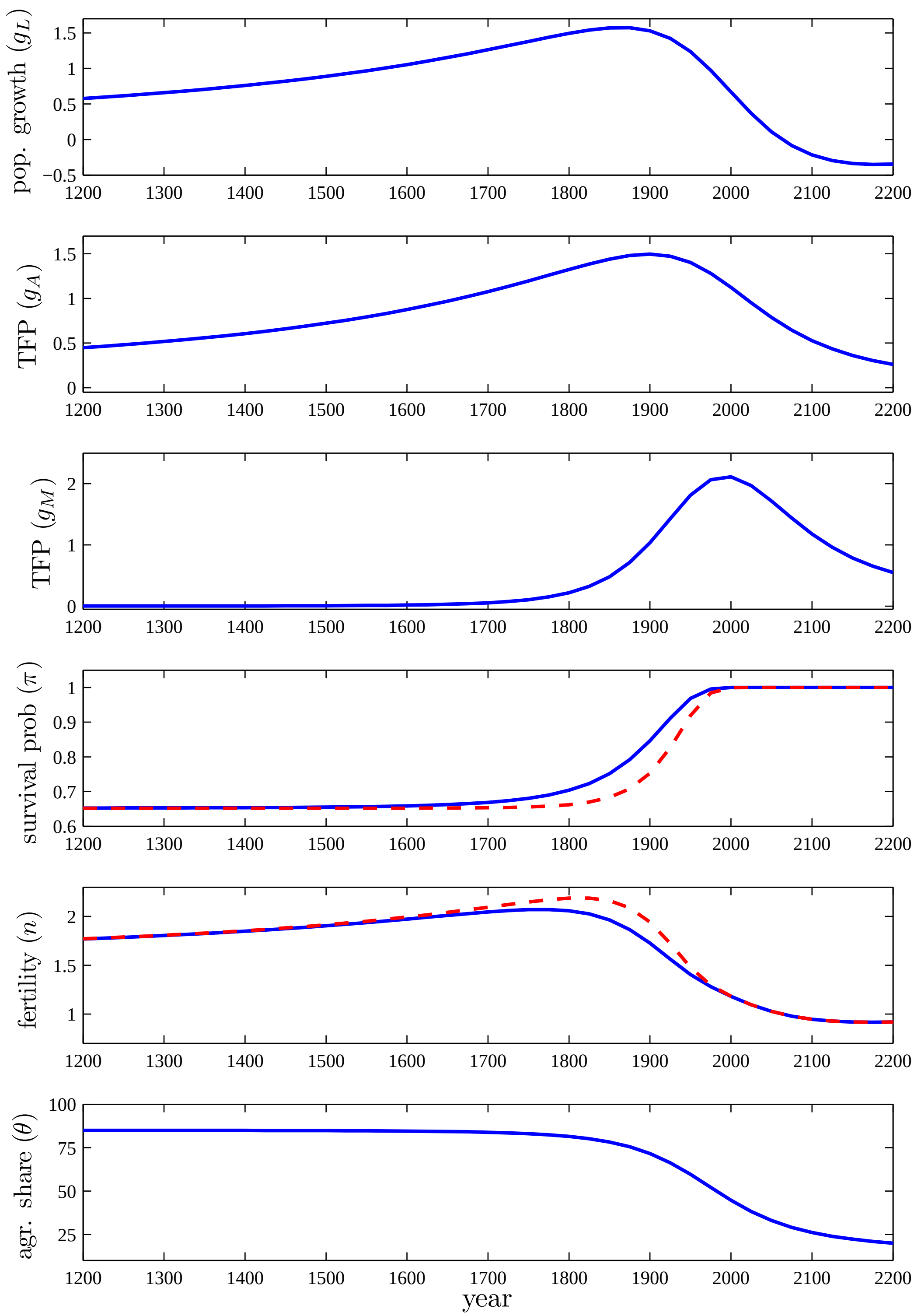
Since the transfer of labor out of agriculture gradually accelerates TFP growth in industry, industrial sector's TFP growth rate eventually surpasses that of agriculture. Henceforth, the price of nutrition gradually increases, and children become increasingly more expensive. All else equal, this leads to a lower rate of birth, and, therefore, a fall in the net rate of reproduction. At the same time, advances in industrial knowledge create a further decline of extrinsic mortality, leading to more investment in the children's nutritional status and, therefore, to a lower intrinsic mortality. Hence, by contrast to the previous periods, declining child mortality is now accompanied by *falling* rates of birth and net reproduction.

Eventually, the child survival rate reaches its maximum. Due to the rising prices of nutrition, however, the birth rate continues to fall. Sooner or later, therefore, the net rate of reproduction reaches that of replacement, and population growth ultimately (and endogenously) comes to a halt.

5.2. **Calibration.** In order to investigate adjustment dynamics we continue by calibrating the model. The parameter values are chosen so that the peak of the demographic transition matches that of 19th-century England and so that the maximum rate of industrial TFP growth (and subsequent slowdown) appears in the late 20th century. For comparative purposes, we use as many parameter values as possible from the benchmark case in Strulik and Weisdorf (2008). The following parameters produce a peak of population growth, at a annual rate of 1.5 percent, in the year 1875, as well as a peak of industrial TFP growth, at a rate of 2 percent, in the late 20th century: $\alpha = 0.8$, $\epsilon = \phi = 0.3$, $\mu = 0.22$, $\delta = 2.5$, and $\gamma = 3.4$. The parameters a, b, λ and β are chosen, so that the child survival probability is about two thirds in the high middle ages (Wrigley *et al.*, 1997), and then reaches a maximum of just below 100 percent in the 20th century.

In the benchmark case we set $\nu = 0$. This textbook-case of neutralizing effects of mortality change and fertility response on net fertility (Galor, 2011, Chapter 4) is helpful here because any changes in the net rate of reproduction are now explained by the relative price of nutrition. For illustrative purposes we begin with the case where there are no negative feedback-effects from urbanization on extrinsic mortality ($d = 0$). The remaining parameter values are thus set to $a = 0.5$, $b = 0.2$, $\beta = 0.5$, and $\lambda = 35$. For better readability of the results, one generation is set to 25 years, approximately the length of the fecundity period. We set $\theta_0 = L_0^A/L_0 = 0.85$ and start values A_0 and L_0, so that the peak in the growth rate of population is reached in 1875 for an economy starting in the year 1 C.E. The value for M_0 is obtained endogenously, and is given by $(\gamma/(\delta\theta_0))^{1/\phi}$.

Figure 2: Long-Run Dynamics: Benchmark Case



Benchmark case, neutralizing effects of mortality on net fertility ($\nu = 0$). Blue (solid lines): no feedback from urbanization to mortality ($d = 0$). Red (dashed) lines: negative feedback from urbanization ($d = 5$).

5.3. **The Benchmark Case.** Figure 2 illustrates the calibrated adjustment path. The period shown runs from the year 1200 to the year 2200. The solid lines show the path for the benchmark economy with $d = 0$. Initially, the labor force is predominately employed to agriculture. In line with the numbers provided by Galor (2005), industrial TFP growth is almost absent during the early stages of development. Productivity growth in agriculture is around 0.4 percent per year during the high middle ages, supporting a population growth rate of 0.5 percent per year.

Increasing knowledge in agriculture, initially, manifests itself in a slowly decreasing price of nutrition, translates almost entirely into population growth, meaning that the standards of living are hardly affected by technological progress. The slowly but steadily growing population gradually increases knowledge in agriculture and agricultural TFP growth builds up, little by little, to eventually reach a 1.4 percentage growth rate in the early 20th century. By then, agricultural TFP growth makes possible a substantial transfer of labor into manufacturing, causing an upsurge in industrial TFP growth. This substantiates a significant decline in the extrinsic child mortality rate, leading parents to increase their spending on child nutrition. With decreasing rates of extrinsic and intrinsic mortality, the child survival probability is on a fast rise.

In the late 19th century, TFP growth in manufacturing surpasses TFP growth in agriculture, and the rate of growth of food prices begins to increase and the relative price of food begins to increase, inducing people to spend more resources on industrial goods while reducing their fertility. By the end of the 20th century the demographic transition is almost complete: the child survival probability has reached its maximum value close to 100 percent and the net rate of reproduction is close to replacement level. TFP growth in industry reaches its peak in the late 20th century and then begins to slow down. This decline, however, is a gradual process, leaving enough momentum for the rate of growth of industrial TFP to exceed one percent per year far into the 22nd century.[8]

The first calibration exercise, with $d = 0$, fails to take into account the "urban penalty", i.e. the negative feedback from urbanization on the survival probability of offspring. This drawback of the model implies that mortality and fertility both start to decline 'too soon'. The red (dashed) lines in Figure 2 show a simulation, which corrects for this shortcoming, obtained by setting $d = 5$. In this case, the onset of the fertility decline is delayed until the early 19th century. One can think of the

[8] Since GDP per worker grows at the rate of wages, which grow at the rate ϕg_M, growth of GDP per worker can be inferred as a scaled version of growth of TFP in manufacturing. For the scaling factor $\phi = 0.3$, the model predicts too little growth of GDP per worker. This shortcoming can only be corrected at the expense of less accurate predictions elsewhere. It is a natural implication of the fact that our simple model neglects other important drivers of growth like, for example, the accumulation of physical and human capital.

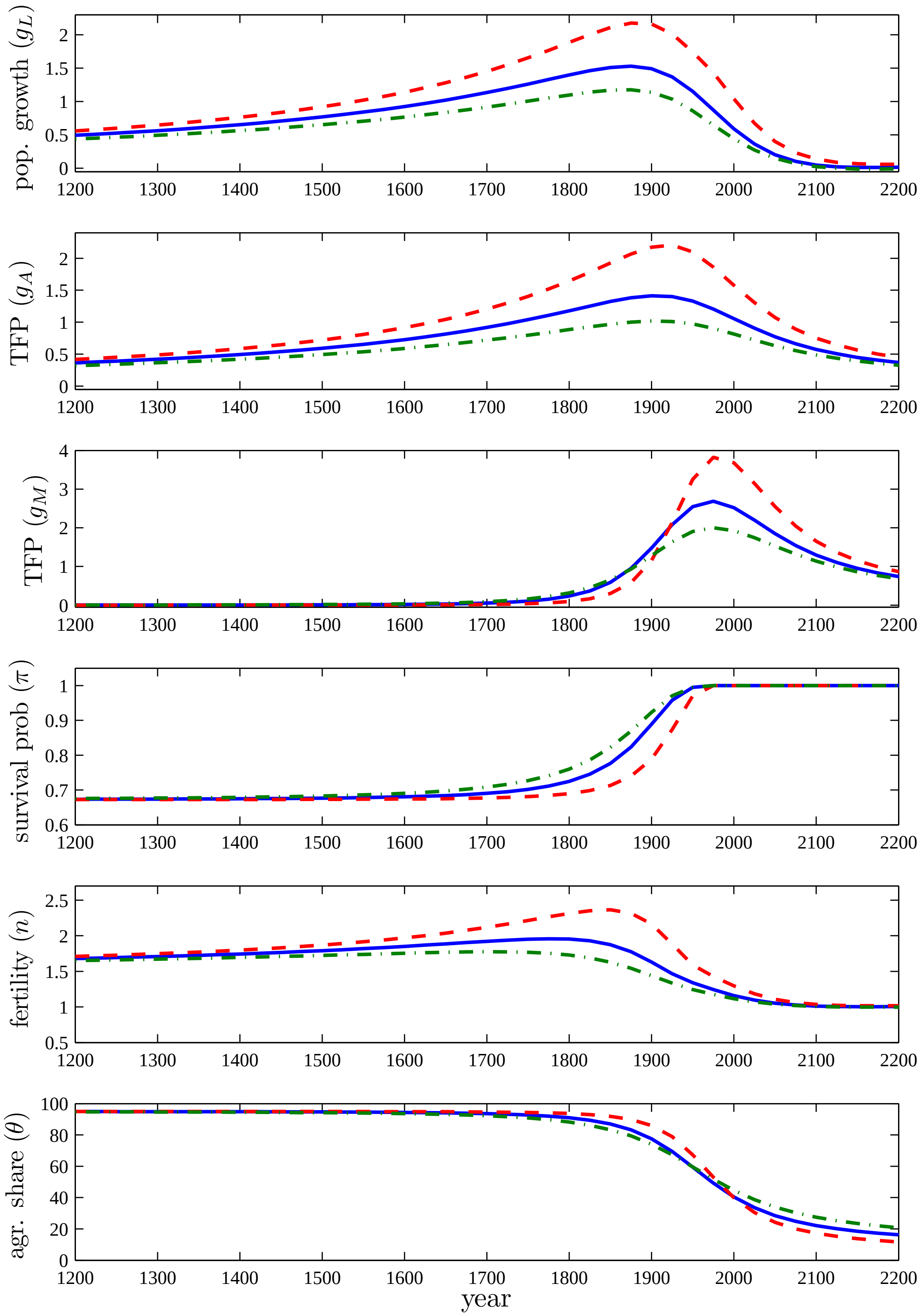
gap between the solid line and the dashed line for child survival as capturing the gap between rural and urban child mortality.

5.4. **The Response of Net Fertility to Extrinsic Mortality and the Shape of Transitional Dynamics.** We now turn to discuss the impact of the sign of cross derivative $g_{\bar{\pi}h}$, as well as the association between the extrinsic child mortality rate and the net rate of reproduction on the transitional adjustment path. The blue (solid) lines in Figure 3 re-iterate the benchmark case with the curvature parameter ν set to 0. The green (dash-dotted) lines show trajectories for $\nu = 0.15$. In this case, we have that $\partial(\pi n)/\partial\bar{\pi} > 0$ (and the cross derivative $g_{\bar{\pi}h}$ is larger than in the benchmark case). As a consequence, parents in the model reduce their fertility, and increase their food expenditure. To see why, one needs to recall that parents prefer to keep the survival elasticity of health expenditure ϵ_h constant at the level of $(\gamma - \beta)/\gamma$. The partial effect of an increase of the curvature parameter ν is an increase of the survival elasticity, $\partial\epsilon_h/\partial\nu = h/(\bar{\pi} + h + \nu)$. The optimal response by parents is to increase the level of food expenditures up to the point at which decreasing returns of nutritional input bring the elasticity back to its desired value.

The main effect of the reduction in net fertility, and hence the lower growth of population, is that the demographic transition is now delayed. Since the population level is now smaller at every stage of development than it was in the benchmark run, the scale-effect of the learning-by-doing is smaller, causing a postponement in the transfer of labor out of agriculture and the demographic transition. In Figure 3, we eliminate this effect by re-adjusting the initial values for the size of knowledge and population, so that every set of trajectories displays a peak of net fertility in 1875. This way the *shape* of the trajectories is comparable across illustrations. Relative to the onset of the decline net fertility, the survival rate begins to improve earlier for $\eta > 0$ because of the induced shift from child quantity to quality.

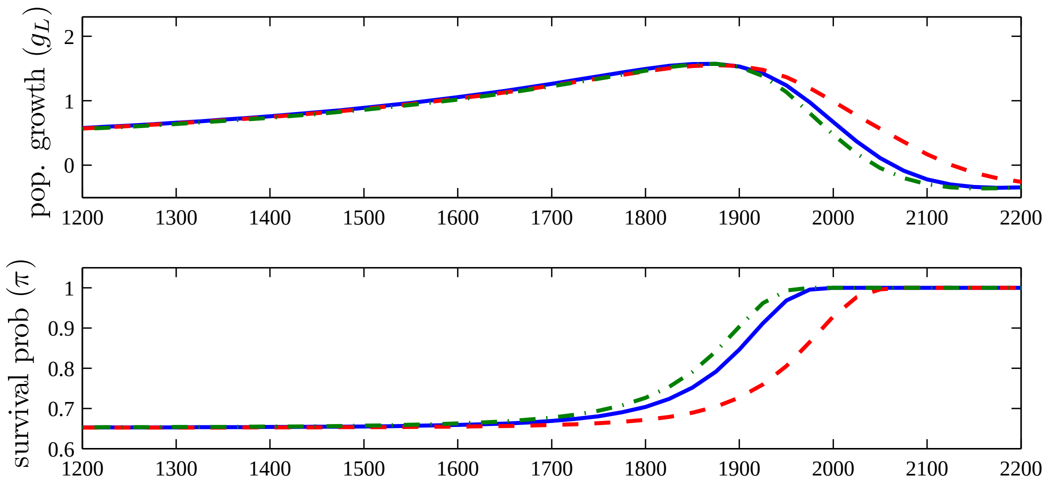
Red (dashed) lines in Figure 3 show a scenario for which η is equal to -0.15, implying a negative association between the extrinsic survival rate and the net fertility rate as well as a negative cross derivative $g_{\bar{pi}h}$ (since $2\beta\pi + \gamma\nu < 0$). Keeping in mind the intuition developed above, parents now react to improving mortality by spending less on nutrition and by demanding more children instead. As a consequence, the population growth rate follows a 'higher' path relative to the benchmark case and it peaks at a higher rate. The survival probability starts to improve at a later point in time, which is caused by the fact that parents are now spending less resources on food.

Figure 3: The Extrinsic Survival – Net Fertility Association and Long-Run Dynamics



Blue (solid lines): benchmark case, neutralizing effects of mortality on net fertility ($\nu = 0$). Green (dash-dotted lines): positive association between survival and net fertility ($\nu = 0.15$). Red (dashed) lines: negative association between survival and net fertility ($\nu = -0.15$). Other parameters as for Figure 2.

Figure 4: The Extrinsic Survival – Net Fertility Association: Scale Adjusted Dynamics



Blue (solid lines): benchmark case, neutralizing effects of mortality on net fertility ($\nu = 0$). Green (dash-dotted lines): positive association between survival and net fertility ($\nu = 0.15$, $\mu = 0.27$). Red (dashed) lines: negative association between survival and net fertility ($\nu = -0.15$, $\mu = 0.155$). Other parameters as for Figure 2.

It is instructive to eliminate the scale in these adjustment dynamics. For that, we adjust the level of agricultural productivity μ, so that every set of trajectories displays a peak of population growth at a rate of 1.5 percent in the year 1875. Some of the results of this adjustment are illustrated in Figure 4. Controlling for the scale reveals that the association between extrinsic child mortality and fertility (governed by the parameter ν) is indeed helpful in explaining the observable gap between the onset of improving child survival and the onset of the fertility decline. The larger ν is, the larger is the predicted gap between mortality and fertility decline. The overall impression, however, is that the results are largely independent of assumptions made about the cross derivative and the partial response of net fertility to changes in the extrinsic child mortality rate. In fact, only details of the predicted shapes of adjustment paths are modified.

5.5. **Timing and Correlations Along the Adjustment Path.** The introduction of an endogenous child mortality rate leads us to qualify the conclusions reached in Strulik and Weisdorf (2008) regarding the timing of the demographic transition. In the latter study, the peak of the population growth rate coincides with that of fertility, both of which, in turn, take place after the growth of industrial TFP has surpassed that of agriculture. Here, by contrast, the break-up of the population growth rate into a fertility and a mortality component permits us to track down separately the occurrence of significant economic and demographic events. For example, in the benchmark case, the fertility rate peaks around 1800, whereas the population growth rate reaches its maximum in

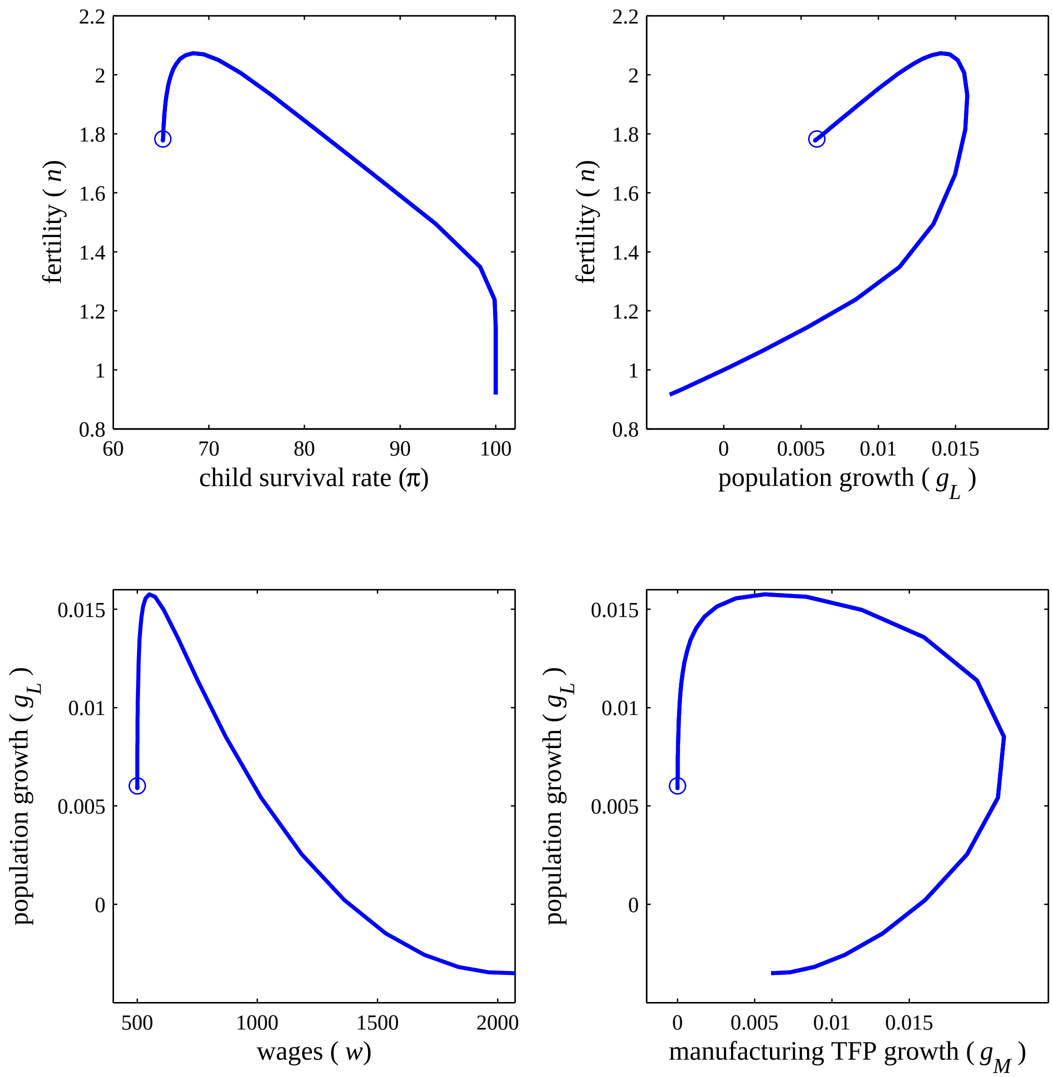
1875. The rate of agricultural TFP growth peaks around 1920, while the rate of industrial TFP growth reaches its maximum in the late 20th century. Moreover, consistent with empirical evidence of most western-world countries (France is an exception), the falling child mortality rate in the present framework precedes the drop in fertility. Hence, the current model correctly predicts the positive relationship between child mortality and net reproduction observed during the early phases of most western-world countries' demographic transitions, a feature missing in previous work on this topic.

The model also predicts that the demographic transition and the economic take-off are accompanied by a structural change, i.e. $\theta \to 0$ for $t \to \infty$. The speed of this structural change during industrialization, however, is underestimated for the set of basic parameters. Since everything is fully endogenous in the present model (i.e. none of the time series are imputed, as in some of the related literature), it is impossible to correct this shortcoming without resorting to less acceptable approximations. By highlighting the interplay between demographic and economic variables, the model is capable of replicating the stylized facts of long-term economic development without resorting other sources of structural changes, such as capital accumulation, international trade (globalization) and R&D-based growth.

Our model also generates some interesting non-linearities along the adjustment path. Based on Figure 5, which shows some of the relevant non-monotonous relationships (for the benchmark economy), the following observations can be made. First, during early stages of development, the model predicts a positive relationship between the child survival probability rate and that of fertility, while, during later stages, the sign of the correlation is reversed (panel 1). Specifically, the lower mortality rate goes together with a rising rate of fertility when the effect of falling food prices on fertility dominates the quantity-quality substitution effect. Oppositely, the lower mortality rate goes together with a lower fertility rate when quantity-quality substitution effect dominates that of falling food prices.

Second, due to a strong preventive-check mechanism during the early phases of development, there is almost no correlation between wages and population growth in the beginning of the period analyzed. At later stages, however, such as during the time of the demographic transition, there is a strong, negative relationship between the two, followed by a weak, negative correlation after the ending of the transition (panel 2).

Figure 5: Correlations Along the Transition: 1200 to 2200

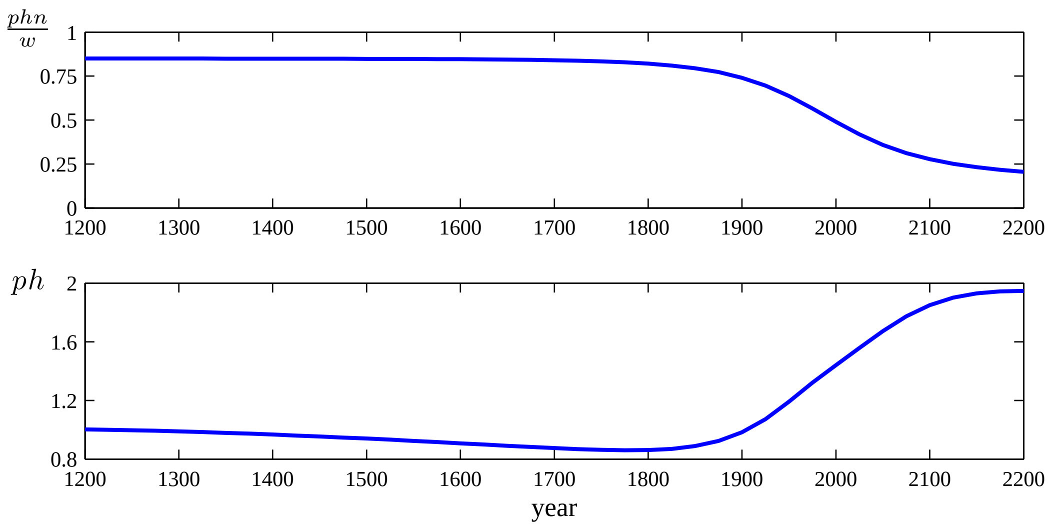


A circle marks the starting point in the year 1200.

Third, the general relationship between the birth rate (total fertility) and the net rate of reproduction is a slightly positive one (panel 3). That is, during the early stages of development, the birth rate and net rate of reproduction are both rising, while both rates are falling during the late stages. For a short period in between, however, the birth rate is declining while the net rate of reproduction still grows.

Finally, the relationship between the net rate of reproduction and industrial TFP growth forms an almost perfect orbit (panel 4). In the early stages of development, there is a positive relationship between TFP growth and population growth. However, during the 'industrial revolution', i.e. the sharp rise in the rate of growth of industrial TFP, the sign of the correlation turns negative. In the post-industrial period, i.e. after the peak in the growth rate of industrial TFP, the sign turns positive once again, as the rates of growth of productivity and population both decreases.

Figure 6: Engel's Law and Expenditure per Child



5.6. **Engel's Law.** A final observation concerns the model's consistency with Engel's Law. Engel's Law states that the share of nutrition expenditure in income, measured by $(p_t n_t h_t)/w_t$, will decline over the course of development (i.e. with economic growth). This is illustrated in Figure 6, which also shows that nutritional expenditure per child, measured by $p_t h_t$, gradually drops until the mid-19th century; then increase steeply throughout the 20th century; after which it continues to increase, but at a somewhat slower pace than earlier (for comparison, the results are shown relative to initial expenditure, i.e. the level of expenditures in 1200 is normalized to one). These results reveal another important insight from endogenizing child mortality: because more children survive at high levels of M, parents are more inclined to increase their spending on child nutrition, irrespective of a rising relative price for food.

6. Extensions

In this section we generalize the utility function and include time of costs of child-rearing. Specifically we assume that parents solve the following problem.

$$\max_{m_t, n_t, h_t} u_t = \frac{m_t^{1-\sigma}}{1-\sigma} + \beta \log h_t + \gamma \log(\pi_t n_t) \tag{23}$$

$$s.t. \quad w_t(1 - \tau n_t) = p_t h_t n_t + m_t. \tag{24}$$

Child survival continues to be given by (2). With $0 < \sigma < 1$ the utility function is now strictly concave but it also preserves the "hierarchy of needs" assumption introduced in Section 3. Manufactured goods are more easily substituted across time than are births and nutrition. The parameter τ controls for the time intensity of child bearing and rearing.

The associated Lagrangian now reads

$$L = \frac{m_t^{1-\sigma}}{1-\sigma} + \beta \log h_t + \gamma n_t + \gamma g(\bar{\pi}_t, h_t) + \omega \left[w_t(1-\tau n_t) - p_t h_t n_t - m_t \right].$$

And the first order conditions are

$$0 = m_t^{-\sigma} - \omega \tag{25}$$

$$0 = \frac{\gamma}{g} g_h + \frac{\beta}{h_t} + \omega p_t n_t \tag{26}$$

$$0 = \frac{\gamma}{n_t} - \omega p_t h_t - \omega \tau w_t. \tag{27}$$

Combining (26) and (27) we get food demand per child determined by (28):

$$\gamma \frac{g_h}{g} \cdot h = \gamma - \beta - m_t^{-\sigma} \tau w_t n_t. \tag{28}$$

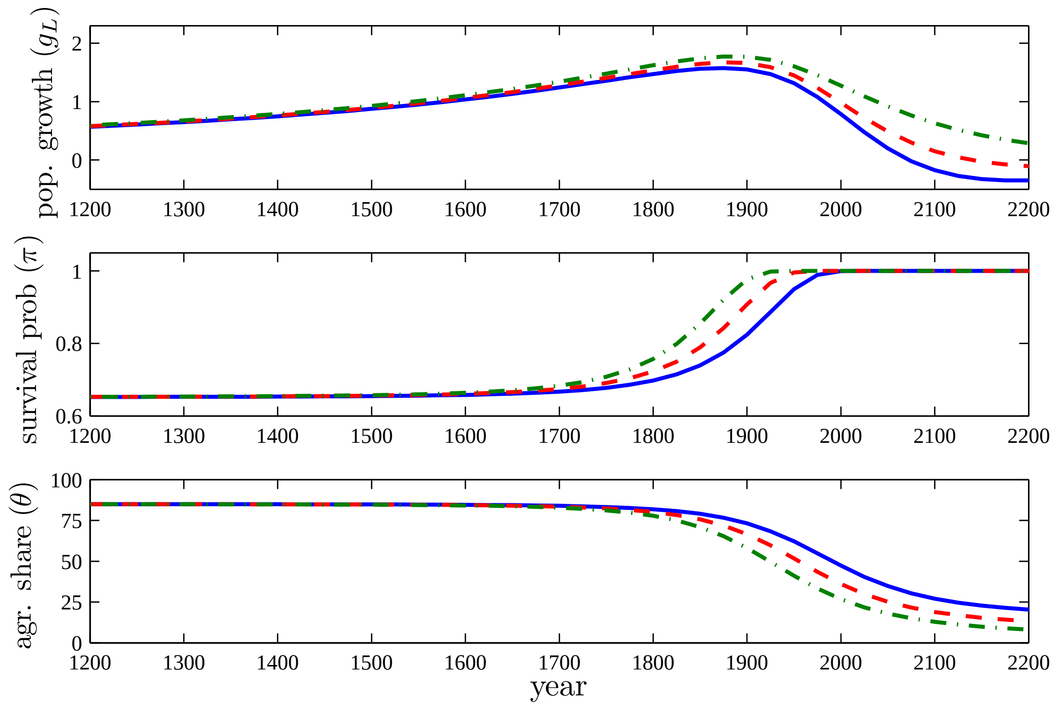
Note that (28) collapses into (7) when $\tau = 0$. Thus, as long as $\tau = 0$, optimal food expenditure per child is the same regardless of the specification of the utility function. Inspection of (25)-(27) furthermore reveals two opposing effects on fertility: one from higher wages, and thus higher opportunity costs of child rearing, which lowers fertility; and one coming from a higher expenditure on manufactured goods through decreasing marginal utility of goods consumption, which increases fertility. In order to obtain the general-equilibrium effect, we solve (24)–(27) together with the equilibrium condition for the goods market,

$$p_t^{\alpha} (h_t n_t)^{\alpha-1} \mu A_t^{\epsilon} = (\delta M_t^{\phi})^{\alpha} L_t^{1-\alpha},$$

and the wage equation $w_t = \delta M_t^{\phi}$ to obtain the equilibrium set $\{h_t, m_t, n_t, p_t\}$ at any point in time, that is for any given set of state variables $\{A_t, M_t, L_t\}$. The model is no longer able to generate an explicit analytical solution, so we solve the problem numerically.

Figure 7 shows an extract of the adjustment paths for the benchmark case ($\sigma = 0$, solid lines), as well as for $\sigma = 0.2$ (dashed lines) and $\sigma = 0.4$ (dash-dotted lines). One effect of iso-elastic utility from manufactured goods is that marginal utility is high when the consumption level is low

Figure 7: Iso-Elastic Utility: Long-run Dynamics

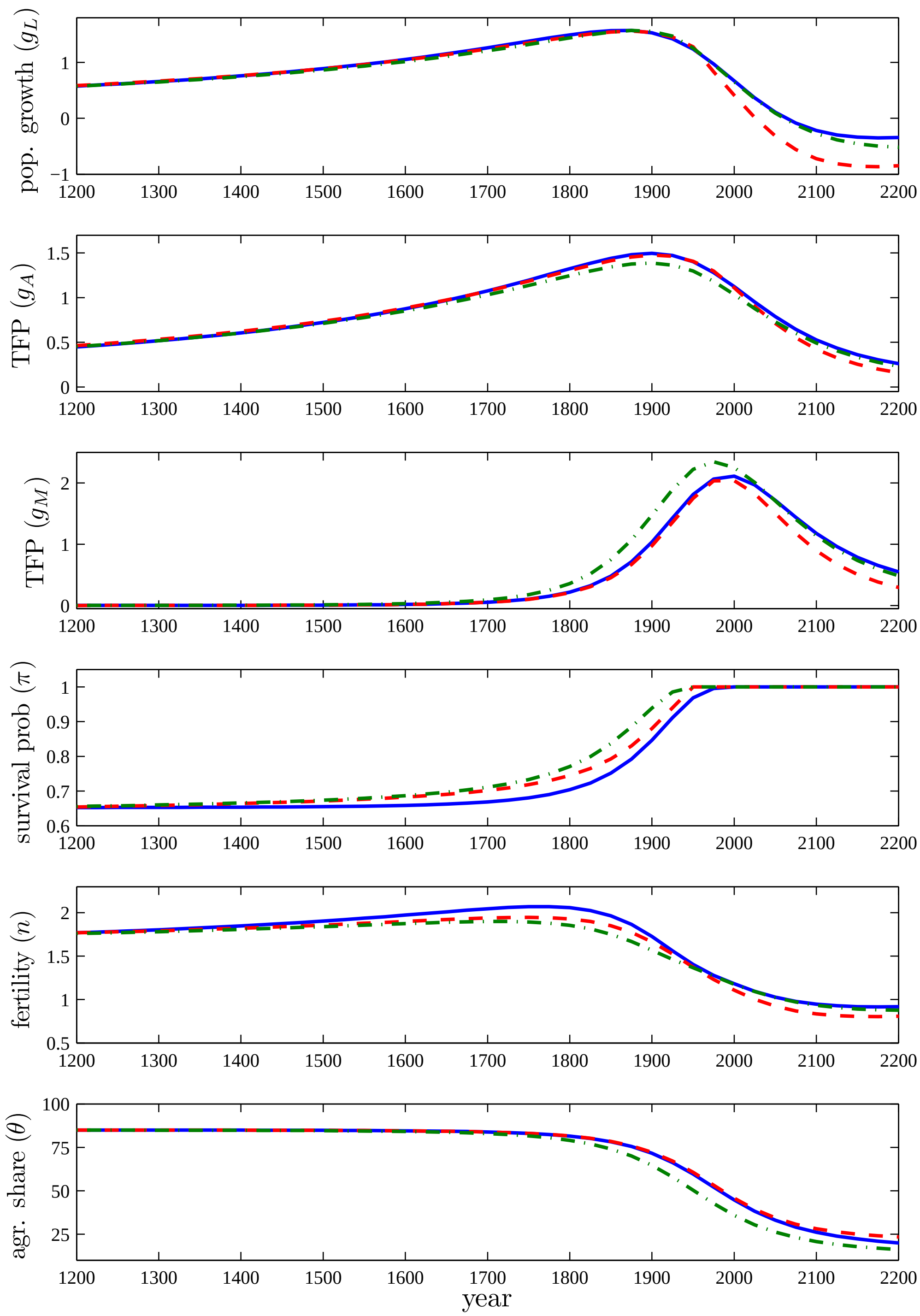


Blue (solid) lines: benchmark run. Red (dashed) lines: $\sigma = 0.2$. Green (dash-dotted) lines: $\sigma = 0.4$.

(before the onset of industrialization). Ceteris paribus, this means that households demand more manufactured goods and fewer children (with food expenditure per child remaining fixed, for reasons explained above). In turn, the fall in fertility causes a slower growth of population, a smaller scale effects through learning-by-doing, and thus a later onset of the demographic transition. In order to eliminate this scale effect and to compare the shape of the adjustment paths we have re-adjusted the initial values for population and knowledge in 1200, so that both sets of trajectories display a peak of population growth in 1875.

Figure 7 illustrates that iso-elastic utility causes a larger gap between the mortality decline and the net fertility decline, as well as an earlier and faster process of industrialization. In the case of England, therefore, the assumption of iso-elastic utility is more realistic in that the predicted trajectories fit better with historical evidence. These effects increase with the size of σ, as the dashed-dotted lines illustrates when $\sigma = 0.4$. At the same time, however, a higher σ causes the model to underperform: the predicted decline of population growth in the 20th century is too small compared to the actual time series. The reason is that diminishing marginal utility generates a low demand for manufactured goods when the level of consumption becomes relatively large. By comparison to the case of linear utility, households thus demand relatively few manufactured goods and relatively many children and react less strongly to falling prices of manufactured goods.

Figure 8: Time Cost of Child Rearing: Long-run Dynamics



Blue (solid) lines: benchmark run. Red (dashed) lines: $\tau = 0.05$ (and $a = 0.29, \mu = 0.26$). Green (dash-dotted) lines: $\tau = 0.05$ and $\sigma = 0.2$ (and $a = 0.29$, $\mu = 0.25$). Other parameters as for Figure 2.

As a final extension we include a cost of rearing children. Figure 8 considers an example where $\tau = 0.05$. Ceteris paribus, the presence of the child-rearing costs reduces the demand for child, and thus delays the demographic transition compared to the benchmark case. Again, initial values are adjusted so that any set of trajectories displays a peak in net fertility in 1875 (in order to better compare the shape of adjustment paths). An additional effect of introducing the child time costs is that it increases child expenditures and consequently child survival rates. In order to match the child mortality statistics of the middle ages—that about one third of children born do not make it to adulthood—we recalibrate the extrinsic survival prospects by setting $a = 0.29$. Furthermore we also set agricultural productivity μ to 0.26 in order to reach a peak rate of population growth at 1.5 percent.

Solid lines of Figure 8 shows the benchmark case, while dashed lines illustrate the re-calibrated case in which a time costs of child rearing is added to the model. The main consequence of including the cost of child rearing is a growing gap between the mortality and the fertility transitions, coming about through an earlier mortality decline. Another visible implication is a very large undershooting of fertility, falling far below the replacement rate in the 21st century. This is caused by the rise in income, which increases the opportunity costs of raising children, causing parents to reduce fertility.

The ideal setting, of course, is to combine the two adjustment paths derived above—that based on the assumption about iso-elastic utility (which rendered fertilty 'too high') and that from the introduction of a cost to raising children (which rendered fertility 'too low'). The dash-dotted lines of Figure 8 take up this idea to the graph, showing the adjustment paths for $\tau = 0.05$ and $\sigma = 0.2$ (and by re-calibrating μ in order to match the peak of net fertility in 1875).

The combined case shows that the model and its predictions are robust to the inclusion of a time cost of child rearing as well as the introduction of an assumption about an iso-elastic utility for manufactured goods. While the times series for fertility, population growth and agricultural productivity virtually coincide, the extended model predicts an earlier onset of the mortality decline; a stronger and steeper fall of agricultural employment during the 19th century; and a higher growth rate of productivity, and thus income per worker, in the 20th century. Taken together, the two extension thus add more realism to the model vis-a-vis the historical evidence.

7. Conclusion

This study provides the first unified growth model to endogenize mortality, fertility and growth, while not relying on human capital accumulation to generate a demographic transition. The result is based on the notion that parents care not only about surviving offspring but also about the nutritional status of the offspring, and that the price of nutritional goods, which thus influences the fertility decision, responds to structural transformations in the productive sectors of the economy. The model provides an economic rationale for the demographic observation that fertility rates decline in response to higher child survival probability and that they do so with a delay. Another major implication of the study, demonstrated by the use of calibrations, is the non-monotonic nature of the relationships between various economic and demographic variables—an important insight for scholars trying to make sense of these relationships empirically. The study also shows that the basic structure of long-run adjustment dynamics is independent of (i) the specific assumptions concerning cross-derivatives; (ii) the direction of impact of extrinsic mortality on net fertility; (iii) more general specifications of the utility function; as well as (iv) consideration of the time costs of child rearing. Alternative specifications will lead to modifications concerning the shape of adjustment paths, which allows a fine-tuning of the peaks and gaps observed during the process of industrialization and the demographic transition.

The robustness tests done in Strulik and Weisdorf (2008) make us confident that the current model is also robust to other modifications. First, the assumption about a constant returns-to-labor production technology in industry can be replaced by one which relies on diminishing returns to labour without affecting the model's qualitative conclusions. Second, the long-run predictions of the model, i.e. the lack of economic growth in steady state, can be modified without changes to the qualitative nature of the results as long as we permit a positive rate of growth of the population on the balanced growth path. Both modifications, however, come at a cost to simplicity.

Appendix A

Sufficient Conditions for Maximum. For the solution (8) and (9) to be a maximum the Hessian evaluated at the extremum has to be negative definite. For that the Hessian determinant has to be positive. Proceeding from (5) and (6) and keeping the survival function for now in its general form $g(\bar{\pi}_t, h_t)$ the second-order derivatives are

$$\frac{\partial^2 u_t}{\partial n_t^2} = -\frac{\gamma}{n_t^2}, \quad \frac{\partial^2 u_t}{\partial h_t^2} = -\frac{\beta}{h_t^2} + \gamma \frac{g_{hh} g - g_h^2}{g^2}, \quad \frac{\partial^2 u_t}{\partial n_t h_t} = \frac{\partial^2 u_t}{\partial h_t n_t} = -p_t.$$

Recall that the solution (5) and (6) fulfils $n_t h_t = \gamma / p_t$, implying that $\partial^2 u_t / \partial(n_t h_t) = -\gamma/(n_t h_t)$. The Hessian determinant is thus obtained as

$$\det H = \frac{\gamma^2}{n_t^2} \left(\frac{\beta}{h_t^2} - \gamma \frac{g_{hh} g - g_h^2}{g^2} \right) - \frac{\gamma^2}{n_t^2 h_t^2} = \frac{\gamma^2}{n_t^2} \left(\frac{\beta - \gamma}{h_t^2} - \gamma \frac{g_{hh} g - g_h^2}{g^2} \right).$$

It is positive if

$$\left(-\frac{g_{hh}}{g} + \frac{g_h^2}{g^2} \right) h_t^2 > \frac{\gamma - \beta}{\gamma}.$$

In words, the survival function has to be sufficiently concave. Inserting (2) the condition becomes

$$\frac{2(\bar{\pi} + \nu) h_t + (\bar{\pi} + \nu)^2}{(\bar{\pi} + h_t + \nu)^2} > \frac{\gamma - \beta}{\gamma}.$$

Inserting the solution h_t from (9) the condition simplifies to

$$1 - \frac{\beta^2}{\gamma^2} > 1 - \frac{\beta}{\gamma} \qquad \Rightarrow \qquad 1 > \frac{\beta}{\gamma}.$$

It is fulfilled for $\gamma > \beta$, that is always when a positive (interior) solution for n_t and h_t exists. The extremum (8) and (9) is a maximum.

Proof of Proposition 1. While a *balanced* growth path involves stagnant levels of population and income, an *unbalanced* growth path, characterized by imploding or exploding growth, may in principle exist. In the following, we explore the two cases of unbalanced growth, starting with the case of imploding growth. Imploding growth implies perpetually negative population growth, i.e. n_t is smaller than one and L_t is decreasing. It is easy to see that imploding growth is not an option since g_t^A and g_t^M are bound to be non-negative. There is no forgetting-by-doing. With $\lim_{L \to 0} g^A = 0$ and $\lim_{L \to 0} g^M = 0$, we have $\lim_{L \to 0} n = const./L^{1-\alpha}$ from (14). As L_t converges to zero, n_t goes to infinity. A contradiction to the initial assumption of n_t being smaller than one. There is no imploding growth. Intuitively, decreasing marginal returns of labor in agriculture ($\alpha < 1$) prevent implosion. As population size decreases agricultural productivity goes up and prices go down so that fertility and thus next period's population increases.

Explosive growth, on the other hand, cannot be ruled out if $\eta > 1$, In this case the relative price of food ultimately goes to zero and fertility to infinity. With growing population growth, productivity growth in both sectors grows hyper-exponentially until the economy reaches infinite fertility in finite time. We can solve the stability condition $\eta < 1$ for the critical ϵ. A sufficient condition for stability of balanced growth is (22). Thus, the learning elasticity in agriculture must not be too large. Otherwise agricultural productivity rises so steeply with population growth that it can sustain further falling food and triggers further population growth. Inspection of (18) shows that the critical ϵ is decreasing in α and increasing in ϕ. Intuitively, the larger the counterbalancing forces of limited land (i.e. the lower α) and of learning in the manufacturing sector (the larger ϕ) are, the higher can learning in the agricultural sector be without leading to explosion. The following proposition summarizes the considerations made above about balanced and unbalanced growth.

Note that $\epsilon < \epsilon_{crit}$ is a *sufficient* condition of stability since it holds for any path along which there is constant population growth. It could be relaxed for the only existing balanced growth path according to which, as shown, population growth is not only constant but also zero. For an intuition of the result inspect (20) again and imagine adjustment dynamics along which an "agricultural revolution" occurs before an "industrial revolution", i.e. a path along which g^A is – because of decreasing returns – already declining whereas g^M is still on the rise. The fact that g^A and thus gL are faster approaching to zero than g^M relaxes the stability condition obtained from analytical considerations, which implicitly assume that all growth rates are in the neighborhood of a balanced growth path.

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