Real exchange rates and economic fundamentals: An investigation based on a Markov-STAR model*

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Abstract

In this paper we introduce a new nonlinear Markov-STAR model to capture both the markov switching and smooth transition dynamics for real exchange rates. The Markov switching part captures the effect of time variations of the equilibrium exchange rates, while the smooth transition part models the nonlinear adjustment to the equilibrium. We describe the model and the estimation algorithm. In an empirical application the Markov-STAR model is applied to the real exchange rates of 18 countries. In an effort to make sense of the switching equilibrium rates, we relate relevant macroeconomic variables, such as output gap, inflation rate, and economic uncertainty to the smoothed probabilities through logit regressions. We find that, consistent with economic models, a deteriorating economy relative to US economy tends to significantly increase the likelihood of the real exchange rate to depreciate relative to the US Dollar for the majority of the countries under investigation. Furthermore, a higher economic uncertainty in the US tends to significantly increase the likelihood of a real exchange rate appreciation for many advanced European economies while it is exactly the opposite for some developing countries. Finally, we also find strong evidence that rising economic uncertainty tends to be associated with a higher exchange rate volatility.

Key words: Real exchange rates, Nonlinearities, Markov Switching, Smooth transition, economic fundamentals
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1 Introduction

It has been well established that the real exchange rate dynamics is featured by nonlinear adjustments (Taylor and Sarno (2003)). In efforts to resolve the purchasing power parity (PPP) puzzle due to Rogoff (1996), researchers find that the tendency for the real exchange rate to revert back to its equilibrium value becomes stronger only when it is further away from its equilibrium value, because of the transaction and transportation costs involved in arbitrage activities. Michael et al. (1997) and Taylor et al. (2001) apply the exponential smooth transition autoregressive (ESTAR) models, due to Teräsvirta (1994), to the real exchange rate and document a stronger tendency for the real exchange rate to revert to its PPP value once the nonlinearity is accounted for. This type of model involves the self-exciting adjustments and contains two autoregressive regimes with a symmetric smooth transition conducted by an exponential transition function between these regimes. In the model the exchange rate is nonstationary when it is close to its PPP equilibrium value but the mean-reversion happens when its deviation from PPP becomes large. An overview about the econometric properties of this model can be found in van Dijk et al. (2002). While Lo (2008) points out that certain nonlinear smooth transition models have the potential of resolving the PPP puzzle, Lo and Morley (2015) further provide empirical evidence that once multiple regimes are taken into consideration the half lives of the PPP deviations are significantly reduced, corroborating findings in the existing literature such as those cited above. Furthermore, Kilian and Taylor (2003) and Rapach and Wohar (2006) show that these smooth transition models can improve the forecast of the real exchange rates relative to linear models. However, one important drawback of these threshold or smooth transition models is that the equilibrium real exchange rates are often assumed to be constant, which is at odds with various economic theories and models that suggest otherwise (see e.g. Engel (2000) and Engel and West (2005)). As such, it is important to introduce additional dynamics to account for the possibly changing equilibrium exchange rates.

Another popular nonlinear model for the real exchange rate is the Markov regime-switching model, due to Hamilton (1989). In this type of model, a set of latent factors that follow a binomial distribution generates distinct regimes, which turn out to be useful to capture sudden but persistent regime shifts in the real exchange rate data. In particular, Engel and Hamilton (1990) demonstrate that an autoregressive model augmented with Markov regime-switching means and volatilities proves successful in capturing the long swings that are often featured in the real exchange rate data. Engel and Kim (1999) decompose the US/UK real exchange rate into permanent and transitory components by allowing for Markov regime-switching variances of the shocks. Further Engel (1994) and Bergman and Hansson (2005) explore the forecast ability of the Markov regime-switching model, and they find some evidence that the Markov switching model can improve upon the forecast performance relative to a standard nonstationary process of the real exchange rates. Bergman and Hansson (2005) further show that the Markov switching model can explain the often documented failure of rejecting a unit root in the real exchange rate (see e.g., Lopez et al. (2005)). These works have shown that the Markov switching dynamics are useful to capture the possibly changing equilibrium real exchange rates.

Although most often the above two different dynamics have been employed separately to model the exchange rate, recently several attempts have emerged to compare and/or combine these two models to study the real exchange rate. For example, Kaufmann et al. (2014) argue that the ESTAR model aims to capture the dynamic adjustment process that is self-excited by large deviations from PPP, but the Markov regime-switching model allows for sudden but persistent regime shifts which are better at capturing large external shocks such as currency devaluations. They apply a statistical model specification test and show that the former tends to hold for countries within the European Union whereas the latter effect rather dominates for less developed countries. Similarly, Ahmad and Lo (2014) study how these two models can be distinguished from each other by conducting Monte Carlo simulations. Given the different nonlinear dynamics and distinct types of nonlinearities these two models aim to capture, it is reasonable to suggest that both of these nonlinear dynamics may be present in the data especially over longer time span. It makes sense to argue that the Markov switching dynamics is better at capturing the possibly changing equilibrium real exchange rate while the threshold or smooth transition dynamics can account for the impact of transaction or transportation cost when the real exchange rate is moving toward its equilibrium value. To allow for both types of nonlinear dynamics, we introduce a model that combines the Markov regime-switching and the ESTAR approaches, in a parsimonious setup. Our new MS-ESTAR model contains an ESTAR process where the first regime is a Markov-Switching Autoregressive (MSAR) process. Our model therefore allows for exogenous shocks by its MS component. However, the size of the shocks is restricted by the forces of the ESTAR component making unrealistic break sizes rather unlikely. (By eye-balling we find that the resulting paths look rather like a real exchange rate process opposite to ESTAR- or MS-processes alone.)

The purpose of this paper is, however, twofold. Following discussions so far, we build on previous literature and propose a new nonlinear Markov-STAR model to capture both types of nonlinear dynamics in the real exchange rates data. More importantly, although the equilibrium exchange rates are related to a set of economic fundamental variables in standard economic models (see Engel and West (2005)) and some previous work have presented supportive empirical evidence (see Rapach and Wohar (2002) and Cerra and Saxena (2010)), such a close connection is all but conclusive (Bacchetta and van Wincoop (2013)). Therefore, our second purpose of this paper is to investigate the connection between the equilibrium real exchange rates and their economic fundamentals after taking into account the realistic nonlinear feature of the exchange rate data through the lens of our newly proposed model. To our knowledge, our work is the first to study the connection between the exchange rate and its economic fundamentals in the context of a highly nonlinear model.

Our approach is in line with Sarno and Valente (2006). They propose a VECM approach and define a so-called nonlinear Markov-Switching-Intercept-Autoregressive-Heteroskedastic-VECM containing an MS component and also an exponential adjustment component. However, their model specification is different from ours. Furthermore, we provide a discussion of the economic motivation of such a general model, and we attempt to relate the regime switchings of the real exchange rate to economic fundamental variables. We present compelling evidence that the Markov regime switchings estimated in our general model are strongly related with the underlying economic fundamental variables, such as the output gap differentials, inflation differentials, and economic uncertainties, in the right direction. Therefore, our work corroborates recent findings in the literature that have found some connections between the exchange rate and its economic fundamentals (see e.g., Engel and West (2005), Rapach and Wohar (2002), Cerra and Saxena (2010), and Balke et al. (2013)). What makes our work distinct from all the above cited work is that our findings are made in a highly nonlinear but also more realistic model that includes both the MS and ESTAR adjustments. Therefore, we identify a stronger connection between the exchange rate and its economic fundamental variables in presence of nonlinear dynamic adjustments often featured in the real exchange rate data.

2 Economic Motivation

The literature that employs the STAR type of models to account for nonlinearity when evaluating the mean-reversions of the real exchange rates often assumes a constant or a stable band for the real exchange rates to converge to. While it is important to account for nonlinearity that could intuitively arise due to transaction or transportation costs (Lo and Zivot (2001)) or aggregation (Taylor (2001)), there are a number of reasons why restricting the equilibrium real exchange rate to a constant may be too restrictive an assumption. For example, Engel (2000) argues that the usual unit root tests often fail to capture the Balassa-Samuelson effect, which refers to time variations of the equilibrium real exchange rates arising from differences of the productivity growth, because of the nontrivial size distortions of these tests in many economically realistic scenarios. As a way to capture these time variations of the equilibrium real exchange rates, Engel and Kim (1999) propose an unobserved components model where they decompose the real exchanges rates into a permanent and a transitory component. The permanent component is taken as an approximation of the time-varying equilibrium exchange rates accounting for structural changes in long observational periods whereas the transitory component governs the convergence towards this equilibrium. Their exchange rate model is given in state space form where the permanent component is modeled as a random walk whereas the transitory component is an AR(2) process.

Other papers also attempted to account for the time variations of the equilibrium real exchange rates using different approaches. For example, Hegwood and Papell (1998) allow for multiple exogenous structural breaks in the mean when testing the PPP and find evidence for shorter half-lives of PPP deviations. On the other hand Papell and Prodan (2006) employ a time trend together with the structural breaks to allow for the Balassa-Samuelson effect and find stronger evidence for PPP after controlling for the time varying equilibrium real exchange rates.

We extend the approach in Engel and Kim (1999) but propose an alternative model to capture the time varying equilibrium real exchange rates using the Hamilton (1989) type Markov regime switching process. Our model allows for shifts in the mean as well as in the variance. Different to Hegwood and Papell (1998) we model shifting regimes in the mean not by a dummy variable but by a Markov Switching stochastic process. Thus, our model also allows for a time varying equilibrium real exchange rate. The difference of our approach to the Engel and Kim model is that our permanent component is constant over some periods with a positive probability whereas it is constantly moving in the Engel and Kim approach. However, one important difference between our model and the Engel and Kim model and the Hedgewood and Papell model is that our transition component towards the equilibrium is nonlinear and depends on the distance between the actual real exchange rate and the switching equilibrium values. In this way, we combine two important features from two major literatures: one that focuses on the time-varying equilibrium real exchange rate but is restricted to linear mean-reversions, and the other that stresses nonlinear mean-reversions but inhibits a changing equilibrium real exchange rate.

We present a simple economic model in the spirit of Engel and West (2005) and Engel (2015) to motivate our econometric specification. Consider the following general interest parity condition:

$$E_t s_{t+1} - s_t = i_t - i_t^* + x_t \tag{1}$$

where s_t is the (log) nominal exchange rate and i_t is the nominal interest rate at time t. E_t denotes the expected value. The exchange rate is quoted as the price of one foreign currency in terms of domestic currencies and the variables with $*$ denote foreign variables. The stochastic variable x is necessary as there has been overwhelming evidence against the strong version of the uncovered interest parity (Fama (1984)). One economic justification of the variable x is that it represents a time-varying risk premium within the rational expectation framework. By definition the (log) real exchange rate q is given by:

$$q_t = s_t + p_t^* - p_t, \tag{2}$$

where p denotes the (log) price level. Also by definition the inflation rate π is:

$$\pi_{t+1} = p_{t+1} - p_t \tag{3}$$

and the real interest rate r_t is defined by:

$$r_t = i_t - E_t \pi_{t+1}. \tag{4}$$

Combining equations (1), (2), (3), and (4), one can easily derive the following stochastic difference equation that determines the real exchange rate:

$$q_t = E_t q_{t+1} + (r_t^* - r_t) - x_t.$$

Iterate this equation forward to obtain:

$$q_t = E_t \sum_{j=0}^{\infty} (r_{t+j}^* - r_{t+j}) - E_t \sum_{j=0}^{\infty} x_{t+j} + \lim_{T\to\infty} E_t q_{t+T}.$$

This present value formulation of the exchange rate determination has been explored by Engel and West (2006) and Balke et al. (2013) among many others. In this particular type of simple model based on the parity condition, the economic fundamentals are the real interest rate differential and the risk premium. The last term will be nontrivial if the rational bubble is present. The real interest rates are primarily affected by productivity, and possibly monetary policies in presence of nominal rigidities and monopolistic competition (Woodford (2003)). The equilibrium real exchange rate can also change due to the second term in the above equation so-called risk premium, which is by consensus most likely stationary, see Engel et al. (2012). At the same time, the rational bubble can emerge in some periods but be contained in others. Given such economic model, it makes sense to employ the stochastic Markov switching variable to approximate the time-varying equilibrium real exchange rate due to changing economic fundamentals and/or the emergence and disappearance of the rational bubbles. Suppose traders collect fundamental news and take advantage of such a model or similar ones to execute her trades. She will not start to trade, however, unless the actual exchange rate differs from her calculated equilibrium value by a margin large enough to make up for the involved transaction or information cost. Therefore, reversions to the equilibrium real exchange rate will be nonlinear.

The rest of the paper is organized as follows. Section 3 defines our model, section 4 describes the estimation of the model parameters, section 5 applies the model on real exchange rate data and section 6 concludes. All Tables and Figures are in the appendix.

3 The Markov-STAR Model

The general ESTAR model is given by two autoregressive regimes connected by a smooth exponential transition function $\mathcal{G}(\cdot;\gamma,c): \mathbb{R} \to [0,1]$. This function governs the transition between the two regimes in a smooth way. Alternatively, an ESTAR model can also be interpreted as a continuum of regimes which is passed through by the process. In general, univariate ESTAR(p) models, $p \geq 1$ and $d \leq p$, are given by

$$\begin{aligned}(q_t - c) &= \left(\sum_{k=1}^{p} \psi_k (q_{t-k} - c)\right) \times (1 - \mathcal{G}(q_{t-d};\gamma,c)) + \left(\sum_{k=1}^{p} \theta_k (q_{t-k} - c)\right) \times \mathcal{G}(q_{t-d};\gamma,c) + \varepsilon_t \\ &= \sum_{k=1}^{p} \psi_k (q_{t-k} - c) + \left(\sum_{k=1}^{p} \phi_k (q_{t-k} - c)\right) \times \mathcal{G}(q_{t-d};\gamma,c) + \varepsilon_t, \quad t \geq 1,\end{aligned} \tag{5}$$

with $\varepsilon_t \overset{iid}{\sim} (0,\sigma^2)$.[1] c can be interpreted as the threshold parameter where $\mathcal{G}(c;\gamma,c) = 0.5$. As we use demeaned data in our application we specify the threshold parameter to be the mean of the respective real exchange rate, i.e. $c = \overline{q}$.

For an ESTAR model the transition function $\mathcal{G}(\cdot)$ is given by

$$\mathcal{G}(\cdot;\gamma,c) = 1 - \exp\left\{-\gamma(q_{t-d} - c)^2\right\}; \quad \gamma > 0\,.$$

This exponential transition function provides a symmetric adjustment towards the equilibrium. Surveys of the broad field of nonlinear time series models in general and STAR models in particular are given by Potter (1999) and van Dijk et al. (2002); see also Teräsvirta (1994).

The most frequently used special case of the general ESTAR model in (5) is the ES-

[1]Throughout the paper we set $d = 1$

TAR(1) model

$$(q_t - c) = \psi(q_{t-1} - c) + \phi(q_{t-1} - c)\left\{1 - \exp\left(-\gamma (q_{t-1} - c)^2\right)\right\} + \varepsilon_t \, .$$

To model real exchange rate behavior, Taylor et al. (2001) and Rapach and Wohar (2006) impose an inner unit root regime, $\psi = 1$. This regime is corrected back by a white noise process for the outer regime, $\phi = -1$, to ensure global stationarity. In general, stationarity is given as long as $|\psi + \phi| < 1$.

Estimation of these models either by nonlinear least squares or maximum likelihood techniques is treated by Klimko and Nelson (1978) and Tjøstheim (1986), respectively. For the Markov switching framework we use the framework based on Lindgren (1978), Engel and Hamilton (1990) and Engel (1994):

$$q_t = \mu_{s_t} + \phi_{1s_t} q_{t-1} + \ldots + \phi_{ps_t} q_{t-p} + \varepsilon_t \, .$$

The values of the autoregressive parameters $\phi_{1s_t}, \ldots, \phi_{ps_t}$ and the mean μ_{s_t} and thus the regime switching is governed by an unobservable Markov chain

$$\mathcal{P}(s_t = j | s_{t-1} = i, s_{t-2} = k, \ldots, q_{t-1}, q_{t-2}, \ldots) = \mathcal{P}(s_t = j | s_{t-1} = i) = p_{ij} \, .$$

The transition probabilities p_{ij} lie in the open unit interval and $\mu_1 \neq \mu_2 \neq \cdots \neq \mu_N$ to ensure a transient Markov chain and clear identification of the N regimes. The $(N \times N)$ transition probability matrix is then given by

$$P = \begin{pmatrix} p_{11} & \cdots & p_{1N} \\ \vdots & p_{ij} & \vdots \\ p_{N1} & \cdots & p_{NN} \end{pmatrix}.$$

s_t is assumed to be an ergodic homogeneous Markov chain with invariant probability measure $\pi = (\pi_i)$ and it is initiated at $t = 0$ to guarantee the independence of $(s_t)_{t>0}$. Extensions of this basic framework are possible, see e.g. Hamilton and Raj (2002) and the papers cited therein.

The MSAR(1)-ESTAR(1) model (henceforward Markov-STAR) considered in this paper is a combination of the MSAR(1)-model in the first regime of an ESTAR(1) process

$$q_t = \mu_{s_t} + \phi q_{t-1} + \psi \mathcal{G}(q_{t-1}; \gamma, c) q_{t-1} + \varepsilon_t, \tag{6}$$

where ε_t is a white noise error term with $E(\varepsilon_t) = 0$ and $Var(\varepsilon_t) = \sigma^2_{s_t}$. This means that we allow only the mean and the variance of the process to switch whereas the autoregressive parameters are hold fixed. Allowing the autoregressive parameters to switch too would make the model difficult to estimate and results hard to interpret. It should also be mentioned that the transition function $\mathcal{G}(\cdot)$ is centered around the switching mean so that the adjustment process depends on the current state of the equilibrium which is one of the desired properties of our model.

Note that via variation of γ our model nests a markov-switching only model as well as a random walk model with a switching drift term. Concretely when $\gamma \to \infty$, the transition function $\mathcal{G}(\cdot)$ gets 1 and the second and third term in (6) drop out. Hence the model becomes a markov switching only model. On the other hand, if $\gamma \to 0$ we have $\mathcal{G}(\cdot) \to 0$ and our model becomes a random walk with switching drift.

Our empirical findings indicate that the half-lifes of a one standard deviation shock depend on the smooth transition part, and thus on γ, quite heavily. Further, our estimates for γ show that a markov-switching only model does not seem appropriate for a vast majority of the countries. Hence we note that the contribution of the smooth transition part in our model is not only given by economic theory alone, but also seems to be important from an empirical point of view. Section 5 describes those considerations and the empirical results in more detail.

4 Model Specification and Estimation

The Markov-STAR model (6) is estimated via maximizing the likelihood function

$$L(\mu, P, \sigma; \gamma, c) = \sum_{i=1}^{N} \sum_{t=1}^{T} \log f_{it} \quad \text{with} \quad f_{it} = \mathbf{1}'(\xi_{it|t-1} \odot \eta_{it}),$$
$$\eta_{it} = \left(2\pi\sigma_i^2\right)^{-1/2} \exp\left(-0.5\varepsilon_{it}^2/\sigma_i^2\right) \quad \text{and} \quad \varepsilon_{it} = q_t - \mu_i - \phi q_{t-1} - \psi \mathcal{G}(q_{t-1}; \gamma, c) q_{t-1} \tag{7}$$

with (ϕ, ψ) being set to $(1, -1)$ and $\odot$ denotes element-wise multiplication. $\xi_{t|t} := Pr(s_t = i|q_t; \theta)$ with $\theta = (\mu_1, \ldots, \mu_N, \sigma_1, \ldots, \sigma_N, \phi, \psi, \gamma, c)$ denotes the conditional probability that the tth observation lies in regime i. $\xi_{t|t}$ then describes the $(N \times 1)$ vector of transition probabilities being constructed by the filter

$$\begin{aligned} \xi_{t|t} &= \frac{\xi_{t|t-1} \odot \eta_t}{\mathbf{1}'(\xi_{t|t-1} \odot \eta_t)} \\ \xi_{t+1|t} &= P \cdot \xi_{t|t} \end{aligned}$$

where the starting values in $\xi_{1|0}$ are set to N^{-1}. The smoothed transition probabilities are calculated via the algorithm of Kim (1993) being given by

$$\xi_{t|T} = \xi_{t|t} \odot \left(P'\left(\xi_{t+1|T} \odot \xi_{t+1|t}^{-1}\right)\right). \tag{8}$$

The iteration starts with $\xi_{T|T}$ and goes backwards until $t = 1$. γ is selected using a grid search with values $\gamma_k = (0.01, 0.03, 0.05, 0.1, \ldots, 2, 2.5, 3, \ldots, 10, 15)$. Hence for each one of the $k = (1, \ldots, 59)$ values of γ_k the likelihood is maximized resulting in $L_k = L(\theta_k)$ with $\theta_k = (\mu_k, P_k, \sigma_k; \gamma_k)$. The final model specification with parameter vector θ_{k^*} then satisfies $k^* = \arg\max_k L_k$.

In the second part of the analysis we specify a logit model in order to explain the behavior of the transition probabilities by macro variables X such as output gap differentials, inflation differentials and economic uncertainty. For this purpose we take the first row of the smoothed probabilities $\xi_{t|T}$, i.e. we fix $i = 1$ in $\tilde{p}_i := Pr(s_t = i | q_T; \theta)$. Then we recode the first row of $\xi_{t|T}$ into a binary variable Y such that

$$Y := \begin{cases} 0 & \text{for } \tilde{p}_1 \leq 0.5 \\ 1 & \text{for } \tilde{p}_1 > 0.5. \end{cases} \tag{9}$$

Hence the markov chain is expected to be in the second state for $Y = 0$.

We then fit the logit model $Y = X\beta + u$ via maximum likelihood where u denotes an error term. The estimated coefficients $\hat{\beta}$ then describe the marginal effect of X on $\ln(\tilde{p}_1/(1-\tilde{p}_1))$. Thus if $\hat{\beta} < 0$, $\tilde{p}_1 < 0.5$ and one would expect an increase of the macro variable leading to a switch from regime 1 to regime 2.

5 Empirical Analysis

We fit the Markov-STAR model (6) to real exchange rate data of 18 countries.[2] For this we use monthly data from 01/60 until 04/14 for the real exchange rates yielding a sample size of at most $T = 652$ (without missing data). Hence for those countries having adopted the Euro in 1999 (Finland, France, Italy, Netherlands, Portugal and Spain) we have data from 01/60 until 12/98, i.e. $T = 468$ observations. An exception marks Germany where data is available only from the time of the German reunification until the Euro adoption.

The real exchange rates are constructed as described in section 2, i.e. $q_t = s_t + p_t^* - p_t$ and demeaned. Hence as q_t is formulated in direct quotation an increasing value of q_t

[2]The selection of the countries is due to data availability of both the exchange rate data as well as the country-specific exogenous explanatory variables.

corresponds to a depreciation of the currency in real terms and vice versa.
For country-specific explanatory variables we compute the output gap and inflation differentials, all relative to the U.S. All data are taken from the *IMF eLibrary*. The output gap differential is constructed as in Engel and West (2006), i.e. as the residuals from quadratically detrended output, where the output is measured as the log of seasonally adjusted industrial production. The output gap differential variable is then constructed as the U.S. output gap minus the output gap of the respective country.
Inflation is measured as the first differences of log *CPI*. Inflation differentials are then constructed as US inflation minus the respective country inflation rate. Finally as a measure for uncertainty we utilize the index by Nick Bloom.[3]
The parameter estimates of the Markov-STAR model (6) are displayed in Table 1. As the real exchange rates are demeaned, a positive (negative) value of $\hat{\mu}$ corresponds to a depreciation (appreciation) of the currency over time. Hence for all countries a regime switch from the first to the second regime marks an appreciation of the currency while for almost two thirds of the countries the probability of residing is slightly higher in the depreciation than in the appreciation regime as indicated by $\hat{p}_1$ and $\hat{p}_2$.
The parameter γ^* dictates the transitional path of the exchange rates toward its equilibrium value. The magnitude of γ^* estimates imply that the transition back to the equilibrium takes place rather smoothly for most of the countries. Here we observe values of $\gamma^* \in [0.01, 0.6]$ for almost 80% of the countries. Exceptions are i.a. Chile and Germany where we observe a high value of γ going along with small sample sizes.
The smooth transition behavior of selected countries is plotted in Figure 1. To get a better idea about the contribution of the smooth transition part we additionally calculate the half-lifes for those countries. For this we apply the methodology for generalized impulse-response functions of Koop et al. (1996), i.e. we conduct the Monte Carlo integration algorithm with a one standard deviation shock to the real exchange rates. The resulting half-lifes are given in Table 2. The first column returns the half-lifes implied by the estimated γ^* given in Table 1. The next two columns describe the resulting half-lifes if a very small (large) γ of .01 (15) is imposed. As one can clearly see, the implied half-lifes differ substantially over different values of γ. The difference for UK e.g. adds up to value of over 13 years. The higher the value of γ the faster the shock fades out due to the fact that the smooth transition part drops out of the model. On the other hand the shock only very slowly fades out if we have a small γ as the transition function becomes smoother in that case. Hence we can say that the contribution of γ seems to be quite decisive concerning the transition behavior and thus the economic implications.

[3]http://www.policyuncertainty.com. Another natural choice would have also been the CBOE Volatility Index (VIX). It is, however, merely available since 1990.

In order to explain the switching behavior of the real exchange rates and relate these switchings to economic fundamental variables, we utilize the smoothed probabilities from the estimation results given by (8) and specify the binary variable Y as described in (9). Then we fit a logit model by regressing the binary variable onto the economic fundamentals such as the output gap differential, inflation differential, and economic uncertainty. The estimation results are displayed in Table 3.[4]

Based on the variable definitions and regression specifications in section 4, an increase in the value of a fundamental variable increases the likelihood of being in regime 1 given a positive β estimate associated with that fundamental variable, but increases the likelihood of being in regime 2 if the β estimate associated with the fundamental variable is negative instead. Turning to the results for the output gap differential first, it appears that in most cases the sign of the estimated coefficients is consistent with what the economic theory or intuitions would predict. Specifically, a higher output gap differential corresponds to a better US economy relative to the studied country and thus leads to an appreciation of the US dollar or the depreciation of the studied currency. Therefore, the sign of $\hat{\beta}$ is expected to be positive so that an increase in the value of the output gap differential will increase the likelihood of being in the regime 1, which is the depreciation regime. For those estimates being significant at the 10% level, expectations are met in almost 90% of the cases.

Concerning inflation, the results are somewhat mixed. Intuitively, an increase in the value of the inflation differential indicates that the US experiences a higher inflation rate than the studied country, leading to an appreciation of the currency against US dollar. As a result, we expect the sign of $\hat{\beta}$ to be negative so that an increasing inflation differential tends to increase the likelihood of being the appreciation regime 2. This, however is true only for 50% of all the cases with significant coefficients.

Finally, economic uncertainty is taken into account by including the Bloom uncertainty index in the regression. This variable is statistically significant in 11 out of total 18 countries in investigation, and thus seems important in describing the switching behavior of the exchange rate. Unfortunately, to our knowledge the existing economic theory has yet to yield a clear prediction of the direction of the impact of the economic uncertainty on exchange rate changes. Intuitively a rise in economic uncertainty affects all currencies and thus results in a higher risk premium in all currencies markets, leaving it unclear whether a particular currency will appreciate or depreciate against the US dollar. However, if it becomes clear that the currency is affected more (less) than its benchmark currency – US dollar in this case – then the currency is expected to de-

[4] The estimation algorithm for Greece did converge only until the convergence tolerance has been set to a higher value. Still the estimates indicate strange results such that the coefficient estimates are all insignificant - yet the McFadden R^2 is very high.

preciate (appreciate) relative to the US dollar. The regression results in Table 3 seem to suggest that a rising economic uncertainty generally appears to affect a number of European countries (namely Denmark, Finland, France, Germany, Italy, Netherlands, Portugal, and Spain) less than the US so that it tends to raise the likelihood of being in the appreciation regime 2 given the negative $\hat{\beta}$. On the other hand, for several other countries (namely Brazil, Canada, Turkey, and Norway), a rising economic uncertainty appears to lead to a depreciation regime of their currencies.[5]

Although the model we estimate so far allows both mean and volatility to switch between two regimes, one drawback of this model is that we have only one latent MS factor to govern both mean and volatility switchings, which is entirely due to a parsimonious purpose.[6] Despite this limitation, our model does allow a possible investigation of the relationship between volatility switchings and economic fundamentals, which is particularly relevant when relating switchings to the Bloom uncertainty index. In other words, one can also interpret the connection between the exchange rate regime switchings and the uncertainty index as reflecting the close association between the general economic uncertainty and the exchange rate market volatility. For example, the estimation results for Brazil in Table 1 show that its currency volatility becomes significantly lower from regime 1 (high volatility) to regime 2 (low volatility). The corresponding significantly positive coefficient estimate associated with Bloom uncertainty index for Brazil in Table 3 thus implies that a rising economic uncertainty index is closely related with an elevating currency risk in the exchange rate market. The same logic applies to several other currencies that appear to experience significant volatility switchings: Canada, Finland, France, and Turkey. However, such a pattern fails to emerge in general. To investigate to what extent this is due to the drawback of mixing switchings of mean and volatility as mentioned above, we modify (6) such that μ is fixed while σ is still allowed to switch to focus on the volatility switching. The model estimation results are displayed in Table 4. Table 5 presents the corresponding logit estimation results for the case of a fixed μ. Following the same specification as before, $\hat{\beta} < 0$ implies an increase in the value of the economic fundamental variable increases the likelihood of being in regime 2 whereas $\hat{\beta} > 0$ corresponds to an increasing likelihood of being in regime 1. Based on the results from Table 4, a switch from regime 1 to regime 2 corresponds to an increase in volatility. If we focus on the results associated with the economic uncertainty index in Table 5 we have statistically significant coefficient estimates in 11 out total 18 countries in investigation. Furthermore, in all these significant cases the coefficient estimates are negative and thus suggest that a rising economic uncertainty tends to increase the likelihood of switching

[5]We notice that the economies of Canada and Norway are featured by a heavy reliance of productions and exports of oil commodity, whereas Brazil and Turkey are developing economies.

[6]We leave it as a future research endeavor to relax this restriction and allow for more general switchings.

from the low volatility regime 1 to the high volatility regime 2, remarkably consistent with the economic intuitions.

Finally, given our discussions of the exchange rate level regime switchings associated with the fundamental variables based on the general model that allows both mean and volatility switchings, but controlled by the same latent MS factor, we want to make sure that the switching is not entirely driven by the volatility switching. Therefore, we estimate (6) by fixing σ and present the results in Table 6. We then use the Likelihood Ratio (LR) test to compare the model with the restricted σ to the general model with unrestricted σ. Hence we calculate $LR = -2\ln(L_{k^{**}}/L_{k^{*}})$ where $L_{k^{**}}$ ($L_{k^{*}}$) denotes the maximized value of (7) in the restricted (unrestricted) model. The test statistics are given in the last column of Table 6. As one can clearly see, all LR values lie between 0 and 1.09. With a critical value of 2.71 for $\alpha = 0.1$ the Null of $H_0 : \sigma_2 = 0$ can never be rejected. Hence we can conclude that it is rather the switch in mean than the switch in volatility that drive the real exchange rate dynamics in the most general model that we estimate.

6 Conclusion

Motivated by popular economic models we propose a new nonlinear Markov-STAR model to capture both the time-varying equilibrium real exchange rates – due to various economic factors – and the smooth transition type of nonlinear adjustment to the equilibrium – due to the economic intuitions that imply arbitrage profits only beyond certain transaction and transportation costs. We find this Markov-STAR model can better capture the time series dynamics of the real exchange rate and the implied half-lifes. More importantly, we aim to evaluate the connection between the real exchange rate and its economic fundamentals through the lens of this newly proposed nonlinear model. The connection between the real exchange rate and its economic fundamentals is featured in most economic models but the empirical evidence to establish such a close relationship has been all but conclusive. To our knowledge, our work is the first one to investigate such a connection in a highly nonlinear context, which takes into account the realistic feature of nonlinearity in exchange rate data. We use US as the benchmark country and apply our model to 18 countries. We find strong evidence that the varying equilibrium real exchange rate is closely related with economic fundamentals predicted by standard economic models. Specifically, we find that a worsening economy relative to US economy tends to significantly increases the likelihood of the real exchange rate to depreciate relative to the US Dollar. Our exercises also cast a light on the role of the economic uncertainty in affecting the equilibrium exchange rates. We find that a higher economic uncertainty index in US significantly increase the likelihood of a real exchange rate appreciation for many advanced European economies while it is the opposite for some developing countries. Lastly, but not least interestingly, we document compelling evidence that a rising economic uncertainty tends to be associated with a higher exchange rate volatility, consistent with the usual economic intuitions. Based on these findings, we reach our conclusion that the connection between the exchange rates and their economic fundamentals becomes much stronger and clearer once the realistic nonlinearity feature of the real exchange rate is explicitly accounted for. Therefore, although our work provides additional empirical support for the fundamental approach to the real exchange rate, it also points out the importance of including the nonlinearity feature into standard economic models of the real exchange rate.

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7 Appendix

7.1 Tables

Country	$\hat{\mu}_1$	$\hat{\mu}_2$	$\hat{p}_1$	$\hat{p}_2$	$\hat{\sigma}_1$	$\hat{\sigma}_2$	γ^*	T
Brazil	0.0021	-0.0023	0.8801	0.9818	0.0618	0.0101	0.01	413
Canada	0.0011	-0.0006	0.8855	0.9908	0.0179	0.0031	0.01	652
Chile	0.0600	-0.0022	0.9990	0.9990	0.0000	0.0211	1.75	64
Denmark	0.0211	-0.0204	0.9499	0.9378	0.0148	0.0151	0.40	568
Finland	0.0000	-0.0014	0.9985	0.9916	0.0000	0.0280	1.15	468
France	0.0000	-0.0008	0.9990	0.9792	0.0000	0.0287	0.01	468
Germany	0.0208	-0.0256	0.9501	0.9438	0.0170	0.0153	15.00	96
Greece	0.0013	0.0000	0.9806	0.9990	0.0258	0.0001	0.55	492
Italy	0.0000	-0.0008	0.9990	0.9749	0.0001	0.0246	0.15	468
Japan	0.0102	-0.0376	0.9627	0.9102	0.0159	0.0196	0.03	652
Mexico	0.0031	-0.0100	0.9774	0.9990	0.0444	0.0001	0.03	652
Netherlands	0.0140	-0.0277	0.9604	0.9485	0.0149	0.0141	0.30	468
Norway	0.0002	-0.0002	0.9990	0.9602	0.0005	0.0241	0.15	652
Portugal	0.0108	-0.0317	0.9588	0.9389	0.0171	0.0142	0.40	468
Spain	0.0170	-0.0215	0.9536	0.9438	0.0169	0.0155	0.30	468
Sweden	0.0001	-0.0005	0.9990	0.9668	0.0003	0.0266	0.60	652
Turkey	0.0196	-0.0039	0.8583	0.9762	0.0869	0.0188	0.01	544
UK	0.0172	-0.0199	0.9156	0.9357	0.0161	0.0130	2.50	316

Table 1: Estimation results of the Markov-STAR model $q_t = \mu_{s_t} + \phi q_{t-1} + \psi \mathcal{G}(q_{t-1}; \gamma, c) q_{t-1} + \varepsilon_t$ with $\phi = 1$ and $\psi = -1$.

Country	$\gamma = \gamma^*$	$\gamma = .01$	$\gamma = 15$	St. Dev.
Denmark	2.2074	11.0146	0.3067	0.1871
Japan	4.4847	6.3560	0.0833	0.3746
Spain	2.4880	10.5711	0.0833	0.2727
UK	1.2280	13.6058	0.6049	0.0835

Table 2: Returns the half-lifes in years of the Markov-STAR model $q_t = \mu_{s_t} + \phi q_{t-1} + \psi \mathcal{G}(q_{t-1}; \gamma, c) q_{t-1} + \varepsilon_t$ with $\phi = 1$ and $\psi = -1$ for a one standard deviation shock. The first column refers to the estimated value of γ from Table 1, the second with an imposed γ of 0.01 and the third with an imposed γ of 15. The empirical standard deviations are given in the last column.

Country	Cons	p-value	Outp gap	p-value	Inflation	p-value	Bloom	p-value	McF R^2	T
Brazil	**-1.2619**	0.0240	2.1166	0.1675	0.2104	0.1265	**0.0083**	0.0178	0.3498	277
Canada	**-2.5149**	0.0000	-0.8764	0.5863	**22.3886**	0.0007	**0.0269**	0.0000	0.2127	638
Chile	-1.4458	0.8621	-5.8024	0.8216	95.0608	0.1064	-0.0180	0.6647	0.3274	50
Denmark	**1.1618**	0.0033	0.1359	0.9115	6.1457	0.2108	**-0.0090**	0.0009	0.1700	482
Finland	**-1.2216**	0.0415	**6.9882**	0.0022	**-11.2645**	0.0435	**-0.0158**	0.0045	0.1860	456
France	**1.4334**	0.0059	**5.2411**	0.0026	-8.7965	0.1933	**-0.0217**	0.0000	0.2611	456
Germany	**5.7656**	0.0042	**30.4195**	0.0001	**-160.3895**	0.0002	**-0.0516**	0.0011	0.3935	84
Greece	3.2912	0.2530	-3.0250	0.8110	-19.7693	0.4640	-0.0011	0.9510	0.9720	71
Italy	**-0.9686**	0.0834	**8.4582**	0.0000	1.7153	0.6583	-0.0025	0.6483	0.1774	456
Japan	**3.1656**	0.0000	**2.7421**	0.0000	3.0448	0.3900	**-0.0115**	0.0000	0.0883	638
Mexico	0.1086	0.8485	0.8032	0.7558	**-2.9040**	0.0016	0.0060	0.1086	0.4300	410
Netherlands	**3.7095**	0.0000	**6.9630**	0.0000	-2.3859	0.6104	**-0.0222**	0.0000	0.2008	456
Norway	**-3.0725**	0.0000	-1.4693	0.1025	**-14.1791**	0.0091	**0.0054**	0.0678	0.0349	638
Portugal	**4.1083**	0.0000	1.6396	0.1463	-3.0523	0.1354	**-0.0264**	0.0000	0.1743	456
Spain	**2.2098**	0.0000	**3.1035**	0.0003	**5.7628**	0.0564	**-0.0147**	0.0004	0.1923	456
Sweden	**-2.3789**	0.0231	**-5.7314**	0.0588	28.2894	0.1957	-0.0015	0.8031	0.7718	206
Turkey	**-2.6346**	0.0007	**7.8153**	0.0000	**1.6519**	0.0714	**0.0078**	0.0573	0.4395	350
UK	0.7189	0.1648	0.4888	0.8291	**20.8430**	0.0176	-0.0026	0.4259	0.0689	302

Table 3: Logit regression results. Significant coefficients for $\alpha = 0.1$ in bold.

Country	$\hat{\mu}$	$\hat{p}_1$	$\hat{p}_2$	$\hat{\sigma}_1$	$\hat{\sigma}_2$	γ^*
Brazil	-0.0020	0.98201	0.8801	0.0101	0.0618	0.0100
Canada	-0.0005	0.9910	0.8854	0.0032	0.0180	0.0100
Chile	-0.0100	0.9990	0.9917	0.0000	0.0279	0.0100
Denmark	0.0004	0.9890	0.8978	0.0072	0.0313	0.0300
Finland	-0.0008	0.9900	0.8905	0.0062	0.0367	0.1000
France	0.0000	0.9990	0.9672	0.0001	0.0288	0.0300
Germany	-0.0064	0.9990	0.9356	0.0013	0.0295	6.5000
Greece	-0.0000	0.9990	0.9792	0.0000	0.0265	0.0300
Italy	0.0000	0.9990	0.9773	0.0000	0.0261	0.0100
Japan	0.0004	0.9832	0.8703	0.0079	0.0372	0.0100
Mexico	-0.0034	0.9782	0.8794	0.0079	0.0832	0.0100
Netherlands	0.0008	0.9822	0.9213	0.0025	0.0292	0.0300
Norway	0.0001	0.9990	0.9553	0.0002	0.0265	0.0100
Portugal	0.0001	0.9990	0.9565	0.0007	0.0265	0.1000
Spain	0.0002	0.9990	0.9377	0.0004	0.0278	0.0100
Sweden	0.0000	0.9988	0.9880	0.0000	0.0256	0.0300
Turkey	-0.0028	0.9751	0.8592	0.0183	0.0869	0.0300
UK	0.0020	0.9733	0.8720	0.0075	0.0323	1.8500

Table 4: Estimation results of the Markov-STAR model $q_t = \mu_{s_t} + \phi q_{t-1} + \psi \mathcal{G}(q_{t-1};\gamma,c)q_{t-1} + \varepsilon_t$ with μ fixed, $\phi = 1$ and $\psi = -1$.

Country	Cons	p-value	Outp gap	p-value	Inflation	p-value	Bloom	p-value	McF R^2	T
Brazil	**1.2199**	0.0290	-2.0712	0.1764	-0.2153	0.1174	**-0.0081**	0.0201	0.3494	277
Canada	**2.3750**	0.0000	0.9886	0.5351	**-21.2466**	0.0011	**-0.0254**	0.0000	0.1994	638
Chile	-3.2782	0.1074	4.5629	0.6382	**82.5774**	0.0310	0.0129	0.1286	0.3728	50
Denmark	**-2.4563**	0.0000	0.9396	0.5976	**17.9266**	0.0268	0.0040	0.2367	0.3990	482
Finland	**4.3886**	0.0000	1.0099	0.4947	-4.7237	0.2255	**-0.0428**	0.0000	0.3330	456
France	**1.4334**	0.0059	**5.2411**	0.0026	-8.7965	0.1933	**-0.0217**	0.0000	0.2611	456
Germany	-0.0354	0.9893	6.7519	0.3806	-34.3305	0.4656	-0.0202	0.3228	0.1486	84
Greece	-3.7843	0.1542	9.0071	0.4445	46.8058	0.1180	0.0128	0.4166	0.9060	72
Italy	0.3635	0.4857	**6.5754**	0.0003	-0.8291	0.8102	**-0.0118**	0.0225	0.2212	456
Japan	**1.5532**	0.0000	**2.8777**	0.0000	**-12.8065**	0.0001	**-0.0142**	0.0000	0.2193	638
Mexico	**2.2328**	0.0000	-3.0232	0.1631	**1.0405**	0.0559	**-0.0077**	0.0119	0.2643	410
Netherlands	**2.3628**	0.0000	1.7446	0.2276	9.7440	0.1002	**-0.0395**	0.0000	0.2843	456
Norway	0.1599	0.5716	**1.7546**	0.0101	**-8.1598**	0.0420	**-0.0116**	0.0000	0.1021	638
Portugal	**-2.3218**	0.0003	1.5063	0.2886	-3.3301	0.2133	0.0005	0.9389	0.0165	456
Spain	0.7946	0.1314	0.5160	0.5240	4.1769	0.1658	**-0.0168**	0.0005	0.1276	456
Sweden	**-2.4018**	0.0206	**-5.9916**	0.0460	34.5575	0.1104	-0.0016	0.7884	0.7798	206
Turkey	**2.5982**	0.0004	**-7.4661**	0.0000	**-1.4762**	0.0814	**-0.0087**	0.0250	0.4256	350
UK	**-1.0304**	0.0557	-2.6436	0.2638	**32.8191**	0.0008	0.0031	0.3610	0.0691	302

Table 5: Logit regression results with μ fixed. Significant coefficients for $\alpha = 0.1$ in bold.

Country	$\hat{\mu}_1$	$\hat{\mu}_2$	$\hat{p}_1$	$\hat{p}_2$	$\hat{\sigma}$	γ^*	LR
Brazil	-0.0067	0.1069	0.9905	0.9322	0.0300	0.1000	0.23
Canada	-0.0055	0.0151	0.9469	0.9369	0.0091	0.5000	0.12
Chile	-0.0140	0.0252	0.9603	0.9467	0.0120	7.0000	0.15
Denmark	-0.0205	0.0210	0.9382	0.9492	0.0150	0.4000	0.00
Finland	-0.0063	0.0406	0.9753	0.9653	0.0194	0.0100	0.36
France	-0.0078	0.0367	0.9710	0.9173	0.0161	0.4000	1.09
Germany	-0.0246	0.0215	0.9387	0.9488	0.0163	15.0000	0.01
Greece	-0.0094	0.0307	0.9522	0.9483	0.0156	0.3500	0.38
Italy	-0.0081	0.0316	0.9726	0.9550	0.0151	0.3000	0.74
Japan	-0.0372	0.0115	0.9283	0.9612	0.0167	0.0500	0.01
Mexico	-0.0018	0.3635	0.9990	0.9990	0.0263	0.2000	0.74
Netherlands	-0.0281	0.0144	0.9428	0.9632	0.0146	0.3500	0.01
Norway	-0.0244	0.0140	0.9292	0.9580	0.0138	0.4000	0.04
Portugal	-0.0298	0.0107	0.9287	0.9567	0.0163	0.3000	0.23
Spain	-0.0084	0.0354	0.9746	0.9360	0.0163	0.0100	0.00
Sweden	-0.0109	0.0279	0.9571	0.9593	0.0154	0.3000	0.26
Turkey	-0.0045	0.1650	0.9970	0.9935	0.0300	0.0500	0.10
UK	-0.0182	0.0187	0.9257	0.9243	0.0148	3.0000	0.01

Table 6: Estimation results of the Markov-STAR model $q_t = \mu_{s_t} + \phi q_{t-1} + \psi \mathcal{G}(q_{t-1}; \gamma, c) q_{t-1} + \varepsilon_t$ with σ fixed, $\phi = 1$ and $\psi = -1$.

7.2 Figures

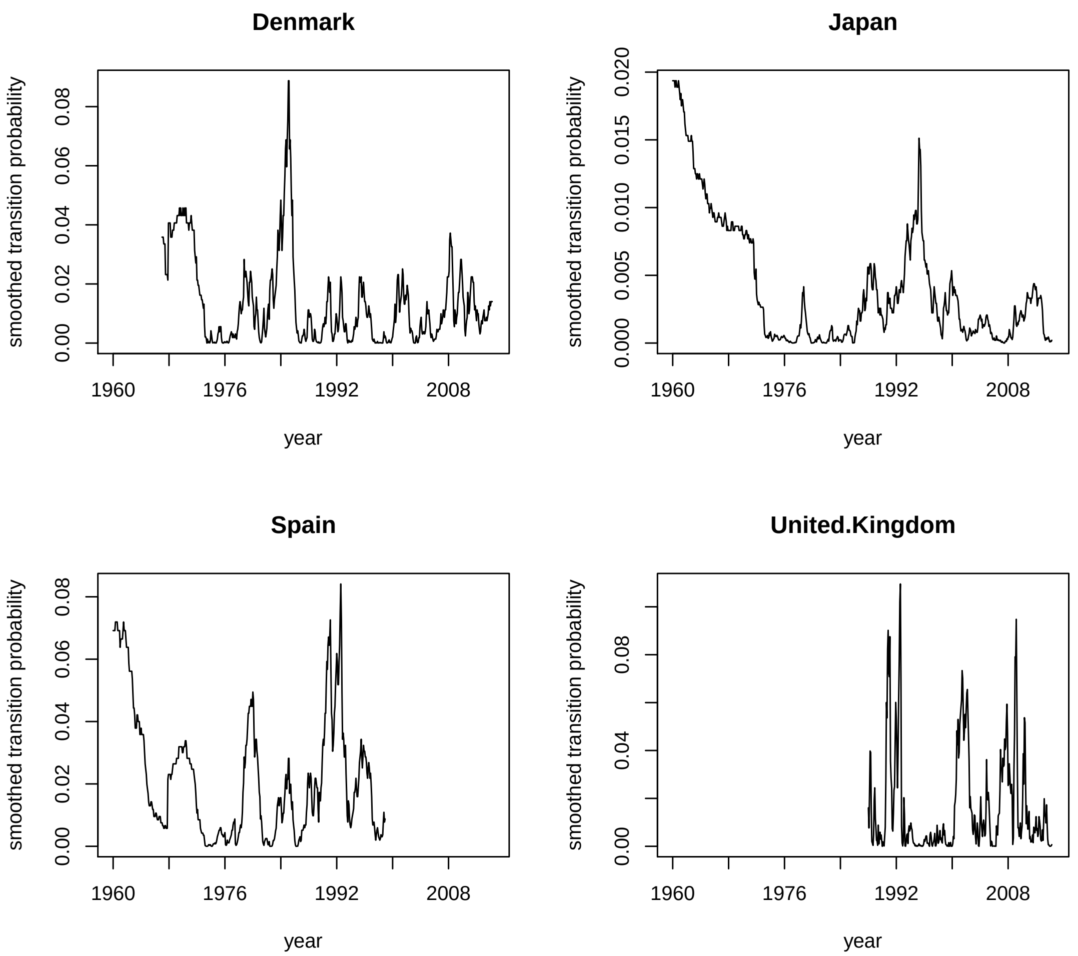

Figure 1: Plots the estimated smoothed transition probabilities $\mathcal{G}(q_{t-1};\gamma^{*},c)q_{t-1}$ of selected countries.