A Principal Component Simulation of Age-Specific Fertility – Impacts of Family and Social Policy on Reproductive Behavior in Germany

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Abstract

This contribution proposes a simulation approach for the indirect estimation of age-specific fertility rates (ASFRs) and the total fertility rate (TFR) for Germany via time series modeling of the principal components of the ASFRs. The model accounts for cross-correlation and autocorrelation among the ASFR time series. The effects of certain measures are also quantified through the introduction of policy variables. Our approach is applicable to probabilistic sensitivity analyses investigating the potential outcome of political intervention. A slight increase in the TFR is probable until 2040. In the median scenario, the TFR will increase from 1.6 in 2016 to 1.68 in 2040 and will be between 1.46 and 1.92 with a probability of 75%. Based on this result, it is unlikely that the fertility level will fall back to its extremely low levels of the mid-1990s. Two simple alternate scenarios are used to illustrate the estimated ceteris paribus effect of changes in our policy variables on the TFR.

Keywords: Fertility**,** Statistical Demography, Forecasting, Family Policy, Principal Component Analysis, Time Series Analysis

1 Introduction

The future development of fertility is of great importance in many areas, particularly regarding planning and evaluation of future needs for political intervention in family and social policy. A shrinking younger working-age population, which is the long-term result of low fertility rates, along with decreasing mortality rates leads to a shift in the age structure in favor of older people. As a result, relatively fewer working-age people shoulder a higher financial burden in terms of social insurance for relatively more elderly people (Bujard 2015; d'Addio and d'Ercole 2005; Vanella 2017a). The fertility level of a country is commonly represented by the total fertility rate (TFR), which is the sum of the age-specific fertility rates (ASFRs) over all ages during a specific year. Therefore, the TFR can be interpreted as the average number of children a woman bears during her reproductive phase, given the current ASFRs remain constant in the future. As this assumption usually does not hold for any woman, the TFR is considered to be a hypothetical measure (Bongaarts and Feeney 1998).

For decades, the TFR in most parts of Europe has been well below replacement level fertility (Eurostat Database 2018a), which is approximately 2.1 for these regions (Bujard 2015; van de Kaa 1987). Although births have no immediate effect on the financial balance of social insurance, they are the most important factor in the long run. Forecasting future ASFRs therefore provides important quantitative information for political decision making

(Zuchandke, Lohse, and Graf von der Schulenburg 2014). For example, political decisions about pension fund reforms must consider the future course of fertility in society and the resulting population structure. Family policy must attempt to address political measures to increase the TFR for low-fertility countries[1], which include the majority of Europe (all but Northern Europe) and Eastern Asia (Bujard 2015).

In economic theory, childbearing is considered to be a personal decision, where the mother maximizes her personal utility (Becker 1960). The decision to bear and raise a child typically subjects the mother to marginal costs. These costs result from temporary or even permanent withdrawal from the labor market. This withdrawal may set back the mother's career opportunities in the long run, not to mention the immediate reduction of income. Furthermore, a pension system, in which the labor force receives future entitlements for their current economic activity, as is the case for Germany, further punishes mothers for leaving the labor market and might be interpreted as an implicit tax on childbearing (Cigno, Casolaro, and Rosati 2003). Therefore, a pension system that compensates mothers for their effort in child raising could ease their marginal costs and stimulate the decision to procreate. We will address this question later (see Section 5).

The feminist movement has led to a postponement of births to an older age (Gustafsson 2001; Lesthaeghe 2010). Whereas this trend is persistent, it is important to determine to which extent these postponed births are being recuperated by births in older age groups. There is a vast discussion in the literature about the effects of family policy on fertility. The conclusions differ strongly depending on the data source, the geographical units or countries under study, the methodology and the variables, as Bujard (2015) has shown. We will discuss this point in the next section with an emphasis on Germany. A superficial analysis of the TFR trends does not account for these shifts. The TFR does not provide information on the actual number of births. High birth numbers result from high fertility probabilities for the cohorts whose female population is large.

This contribution proposes a conditional forecast approach for the future course of ASFRs. The model framework is based on the Lee–Carter model for fertility (see Lee 1993), which makes use of principal component analysis (PCA) and time series analysis. We include family policy variables in the PCA, thereby allowing for stochastic estimation of the potential future impact of political measures aimed at increasing a country's reproductive level. The methodology enables the integration of the correlations among the ASFRs and the autocorrelations among each set of ASFRs. Simulations of Wiener processes enable stochastic quantification of the future course of the ASFRs through prediction intervals (PIs). The trajectories of the ASFRs can be cumulated to stochastic forecasts of the TFR, which will be illustrated with 75% and 90% PIs.

In the next section, we give an overview of past studies on the impact of family policy on the TFR in Germany and other low-fertility countries. Section 3 provides a short review of past and present approaches for projecting or forecasting fertility in Germany, although many of these models are commonly applied internationally as well. In Section 4, we propose our forecast model. The model is in essence a two principal component time series model that avoids the variance bias resulting from the Lee–Carter model, as proposed by Vanella (2017a). Our model indicates that fertility trends are strongly affected by the tempo effect in fertility (Bongaarts and Feeney 1998). Moreover, we conclude that the quantum of fertility can be influenced by adequate political measures. Our model

[1] Low-fertility countries are those with a TFR of less than 2.1 children per woman (United Nations 2015).

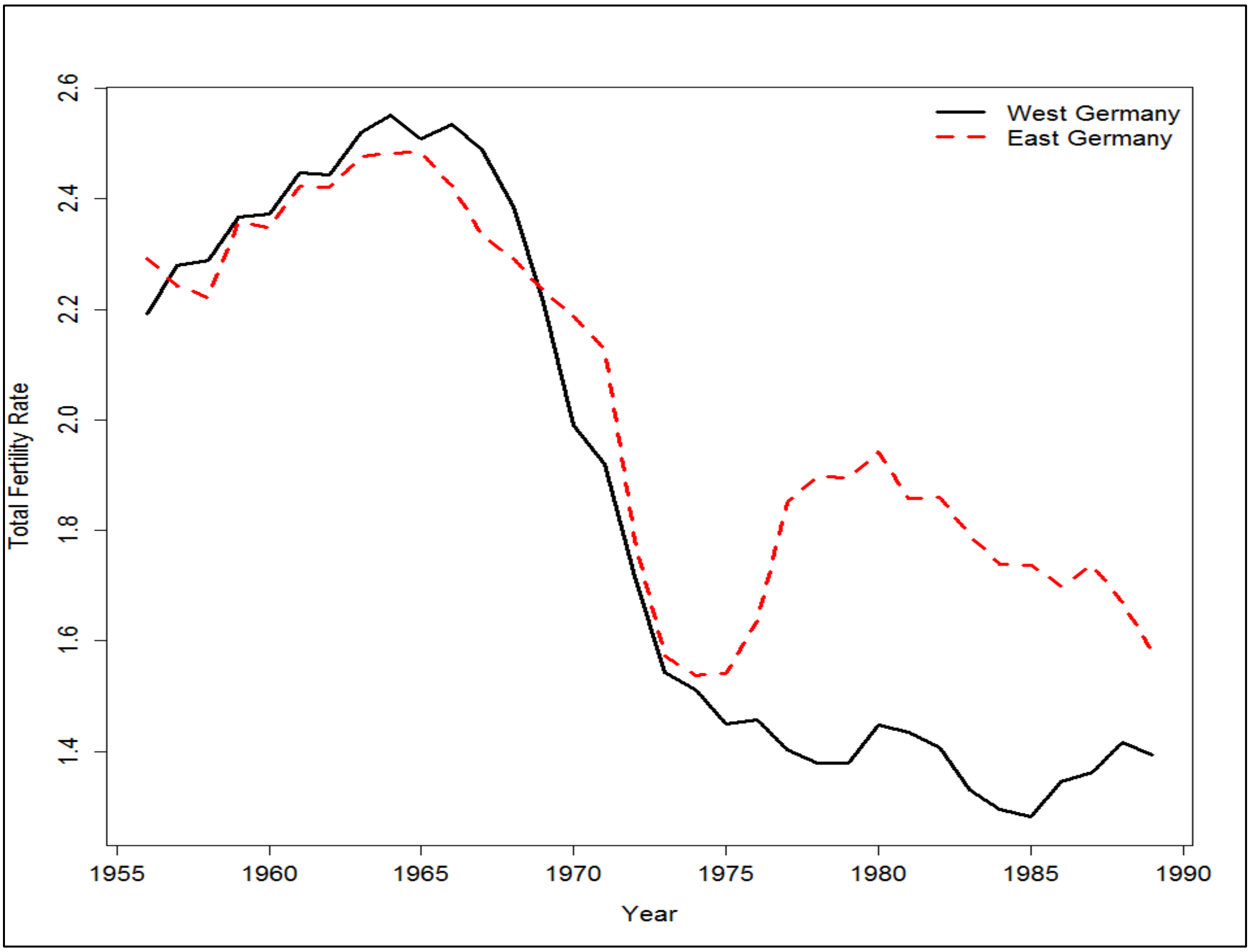
is applied for probabilistic sensitivity analyses of reforms in family policy based on a selection of variables available for Germany. However, the framework can be applied to other countries and works even better for countries with longer time series and better data.

2 Studies on the Link between Family Policy and Fertility Level

The importance of family policy to fertility is often debated. The effects of certain measures are difficult to specify because their impact can be observed directly only in the long run or not at all (Bujard 2011; d'Addio and d'Ercole 2005). For Germany, statistical testing of the impact of policy is extremely hard for several reasons. First, German reunification in 1990 caused a structural break. Second, time series (TS) of political variables are not always available for long past horizons, for example, due to reforms and the resulting structural breaks in the data. The West German (*Bundesrepublik Deutschland or BRD)* government realized the issue of low fertility and the importance of family policy and implemented many measures beginning in the late 1970s.

Fig. 1 illustrates the course of the TFRs for the two German entities from 1956 to 1989.

Fig. 1 Total fertility rates in West and East Germany

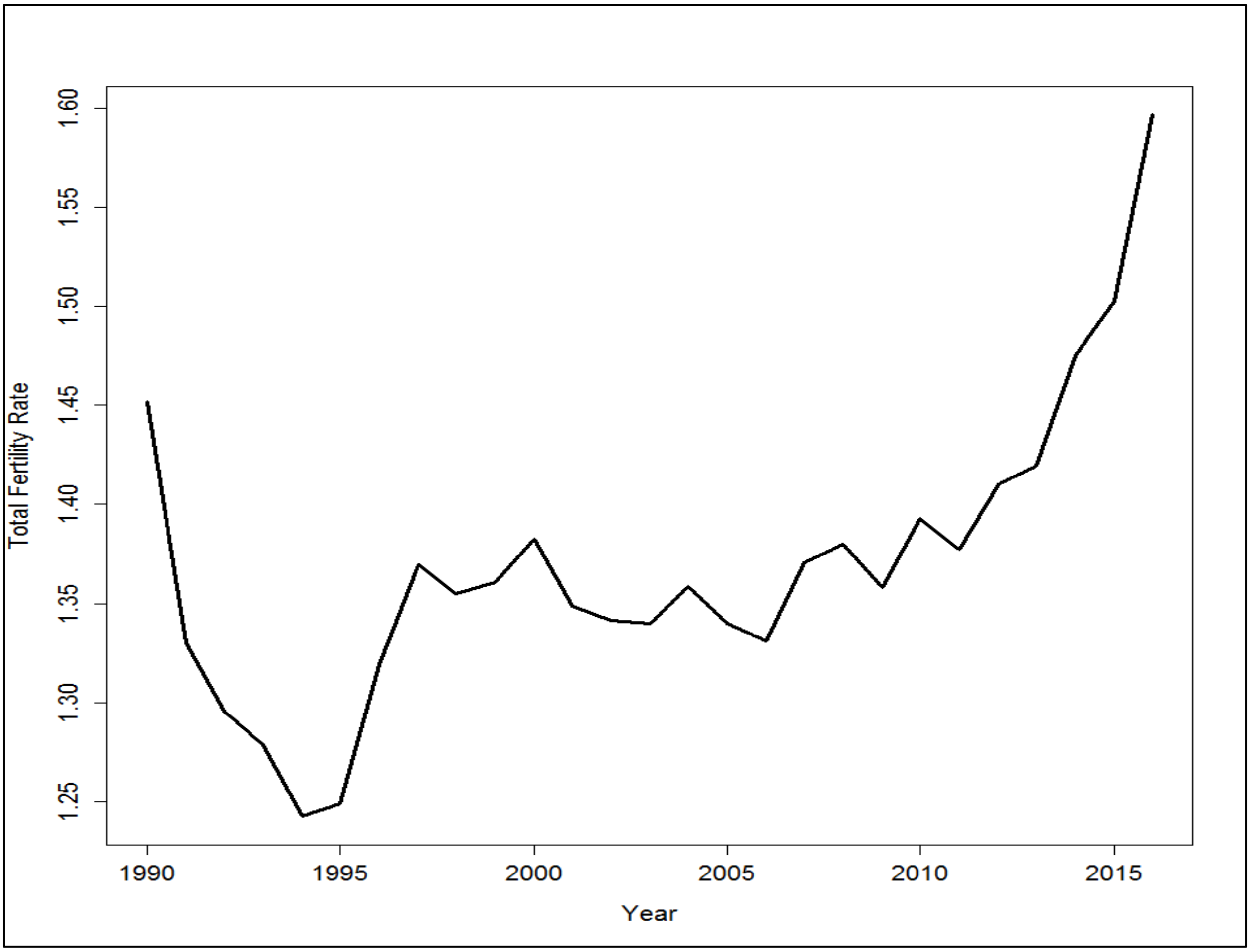


Sources: Human Fertility Database 2016a, 2016b; Own design

We observe increasing TFRs for both parts of Germany after World War II, reaching a climax in the mid-1960s, after which the TFRs decrease quite heavily until the early 1970s. The main reason for this was the second wave of the Women's Rights Movement beginning in 1968, which was especially distinct in West Germany (Hertrampf 2008). Although the introduction of the birth control pill is not causally linked with the decline in births, the active decision to conceive children was facilitated by its market release at the beginning of the 1960s (Bundeszentrale für politische Bildung 2015). This trend persisted in the BRD until the late 1970s, and the TFR decreased to 1.38 in 1978 after reaching 2.54 in 1966. By contrast, the TFR in East Germany *(Deutsche Demokratische Republik or DDR)* declined to 1.54 children by 1974, after which it increased to 1.94 in 1980. This change might be traced back to strong reforms focused on a combination of financial, temporal and infrastructural support in family policy in the DDR since 1972, whose effects nevertheless appeared to vanish after 1980 (Büttner and Lutz 1990; Höhn and Schubnell 1986). In 1979, the BRD, whose family policy had previously been restricted to financial support, began to alter its policy with the passing of the Maternity Leave Act (*Gesetz zur Einführung eines Mutterschaftsurlaubs)*. This law ensured maternity leave of up to six months after childbirth for employed women. Furthermore, the law forbade employers to release female employees during their maternity leave (BGBl I 1979/32). The next milestone in the undertaking of raising the TFR was the Federal Child-Raising Allowance Act *(Bundeserziehungsgeldgesetz or BErzGG*) in 1985. The BErzGG gave one parent, independent of sex, the possibility to take paid parental leave of up to ten months after the birth of a child (BGBl I 1985/58). This period was lengthened in the following years up to the first three years after birth (BGBl I 1989/32; BGBl I 1991/64). Fig. 1 shows an increase in TFR in 1980 and after 1985, the years after the mentioned reforms, implying a positive effect of the family policy measures on reproductive behavior, although the effect might weaken after some period. Fig. 2 shows the TFR for reunited Germany since 1990. After reaching its minimum in 1994 at approximately 1.24, the TFR has since recovered slowly.

In 2007, the BErzGG was annulled with the introduction of the *Bundeselterngeld- und Elternzeitgesetz (BEEG).* Whereas the BErzGG offered a constant amount as financial compensation for the time spent raising a child, the parental allowance *(Elterngeld)* varies depending on the wage the person taking parental leave earned during the last twelve months before the parental leave and can be up to two-thirds of the average wage of the person during that time span. The *Elterngeld* can be paid for up to 14 months for each couple, whereas the payment for one of the partners is limited to twelve months. An additional innovation was that the parental leave can be split among the first years of life of the child. Currently, the total allowable parental leave is up to three years taken over the first eight years after childbirth (BGBl I 2006/56). While classic measures in family policy have focused on financial compensation, measures since the late 1970s identified time as another important factor. The goal has shifted to giving employed potential parents the option to take some time off work to raise a child without the risk of losing their jobs. Bujard and Passet (2013) estimated the effects of the *Elterngeld* reform in 2006 on fertility using SOEP data. They did not find a general statistically significant impact of the *Elterngeld* but concluded that it had a positive effect on the fertility of females over 35 years of age as well as for females with an academic degree. The reform therefore appeared to have had an encouraging effect for recuperation of births in an older age group, especially for those with a high level of education, whose schooling takes a long time.

Fig. 2 Total fertility rate in Germany



Sources: Destatis 2015a, 2015b, 2018; GENESIS-Online 2018a, 2018b; Own calculation and design

Following the *Trias of Family Policy* concept (Bujard 2011), family policy started to offer infrastructural support in childcare in addition to financial compensation and time for child raising. Since 1996, all parents in Germany have a claim for a childcare placement for their children over three years old (Spieß, C.K. 2014). Whereas child care opportunities were rare in the past, especially for very small children under three years of age, in 2008 the *Kinderförderungsgesetz (KiföG)* was passed. Since August 2013, the *KiföG* guarantees childcare placement for children under three years of age if both parents are employed or in school (BGBl I 2008/57). Moreover, the government ran the *Zukunft Bildung und Betreuung* (Future Education and Care), which between 2003 and 2009 subsidized the construction of full-time schools and the evolution of normal schools to full-time schools (Bundesministerium für Bildung und Forschung 2018). These initiatives demonstrate the increasing awareness of the importance of giving both parents the flexibility of returning to work relatively quickly after childbirth, and therefore, the high priority of coping with low fertility in family politics. The TFR has recently reached 1.60, its highest level since the early 1970s (for both parts of Germany combined). One might wonder whether this trend stems from effective political measures.

This overview illustrates the complexity of family policy and the resulting difficulty of evaluating it. Past studies have addressed these questions intensively. Bujard (2015) provided an overview of a range of international studies concerning the effects of family policy on fertility. Mostly, these studies attempt to estimate the impact via econometric models with the TFR as the endogenous variable. Because a vast literature on this topic exists, we keep our focus on studies on fertility in Germany, especially in the light of the lack of comparability for the impact of family policy across countries (see Kalwij 2010 on this). Cigno and Rosati (1996) investigated the influence of macroeconomic factors (income, male and female wage, interest rates) and family transfers in the form of child benefits and social security coverage on the TFR for Italy, the BRD, the USA, and the UK using cointegration analysis. However, the analysis was largely incomplete because the model for the BRD did not include child benefits as an explanatory variable. One interesting result of the analysis was that social security coverage was negatively correlated with the TFR, indicating that a greater supply of social insurance has a negative effect on reproductive behavior. In a follow-up paper, Cigno, Casolaro, and Rosati (2003) elaborated on the earlier model with a vector autoregressive model using child benefit, social security coverage, pension gap, interest rate and mean real wage for both genders as explanatory variables. They identified positive correlations between the TFR and the lagged social security deficit. Moreover, the study supported the results of the earlier study; therefore, the generally good social security coverage in Germany leads to a reduction in the TFR, whereas a larger gap between working-age income and the received retirement pension supports the decision to procreate. In an investigation of micro-level data for 16 Western European countries, Kalwij (2010) applied a proportional hazard model to estimate the age-specific risk of child conception based on demographic, economic, and country-specific explanatory variables. He concluded that, among other effects, parental leave had a positive impact on the probability of bearing a first child, while subsidized daycare opportunities for employed mothers increased the likelihood to conceive an additional child if the mother had previously given birth. The results for financial benefits where ambiguous, which Kalwij traced to spurious regression when not controlling for the overall fertility, represented by the TFR and the crude birth rate, in the population under study.

Gauthier and Hatzius (1997) conducted regression of the TFR on economic and family policy variables for a pool of 22 industrialized countries and concluded that cash benefits had a positive effect on the TFR, whereas maternity leave opportunities had no effect on fertility. They attributed these results to the small variation in the maternity leave data. Adserà (2004) estimated a series of panel data models for data on 23 OECD countries for the years 1960-1997. She identified, among other results, a highly significant positive effect of the length of maternity leave after birth on the TFR. Bujard (2011) performed a series of regression analyses for all OECD countries and found that the financial benefits, the length of paternal leave and the rate of children under three years of age in daycare (lagged by one year) had a positive effect on the TFR. Furthermore, the analysis indicated that the costs of daycare were negatively correlated with the TFR, indicating that the marginal costs of children had a negative effect on birth rates (d’Addio and d’Ercole 2005). Bauernschuster, Hener, and Rainer (2013) found, based on German panel data for the years 1998-2010, that a high degree of childcare coverage was associated with increased fertility rates for all age groups.

3 Fertility Forecasts and Projections for Germany

Future projections for fertility in Germany are often based on deterministic scenario analyses. Germany’s federal statistical office *Destatis* assumes a constant TFR in its 13th coordinated population forecast for Germany. The underlying assumptions are that ASFRs will be decreasing for younger women under 30 years of age and that those losses will be balanced by increasing ASFRs for older women. This effect is known as the tempo effect (Bongaarts and Feeney 1998). These assumptions are based on survey data of the 2008 and 2012 micro censuses on childlessness and the number of children mothers conceive in combination with historical trends of the final number of children for older cohorts. The trends between these two points in time represent the trends mentioned above. In an alternate scenario, which the authors classify as realistic, based on expert opinion rather than on empirical facts, they assume a slight increase in the TFR. This increase would result from a larger increase in the ASFRs of women over 30 year of age and constant fertility rates for younger women. In this case, we would not only see the tempo effect but also a quantum effect, resulting in an increasing TFR. The TFR in this scenario will increase to 1.6 in 2028 and remain constant thereafter[2] (Pötzsch and Rößger 2015). While the model assumptions might be based on reasonable arguments, they appear to be too restrictive for deriving long-term trends through 2060. Furthermore, the large range of possible scenarios is neither identified nor quantified with individual probabilities.

The United Nations (UN) propose a Bayesian hierarchical model (BHM) for stochastic projection of TFRs (United Nations 2015 and 2017; Raftery et al. 2014; Alkema et al. 2011). The BHM is a global model composed of 158 countries, which are classified into one of three possible cases: high-fertility countries (Phase I), countries transitioning from high to low fertility (Phase II), and low-fertility countries (Phase III). Germany is classified as a Phase III country because its TFR has been below the replacement level of 2.1 children since the early 1970s, as illustrated in Fig. 1. The TFR for Germany is assumed to slowly recover and converge toward 2.1 in the long run and is modeled by an autoregressive model of order one [AR(1)]. The quinquennial TFR is stochastically simulated 60,000 times with Markov chain Monte Carlo algorithms to identify the median scenarios with PIs through 2100. In the median scenario, the TFR in Germany is expected to exceed 1.6 through the mid-21st century and to exceed 1.7 by the end of the century. The trajectories for the TFR are thereafter distributed over the reproductive ages leading to trajectories of the ASFRs. These schedules are weighted averages of the past experience of low-fertility countries and Germany's latest historical development with respect to age-specific fertility. The fertility schedule is assumed to converge in the long term toward the global age-specific fertility schedule (Ševčíková et al. 2015). The UN model has some interesting features. It quantifies uncertainty via stochastic simulations while including both national and international trends. One might wonder whether the mathematical assumptions in the model are too restrictive in assuming a global convergence of international fertility trends. Similar points can be made about the ASFRs. The model proposed in our paper takes correlations among age groups into consideration without imposing excessively strict assumptions about their future behavior.

Alders et al. (2007) attempted to combine the strengths of quantitative and qualitative models with TS models and performed TFR forecasting for 18 European countries, including Germany. For Germany, they used data from 1950-2000 to estimate a generalized autoregressive conditional heteroscedasticity model resulting in point forecasts and 80% PIs from 3,000 trajectories until 2050. Given the estimated TFR, age schedules were used to estimate the ASFRs. The plausibility of the results of the quantitative forecasts were qualitatively assessed by two fertility experts. Although the technique appears to consider all necessities, the estimated 80% PI for the TFR in Germany is between 0.88 and 2.21, which is too wide for valuable policy implications. Another caveat is the assumption of an age schedule, which ignores the tempo effect.

Alho (1990) proposed indirect estimation of the TFR through forecasting the average ASFR. He constrained the average ASFR to upper and lower bounds through a modified logistic transformation. Lee (1993) proposed a

[2] Note, that the TFR in Germany in 2016 has already reached this level, as explained in Section 2.

fertility index for indirect estimation of the ASFRs, which he derived from a PCA for the ASFRs. He integrated Alho's transformation into his forecast model to constrain the TFR to certain bounds. To include uncertainty in the forecast, Lee applied a simple autoregressive moving average model [ARMA(1,1)], with which he simulated 1,000 trajectories for the fertility index. Approximate 95% PIs for the TFR can be derived with this process. Härdle and Myšičková (2009) applied Lee's model to forecast the TFR in Germany until 2060 with PIs. Their forecast has some major flaws. First, they assume that the mean TFR remains constant at its last observed level, thereby ignoring the current fertility trends. Second, the PIs are excessively narrow and have an illogical structure because they are wider for the first few periods and stagnate thereafter. Realistically, the risk in future predictions should increase for more distant points in the future. Moreover, the restriction to one simple fertility index completely ignores the uncertainty associated with the remaining PCs, which leads to an overall underestimation of future risks. Fuchs et al. (2018) used a similar approach to forecast the labor force in Germany until 2060. They distinguished between nationals and foreigners, thereby including possible effects of international migration on fertility. Hyndman and Ullah (2007) proposed a robust adjustment to Lee's model that is insensitive to past outliers due to extreme events, e.g., wars or epidemics. Deschermeier (2015) applied this approach to forecast the population of Germany until the year 2030. A major flaw of this approach is that outliers are assumed to be one-time events that cannot be repeated in the future. Because events that occurred in the past cannot logically be ruled out for the future and assumed to have zero probability, this approach leads to underestimation of future uncertainty in the forecast. Vanella's (2017a) proposed approach for addressing the estimation error of the future variance is applied in this paper for a more accurate estimation of the PIs for the TFR and the ASFRs.

Since the 1980s, forecast models for cohort, rather than period, fertility have gained popularity. One such approach is the cohort autoregressive integrated moving average model of de Beer (1985). A modern approach based on Bayesian statistics was presented by Schmertmann et al. (2014). Although the cohort perspective has its advantages and is well justified, we prefer a period perspective because period effects can be observed in a more timely manner. Summary measures, such as the cohort fertility rate, provide clear information about the average number of children a cohort of females has given birth to. We are able to analyze these results only *after* the cohort's reproductive phase; therefore, there is a large time lag to consider. In addition, it is riskier to extrapolate future trends over cohorts than over ages because the trends are less stable (Bell 1988).

4 Method and Data

The data used for this study are cumulated from a multiple data sources to obtain a broad basis for our modeling approach. First, the life birth numbers by single years of age of the mother were obtained. Data for births since 1992 can be downloaded from Destatis's database, GENESIS-Online (GENESIS-Online 2018b). For the years 1968-1990, the age-specific birth numbers for the two parts of Germany were provided by Destatis on request (Destatis 2015b). The data for 1991 were also provided by Destatis for East Germany (Destatis 2018), and the data for West Germany were taken from the Statistical Yearbook of the Federal Republic of Germany (Destatis 1993). The birth numbers for the years 1968 to 1991 were merged to avoid a structural break in the TS due to different geographical bases in the data. The population data by age and sex for Germany as a whole were provided by Destatis on request for the years 1967-2011 (Destatis 2016a). The data for 2012-2015 were downloaded from

GENESIS-Online (2018a). All the demographic statistics used were provided by Destatis, ensuring that no error in the data due to different sources exists.

Birth numbers in the data are broken down by single years of age and are cumulated for the upper age group of “50 years and older”. To take very late births into account without overestimating the ASFRs for 50-year-old mothers, the births for this age group are assumed to follow a geometric decrease:

$$\tilde{B}_{49+\alpha,y} = B_{49,y} * \gamma_y^{\alpha}, \alpha = 1,2,..,5 \quad (1)$$

Past live births $\tilde{B}$ for 50- to 54-year-old mothers in year y were estimated in this way. In this case, γ is the growth rate resulting from the condition:

$$\sum_{\alpha=1}^{5} B_{49,y} * \gamma_y^{\alpha} = B_{50+,y}, \quad (2)$$

where $B_{50+,y}$ denotes the cumulated number of births to women aged 50 years or older. In a similar manner, live births to 13-year-olds were approximated in the cases where they were not available. The γ_y values were calculated mathematically under the condition that the estimators of the live births, which were rounded to zero decimal places, were equal the known number of births in the age group.

The ASFR for females aged a in year y was estimated as the ratio of live births to mothers aged a $(B_{a,y})$ over the mean female population aged a during year y. We have no daily only annual age-specific population data for. To address this limitation, we assumed constant probabilities for death and migration over the course of the year, which allowed us to estimate the mean female population aged a in y $(\bar{F}_{a,y})$ as the mean between female population aged a-1 at the end of year y-1 $(F_{a-1,y-1})$ and the female population aged a at the end of year y $(F_{a,y})$:

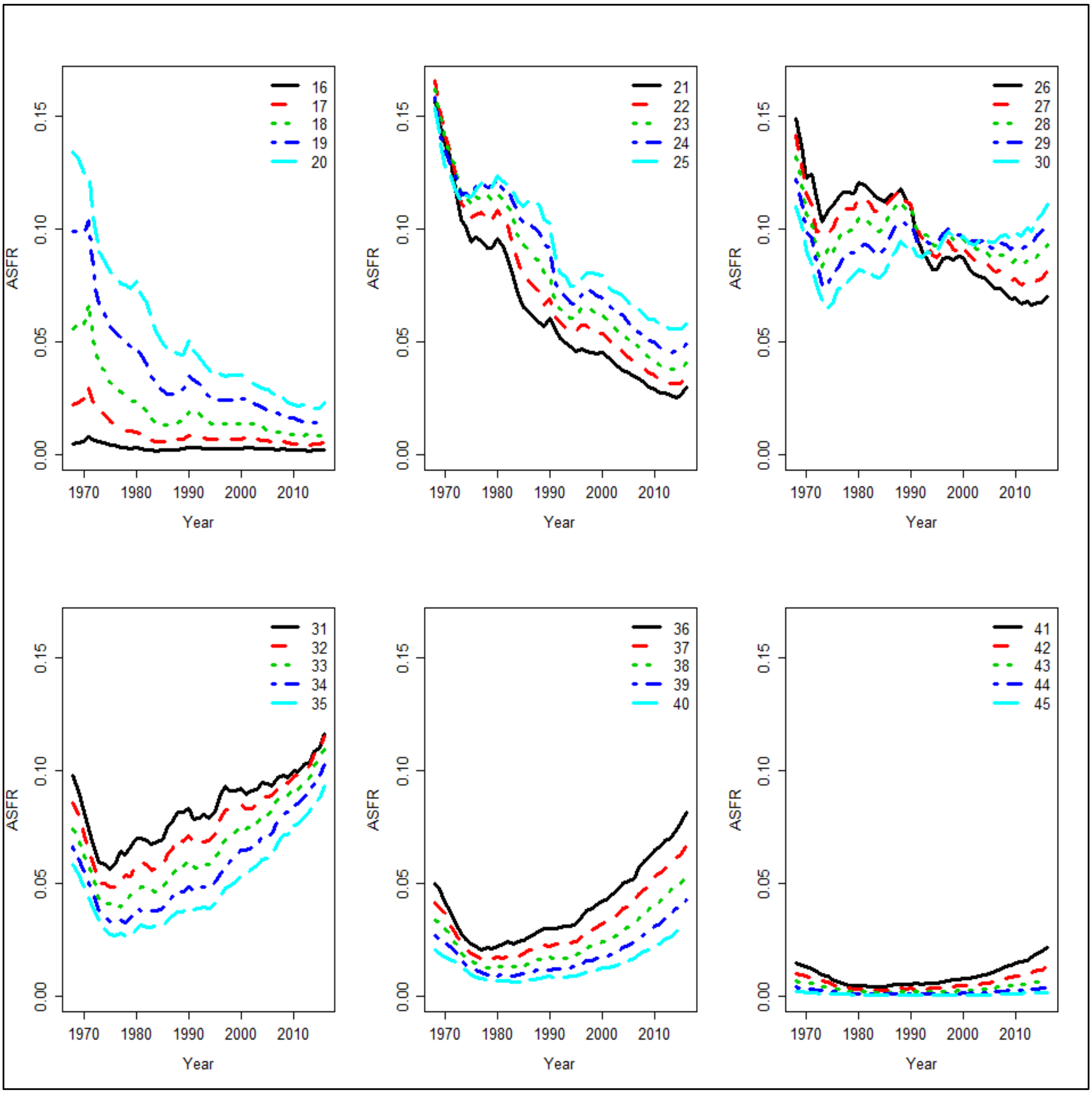
$$ASFR_{a,y} = \frac{B_{a,y}}{\bar{F}_{a,y}} = \frac{B_{a,y}}{\frac{1}{2}(F_{a,y} + F_{a-1,y-1})} \quad (3)$$

Fig. 3 illustrates the ASFRs for 16- to 45-year-old females derived in this way[3].

We observe a general negative trend in fertility until the middle of the 1970s. Thereafter, the ASFRs for females in their mid-20s and older increased slightly, which might be associated with the reforms of 1972 and 1976 in the DDR (Büttner and Lutz 1990; Höhn and Schubnell 1986). The effect of these reforms appears to vanish after 1980. Since 1979, family policy in the BRD started to follow the concept of the *Trias* instead of providing pure financial compensation. Fig. 3 shows that the positive trend in reproduction for females aged 30 years and older continued, but it remains unclear whether this is a result of the policy reforms since 1979. Our PCA will help to separate this impact from other contemporaneous trends.

[3] Actually, the ASFRs were derived for 13-54 year olds. For the sake of comparability, not all time series are shown here. ASFRs for very young and old females are too small for graphical illustration at the chosen scales.

Fig. 3 Age-specific fertility rates for females in Germany aged 16-45 years since 1968



Sources: Destatis 1993: 79; Destatis 2015b, 2016, 2018; GENESIS-Online 2018a, 2018b; Own calculation and design

PCA was performed as the next step. The use of the log or logit transformation of ASFRs and TFRs is popular in the literature (see, e.g., Alho 1990; Lee 1992) because these transformations ensure that future forecasts or projections remain within certain limits, as explained in Section 3. A standard logit transformation produces values in the interval (0,1) for the original variable[4]. A log transformation has no upper bound, but ensures a positive variable of interest. We selected a square root transformation, which is a valid transformation because it is a monotonously increasing and continuous link function. The features of the square root transformation are similar to those of the log transformation, but the square root transformation also allows the original variable to be zero, which makes sense for ASFRs because ASFRs can realistically be zero for very young or old age groups. The square root

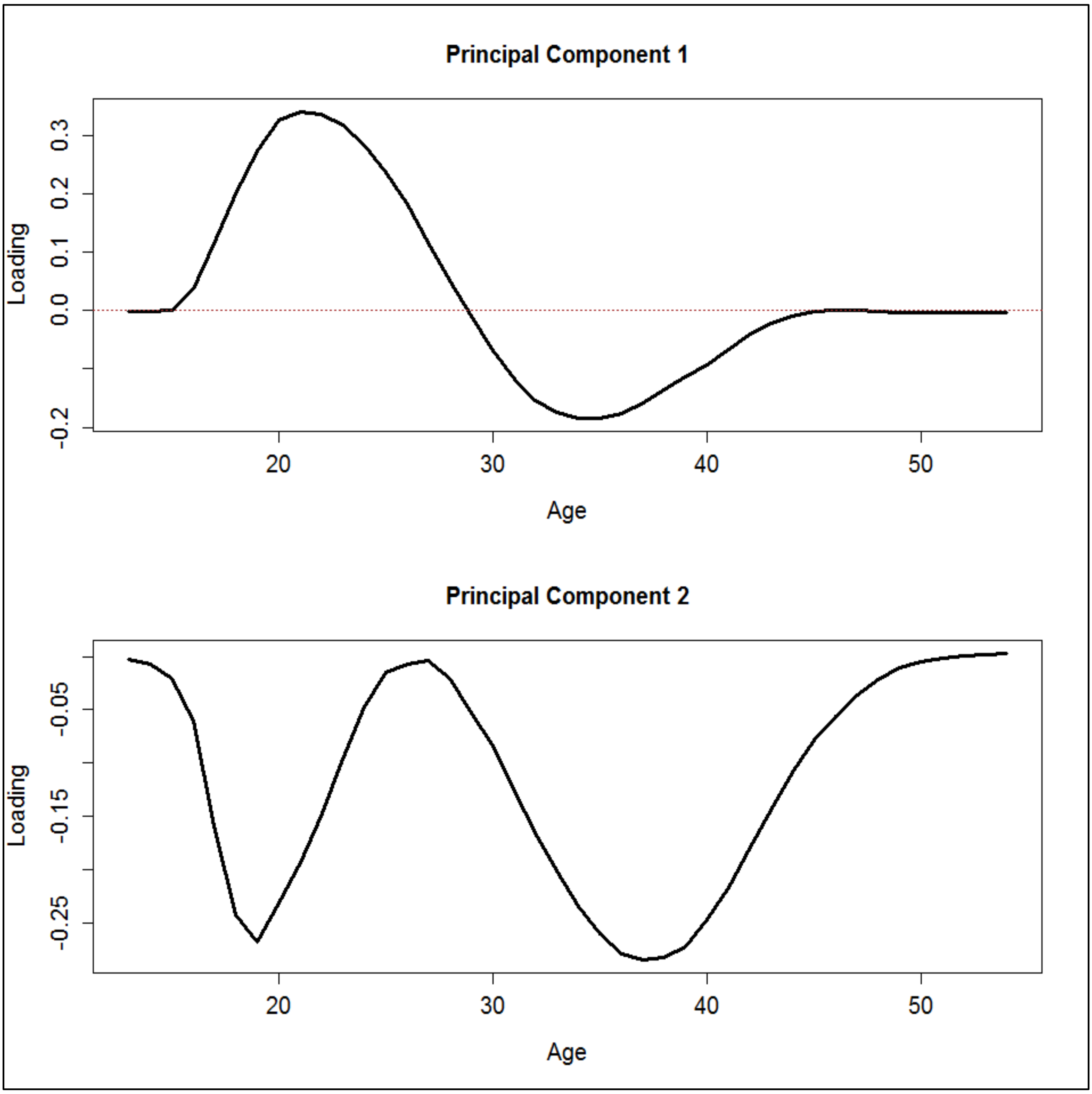
[4] The standard logit of a variable x is $ln\left(\frac{x}{1-x}\right)$, see, e.g., Johnson (1949).

transformation allows us to keep these age groups in the analysis; otherwise, we would systematically underestimate the TFR by omitting fertile age groups. Algebraically, the transformation is:

$$\boldsymbol{C} = \boldsymbol{F}^{1/2} \times \boldsymbol{E} \tag{4}$$

where $\boldsymbol{F}$ is a 49x42 matrix of ASFRs (49 periods in the rows, 42 years of age in the columns), $\boldsymbol{E}$ is a 42x42 matrix of the loadings (each column is one eigenvector) and $\boldsymbol{C}$ is the theoretical TS of the principal components (a 49x42 matrix). Fig. 4 shows the loadings of the first two PCs.

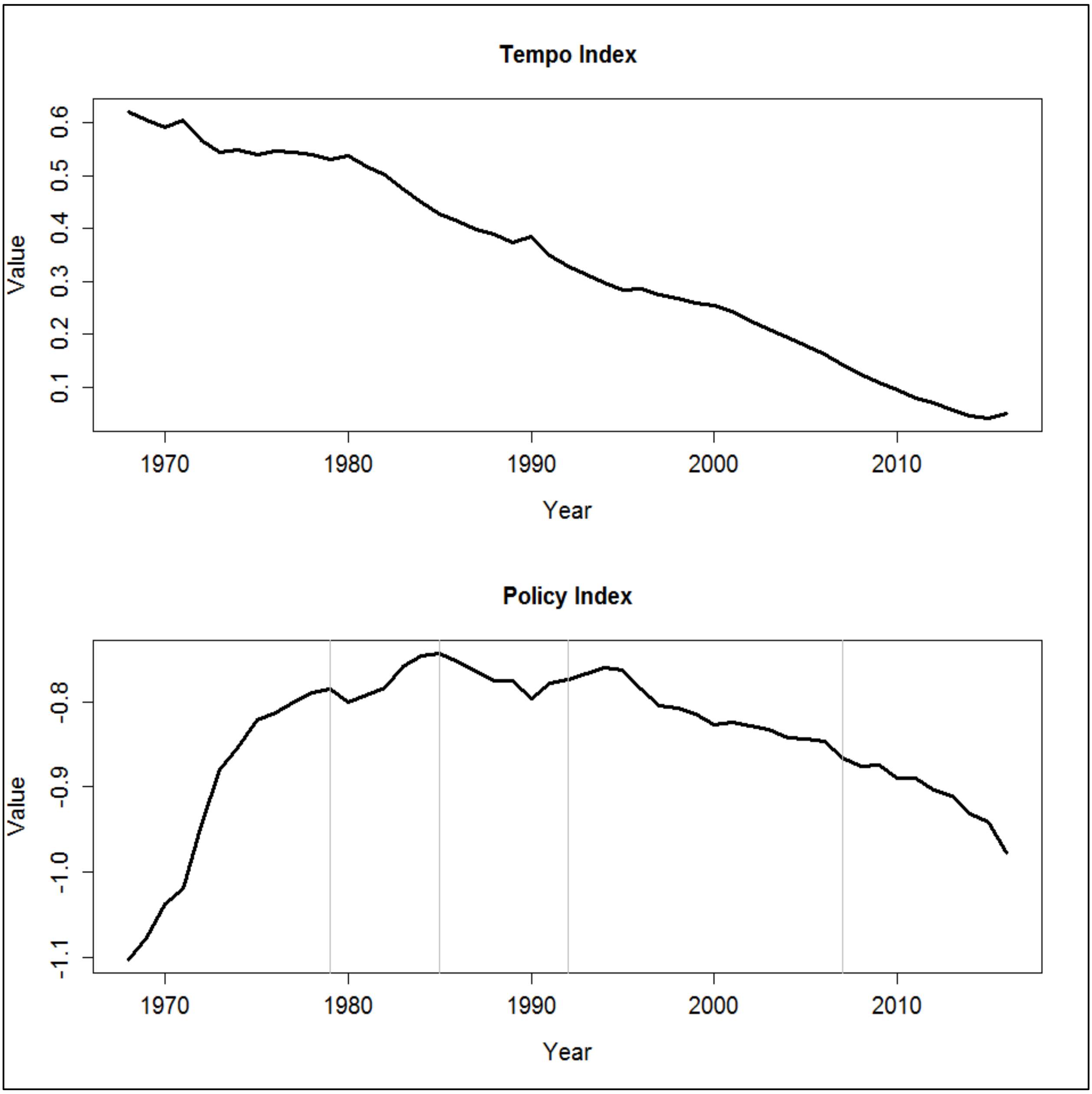
Fig. 4 Loadings of the first two principal components for the square roots of the age-specific fertility rates



Source: Own calculation and design

The first PC is loaded negatively for females under 29 years, and the correlations become positive for females 30 years and over before converging to zero. Increases in PC 1 are therefore associated with decreasing ASFRs for younger females and increasing ASFRs for women over 30 years. Therefore, this PC is associated with the tempo effect. Increasing values for PC 1 show a strong trend toward shifting births from younger ages to older ages. Therefore, throughout the rest of this paper, PC 1 will be called the *Tempo Index*. PC 2 is generally nonpositive; it is especially strongly negatively loaded for very young females and again becomes very small for women between 30 and approximately 45 years of age. This PC therefore represents changes in overall fertility. To put the PCs into historical context, the hypothetical past courses of the two PCs are plotted in Fig. 5.

Fig. 5 Past courses of the first two principal components



Source: Own calculation and design

Fig. 5 shows a positive trend in the Tempo Index, which represents the persistent trend of birth postponement since the late 1960s. As described earlier, the postponement was a result of the second wave of the Women's Rights Movement since 1968, manifesting as a female desire for emancipation and self-participation in the labor market. Gustafsson (2001) conducted a literature review and concluded from the analyzed studies and the TS of mean age at birth that the timing of births may have a large impact on the lifetime earnings and lifetime human capital of the mother. She concluded that the mother's childbearing tax was lower in cases of either a high or very low age of first conception in comparison to cases of intermediate ages of first conception. Therefore, giving birth at an age over 30 years is more attractive than giving birth at approximately 25 years if the mother chooses to pursue her own career.

Fig. 5b shows the course of PC 2; the years 1979, 1985, 1992 and 2007 are marked. Major family policy reforms were introduced during these years. We observe a short-term decrease in PC 2 in 1980 and in the years after 1985. After reunification in 1990, the PC decreased again until 1994, after which it mostly decreased to the current level. PC 2 thus appears to be associated with family policy reforms and is therefore called the *Policy Index* hereafter.

5 Results

We performed a regression analysis with eight possible explanatory variables to estimate the association of the Policy Index and policy measures. The first category of explanatory variables includes financial benefits: the fixed monthly child benefit *(Kindergeld)* for the first child ($\boldsymbol{k_1}$), second child ($\boldsymbol{k_2}$), third child ($\boldsymbol{k_3}$), and fourth or greater child ($\boldsymbol{k_4}$). Moreover, financial benefits associated with parental leave are also included: the old *Erziehungsgeld* ***(ErzG)***, which was discontinued in 2007, and its successor, the *Elterngeld*. The *Elterngeld* is difficult to operationalize in a model because it depends on the previous wages of the persons receiving it but is also subject to upper and lower bounds. We attempt to measure its effect as a binary variable (**EG**) that takes the value of one for every year since the *Elterngeld*'s introduction and a value of zero otherwise. The amounts are calculated in Euros for the first year after birth, inflation-adjusted to 2010 prices. Finally, the pension entitlements for child raising are considered as indirect financial benefits (*Rentenpunkte;* **RP)**.

We also tested an infrastructural variable. On the basis of the studies mentioned in Section 2, we assume daycare to have an important impact on fertility. Whereas complete data on daycare are rare, TS on statutory expenditures for daycare coverage exist back to at least 1992. We use the real expenditures in billion Euros, adjusted to 2010 prices (*Betreuungsausgaben;* **BA***)*. The Policy Index is regressed on the first lags of the explanatory variables because we can assume a time horizon of approximately one year between a couple's decision to conceive a child and birth (see, e.g., Bujard 2011). One caveat of the chosen time horizon is that the possible parental leave, which might have a significant impact on fertility, does not vary. Therefore, we lack a variable for the time dimension. This may not be too grave a problem because significant policy reforms for parental leave do not appear probable in the near future[5].

We start the econometric fitting of the Policy Index regression model with a model including the eight explanatory variables and iteratively omit one of the variables in each iteration before stopping at the best model in terms of two criteria. First, the coefficient of each explanatory variable should be negative and have a statistically significant effect. The Policy Index is negatively correlated with ASFRs; thus, a negative effect of a regressor on the Policy

[5] In the current political debate, the total length of parental leave is not under discussion, but rather how to split it.

Index leads to a positive effect of the same regressor on the ASFRs. Second, the final model should ideally minimize Akaike's Information Criterion (AIC)[6]. Table 1 shows the results of the range of models tested iteratively in the regression analysis. The estimated coefficients are presented for each variable, along with their associated standard errors in brackets. The last two rows show the $\boldsymbol{R^2}$ and the AIC of the respective model.

Table 1. Predictive Model Specifications for Policy Index

Variable	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
(Intercept)	-0.885086*** (0.0974)	-0.918397*** (0.08526)	-0.650603*** (0.03319)	-0.629314*** (0.03105)	-0.629585*** (0.03015)	-0.660620*** (0.0154)
$\boldsymbol{k_1}$	-0.000121* (0.000090)	-0.000179*** (0.000042)	-0.000109** (0.000046)	-0.000041*** (0.000009)	-0.000040*** (0.000008)	-0.000048*** (0.000006)
$\boldsymbol{k_2}$	0.000071 (0.000105)	0.000134 (0.000061)	0.000115 (0.000077)	-	-	-
$\boldsymbol{k_3}$	0.000189 (0.000255)	-	-	-	-	-
$\boldsymbol{k_4}$	-0.000070 (0.000230)	0.000099 (0.000030)	-	-	-	-
ErzG	0.000002 (0.000006)	0.000002 (0.000006)	-0.000003 (0.000008)	-0.000001 (0.000008)	-	-
EG	-0.007624 (0.03963)	0.000076 (0.03771)	-0.041066 (0.04478)	-0.031200 (0.04582)	-0.025823*** (0.009183)	-0.028422*** (0.009015)
RP	0.009285 (0.02166)	0.008614 (0.02134)	-0.030159* (0.02242)	-0.014179 (0.02042)	-0.015987 (0.0134)	-
BA	-0.009357*** (0.001354)	-0.008705*** (0.001017)	-0.007623*** (0.001212)	-0.006696*** (0.001079)	-0.006704*** (0.001048)	-0.006663*** (0.001059)
$\boldsymbol{R^2}$	*0.9785*	*0.9778*	*0.9625*	*0.9575*	*0.9575*	*0.9543*
AIC	*-142.01*	*-143.14*	*-132.67*	*-131.67*	*-133.65*	*-133.92*

Source: Own calculation and design

A single asterisk next to an estimated coefficient means the probability for the real coefficient to take a value less than zero is greater than 90%, two asterisks indicate a probability greater than 95% and three asterisks denote a probability greater than 99%.

Our iterative procedure identifies Model 6 as the best model. Models 1-3 are not reasonable because some of the coefficients are positive, which probably stems from spurious regressions. Ordinary least squares regression assumes no correlation among the regressors. The regressors are exogenous because they are all determined politically. However, different child benefits are typically adjusted simultaneously, which results in multicollinearity in our data. Comparison of Models 4-6 shows a decreasing AIC and increasing evidence for positive effects of the remaining regressors on the ASFRs. The statistical evidence of the pension entitlement's effect on fertility in

[6] The AIC is a standard criterion in model fitting that takes into account the fit of the model to the data while punishing for model complexity.

Model 5 is weak at approximately 12.9%. We could also argue for accepting this model. Because the statistical evidence is weak for the data and the AIC is slightly smaller, we select Model 6 as the best specification. Model 6 has a high $\boldsymbol{R^2}$ of approximately 0.9543, which means the model explains 95.43% of the variation in the Policy Index over the base time horizon. Clearly this does not mean that the Policy Index explains all the changes in the quantum of fertility because the time horizon of the regression is restricted to 1993-2016 due to data limitations and the German reunification. Nevertheless, a large portion of the increase in the TFR observed during the last 20 years can be attributed to effective family policy. Therefore, the Policy Index can be explained well by the daycare supply and financial benefits, although we find evidence for an effect of these factors only on first births. The reform in 2006, including the temporal component of the *Elternzeit* allowing for parental leave and the financial component of the *Elterngeld*, had a stimulating effect on the ASFRs. Although similar opportunities were provided previously, the nature of the *Elterngeld* included greater financial benefits than the fixed *Erziehungsgeld* because it is generally linked to personal income. Pension entitlements might also have a positive effect because they encourage taking time off from work since they reward mothers with higher pension payments as if they had continued participating in the labor market during the first three years after birth. The selection of Model 6 does not account for the possible effects of pension entitlements.

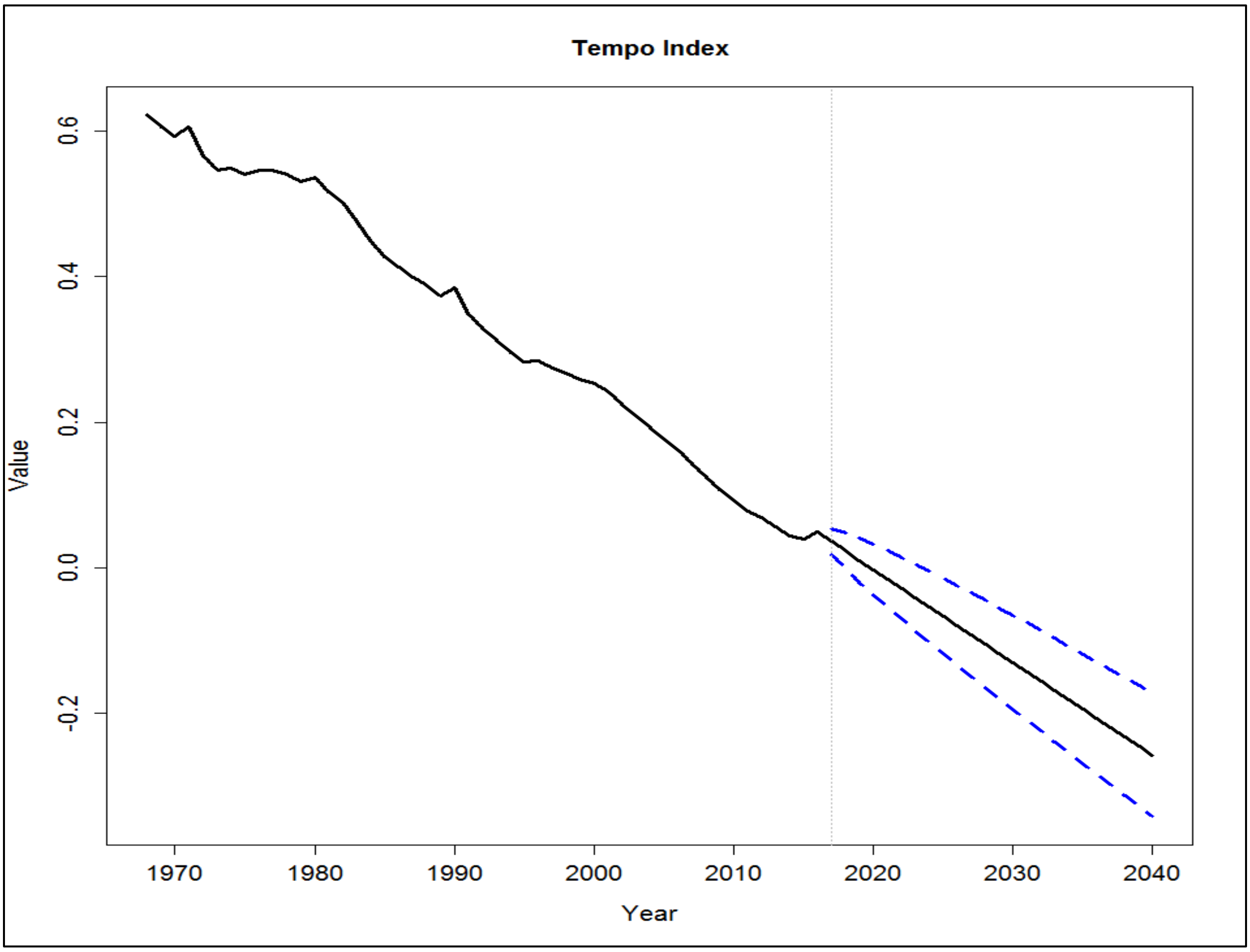
We simulate the ASFRs indirectly through conditional forecasting of the PCs. The Tempo Index has a stable positive long-term trend, which is statistically quantified as linear. The remaining disturbance after fitting a linear trend to the data is modeled with an ARIMA model[7]. The degrees of the ARIMA model are determined by graphical analysis of the hypothetical TS and its differences, the autocorrelation function, the partial autocorrelation function, the augmented Dickey–Fuller test and the ARCH-LM test for conditional heteroscedasticity.[8] The fitted model for the Tempo Index t is:

$$t_y = 25.74066 - 0.01275y + e_{y-1} + \varepsilon_y, \varepsilon_y \sim \mathcal{NID}(0, 0.01784^2), \quad (5)$$

where e is a random walk process[9] and y is the year. The uncertainty of the forecast is accounted for by simulating 10,000 paths of a Wiener process until the year 2040, following Vanella (2017b). We can then derive quantiles from the simulation results. Fig. 6 shows the median simulation as well as the estimated annual 90% PIs of the Tempo Index.

[7] See, e.g., Box et al. 2016; Shumway and Stoffer 2011; Vanella 2017b on ARIMA models.
[8] See, e.g., Vanella 2017b on these tests.
[9] A random walk is an ARIMA(0,1,0) process.

Fig. 6 Forecast of the Tempo Index



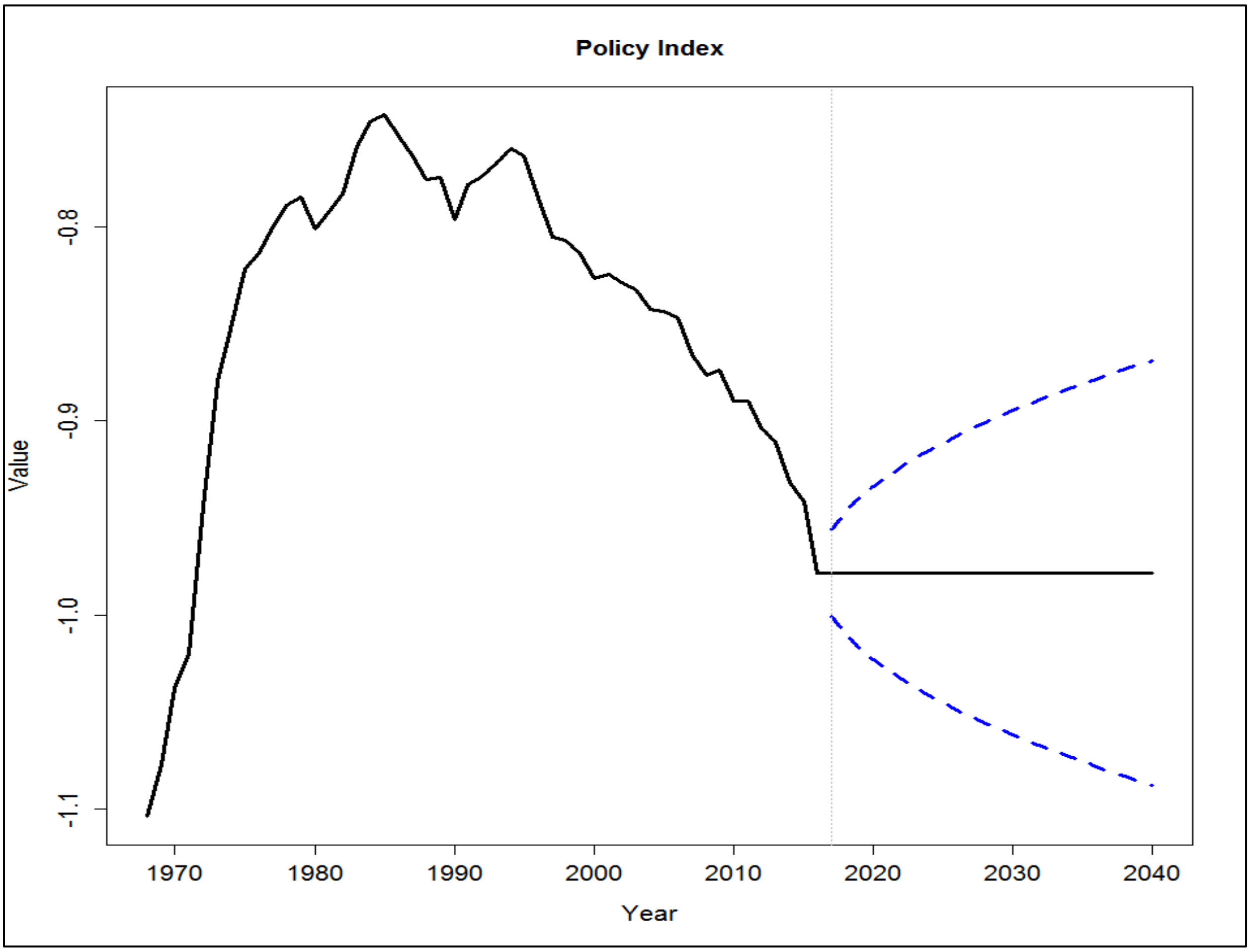
Source: Own calculation and design

We now use the previously fit explanatory model to simulate the Policy Index. In this case, the nuisance parameter is again best fit by a random walk. Therefore, the forecast model of the Policy Index is:

$$p_y = -0.66062 - 0.00005k_1 - 0.02842eg - 0.00666b + f_{y-1} + \xi_y, \quad \xi_y \sim \mathcal{NID}(0, 0.01207^2), \qquad (6)$$

where f is a random walk process, k_1 is the annual first child benefit inflation-adjusted to 2010 prices, eg is the *Elterngeld* dummy variable, and b is inflation-adjusted money spent on subsidizing daycare in billion Euros. All explanatory variables are lagged by one year.. Assumptions about the four explanatory variables are required to simulate the Policy Index. We estimate a conditional forecast assuming that the child benefit and daycare subsidization are held constant truly at their last observed levels in 2015. This assumption appears to be plausible and allows for further sensitivity analyses. The forecast resulting from the simulation study of the Policy Index is shown in Fig. 7, including the median outcome and the 90% PIs.

Fig. 7 Forecast of the Policy Index



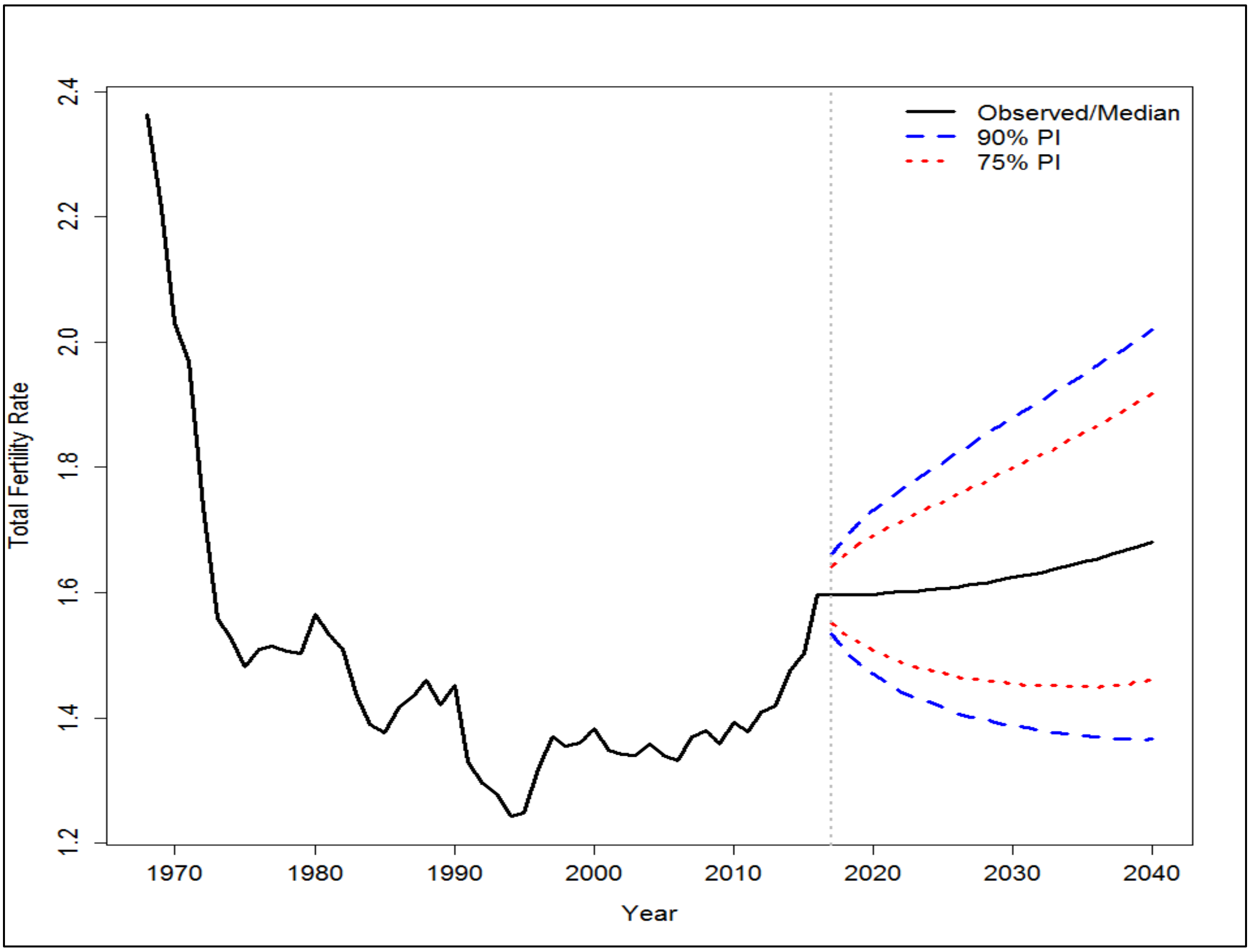
Source: Own calculation and design

The first two PCs explain approximately 96.3% of the variance in the ASFRs for the base time horizon of 1968-2016. The Tempo Index explains the largest share (78.7%), whereas the Policy Index explains 17.6% of the variance in the ASFRs. The remaining 40 PCs are simulated as random walk processes to consider their risk. A smaller number of PCs would lead to tighter PIs because a portion of the variance is assumed to be zero. Because the Tempo Index and the Policy Index explain a large proportion of the variance in the data, the possible error arising from this simplification is negligible. The resulting simulations of the PCs can be plugged back into (3) and solved for $\boldsymbol{F}$ to simulate the ASFRs. The simulation matrix of the ASFRs in year y is:

$$\widehat{\boldsymbol{F}}_y = \left(\widehat{\boldsymbol{C}}_y \times \widehat{\boldsymbol{E}}^{-1}\right)^2 \tag{7}$$

The hats over the matrix notations denote empirical matrices derived from simulations rather than theoretical matrices. The TFR is a synthetic measure that provides an idea about the general fertility level during a certain period. Therefore, we use our ASFR simulations to forecast the TFR, which is simply the sum of all the ASFRs for a specific year. Thus, summing over the rows of $\widehat{\boldsymbol{F}}_y$ yields 10,000 trajectories for the TFR in year y, from which we can obtain arbitrary quantiles. Fig. 8 shows the median trajectory for the TFR until 2035, along with 75% and 90% PIs.

Fig. 8 Forecast of the total fertility rate

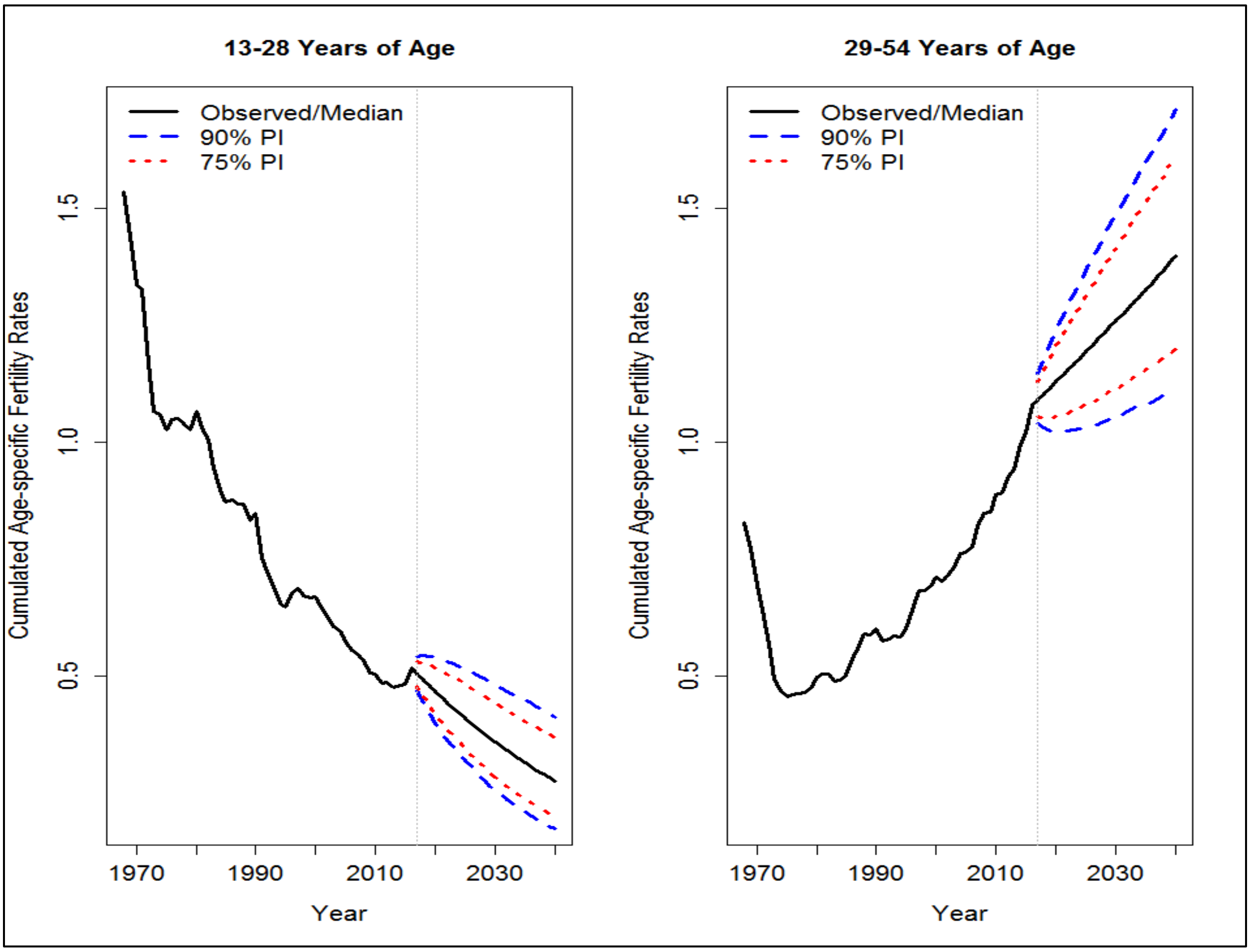


Sources: Destatis 1993: 79; Destatis 2015b, 2016, 2018; GENESIS-Online 2018a, 2018b; Own calculation and design

Under the assumed family policy regime, an additional slight increase in the TFR is probable over the forecast horizon whereas future changes in daycare supply are also taken into account. In the median scenario, the TFR increases from 1.6 in 2016 to 1.68 in 2040. Furthermore, the TFR will be between 1.37 and 2.02 with a probability of 90% and between 1.46 and 1.92 with a probability of 75%. Based on this result, it is unlikely that fertility will fall back to its extremely low level of the mid-1990s. An increase to the replacement level of 2.1 also appears to be unlikely, but an increase toward the Northern European level of approximately 1.8 (Eurostat Database 2018a) is realistic, though improbable.

Fig. 3 showed that the long-term trends of birth rates are negative for women under 29 years of age and positive for women over 29 since the mid-1970s. These trends are expected to continue in the future, as shown by our forecast. Fig. 9 presents the forecasts of the cumulated ASFRs for two age groups: 13-28 years of age and 29 years or older.

Fig. 9 Forecast of fertility by age



Sources: Destatis 1993: 79; Destatis 2015b, 2016, 2018; GENESIS-Online 2018a, 2018b; Own calculation and design

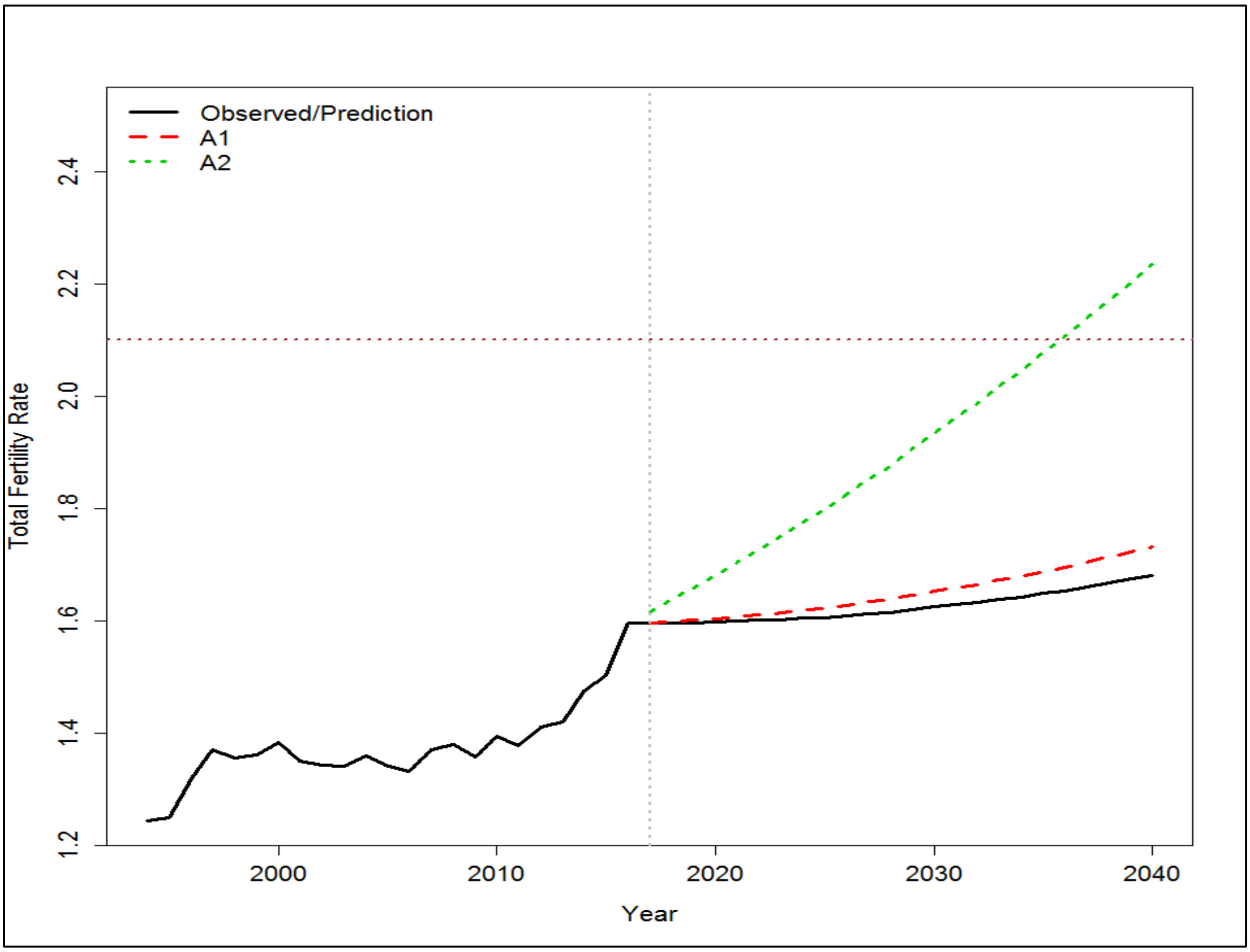
We see that the postponement of birth is likely recuperated at older ages, and the median result indicates an increase in the TFR. This development is influenced strongly by effective family policy, as shown earlier. The slight increase in fertility for the younger group during the last two years is mainly associated with the high international immigration into Germany since 2014. In 2014 (Eurostat Database 2018b), 60,873 children in Germany were born to foreign mothers under 30 years of age compared to the total of 236,413 mothers in that age group (a ratio of approximately 0.257). That same ratio increased to 71,146 to 231,918 (0.307) in 2015 and even further to 92,581 to 232,476 in 2016 (0.398). Because immigration is expected to decrease slowly in the future (see, e.g., Fuchs et al. 2018; Vanella and Deschermeier 2018), this increase in the ASFRs for younger females is expected to quickly revert to its long-term trend and the ASFRs will continue to decrease.

The estimated model provides the opportunity to derive the sensitivity of the TFR to changes in family policy. Therefore, we simulate alternate family policy scenarios by constraining model variables. Compared to the median TFR forecast, we assume two ceteris paribus scenarios.

- A1: The inflation-adjusted *Kindergeld* is increased annually by 1% from 2018 onwards.
- A2: The inflation-adjusted investment into daycare supply is increased by approximately 1.568 billion Euros annually (mean annual increase between 2007 and 2015).

The mean courses of the future TFR under these two assumed scenarios in addition to the baseline scenario resulting from our model are plotted in Fig. 10.

Fig. 10 Simulated total fertility rate under three different scenarios



Sources: Destatis 2015a, 2015b, 2018; GENESIS-Online 2018a, 2018b; Own calculation and design

The PCA produces a nonparametric estimate of the TFR; therefore, no simple formula can be derived. Our sensitivity analysis shows that a 1% annual increase in the inflation-adjusted child benefit leads to an increase in the TFR by approximately 0.05 by 2040. The second alternate scenario assumes an approximately 1.568 billion Euro annual increase in real daycare subsidization, which is the mean increase over the span of 2007 to 2015. Under this assumption, the TFR in the median scenario increases substantially and reached the replacement level by 2036. Clearly, this increase would require an enormous financial investment. Furthermore, the trends might not be linear but rather converge at some point. Our primary goals are to illustrate how the abstract PCA approach produces interpretable results, such as the TFR, and how our model can be used for political sensitivity analyses. We might (and should) integrate the results into cost–utility analyses to adjust family policy.

6 Conclusions, Limitations and Outlook

The future evolution of fertility is the strongest driver for long-term stability of social security systems and labor markets. Therefore, the importance of good fertility forecasts as a quantitative basis for political planning should not be underestimated. Possible political measures in family policy must be planned carefully, weighing the possible effects on fertility and the direct costs. Stochastic TS modeling combined with a simulation approach can help to visualize uncertainty via PIs. Official fertility projections usually implement a scenario technique, which does not provide information on the probability of occurrence. Political planning based on scenarios is limited to choosing the alternative that is most in line with the political agenda. The quantification of uncertainty makes the forecasting results of stochastic approaches less vulnerable to be treated subjectively.

We proposed a principal component TS model for conditional forecasting of future ASFRs. The method is rooted in Lee–Carter modeling and accounts for autocorrelation and cross-correlation among the variables, thereby taking trends among the ASFRs and over time into account. We have shown that a major portion of the fertility trends during the last decades can be attributed to the postponement of birth due to the second wave of the Women's Rights Movement in combination with the development of the birth control pill. The postponement has also resulted in a recuperation of births in older age groups. Moreover, although there have been some effective reforms in Germany during the last 40 years, we conclude that good family policy measures have the power to compensate for social trends and can increase the probability of birth recuperation for women over 30 years of age. Our model also included family policy variables in the analysis and indirectly estimated their effect on fertility by age of the females via PCA. Our approach provides the opportunity for sensitivity analyses of possible family policy measures, as illustrated in Section 4.

We want to stress that the analysis in not conclusive. The TS for the political variables are relatively short, and the model fit will naturally improve as more data become available. Some variables, such as daycare supply, that would provide a more direct estimate of the effects of political measures on fertility are simply not available as regularly as needed. Furthermore, we could not test the effect of parental leave because the overall total has not changed over the base horizon. Other reforms, such as the BEEG in 2006, are difficult to operationalize because their structures are too complex to be covered completely in an econometric model.

Although we use Germany as an example, the model framework is applicable to other industrialized countries. Furthermore, the analysis might produce better results for countries with more complete data (and without a structural break due to reunification).

REFERENCES

Adserà A (2004) Changing fertility rates in developed countries. The impact of labor market institutions. Journal of Population Economics 17:17-43

Alders M, Keilman N, Cruijsen H (2007) Assumptions for long-term stochastic population forecasts in 18 European countries. European Journal of Population 23:33-69

Alho JM (1990) Stochastic methods in population forecasting. International Journal of Forecasting 6: 521-530

Alkema L, Raftery AE, Gerland P et al. (2011) Probabilistic Projections of the Total Fertility Rate for All Countries. Demography 48:815-839

Bauernschuster S, Hener T, Rainer H (2013) Does the Expansion of Public Child Care Increase Birth Rates? Evidence from a Low-Fertility Country. Ifo Working Paper 158

Becker GS (1960) An Economic Analysis of Fertility. In: Roberts GB (ed) Demographic and Economic Change in Developed Countries. Columbia University Press, New York, pp 209-240

Bell W (1988) Applying Time Series Models in Forecasting Age-specific Fertility Rates. Bureau of the Census, Statistical Research Division, Washington, D.C.

BGBl I 1979/32 Gesetz zur Einführung eines Mutterschaftsurlaubs. Bundesanzeiger Verlag, Bonn, pp 797-799

BGBl I 1985/58 Gesetz über die Gewährung von Erziehungsgeld und Erziehungsurlaub. Bundesanzeiger Verlag, Bonn, pp 2154-2163

BGBl I 1989/32 Gesetz zur Änderung des Bundeserziehungsgeldgesetzes. Bundesanzeiger Verlag, Bonn, pp 1297-1300

BGBl I 1991/64 Zweites Gesetz zur Änderung des Bundeserziehungsgeldgesetzes und anderer Vorschriften. Bundesanzeiger Verlag, Bonn, pp 2142-2145

BGBl I 2006/56 Gesetz zur Einführung des Elterngeldes. Bundesanzeiger Verlag, Bonn, pp 2748-2758

BGBl I 2008/57 Gesetz zur Förderung von Kindern unter drei Jahren in Tageseinrichtungen und in Kindertagespflege. Bundesanzeiger Verlag, Bonn, pp 2403-2409

Bongaarts J, Feeney G (1998) On the Quantum and Tempo of Fertility. Population and Development Review 24:271-291

Box GEP, Jenkins GM, Reinsel GC et al. (2016) Time Series Analysis: Forecasting and Control. Wiley, Hoboken

Bujard M (2011) Familienpolitik und Geburten: Ein internationaler Vergleich. Bundesministerium für Familie, Senioren, Frauen und Jugend, Berlin

Bujard M (2015) Wirkungen von Familienpolitik auf die Geburtenentwicklung. In: Niephaus Y, Kreyenfeld M, Sackmann R (eds), Handbuch Bevölkerungssoziologie. Springer Fachmedien, Wiesbaden, pp 619-646

Bujard M, Passet J (2013) Wirkungen des Elterngelds auf Einkommen und Fertilität. Zeitschrift für Familienforschung 25:212-237

Bundesministerium für Bildung und Forschung (2018) Gute Bildung in Ganztagsschulen. https://www.bmbf.de/de/gute-bildung-in-ganztagsschulen-83.html. Accessed 16 February 2018

Bundeszentrale für politische Bildung (2015) 55 Jahre “Pille”. http://www.bpb.de/politik/hintergrund-aktuell/210997/55-jahre-pille-. Accessed 15 February 2018

Büttner T, Lutz W (1990) Estimating Fertility Responses to Policy Measures in the German Democratic Republic. Population and Development Review 16:539-555

Cigno A, Casolaro L, Rosati FC (2003) The Impact of Social Security on Saving and Fertility in Germany. FinanzArchiv 59:189-211

Cigno A, Rosati FC (1996) Jointly determined saving and fertility behavior: Theory, and estimates in Germany, Italy, UK and USA. European Economic Review 40:1561-1589

d’Addio AC, d’Ercole MM (2005) Trends and Determinants of Fertility Rates: The Role of Policies. OECD Social, Employment and Migration Working Papers 27. OECD Publishing, Paris

de Beer J (1985) A Time Series Model for Cohort Data. Journal of the American Statistical Association 80:525-530

Deschermeier P (2015) Die Entwicklung der Bevölkerung Deutschlands bis 2030 – ein Methodenvergleich. IW Trends - Vierteljahresschrift zur empirischen Wirtschaftsforschung 42:97-111

Destatis (1993) Statistisches Jahrbuch 1993 für die Bundesrepublik Deutschland. Statistisches Bundesamt, Wiesbaden

Destatis (1994) Statistisches Jahrbuch 1994 für die Bundesrepublik Deutschland. Statistisches Bundesamt, Wiesbaden

Destatis (1995) Statistisches Jahrbuch 1995 für die Bundesrepublik Deutschland. Statistisches Bundesamt, Wiesbaden

Destatis (1996) Statistisches Jahrbuch 1996 für die Bundesrepublik Deutschland. Statistisches Bundesamt, Wiesbaden

Destatis (1997) Statistisches Jahrbuch 1997 für die Bundesrepublik Deutschland. Statistisches Bundesamt, Wiesbaden

Destatis (1998) Statistisches Jahrbuch 1998 für die Bundesrepublik Deutschland. Statistisches Bundesamt, Wiesbaden

Destatis (1999) Statistisches Jahrbuch 1999 für die Bundesrepublik Deutschland. Statistisches Bundesamt, Wiesbaden

Destatis (2000) Statistisches Jahrbuch 2000 für die Bundesrepublik Deutschland. Statistisches Bundesamt, Wiesbaden

Destatis (2001) Statistisches Jahrbuch 2001 für die Bundesrepublik Deutschland. Statistisches Bundesamt, Wiesbaden

Destatis (2002) Statistisches Jahrbuch 2002 für die Bundesrepublik Deutschland. Statistisches Bundesamt, Wiesbaden

Destatis (2003) Statistisches Jahrbuch 2003 für die Bundesrepublik Deutschland. Statistisches Bundesamt, Wiesbaden

Destatis (2004) Statistisches Jahrbuch 2004 für die Bundesrepublik Deutschland. Statistisches Bundesamt, Wiesbaden

Destatis (2005) Statistisches Jahrbuch 2005 für die Bundesrepublik Deutschland. Statistisches Bundesamt, Wiesbaden

Destatis (2006) Statistisches Jahrbuch 2006 für die Bundesrepublik Deutschland. Statistisches Bundesamt, Wiesbaden

Destatis (2007) Statistisches Jahrbuch 2007 für die Bundesrepublik Deutschland. Statistisches Bundesamt, Wiesbaden

Destatis (2008) Statistisches Jahrbuch 2008 für die Bundesrepublik Deutschland. Statistisches Bundesamt, Wiesbaden

Destatis (2009) Statistisches Jahrbuch 2009 für die Bundesrepublik Deutschland. Statistisches Bundesamt, Wiesbaden

Destatis (2010) Statistisches Jahrbuch 2010 für die Bundesrepublik Deutschland mit >>Internationalen Übersichten<<. Statistisches Bundesamt, Wiesbaden

Destatis (2011) Statistisches Jahrbuch 2011 für die Bundesrepublik Deutschland mit >>Internationalen Übersichten<<. Statistisches Bundesamt, Wiesbaden

Destatis (2013) Statistisches Jahrbuch 2013: Deutschland und Internationales. Statistisches Bundesamt, Wiesbaden

Destatis (2014) Statistisches Jahrbuch 2014: Deutschland und Internationales. Statistisches Bundesamt, Wiesbaden

Destatis (2015a) Lebendgeborene nach dem Alter der Mütter – Insgesamt Deutschland 1991-2013. Statistisches Bundesamt (data provided on request on 18 June 2015)

Destatis (2015b) Lebendgeborene nach dem Alter der Mütter – Insgesamt Alte Bundesländer 1960-1990. Statistisches Bundesamt (data provided on request on 18 June 2015)

Destatis (2015c) Statistisches Jahrbuch 2015: Deutschland und Internationales. Statistisches Bundesamt, Wiesbaden

Destatis (2016a) Bevölkerung am 31.12.1968-2011 nach Alters- und Geburtsjahren – Deutschland. Statistisches Bundesamt (data provided on request on 17 March 2016)

Destatis (2016b) Statistisches Jahrbuch 2016: Deutschland und Internationales. Statistisches Bundesamt, Wiesbaden

Destatis (2017) Statistisches Jahrbuch 2017: Deutschland und Internationales. Statistisches Bundesamt, Wiesbaden

Destatis (2018) Lebendgeborene nach dem Alter der Mütter – Neue Länder und Berlin-Ost – Berichtsjahr: 1990-2000. Statistisches Bundesamt (data provided on request on 13 February 2018)

Eurostat Database (2018a) Fertility indicators. Eurostat. Available at http://ec.europa.eu/eurostat/data/database (data downloaded on 16 February 2018)

Eurostat Database (2018b) Live births by mother's age and citizenship. Eurostat. Available at http://ec.europa.eu/eurostat/data/database (data downloaded on 06 April 2018)

Fuchs J, Söhnlein D, Weber B et al. (2018) Stochastic Forecasting of Labor Supply and Population: An Integrated Model. Population Research and Policy Review 37:33-58

Gauthier AH, Hatzius J (1997) Family Benefits and Fertility: An Econometric Analysis. Population Studies 51:295-306

GENESIS-Online (2018a) Bevölkerung: Deutschland, Stichtag, Nationalität/Geschlecht/Familienstand. Destatis. Available at https://www-genesis.destatis.de/genesis/online/data;jsessionid=A08E5AB1C4B631C768EA93AF7D812FC0.tomcat_GO_1_2?operation=abruftabelleAbrufen&selectionname=12411-0006&levelindex=1&levelid=1518776848131&index=6 (data downloaded on 05 February 2018)

GENESIS-Online (2018b) Lebendgeborene: Deutschland, Jahre, Alter der Mutter, Geschlecht der Lebendgeborenen, Familienstand der Eltern. Destatis. Available at https://www-genesis.destatis.de/genesis/online/data;jsessionid=A08E5AB1C4B631C768EA93AF7D812FC0.tomcat_GO_1_2 ?operation=abruftabelleAbrufen&selectionname=12612-0007&levelindex=1&levelid=1518776998635&index =7 (data downloaded on 03 February 2018)

Gustafsson S (2001) Optimal Age at Motherhood. Theoretical and Empirical Considerations on Postponement of Maternity in Europe. Journal of Population Economics 14:225-247

Härdle W, Myšičková A (2009) Stochastic Population Forecast for Germany and its Consequence for the German Pension System. SFB 649 Discussion Paper 2009-009, Humboldt-Universität zu Berlin

Hertrampf S (2008) Ein Tomatenwurf und seine Folgen. Eine neue Welle des Frauenprotestes in der BRD. Bundeszentrale für politische Bildung. http://www.bpb.de/gesellschaft/gender/frauenbewegung/35287/neue-welle-im-westen. Accessed on 15 February 2018

Höhn C, Schubnell H (1986) Bevölkerungspolitische Maßnahmen und ihre Wirksamkeit in ausgewählten Industrieländern (II). Zeitschrift für Bevölkerungswissenschaft 12:185-219

Human Fertility Database (2016a) East Germany, Period total fertility rates and period fertility rates by age 40. Max Planck Institute for Demographic Research (Germany) and Vienna Institute of Demography (Austria). Available at http://humanfertility.org/cgi-bin/country.php?country=DEUTE&tab=si (data downloaded on 15 February 2018)

Human Fertility Database (2016b) West Germany, Period total fertility rates and period fertility rates by age 40. Max Planck Institute for Demographic Research (Germany) and Vienna Institute of Demography (Austria). Available at http://humanfertility.org/cgi-bin/country.php?country=DEUTW (data downloaded on 15 February 2018)

Hyndman RJ, Ullah MS (2007) Robust forecasting of mortality and fertility rates: A functional data approach. Computational Statistics & Data Analysis 51:4942-4956

Johnson NL (1949) Systems of Frequency Curves generated by Methods of Translation. Biometrika 36:149-176

Kalwij A (2010) The Impact of Family Policy Expenditure on Fertility in Western Europe. Demography 47:503-519

Lee RD (1992) Stochastic demographic forecasting. International Journal of Forecasting 8:315-327

Lee RD (1993) Modeling and forecasting the time series of US fertility: Age distribution, range, and ultimate level. International Journal of Forecasting 9:187-202

Lesthaeghe R (2010) The Unfolding Story of the Second Demographic Transition. Research Report 10-696. University of Michigan, Population Studies Center, Institute for Social Research, Ann Arbor

Pötzsch O, Rößger F (2015) Bevölkerung Deutschlands bis 2060: 13. Koordinierte Bevölkerungsvorausberechnung. Statistisches Bundesamt, Wiesbaden

Raftery AE, Alkema L, Gerland P (2014) Bayesian Population Projections for the United Nations. Statistical Science 29:58-68

Schmertmann C, Zagheni E, Goldstein JR, Myrskylä M (2014) Bayesian Forecasting of Cohort Fertility. Journal of the American Statistical Association 109:500-513

Shumway RH, Stoffer DS (2011) Time Series Analysis and Its Applications: With R Examples. Springer Science+Business Media, New York, Dordrecht, Heidelberg, London

Spieß CK (2014) Familienbezogene Infrastruktur. Bundeszentrale für politische Bildung. http://www.bpb.de/politik/innenpolitik/familienpolitik/194465/familienbezogene-infrastruktur. Accessed on 15 February 2018

Ševčíková H, Li N, Kanterová V et al. (2015) Age-Specific Mortality and Fertility Rates for Probabilistic Population Projections. Working Paper No. 150, Center for Statistics and the Social Sciences at the University of Washington

United Nations (2015) World Population Prospects: The 2015 Revision, Methodology of the United Nations Population Estimates and Projections. Department of Economic and Social Affairs, Population Division: Working Paper No. ESA/P/WP.242, New York

United Nations (2017) World Population Prospects: The 2017 Revision, Key Findings and Advance Tables. Department of Economic and Social Affairs, Population Division Working Paper No. ESA/P/WP/248, New York

van de Kaa DJ (1987) Europe's second demographic transition. Population Bulletin 42: 1-59.

Vanella P (2017a) A principal component model for forecasting age- and sex-specific survival probabilities in Western Europe. Zeitschrift für die gesamte Versicherungswissenschaft (*German Journal of Risk and Insurance*) 106:539-554

Vanella P (2017b) Stochastische Prognose demografischer Komponenten auf Basis der Hauptkomponentenanalyse. Hannover Economic Papers 597. Gottfried Wilhelm Leibniz Universität Hannover, School of Economics and Management

Vanella P, Deschermeier P (2018) A stochastic Forecasting Model of international Migration in Germany. In: Kapella O, Schneider NF, Rost H (eds) Familie – Bildung – Migration. Tagungsband zum 5. Europäischen Fachkongress Familienfoschung. Verlag Barbara Budrich, Opladen, Berlin, Toronto [forthcoming]

Zuchandke A, Lohse U, Graf von der Schulenburg JM (2014) Financial sustainability of the German statutory pension scheme through 2060 – Can higher fertility rates and immigration mitigate the financial pressure? Zeitschrift für die gesamte Versicherungswissenschaft (*German Journal of Risk and Insurance*) 103:283-299