Measuring Macroeconomic Convergence and Divergence within EMU Using Long Memory

Lena Dräger* Theoplasti Kolaiti** Philipp Sibbertsen§

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Abstract

This paper tests for convergence of EMU inflation rates and industrial production growth rates by testing for the existence of fractional cointegration relations. The notion of fractional cointegration allows for long-term equilibria with a higher degree of persistence than allowed for in the standard cointegration framework. Over the full sample from 1999:1-2019:12, we find evidence of fractional cointegration in both inflation and industrial production among many country pairs, where nominal convergence does not necessarily imply real convergence and *vice versa*. Our results suggest some evidence for "convergence clusters" among either core or periphery countries in the case of inflation. By contrast, we find more evidence of mixed core-country cointegration pairs for industrial production, where convergence may be driven by trade links. Testing for a break in the persistence structure, the results suggest evidence of a break in the persistence of both inflation and industrial production during the beginning of the financial crisis in a number of countries. In all cases, persistence is substantially higher after the break, suggesting a higher potential for diverging processes during economic crises. However, in some cases we find a second break marking the end of the crisis period, with persistence back at pre-crisis levels after the second break.

Keywords: EMU inflation rates, industrial production, fractional cointegration, persistence breaks

JEL classification: F15, F45, C32

*Leibniz University Hannover. Email: draeger@gif.uni-hannover.de.
**Leibniz University Hannover. Email: kolaiti@statistik.uni-hannover.de.
§Leibniz University Hannover. Email: sibbertsen@statistik.uni-hannover.de.

1 Introduction

Ever since the idea of a European Monetary Union (EMU) was born, economists have discussed the need for member countries to converge in nominal and real terms in order to ensure the stability of the union. This follows from the concern that inflation differentials between member countries with a single monetary policy may lead to differences in real interest rates and in international competitiveness, thereby creating real divergence as measured in current account imbalances. These imbalances are difficult to address if the affected countries share a common currency and thus have permanently fixed nominal exchange rates and a single monetary authority (Mundell, 1961).

Early studies before the start of EMU demonstrate some success in terms of nominal convergence of the member states (Beliu and Higgins, 2004), whereas others use cointegration analysis to demonstrate potential long-run stability problems with respect to macroeconomic dynamics in the so-called "periphery" countries Italy, Spain and Portugal (Haug et al., 2000). However, the initial convergence in inflation rates was somewhat reversed after the start of EMU, resulting in persistent inflation differentials, where groups of countries showed inflation rates persistently above or below the EMU average (ECB, 2003). Some authors argued that these dynamics were signs of economic catching-up processes (Blanchard and Giavazzi, 2002). However, the European sovereign debt crisis following the financial crisis of 2008 revealed potential large economic costs of macroeconomic divergence within a monetary union (Borio and Disyatat, 2011; Gnimassoun and Mignon, 2016).

In this paper, we thus use fractional cointegration methods accounting for long-memory equilibrium processes as a tool to measure nominal and real convergence or divergence since the start of EMU. In this set-up, economic convergence does not necessarily imply convergence in levels. Rather, the existence of a stable long-run equilibrium between macroeconomic variables across EMU countries is regarded as evidence of close and stable macroeconomic relationships among those countries. This is what we term macroeconomic convergence. Testing for the existence of a long-run equilibrium with fractional cointegration has the advantage of allowing the degree of integration to take any real number on the unit interval. Therefore, the framework allows for higher flexibility by allowing the long-run equilibrium to be a mean-reverting rather than a short-memory stationary process. Evidence for fractional cointegration with long memory in the equilibrium errors then indicates the existence of a long-run equilibrium, while at the same time deviations from this equilibrium may be very persistent. In contrast to the previous literature, we account for the full EMU period including the financial crisis period from 2008 onwards. However, this makes our analysis vulnerable to spurious results through changes in the convergence mechanism and thus in the cointegrating relation. We therefore apply the regression-based Lagrange Multiplier test introduced by Martins and Rodrigues

(2014) to test for structural breaks in the order of integration and estimate the break dates.

We estimate the degree of fractional long memory for inflation and industrial production growth rates (IP) in 11 EMU countries for the period from January, 1999, to December, 2019, where we distinguish between "core" and "periphery" countries. Our results suggest breaks in the persistence of both inflation and industrial production around the beginning of the financial crisis in 2007-08 for a majority of countries in our sample. In some countries, we find a second break marking the end of the crisis period. Inflation and industrial production are generally found to be more persistent in the periphery countries than in the core countries. In addition, persistence is estimated to be higher in the crisis period. This implies that both inflation rates and industrial production were more integrated in the EMU before the financial crisis of 2008, while the higher persistence during the crisis period carries the danger of diverging processes in case of asymmetric shocks.

We further test for the existence of fractional cointegration relationships for inflation and IP among all country pairs of our sample and estimate the memory of the residuals d_v by using the Narrow Band Least Squares estimation for the cointegrating vector β. We find evidence of fractional cointegration for both inflation and industrial production over the whole sample period. In the case of inflation, we find evidence of fractional cointegration among core and among periphery countries, but fewer evidence of mixed core-periphery cointegration pairs. This result suggests a certain risk of "convergence clubs" regarding inflation, which could lead to current account disequilibria if the development is sustained over longer periods of time. By contrast, in the case of industrial production, we find evidence of cointegration among core country-pairs, but also among mixed core-periphery pairs, with fewer long-run equilibria among periphery countries. In summary, it seems that real convergence does not necessarily imply nominal convergence and *vice versa*.

Our paper builds on previous work by some of the authors in Leschinski et al. (2018). Using the fractional cointegration methodology, Leschinski et al. (2018) provide empirical evidence for periods of convergence and divergence for long-term EMU government bonds that coincide with bull- and bear-market periods in the stock market. Specifically, stronger market integration is associated with bull-market periods and is more intense among core countries than among periphery countries. Periods of disintegration coincide with bear-market periods. Their results thus imply time-variation in the degree of convergence of EMU government bonds, with the possibility of divergence even before the financial crisis of 2008. Moreover, the authors report evidence of disintegration in government bonds for all countries during the period of the financial crisis and the European sovereign debt crisis from 2008 onwards.

Our paper further relates to the previous literature using fractional cointegration methods to measure macroeconomic convergence between European economies. In an early study, Beliu and Higgins (2004) use fractional cointegration tests to evaluate the convergence of inflation, long-term interest rates and industrial production of 14 EU countries vis-à-vis Germany. The sample (1957-2001) covers the period until the Euro cash changeover, whereas we focus on the period after the start of EMU in 1999. The authors present evidence of nominal convergence in inflation and long-term interest rates, as these series are fractionally cointegrated with the German counterpart. However, the equilibrium errors display long memory, so that any deviation from equilibrium will be persistent. The authors find no evidence of fractional cointegration in industrial production and, thus, no evidence for real convergence. Meller and Nautz (2012) test for differences in inflation dynamics among European countries before and after EMU using panel estimates of fractional cointegration. Their results suggest that inflation persistence converged and was significantly reduced with the introduction of EMU. More recently, Hualde and Iacone (2017b) analyze both level inflation as well as inflation differentials between EMU country pairs allowing for long memory and cointegration with the test procedure derived in Hualde and Iacone (2017a). Their results suggest that the "core" economies of EMU are more integrated than the "periphery" countries, as the latter show more persistent inflation differentials. Similarly, Karanasos et al. (2016) study convergence of inflation among EMU countries for the period 1980-2013 using a broad range of test methods, which includes tests allowing for long memory and for structural breaks. Similar to the study by Hualde and Iacone (2017b), the authors present evidence for three clubs of convergence consisting of "core" EMU countries, while there is evidence of divergence in inflation for the remaining countries. We extend these previous studies, as we account for the effects on nominal and real convergence of the recent period of economic turmoil following the financial crisis in 2008. In addition, we compare the results from several semiparametric tests for fractional cointegration by Souza et al. (2018), Wang et al. (2015) and Chen and Hurvich (2006).

Other studies applying the concept of fractional cointegration evaluate, for instance, the stability of money demand functions (Caporale and Gil-Alana, 2005), the effect of inflation targeting regimes on inflation persistence (Canarella and Miller, 2017) or the effect of a monetary policy shock when long memory in the output gap and inflation is accounted for (Lovcha and Perez-Laborda, 2018).

The remainder of the paper is structured as follows. The econometric methodology of fractional cointegration with long memory is detailed in section 2. Section 3 describes the data set. The results of our anaylsis are presented and discussed in section 4. Finally, section 5 summarizes and concludes.

2 Methodology

Inflation and industrial production persistence in Europe has attracted renewed attention since the foundation of EMU and the implementation of a common monetary policy among the member countries. Persistence is here measured by the order of integration of the inflation series. If this order of integration d fulfills $0 < d < 0.5$, the series is said to exhibit stationary long memory or stationary long-range dependence resulting in a hyperbolic decay of the autocorrelation function and therefore in a slow decay of dependencies between observations far away from each other. Also the impulse-responses show a hyperbolic decay. For $0.5 < d < 1$, the series is non-stationary, but still mean reverting in the sense that the expected time for returning to its mean is finite. Therefore, the concept of long memory or fractional integration widens the traditional $I(1)/I(0)$ duality by allowing for highly persistent equilibrium processes.

A significant number of empirical studies that have been adopted in recent years support the existence of long-range dependence in inflation rates. Baillie et al. (2002), Canarella and Miller (2016), Canarella and Miller (2017) Hassler and Wolters (1995), Kumar and Okimoto (2007) and Meller and Nautz (2012) among others, measure persistence as a fractionally integrated process. Moreover, fractional integration techniques are able to capture convergence among inflation policies and detect persistence shifts more consistently than the $I(1)/I(0)$ approach, as assumptions about the $I(0)$ process of inflation process or the short-run persistence structure are not required (Kumar and Okimoto (2007)). Inflation convergence requires the existence of a stable equilibrium relationship, but not exact equality between the inflation rates. A similar definition of convergence can also be applied to real economic variables, such as industrial production (IP).

In general, (fractional) cointegration is an equilibrium concept where the persistence of the cointegrating residual d_v determines the speed of adjustment towards the cointegration equilibrium $\beta' X_t$, and shocks have no permanent influence on the equilibrium as long as $d_v < 1$ holds. We therefore allow for fractional cointegration and consider a bivariate system of the form

$$X_{1t} = c_1 + \xi_1 Y_t + \Delta^{-(d-b_1)} u_{1t} \mathbb{1}_{\{t>0\}} \tag{1}$$

$$X_{2t} = c_2 + \xi_2 Y_t + \Delta^{-(d-b_2)} u_{2t} \mathbb{1}_{\{t>0\}} \tag{2}$$

$$Y_t = \Delta^{-d} e_t \mathbb{1}_{\{t>0\}}, \tag{3}$$

where the coefficients c_1, c_2, ξ_1, and ξ_2 are finite, $0 \leq b_1, b_2 \leq d$, L is the lag-operator, the fractional differences $\Delta^d Y_t = (1-L)^d Y_t$ are defined in terms of generalized binomial coefficients such that

$$(1-L)^d = \sum_{k=0}^{\infty} \binom{d}{k} (-1)^k L^k = \sum_{k=0}^{\infty} \pi_k L^k,$$

$$\text{with} \quad \binom{d}{k} = \frac{d(d-1)(d-2)\ldots(d-(k-1))}{k!},$$

and e_t and $u_t = (u_{1t}, u_{2t})'$ are martingale difference sequences. The memory of both X_{1t} and X_{2t} is determined by Y_t so that they are integrated of the same order d, denoted by $X_t \sim I(d)$, where the memory parameter is restricted to $d \in (0, 1]$ and $X_t = (X_{1t}, X_{2t})'$. Since it is assumed that $u_{1t} = u_{2t} = e_t = 0$ for all $t \leq 0$, the processes under consideration are fractionally integrated of type-II. For a detailed discussion of type-I and type-II processes we refer to Marinucci and Robinson (1999). The spectral density of X_t can be approximated by

$$f_X(\lambda) \sim \Lambda_j(d)\, G\, \overline{\Lambda_j(d)}, \quad \text{as } \lambda \to 0^+, \tag{4}$$

where G is a real, symmetric, finite, and non-negative definite matrix, $\Lambda_j(d) = \text{diag}\left(\lambda^{-d} e^{i\pi d/2}, \lambda^{-d} e^{i\pi d/2}\right)$ is a 2×2 diagonal matrix and $\overline{\Lambda_j(d)}$ is its complex conjugate transpose. The periodogram of a process X_t is defined through the discrete Fourier transform $w_X(\lambda_j) = \frac{1}{\sqrt{2\pi T}} \sum_{t=1}^{T} X_t e^{i\lambda_j t}$ as $I_X(\lambda_j) = w_X(\lambda_j)\overline{w_X(\lambda_j)}$, with Fourier frequencies $\lambda_j = 2\pi j/T$ for $j = 1, ..., \lfloor T/2 \rfloor$, where the operator $\lfloor \cdot \rfloor$ returns the integer part of its argument.

The two series X_{1t} and X_{2t} are said to be fractionally cointegrated, if there exists a linear combination

$$\beta' X_t = v_t,$$

so that the cointegrating residuals v_t are fractionally integrated of order $I(d-b)$ for some $0 < b \leq d$. Obviously, for the model in equations (1) to (3), this is the case for every multiple of the vector $\left(1, -\frac{\xi_1}{\xi_2}\right)'$ and $b = \min(b_1, b_2)$.

We restrict ourselves to a bivariate set-up as is common in the literature to avoid identification problems.

According to the definition above, we thus test if among all pairs of EMU countries there exists an equilibrium relationship between the inflation rates or between IP growth rates (X_{1t} and X_{2t}), such that the persistence of deviations from the equilibrium denoted by v_t is reduced compared to that of the individual series. The degree of long memory $d - b$ in the cointegrating residual determines the persistence of deviations from the long-run equilibrium. The existence of a fractional cointegration relationship is then taken as evidence for economic integration.

3 Data

We conduct the analysis of macroeconomic convergence for 11 EMU countries: Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Portugal and Spain. As is common in the literature, we term the group of Austria, Belgium, Finland, France, Germany and the Netherlands the 'core countries' of EMU, while we call the group of Greece, Italy, Ireland, Spain and Portugal the 'periphery countries'. The data sample ranges from 1999:1 to 2019:12. Our sample thus starts with the official start of the EMU with the ECB acting as single central bank for the monetary union.

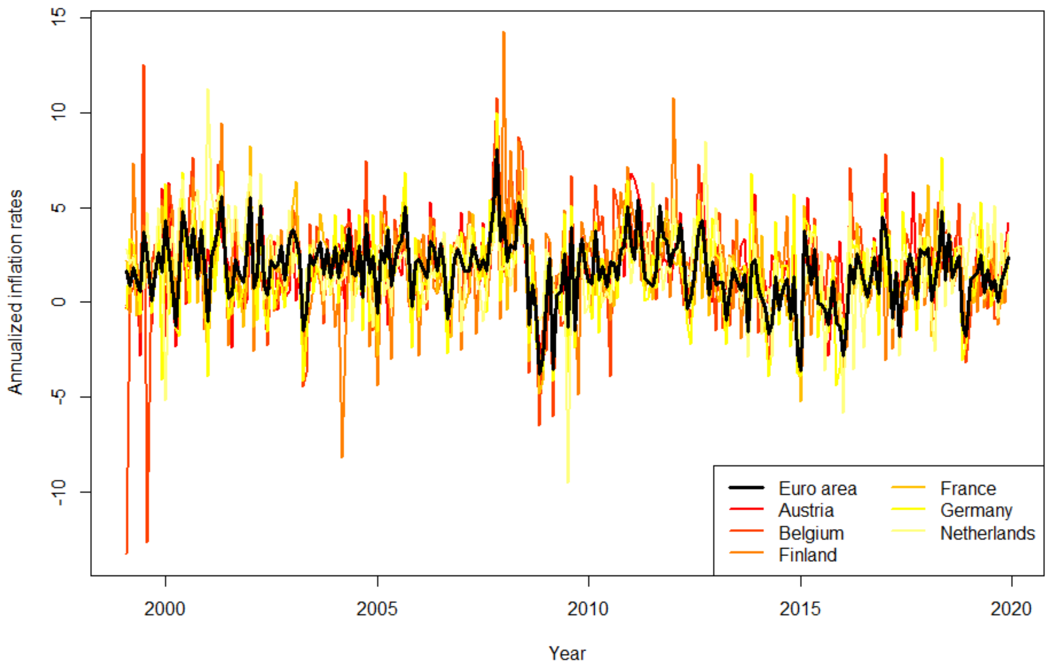
Data for inflation is obtained from Eurostat and measured with the monthly seasonally unadjusted Harmonized Indices of Consumer Prices for all items, with 2015 as base year

(2015 = 100). We seasonally adjust each series by using the X-13 R package, developed by the United States Census Bureau. Then, we define annualized inflation rates for each country i as

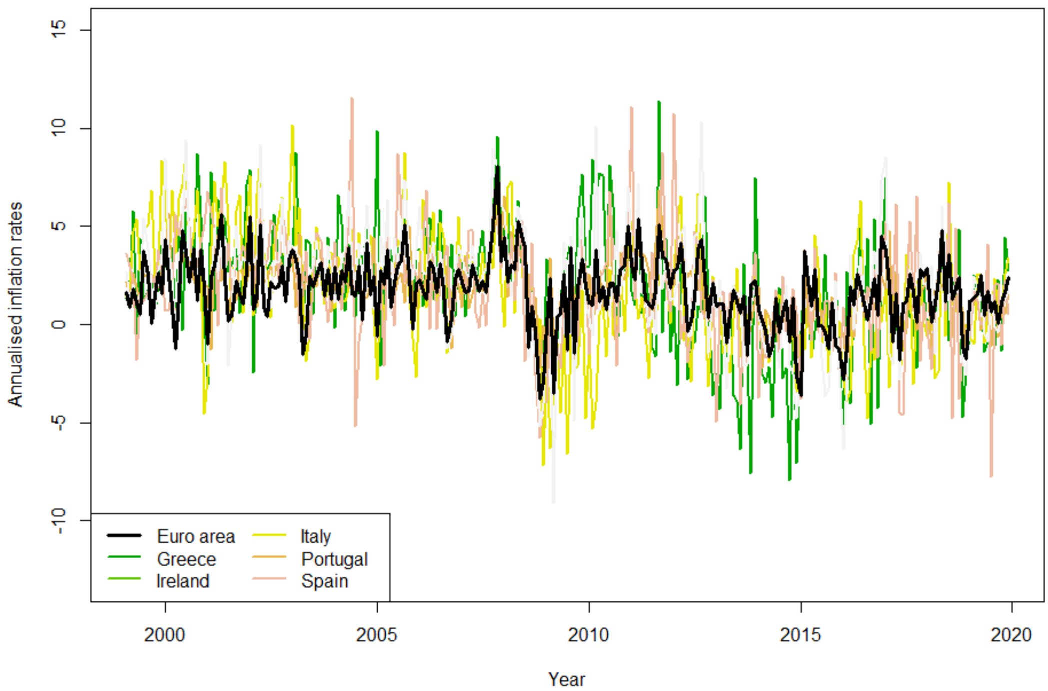
$$\pi_{it} = 1200(log(HCPI_{it}) - log(HCPI_{it-1}))$$

The series are shown for all countries in our sample in Figure 1. Visual inspection suggests that all EMU countries in our sample experienced a drop in inflation at the start of the financial crisis in 2008, with somewhat more volatile inflation rates after 2008. Overall, inflation rates in the periphery countries seem more volatile than those in the core countries.

Data for quarterly seasonally adjusted industrial production growth rates are obtained from the Federal Reserve Economic Database (FRED) by the St. Louis Fed. The data is shown in Figure 2. As in the case of inflation rates, the pronounced drop in industrial production in 2008 is clearly visible in all countries of our sample. Before the crisis, countries like France, Germany and Spain experienced very stable IP growth, while countries like Finland, Greece and Portugal exhibit higher IP volatility. After the financial crisis, the negative effects of the European sovereign debt crisis on industrial production growth rates are clearly observable in Greece and Portugal. Here, we exclude Ireland from our sample because of data irregularities.

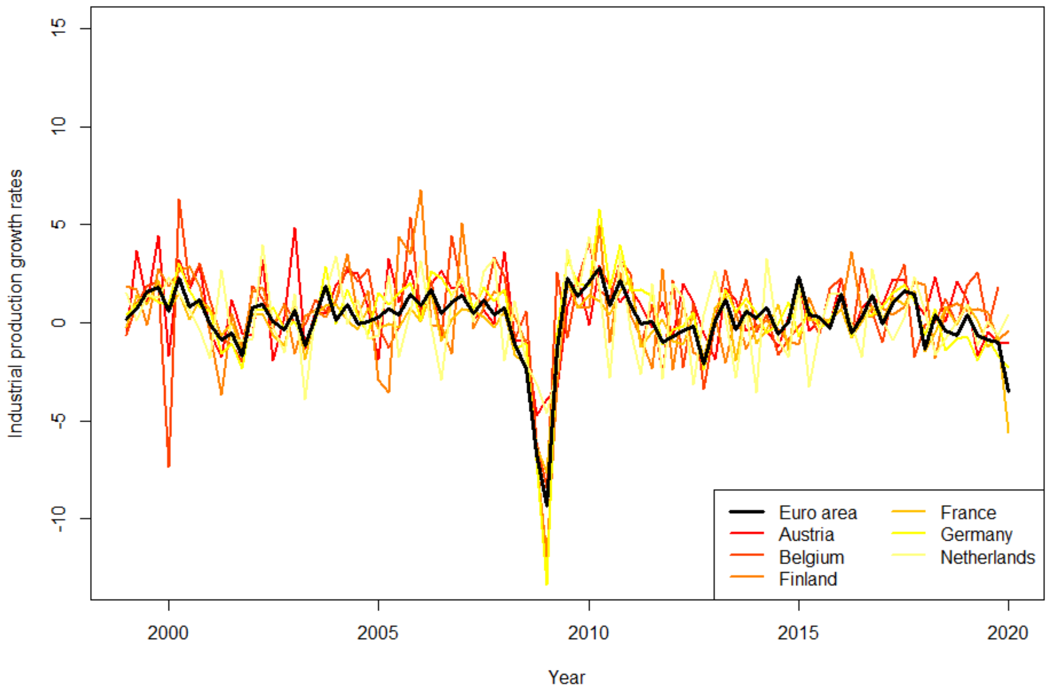


(a) Annualized inflation rates in core countries

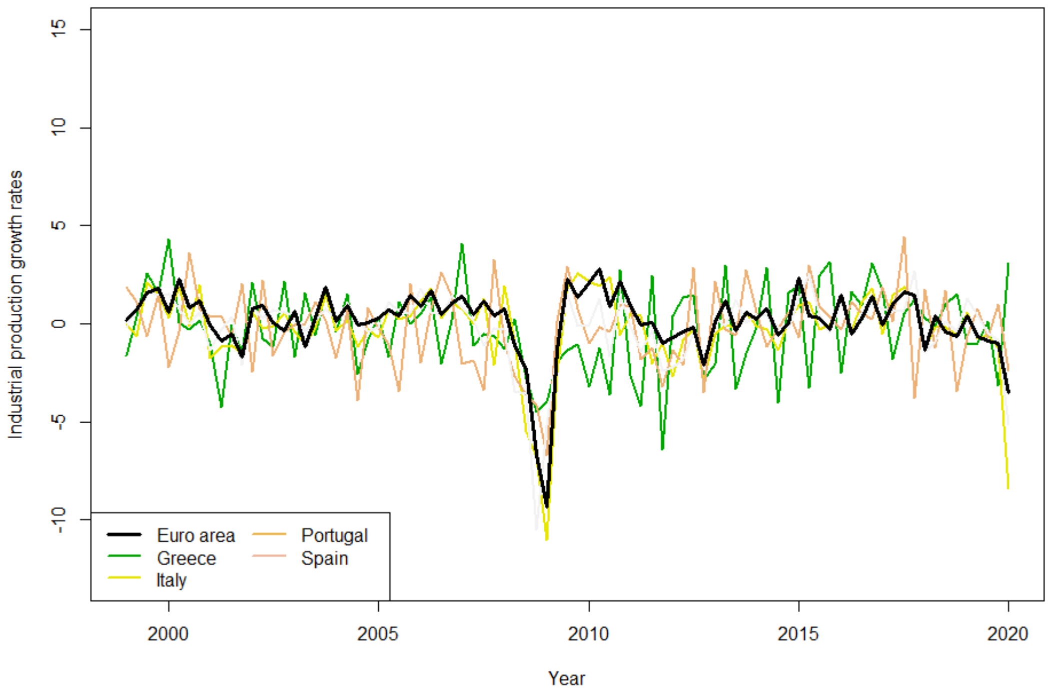


(b) Annualized inflation rates in periphery countries

Figure 1: Annualized inflation rates of EMU countries.



(a) Industrial production growth rates in core countries



(b) Industrial production growth rates in periphery countries

Figure 2: Industrial production growth rates of EMU countries.

4 Empirical Results

4.1 Analysis of Convergence and Divergence of EMU Inflation Rates

4.1.1 Measuring the degree of long memory in EMU inflation rates

In this section, following Kumar and Okimoto (2007), we model inflation rates as a fractionally integrated process and estimate the memory parameters as a measure of persistence for the largest economies in the EMU. Memory parameters for each country are shown in Table 1, where we differentiate between the so-called "core" economies of Austria, Belgium, Finland, France, Germany and the Netherlands, and the so-called "periphery" countries Greece, Ireland, Italy, Portugal and Spain. For our estimation, we use the exact local Whittle estimator of Shimotsu (2010) with a 0.75 bandwidth d-value ($m = T^{0.75}$). As a direct extension of Shimotsu et al. (2005), this estimator has the advantage of allowing for non-zero means, while the properties of consistency and asymptotically normal distribution for all values of d continue to hold. Similar to other findings in the literature (Hualde and Iacone, 2017a; Meller and Nautz, 2012), inflation persistence is higher for the periphery countries. Overall, the range of memory parameters is larger in the periphery countries, ranging from 0.26 in Spain to 0.44 in Italy. In fact, the mean values of the order of fractional integration are 0.24 and 0.34 for core and periphery countries, respectively.

Table 1: Memory estimates for inflation rates

Austria	Belgium	Finland	France	Germany	Netherlands	Mean
0.23	0.24	0.3	0.21	0.12	0.33	0.24
Greece	Ireland	Italy	Portugal	Spain		Mean
0.35	0.36	0.44	0.28	0.26		0.34

Note: Shimotsu (2010) estimates of d with bandwidth $m = T^{0.75}$.

4.1.2 Testing for fractional cointegration in EMU inflation rates

We apply a number of semiparametric tests for the null hypothesis of no fractional cointegration. The advantage of semiparametric methods is that we do not impose any assumptions on the short-run behavior of the series, apart from mild regularity conditions. Thus, we can avoid spurious findings that might arise due to misspecification. Research on semiparametric tests for fractional cointegration has been an active field in recent years and there exist a variety of competing approaches. Whereas some approaches rely on the spectral representation of multivariate long memory processes and test whether the spectral matrix G has full rank or not, other tests are residual-based and test for the strength of integration of the cointegration residuals. To make sure that our results are robust to the way of testing, we apply tests from both strands of the literature.

Souza et al. (2018) use the fractionally differenced process $\Delta^d X_t$ and the fact that the determinant $D_{\Delta^d}(\lambda)$ of $f_{\Delta^d X}(\lambda)$ is of the form $D_{\Delta^d}(\lambda) \sim \tilde{G}|1 - e^{-i\lambda}|^{2b}$, where $\tilde{G}$ is a scalar constant and $0 < \tilde{G} < \infty$. An estimate of b can therefore be obtained via a log-periodogram regression and the null hypothesis that $b = 0$ can be tested based on the resulting estimate. We denote this spectral-based test in the following as SRFB18.

The test of Wang et al. (2015) (denoted by WWC15) is based on the sum over the fractionally differenced process $\Delta^{\hat{d}_v} X_{2t}$, where $\hat{d}_v$ is an estimate of the memory from the cointegrating residuals obtained using a consistent estimator for the cointegrating vector β such as the narrow-band least squares estimator of Robinson (1994), Robinson and Marinucci (2003), and Christensen and Nielsen (2006), among others. In contrast to that, the test of Chen and Hurvich (2006) (denoted by CH06) is directly based on $\hat{d}_v$, but the cointegrating space is estimated by the eigenvectors of the averaged and tapered periodogram matrix local to the origin. Obviously, these two tests are residual-based.

Leschinski et al. (2020) suggest that the three testing procedures have better performance among a group of eight semiparametric tests, particularly when testing for fractional cointegration. In bivariate cases, SRFB18 presents the best performance. Following their finding, we perform all tests at the 5% significance level. The bandwidth is selected as $m = T^{0.75}$ for all three testing procedures. The trimming parameter r is set to 3 for SRFB18 and the integer for averaging the periodogram is 25 for CH06. As fractional cointegration needs as a core assumption that the order of integration is equal between the series, we use pairwise tests as suggested by Robinson and Yajima (2002) to test for the equality of the memory parameters. The results shown in Table 2 suggest that the hypothesis of a common memory parameter is rejected for the case of Germany with Greece, Ireland and Italy, as well as for the pair of France-Italy at the 5% significance level. Therefore, these pairs are not included in our analysis. Next, we test for pairwise cointegrating relationships under the null hypothesis of no fractional cointegration.

Table 2: Pair-wise testing for common memory integration in inflation rates

EMU countries	Belgium	Finland	France	Germany	Netherlands	Greece	Ireland	Italy	Portugal	Spain
Austria	0.11	0.65	0.08	0.99	0.93	1.10	1.21	1.89	0.55	0.48
Belgium		0.59	0.2	1.04	0.8	1.06	1.16	1.77	0.43	0.36
Finland			0.77	1.58	0.28	0.43	0.60	1.22	0.1	0.25
France				0.93	0.97	1.26	1.38	2.15**	0.63	0.58
Germany					1.78	1.98**	2.22**	2.77**	1.39	1.42
Netherlands						0.15	0.26	0.81	0.4	0.53
Greece							0.11	0.75	0.56	0.76
Ireland								0.61	0.68	0.86
Italy									1.35	1.61
Portugal										0.14

Note: Results of Robinson and Yajima (2002) test. ** denotes significance of the test statistic at the 5% level where the critical value is 1.9599.

An example of our analysis is in Table 9 in the appendix where we report the test results for Germany as the reference country, since it is the largest economy within the EMU. There is evidence for fractional cointegration of inflation rates with the German counterpart in all seven countries we were allowed to test. For the case of Austria, the null hypothesis is rejected for all three testing procedures. France, Netherlands and Spain suggest inflation convergence only for the WWC15 test. We repeat the testing procedures for all possible pairs.

As a second step, we estimate the cointegration vector β as well as the memory parameters d_v by using Narrow Band Least Squares estimation. Robinson (1994) shows that NBLS estimation is consistent under stationary cointegration whereas an OLS approach might not retain consistency.

Table 3: Pair-wise memory estimates on the cointegrating residuals of inflation rates

EMU countries	Austria	Belgium	Finland	France	Germany	Netherlands	Greece	Italy	Portugal	Spain
Austria		**0.04**		**0.12**	**0.03**					
Belgium	**0.18**			**0.18**	**0.19**			**0.16**	**0.1**	**0.15**
Finland						**0.24**		**0.22**		**0.23**
France	**0.14**	**0.1**			**0.1**					
Germany	**-0.06**	**0.03**		**0.01**						
Netherlands			**0.27**					**0.28**	**0.19**	**0.24**
Greece									**0.26**	**0.24**
Italy		**0.13**	**0.23**			**0.29**			**0.09**	**0.12**
Portugal		**0.2**				**0.23**	**0.12**	**0.21**		**0.05**
Spain		**0.21**	**0.24**			**0.24**	**0.11**	**0.2**	**0.11**	

Note: Exact Local Whittle estimates, d_v, with a $m = T^{0.75}$ bandwidth based on the Narrow Band Least Squares estimation of the cointegration vector β.

We thus repeat the NBLS estimation between all EMU country pairs and we present the estimated values of the memory of the residuals in Table 3. Only the cases where the hypothesis of a common memory parameter is not rejected, at least one of the testing procedures are statistically significant and the memory of d_v is less than the memory of both of the individual series are presented (e.g. Ireland is not cointegrated with any of the countries, therefore is not presented in the following table). As can be seen in Table 3, results are not symmetric as it is expected for cointegration relations. This is due to the irregular behavior of the inflation series during the crisis period. It is well known that such data irregularities can lead to asymmetric cointegration (see for example Enders and Siklos (2001)).

For the core countries, inflation rates between Germany, Austria and the rest of the countries are estimated to have the strongest cointegration relation, as the adjustment to equilibrium is achieved faster after potential shocks (d_v ranges between -0.06 and 0.04). Belgium, France, Finland and the Netherlands are all estimated to have fractional cointegration relationships, but with slower adjustment to equilibrium overall.

For the periphery countries (see the lower part of Table 3), the memory parameters that belong to the pairs of Italy, Portugal and Spain are lower than the parameters among the core-periphery or periphery-core pairs. On average, we observe that cointegration pairs cluster among core and periphery countries, with fewer mixed core-periphery cointegration pairs. This suggests that over the whole estimation period, separate 'convergence clubs' for EMU inflation rates may be observed. This could be worrisome as it might be indicative of macroeconomic divergence processes, which could lead to current account disequilibria.

4.1.3 Testing for a break in fractional integration of EMU inflation rates

In addition to our analysis of fractional cointegration, we test for a break in the order of fractional integration in inflation rates. This allows us to test whether persistence and thereby potential convergence or divergence processes in inflation changed over time. This could be due, for instance, to the disruptions of the global financial crisis and the European sovereign debt crisis. Hence, we perform a regression-based Lagrange Multiplier test introduced by Martins and Rodrigues (2014) that generalizes the conventional integration approaches to the fractional integrated process context. In what follows we will denote by $\hat{\tau}$ the estimated point of the persistence shift and by d_1 the order of integration before the shift and by d_2 the order of integration after the break. Table 4 suggests evidence of a structural break in inflation persistence for all EMU countries. Specifically, Austria, Belgium, Finland, France, Greece, Ireland and Italy's estimated breakpoints ($\hat{\tau}$) occur from October, 2006 to October, 2008. This corresponds to the time period covering the beginning of the subprime crisis, i.e. a heightened level of financial stress, to the beginning of the global financial crisis following the default of Lehman Brothers on September 15, 2008. Comparing d_1 and d_2, the results suggest a shift from $d \leq 0$ to $d > 0$ at the time of $\hat{\tau}$, that is an increase in inflation persistence after the break. The alternative hypothesis of decreasing memory holds for a smaller number of countries. For Finland, Greece, Ireland, Italy, Portugal and Netherlands we find (additional) memory shifts between April, 2014 and October, 2016, with inflation persistence returning to its pre-crisis lower levels.

In summary, we find that inflation persistence since the beginning of EMU is stationary mean-reverting. A test in persistence change is showing evidence of breaks in the beginning of the financial crisis where we observe memory parameters around zero in the pre-crisis period for the majority of the sample, and substantially higher values around 0.4 in the post-crisis period. Moreover, the inflation persistence during the crisis period increased in particular in the periphery countries, implying potentials for inflation divergence. Finally, while we find evidence of fractional cointegration in inflation among the full sample, the core countries tend to be fractionally cointegrated with a faster speed of adjustment, see also Hualde and Iacone (2017b) and Karanasos et al. (2016).

Table 4: Persistence change test in inflation rates

EMU country	Test-statistic	Date	d_1	d_2
Austria	-3.84***	10/2006	-0.25	0.27
Belgium	-2.68***	09/2007	-0.01	0.35
Finland	-1.83**	10/2007	0.02	0.34
	-2.67*	02/2016	0.28	-0.18
France	-1.84*	07/ 2007	-0.21	0.39
Germany	-2.92***	06/2005	-0.09	0.17
Netherlands	-1.9317*	12/2014	0.35	0.10
Greece	-2.96***	05/2007	-0.19	0.37
	-3.87***	10/2014	0.36	-0.1
Ireland	-1.98**	10/2008	0.13	0.32
	-2.60**	10/2016	0.40	-0.15
Italy	-4.79***	08/2007	-0.02	0.45
	-2.28*	12/2014	0.46	0.14
Portugal	-3.82***	04/2014	0.32	-0.15
Spain	-1.75*	06/2005	-0.06	0.33

Note: Results of Martins and Rodrigues (2014) test where *, **, *** indicate levels of significance at 10%, 5%, 1%, respectively. "d_1" and "d_2" refer to the memory parameters of the respective subperiod of the estimated breakpoint.

4.2 Analysis of Convergence and Divergence of EMU Industrial Production

4.2.1 Measuring the degree of long memory of EMU industrial production

Following the analysis of convergence in EMU inflation rates in the previous section, we next investigate the convergence of industrial production rates between the same countries, except Ireland. First, as in subsection 4.1.1 we estimate the memory parameter of each country. Results are presented in Table 5.

Table 5: Memory estimates for industrial production growth rates

Austria	Belgium	Finland	France	Germany	Netherlands	Mean
0.31	0.08	0.27	0.39	0.47	-0.12	0.23
Greece	Italy	Portugal	Spain			Mean
0.11	0.68	0.22	0.56			0.39

Note: Shimotsu (2010) estimates of d with bandwidth $m = T^{0.75}$.

We observe that there are more cases of memory estimates with $d > 0.5$ in the periphery countries (Italy and Spain), while in the core countries almost all memory estimates are stationary mean-reverting with $0.5 < d < -0.5$. In line with our previous results for inflation, we find that long memory parameters of industrial production are on average higher in the periphery countries of EMU.

4.2.2 Testing for fractional cointegration of EMU industrial production

Next, we test for fractional cointegration in EMU industrial production among all possible country pairs. In Table 6, the hypothesis of memory equality is rejected for almost all cases in Italy and the Netherlands. Therefore, they are excluded from our analysis, as before. Similarly to the previous section, we present in Table 10 in the appendix the test results for cointegration between industrial production in Germany and the remaining EMU countries.

Table 6: Pair-wise testing for common memory integration in industrial production growth rates

EMU countries	Belgium	Finland	France	Germany	Netherlands	Greece	Italy	Portugal	Spain
Austria	1.208	0.041	0.389	0.742	1.965**	0.475	1.972**	0.510	1.356
Belgium		1.231	1.562	1.9496	0.9133	0.583	2.978**	0.577	2.334
Finland			0.456	0.784	2.094**	0.444	2.033**	0.453	1.380
France				0.415	2.420**	0.834	2.058**	0.929	1.224
Germany					2.827**	1.091	1.652	1.259	0.798
Netherlands						1.422	3.729**	1.348	3.089**
Greece							2.285**	0.032	1.761
Italy								2.649**	0.776
Portugal									2.008**

Note: Results of Robinson and Yajima (2002) test. ** denotes significance of the test statistic at the 5% level where the critical value is 1.9599.

Table 7 reports the NBLS estimates of the memory parameters, d_v, of the IP fractional cointegration relation in all country-pairs. Here, asymmetric cointegration for industrial production country-pairs is even more pronounced than in the case of inflation. This is again due to the drop of industrial production during the financial crisis. We also exclude the Netherlands since it is not fractionally cointegrated with any of the remaining countries. Comparing our results for fractional integration of EMU inflation and IP, inflation convergence was suggested mostly between core and between periphery countries, whereas convergence in IP growth rates appears stronger between mixed groups of core/periphery countries. For instance, German IP is fractionally cointegrated with Austria and Finland, but also Italy and Spain. This result could mirror the strong trade links between these countries. Overall, it appears that nominal convergence does not necessarily imply real convergence and *vice versa*. The fact that we observe more bilateral cointegration relationships among core or between core and periphery countries, but few among periphery countries, suggests that convergence in industrial production might be driven by the core countries.

Table 7: Pair-wise memory estimates on the cointegrating residuals of industrial production growth rates

EMU countries	Austria	Belgium	Finland	France	Germany	Greece	Italy	Portugal	Spain
Austria		**-0.07**	**-0.26**	**0.04**	**0.06**	**0.06**			
Belgium	**-0.12**		**-0.19**	**-0.03**					
Finland	**0.09**	**0.06**		**0.18**	**0.09**				**0.08**
France	**-0.02**	**0.00**	**0.09**			**0.09**		**0.02**	
Germany	**0.29**		**0.15**				**0.21**		**0.32**
Greece	**0.05**			**0.07**				**0.07**	
Italy					**0.2**				**0.12**
Portugal				**0.07**		**0.09**			
Spain			**0.04**		**0.28**		**0.07**		

Note: Exact Local Whittle estimates, d_v, with a $m = T^{0.75}$ bandwidth based on the Narrow Band Least Squares estimation of the cointegration vector β.

Table 8: Persistence change test in industrial production growth rates

EMU country	Test-statistic	Date	d_1	d_2
Austria	-2.57***	2005.03	-0.18	0.37
Belgium	-2.13**	2004.04	-0.5	0.16
France	-2.06**	2008.03	0.19	0.31
	-2.56***	2011.01	0.59	-0.4
Italy	-2.53***	2008.03	0.31	0.82
Portugal	-2.05**	2008.02	-0.32	0.43
Spain	-3.69***	2007.01	0.38	0.58
	-4.98***	2009.03	0.7	0.37

Note: Results of Martins and Rodrigues (2014) test where *, **, *** indicate levels of significance at 10%, 5%, 1%, respectively. "d_1" and "d_2" refer to the memory parameters of the respective subperiod of the estimated breakpoint.

4.2.3 Testing for a break in fractional integration of EMU industrial production

Finally, we apply the test by Martins and Rodrigues (2014) to test for a break in the order of fractional integration in EMU industrial production. The results are shown in Table 8 . As can be seen, we find a significant break in IP persistence in most, but not all countries. In France, Italy, Portugal and Spain the test suggests a significant breakpoint in presistence of industrial production around the beginning of the global financial crisis, with higher persistence in the post-crisis period. This suggests that the large drop in IP observed in all countries of our sample led to changes in the long memory of the series in these countries. However, for France and Spain we find a second breakpoint in 2011 and 2009, respectively. In these two countries, after the second break estimated persistence is at pre-crisis levels or even lower, suggesting that the change in persistence during the crisis was only temporary in these cases.

5 Conclusion

In this paper, we apply methods of fractional cointegration to investigate the degree of real and nominal convergence between EMU countries. Specifically, we model both inflation rates and industrial production growth rates as fractionally integrated processes and estimate the memory parameters as a measure of persistence. The analysis covers the full EMU period from January, 1999, to December, 2019 for 11 EMU countries consisting of both "core" and "periphery" countries. Moreover, we test for breaks in persistence with the test by Martins and Rodrigues (2014).

To test for the existence of fractional cointegration relationships for inflation and IP among all country pairs of our sample, we estimate the memory parameters of the residuals d_v of the fractional cointegration relation using the NBLS estimation on the cointegrating vector β. We find evidence of fractional cointegration among EMU countries for both inflation and real IP growth. In the case of inflation, the results suggest more cointegration relations among core or among periphery countries. This implies the existence of separate "convergence clubs" within EMU inflation rates over the whole estimation period. This could be an indicator of macroeconomic divergence processes. In the case of EMU industrial production growth rates, we find evidence of fractional cointegration either among core or among mixed core-periphery country pairs. This could be indicative of real convergence processes being driven by trade links among the EMU countries. Overall, we find that nominal convergence does not necessarily imply real convergence and *vice versa*.

In addition, our results suggest breaks in the persistence of both inflation and industrial production growth rates around the beginning of the financial crisis in 2007-08 for a majority of countries in our sample. For some countries, we find a second break in 2009-11 (IP) or 2014-16 (inflation) marking the end of the crisis period. Generally, we find lower persistence of inflation and IP growth rates before the crisis and substantially higher persistence in the crisis period. In case of a second break, persistence reverts back to pre-crisis levels. The higher persistence of the series during the crisis period carries the danger of diverging processes in case of asymmetric shocks.

To sum up, our analysis gives a detailed picture of time-variation in real and nominal convergence processes since the start of the EMU. While the financial crisis and the European sovereign debt crisis was a period of potential divergence in both inflation and industrial production, there is nevertheless some evidence that a) the crisis period ended in some cases before the end of our sample and b) we still find evidence of fractional cointegration in many country pairs for the full sample period. Still, in light of the current crisis due to the COVID-19 pandemic our results tell a cautious tale about the potential vulnerability of macroeconomic convergence within EMU during economic crisis periods.

6 Appendix

The results of the three semiparametric tests we used in our analysis of pair-wise fractional cointegration, with Germany as a reference country, are presented in the next tables. The tests by Souza et al. (2018),Wang et al. (2015),Chen et al. (2006) are denoted as SRFB18, WWC15 and CH06 respectively. Here, the null hypothesis is no fractional cointegration.

Table 9: Pair-wise testing for stationary fractional cointegration in inflation rates

Germany/	SRFB18	WWC15	CH06
Austria	2.2855**	23.355**	2.4783**
Belgium	1.032	7.368**	1.7854**
Finland	1.3497	9.0378**	1.3954**
France	1.5783	10.9783**	1.0047
Netherlands	0.056	9.7593**	1.0851
Portugal	0.9944	10.1987**	1.9513**
Spain	-0.6128	7.8308**	1.3307

Note: Critical values at $\alpha = 5\%$ are 1.960 for both SRFB18 and WWC15, as well as 1.386 for CH06. ** denotes significance of the test statistic at the 5% level. Here, H_0: no fractional cointegration.

Table 10: Pair-wise testing for stationary fractional cointegration in industrial production growth rates

Germany/	SRFB18	WWC15	CH06
Austria	5.382**	3.377**	2.253**
Belgium	2.004**	1.37	2.205**
Finland	3.760**	0.505	1.310
France	4.448**	0.001	1.264
Greece	2.301**	0.348	1.091
Italy	5.884**	0.42	1.744**
Portugal	2.654**	0.13	0.932
Spain	3.398**	0.191	1.007

Note: Critical values at $\alpha = 5\%$ are 1.960 for both SRFB18 and WWC15, as well as 1.386 for CH06. ** indicates significance at the 5% level. Here, H_0: no fractional cointegration.

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