# The Impact of Population Aging on the German Statutory Pension Insurance – A Probabilistic Approach [i]


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## Abstract

The demographic transition is a phenomenon affecting many industrialized societies. These economies are experiencing a decline in mortality alongside low fertility rates – a situation that puts social security systems under severe pressure. To implement appropriate reform measures, adequate forecasts of the future population structure, specifically in pay-as-you-go systems are needed. We propose a probabilistic approach to forecast the numbers of pensioners in Germany up to 2040, considering trends in population development, labor force participation, and early retirement as well as the effects of further pension reforms. A principal component analysis is used for dimensionality reduction and consideration of cross-correlational effects between age- and sex-specific pension rates for both old-age and disability pensions. Time series methods enable the inclusion of autocorrelation effects in the model and the simulation of future uncertainty. The model predicts that, in the median, the numbers of old-age pensioners will increase by almost 5 million individuals from 2017 to 2036, alongside increases in disability pensions by 2036, given the raising of the legal retirement ages following the introduced regulations. After that point, a moderate decrease can be expected. The results show a clear need for further reforms, if the German statutory pension system is to be sustainable in the long run.



**Keywords:** Population Aging; Stochastic Forecasting; Principal Component Analysis; Time Series Analysis; Applied Econometrics; Public Pension Systems; Social Policy.

**JEL:** C53, H55, J11.


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# 1 Introduction

Countries with low fertility and decreasing mortality rates struggle with an aging population structure and negative natural population growth (OECD 2018). Decreasing mortality means longer periods of pension claims as long as retirement ages are not adjusted proportionally to the increases in life expectancy. C.p., low fertility results in a smaller workforce in the long run (Zuchandke et al. 2014). In Western Europe, for instance, mortality has been decreasing almost monotonically since the 1970s (Vanella 2017), whereas replacement-level fertility has not been reached yet (Vanella and Deschermeier 2019). For countries applying a Bismarck-type pension system (pay-as-you-go)[1], this particular demographic development results in double financial distress: The elderly are at increased risk of suffering from old-age poverty, while a growing share of labor income generated by the working population has to be transferred to the elderly (Goffart 2018). Demographic aging combined with pay-as-you-go schemes thus affects the financial sustainability of pension systems if that trend is not averted by policy reforms.

Demographic forecasts are of great importance for both researchers and policymakers, to ensure that pay-as-you-go systems, in particular, remain financially viable in the long run. However, population forecasts are rarely of a probabilistic nature; instead, deterministic projections are mostly conducted (see, e.g., European Commission 2018; OECD 2018; Pötzsch and Rößger 2015). These approaches result in equally deterministic pension projections because they rely on the underlying population predictions (see, e.g., Vogt 2017; Werding 2011; Wilke 2009). Population and pension forecasts should be probabilistic, since those models can quantify the uncertainty of the prediction, and thus, an assessment of the expected precision of the predicted development is possible (see, for instance, Keilman et al. 2002; Dunstan and Ball 2016).

[1] Pension payments are redistributed from the labor force to the pensioners within the same period (see, e.g., Graf von der Schulenburg and Lohse 2014 on this).

In the case of Germany, payments out of the statutory pension insurance *(DRV)* are based on this original Bismarckian principle (Wilke and Börsch-Supan 2009). Public pension payments constitute the largest share of retirement income in Germany at approximately 63% (Federal Ministry of Labour and Social Affairs 2019). Therefore, future old-age income will depend heavily on changes in the size and structure of the population, which are essential for the financial stability of the DRV. The DRV should ensure a certain living standard for its pensioners while not overloading the working population with excessively heavy contributions to the pension system (Vogt 2017).

Since potential pension reforms should be based on new and adequate forecasts of the future development of the DRV (Zuchandke et al. 2014), the present contribution provides a stochastic forecast of the year-end old-age and disability pensioner numbers receiving payments from Germany through the year 2040. We use principal component analysis (PCA) for dimensionality reduction and consideration of cross-correlation between the age- and sex-specific pension rates (ASSPRs), and the connections among retirement, disability, and legal retirement age are covered as well. Time series models include autocorrelation of the ASSPRs, providing the methodological framework for quantification of future uncertainty in these predictions. Combined with a fully probabilistic population forecast model developed in earlier studies (Vanella 2017; Vanella and Deschermeier 2018, 2019, 2020), a forecast of the future numbers of pensioners is elaborated. The model takes trends in labor force participation and early retirement, along with demographic trends such as decreasing mortality and morbidity, into consideration implicitly by time series analysis. The effects of previous pension reforms are captured to some extent by an econometric model in the forecast. The simulation not only returns the median age- and sex-specific retired and officially disabled population up to the year 2040 but also quantifies the uncertainty in the forecast, illustrated with 75% and 90% prediction intervals (PIs) for each year, age and sex.

The remainder of this paper is structured as follows. The next section presents an overview of German pension reforms since the 1980s. Section 3 gives a literature review on forecasts and projections for statutory pension systems with special emphasis on stochastic approaches on the one hand and studies of Germany on the other hand. We will then describe the method and data used for our analysis and present a selection of the results generated by our forecast. The model is applied to Germany but is in principle applicable to other countries as well, especially those that apply a Bismarck-type social security system. The paper will conclude with a discussion of the results and limitations, giving an outlook of opportunities for further research.

## 2 The German Pension Insurance and its demography-related Reforms

Demographic aging puts pension systems under pressure, which is not solely a problem of the German economy, but is also recognized by other industrialized countries, which are hit similarly hard or even heavier than Germany by the demographic trends. Countries with low fertility and decreasing mortality, such as Italy (Baldacci and Tuzi 2003), Japan (Ogawa 2005), Finland (Koissi 2006), China (Wang et al. 2019) or Croatia (Tomaš 2020) have recognized the need for pension reforms as well and are discussing possible potential reforms.

The timing of pension claims is an individual decision. However, there is strong evidence for the effects of policy reforms on social security and retirement decisions or expectations (Börsch-Supan 1992, 2000; Coppola and Wilke 2014; Buchholz et al. 2013). Moreover, social policy can try to influence retirement decisions by bonus-malus systems to affect retirement behavior as well as labor force supply (Gruber and Wise 2000).

Since the late 1980s, the German government has passed a series of pension reforms as countermeasures to the demographic aging process. In 1989, the *Rentenreformgesetz 1992 (RRG 1992)* was the first reform with a clear demographic agenda: it raised the legal retirement age for female and unemployed persons from 60 years to 65 (at that time the standard legal retirement age for men) until 2008 and was one important measure for containing the number of future retirees. Moreover, the exceptional early retirement age of 63 years for persons who had been employed for at least 35 years was abolished[2] (RRG 1992). Furthermore, whereas early retirement without monetary "sanctions" had previously been possible, the reform introduced a financial bonus-malus system for the individual retirement decision. Since that reform, every month of premature pension claims reduces the monthly pension payments by 3‰. On the contrary, each month of delayed pension claims beyond the individual legal retirement age is rewarded with an increase of 5‰ in monthly pension payments (Wilke 2009).

The *Wachstums- und Beschäftigungsförderungsgesetz (WFG)* in 1996 *(WFG 1996)* accelerated the increases in the legal retirement ages for unemployed and females even further, underlining the urgency of the policy measures. Due to the RRG 1992, the increase in the retirement age of these two groups would have ended at the target age of 65 in 2018 (RRG 1992, §41 I). However, the WFG required that mark to be reached in 2007 for unemployed men and 2010 for women. The legal retirement age for severely disabled persons was increased from 60 to 63 years between the years 2000 and 2006 following the *Rentenreformgesetz 1999* (RRG 1999).

According to the so-called *Riester reform* of 2001, a new pension formula was introduced that linked individual pension payments to the overall development of labor income and old-age saving rates in society over time. In 2004, this new pension formula was enhanced by adding the so-called *sustainability factor,* which is directly connected to the system dependency ratio.[3] Therefore, this adjustment considers directly the overall demographic and labor market development when determining pension payments (Wilke 2009). The effects of taking into account

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[2] In 2014, the German government returned to a similar measure, with a legal retirement age of 63 years for persons who have 45 years of social security payments (Bundesregierung 2013). This change is considered in our model as well.

[3] The ratio of number persons exceeding a certain age (mostly 65 years) over the number of persons in the assumed working-age, e.g. 15-64 years (Wilke 2009).

the system dependency ratio into the benefit calculation compared to considering the life expectancy as alternative reform options, was analyzed in a simulation study (see Fehr and Habermann 2006).

The *RV-Altersgrenzenanpassungsgesetz* from 2007 was the latest reform aimed at responding to the demographic aging process in Germany. The standard legal retirement age will now increase gradually to 67 years until 2031. The legal retirement age for severely disabled individuals is adjusted accordingly from 63 to 65 years and for the small group of mineworkers from 60 to 62 years (RV-AltAnpG 2007). These differing retirement ages find direct consideration in our forecast model. Therefore, the average annual retirement ages for different retirement groups are illustrated in Appendix A for historical, current, and future time horizons.

## 3 Forecasts of Pension Demand with Special Emphasis on Germany

Wilke and Börsch-Supan (2009) simulate the labor force in Germany until 2050 using scenario analyses of the development of the population and trends in labor force participation. The combination of two demographic and four different labor market scenarios gives eight trajectories of the labor force in Germany by 2050. Börsch-Supan and Berkel (2004) estimate individual probabilities of retirement under different socio-economic criteria in an econometric framework using data from the German Socio-Economic Panel Study (SOEP). Bucher-Koenen and Wilke (2009) apply the results of these two studies to estimate the long-term effect of the RV-AltAnpG 2007 (see Section 2) by simulating different scenarios for the labor force participation rates and the population's adjustment of its average retirement age to the increasing legal retirement ages of the reform.

Estimation of future financial development and thus the potential need for intervention in the DRV is generally based on deterministic projections of the population, combined with the legal retirement age and assumptions on the development of the labor market. Mostly, simple statistics such as the old-age dependency ratio are consulted for this (e.g., Pötzsch and Rößger 2015; European Commission 2018). Wilke (2009) and Holthausen et al. (2012) propose a detailed model for long-term projections of the future financial outlook for a wide range of German social security reforms until the year 2100. These analyses include many factors, such as population development, labor force participation, and policy reforms. They introduce a complex model that can show the future demand for social security based on subjective assumptions about future demographic and economic development. Werding (2011, 2013) projects the financial outlook of the DRV using the population projections by Destatis, deriving possible trends in labor force participation from micro census data and modeling future macroeconomic growth with a Cobb-Douglas production function. From these partial models, the scenarios for the future old-age dependency ratio and the resulting financial expenses for old-age pensions until 2060 are derived. The EU and the OECD offer similar projections for their respective member countries on an annual basis (see, e.g., European Commission 2018; OECD 2018).

All models are very detailed and provide suggestions for further model advances. They are quite restrictive in their assumptions, however, which is inevitable for deterministic models. Furthermore, the assumptions on fertility and migration development are in many cases questionable because the total fertility rate (TFR) is generally assumed to be constant. Vanella and Deschermeier (2019) show that a naïve forecast of the TFR for Germany performs rather poorly, when assuming a constant TFR. Drawbacks of some simulations are that they do not include the increase in the legal retirement ages as a result of the pension reform of 2007, thereby overestimating the number of old-age retirees. Deterministic methods generally have some limitations because they are restricted to a limited number of scenarios whose respective probabilities of occurrence are mostly not quantified. Thus, stochastic forecasts in demographic research are gaining popularity as an alternative to common deterministic projections that use scenarios to address future uncertainty (Istat 2018; Keilman et al. 2002; Lee 1998). Stochastic forecasts based on simulations are less prone to subjective decision-making and provide a huge number of possible future outcomes while being able to quantify their likelihood. Keilman et al. (2002) propose a probabilistic population forecast model, which is applied to Norway until 2050. Fertility is forecast using a multivariate autoregressive moving average (ARIMA) model, including the TFR, the mean age at childbearing (MAC), the variance in the MAC, and the minimum reproductive age as four parameters. Alho and Spencer (2005) propose a probabilistic forecast approach to the old-age dependency ratio, based on a stochastic model of population forecasting for Finland. Their method could help formulate social policy reforms that include flexible adjustments of the legal retirement age. Li et al. (2009) estimate the aging effect in the Chinese population as a proxy of the pension demand by deriving the future old-age dependency ratio. They do so by constructing a probabilistic population forecast through stochastic modeling of the demographic components fertility, mortality, and international migration. These partial forecasts are performed through a combination of quantitative and qualitative model assumptions as baseline scenarios for future development. Uncertainty is quantified by assuming a similar future risk for the demographic components in China compared to a pool of European countries over the distant past, as proposed in Alho and Spencer (2005). The net migration in the mean is assumed according to the UN projection (see United Nations 2007). The uncertainty of future migration is assumed to be similar to past trends for Europe, as given in Alho and Nikander (2004). The resulting population forecast is used to estimate simulations of the future old-age dependency ratio. Ahn et al. (2005) apply a similar method for a stochastic projection of the financial outlook of the Spanish pension insurance through 2050. Giang and Pfau (2008) generate a partially probabilistic projection of the financial pension outlook for Vietnam until 2100. Fertility and mortality are estimated by the popular Lee-Carter models for these two components (see Carter and Lee 1992; Lee and Carter 1992; Lee 1993), whereas the modeling procedure for international migration is not clearly described in the paper and appears to be deterministic. Assuming stationarity for the labor force participation rates, the authors derive estimates for the age- and sex-specific numbers of contributors to the social security system. Giang and Pfau (2008) extract the projections of future pensioner numbers. These entities are used for a stochastic estimation of the future old-age dependency ratio. Forecasting a range of economic factors, the future outlook for contributions into the pension insurance as well as the demand for pension entitlements is approximated. Tomaš (2020) generates a stochastic cohort-component forecast of the population in Croatia until the year 2060. He estimates a time series model of the new annual pensioners, which leads to a stochastic forecast of the pensioner old-age pension numbers. He then computes different scenarios to illustrate the effect of the population development on the pension payments.

Lipps and Betz (2005) forecast the population in Germany until 2050 stochastically by running 500 trajectories. Mortality and fertility are estimated for West and East Germany separately. Age-specific mortality rates are forecast using the Lee-Carter model for mortality, while the TFR is assumed to be a random walk process. The age schedule for the fertility rates is assumed Gaussian, with a converging MAC over the long term. Under these assumptions, the distribution of age-specific fertility rates (ASFRs) is simulated. The total net migration is assumed to be an autoregressive process of order one (AR(1)) (see, e.g., Shumway and Stoffer 2011 on AR processes). Simulating the net migration and the age distribution, the future population is estimated as well. The trajectories for the population are used for computation of the old-age dependency ratio, in the respective paper defined as the ratio of people over 60 years of age to the population between 20 and 59. Härdle and Myšičková (2009) propose a probabilistic cohort-component forecast for the population in Germany through 2058. Age-specific mortality and fertility rates are forecast by applying the respective Lee-Carter models. International migration is modeled separately for immigration and emigration, where the total numbers for both statistics are estimated by AR(1) models. The age structures of the migrants are approximated by

Kernel density estimation.[4] As a result of the population forecast, the authors forecast the old-age dependency ratio for retirement ages 65 and 67. Using a status quo assumption, the authors derive a stochastic projection of the future social insurance premium rate and the average replacement rate.

More details on the data and methods used in the more significant studies on pension forecasting mentioned in this section are given in Appendix C. One common merit of the presented studies is the indirect derivation of the retired population over the labor force participation and the resident population. That does not include the population living abroad while receiving pension payments in the country under study. In the case of a country such as Germany, where the net migration of the native population above retirement age is mostly negative (Federal Ministry of Labour and Social Affairs 2018; Vanella and Deschermeier 2018), this leads to systematic underestimation of the retired population. Our model is not only based on a fully probabilistic population forecast, but includes trends in prevalence rates of old-age and disability pensions as well stochastically. The combination of these two pension types in a joint model is another innovative feature of our model. Details will be presented in the next section.

## 4 Method and Data

In this section, we propose a joint probabilistic forecast model for the number of old-age pensions and disability pensions by sex and age of the pensioners. In the first step, past age- and ASSPRs for old-age and disability pensions are estimated. The data have been accumulated from three sources: the German Federal Statistical Office *Destatis*, the *DRV,* and the Federal Health Reporting Service *(gbe-bund),* provided by Destatis and the Robert Koch Institute

[4] See, e.g., Härdle et al. (2004) on that.

(RKI). Thus, we used the year-end sex-specific stocks of old-age pensioners by age (in years)[5] for the years 1992-2009 from the gbe-bund database (Destatis 2018a).

It is not advisable to use data before 1992 because the integration of pensions for citizens from the former German Democratic Republic *(DDR)* into the DRV after the German reunification did not happen before 1992 (RÜG 1991). Therefore, data until 1991 are available for West Germany only. Furthermore, the DRV was reformed in 1992, transforming disability pensions for persons who had already passed their subjective legal retirement age into old-age pensions (RRG 1992). The data for 2010-2017 was downloaded from the statistics portal of the DRV (Statistikportal der Rentenversicherung 2020a). Because the gbe-bund data originate from the DRV as well, we ensure that our data sets are consistent.

We estimate the pension rates for ages 60 to 64 for both genders. Old-age pensions for persons above these age limits are cumulated for consistency reason. This does not bias the results because retirement risk does not change significantly among these age groups (Börsch-Supan and Berkel 2004). However, there is still some difference due to the undercounting of international migration (Vanella and Deschermeier 2018), but the grouping decreases the dimensionality of the data and mitigates the error naturally arising from the population updating in the old-age population (Vanella 2017).

One advantage of our approach is that we take into account the numbers of persons residing abroad who receive pension payments from the DRV; previous approaches have not done this. Disability pensions are not discriminated by age, but simply by sex and type of disability (full or partial). The reason for this is, that separating by age would lead to implausible estimates in some cases. The data for the years 2010-2017 were downloaded from the statistical database of

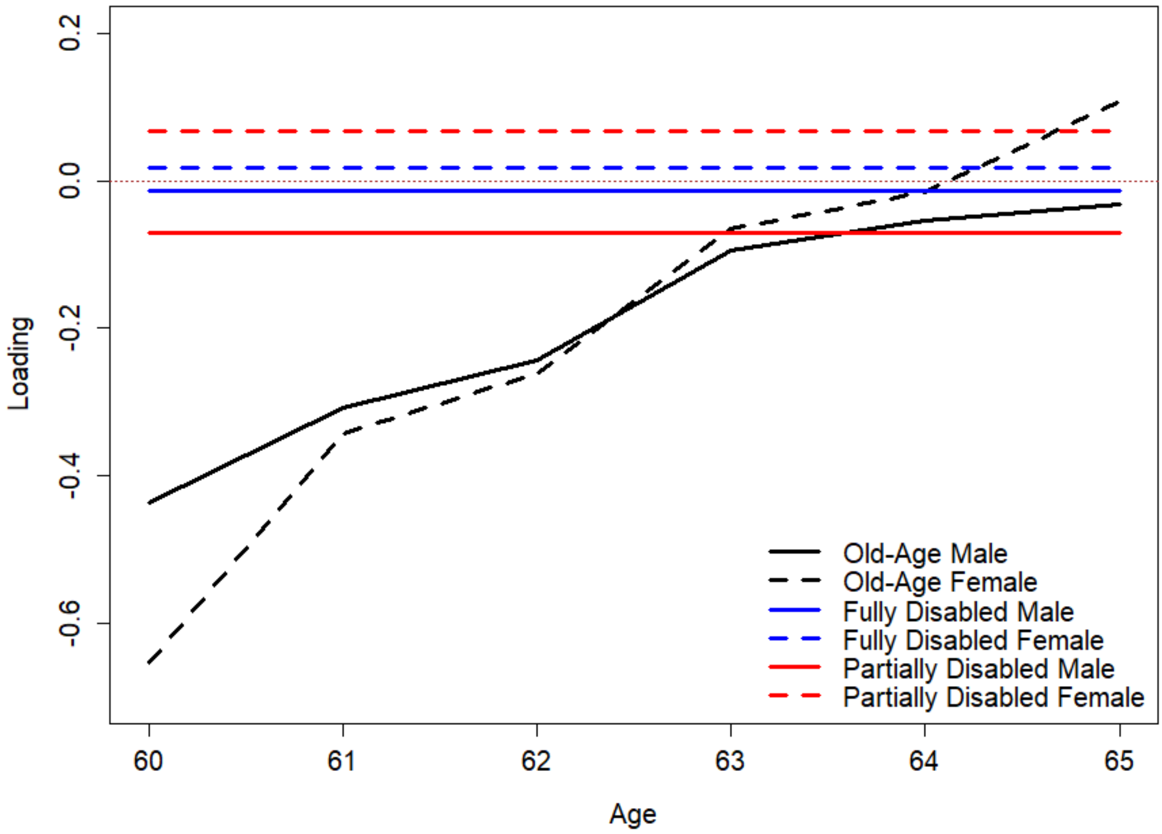
[5] The ages 60-99 are annually and the ages above 99 are grouped.

the DRV (Statistikportal der Rentenversicherung 2020b), and the data for 2000-2009 are available at the DRV research homepage (Forschungsportal der Deutschen Rentenversicherung 2018). The data for 1992-1999 was provided by the DRV on demand (Deutsche Rentenversicherung Bund 2018). Year-end population estimates for 1992-2017 by sex and age based on the 2011 census have been downloaded from the Human Mortality Database (2019).

The pension counts are divided by the population estimates, allowing us to calculate annual ASSPRs for the period 1992-2017. The resulting data matrix has 16 columns as conglomerates of 16 time series of ASSPRs. Basing the model on the ASSPRs has the advantage of including the possibility of a return into the labor force indirectly in our data. Another advantage of our approach is that we take into account the numbers of persons receiving pension payments from the DRV while residing abroad, which, to our knowledge, previous studies did not. As stated above, earlier approaches tend to estimate labor force participation rates first and derive pension rates from those. That approach has a major limitation, since it ignores the population receiving pension payments while living abroad. For Germany, this leads to a systematic underestimation of the pension numbers because the number of persons living abroad after retiring is certainly larger than vice versa (Deutsche Rentenversicherung Bund 2017; Vanella and Deschermeier 2018).

We apply PCA (see, e.g., Chatfield and Collins 1980; Handl 2010; Vanella 2018 for a comprehensive description and application of the method) to the matrix of the logistically transformed ASSPRs with 1.03 as the upper limit, which is approximately the historical maximum rate for Germany. This transformation prevents the simulations for the ASSPRs to take unrealistically high values (see Vanella and Deschermeier 2019 for a similar application for age-specific fertility rates). The PCA approach allows us to minimize the effective dimension of the data while also covering the correlations between the time series in our model (Vanella 2018). The principal components (*PCs)* are linear combinations of all ASSPRs, which are correlated to these while being uncorrelated to each other (Chatfield and Collins 1980; Vanella 2018). The correlations (or *loadings*) between the first PC and the logistically transformed ASSPRs are illustrated in Figure 1.

**Figure 1. Loadings of the first Principal Component**

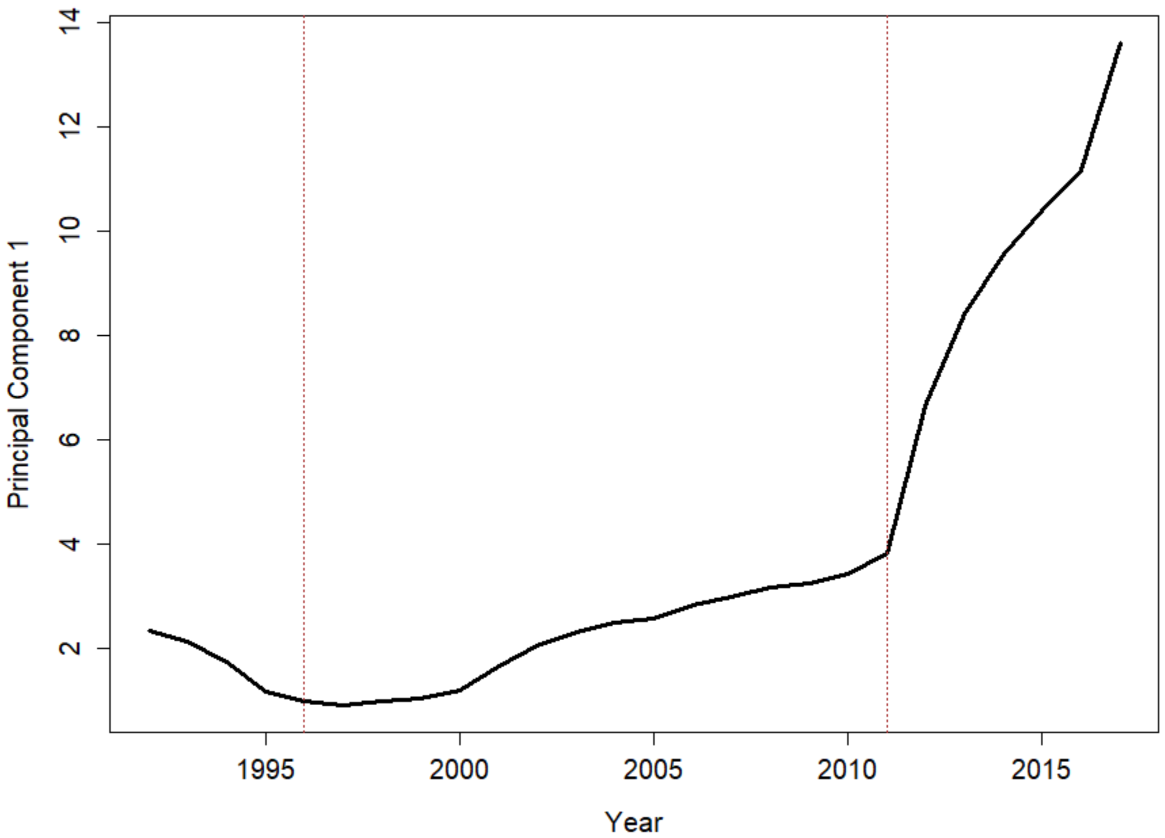



Source: Own calculation and design.

Principal Component 1 *(PC1)* is negatively correlated with the rates of old-age pensioners in the pre-legal retirement ages and with the rates of disabled males. Moreover, its loadings are positive for disabled females. Positive trends in PC1 are therefore, c.p., associated with decreases in early retirement rates. Disability pension rates of males decrease c.p. with increases, while the females rates increase instead. PC1 explains approximately 91% of the total variance in the logistically transformed ASSPRs. Figure 2 shows the historical course of PC1. The years

1996 and 2011 are marked by vertical lines to stress that in the following years the effects of the RRG 1992 and the RV-AltAnpG 2007 started to kick in (see Appendix A).

**Figure 2. Past Course of Principal Component 1**




Source: Own calculation and design.

PC1 has a decreasing trend until the late 1990s. It increases almost monotonically shortly after 1997 and has an even steeper slope since 2012, strongly implying a connection of PC1 to the past pension reforms that introduced raises in the legal retirement ages, as explained in Section 2.

To test our a priori stated hypothesis about PC1 to some extent and to integrate the effects of the legal retirement ages on it for our forecast model of the future pensioner numbers, we iteratively fit an explanatory model for PC1 with the mean annual retirement ages as exogenous variables. Those variables are derived from the sources presented in Section 2. The results of the different iterations are given in Table 1, with standard errors for the coefficients in brackets. For informative purposes only, we also report the $R^2$ and adjusted $R^2$ for each model.

**Table 1. Model Estimates for Principal Component 1**[6]

| Retirement Age | Model 1.1 | Model 1.2 | Model 1.3 | Model 1.4 |
|---|---|---|---|---|
| **Intercept** | - 4,426*** (709) | -4,326*** (625) | - 4,551*** (555) | - 3,907*** (474) |
| **ln(Standard)** | 711** (258) | 669*** (218) | 732*** (202) | 579*** (198) |
| **35 Years Long Insured** | 0.52 (0.65) | 0.37 (0.46) | - | - |
| **45 Years Long Insured** | 0.31 (0.26) | 0.3 (0.25) | 0,41* (0.21) | - |
| **Severely Disabled** | - 0.21 (0.63) | - | - | - |
| **Unemployed** | - 0.43* (0.23) | - 0.44* (0.22) | - 0.32* (0.16) | - 0.14 (0.14) |
| **Women** | 0.64*** (0.21) | 0.58*** (0.14) | 0.58*** (0.14) | 0.58*** (0.15) |
| **ln(Mineworkers)** | 344*** (107) | 363*** (88) | 356*** (86) | 358*** (92) |
| $R^2$ Adj. $R^2$ | 0.9912 0.9878 | 0.9911 0.9883 | 0.9908 0.9885 | 0.9891 0.987 |
| **AIC** **BIC** | 34.51 45.83 | 32.66 42.73 | 31.54 40.35 | 34.06 41.6 |

Source: Own calculation and design.

[6] One asterisk means statistical significance on a 10% level against $H_0: \beta_x = 0$, with $\beta_x$ being the x[th] coefficient. Two asterisks indicate a 5% significance level and three asterisks mean 1%.

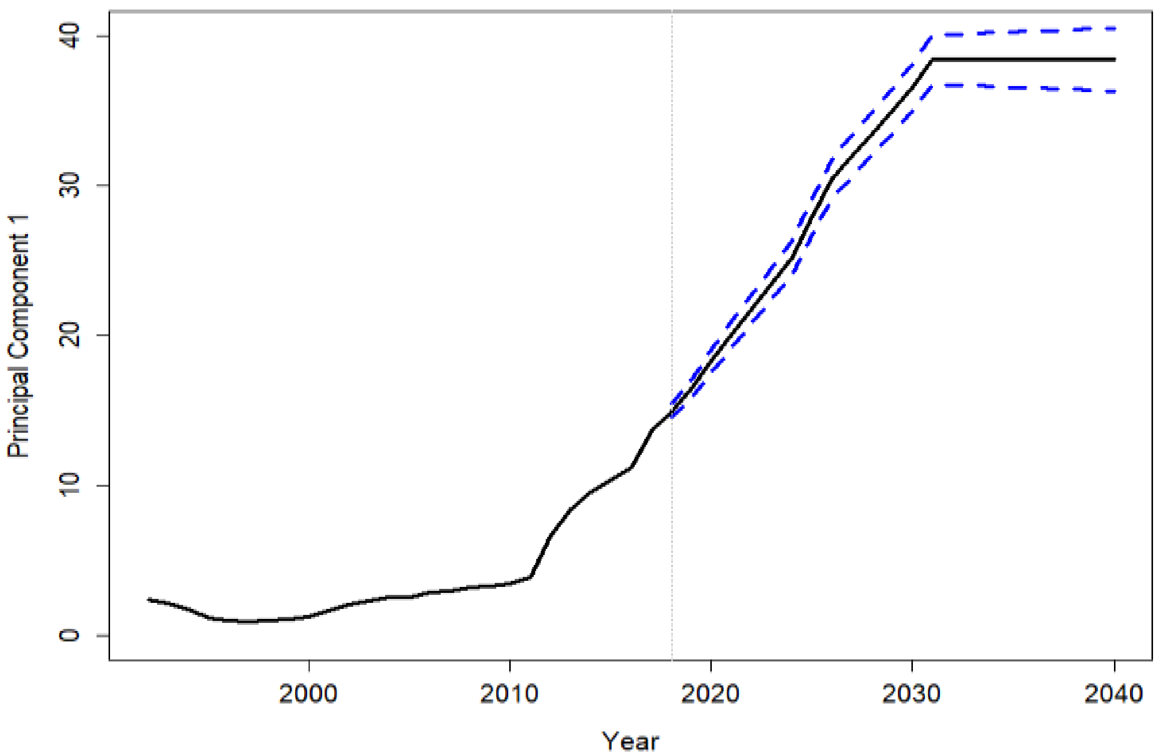
The models all show high joint significance. For the standard legal retirement age and the legal retirement age of mineworkers, the natural logarithms are put into the model, since the scatter-plots suggest a logarithmic connection between the two variables and PC1. We optimize the model iteratively by omitting the variable with the smallest individual significance in each iteration. We finally choose the model, which minimizes Akaike's Information Criterion (AIC) and the Bayesian Information Criterion (BIC). Both are minimized by Model 1.3. Omitting more variables leads to worsening model fits, as indicated by increases in the AIC and the BIC in Model 1.4. The final model accentuates the effects of the standard legal retirement age, the earlier legal retirement age for persons with 45 years of social insurance payments alongside the legal retirement ages of females and mineworkers on PC1. The PC stresses developments in very early retirement between age 60 and 62, an age group where most retirement stems from mineworkers. For example, in 2016, over 70% of the pensioner numbers among male 60-year-olds were mineworkers (Deutsche Rentenversicherung Bund 2017a). The coefficient for the legal retirement age of unemployed persons is negative, which appears strange at first. One possible explanation might be, that an increasing legal retirement age for unemployed might set incentives for them to retire early instead of applying for welfare services for longer periods (Brussig 2012).

After smoothing the data to the quantified model, we fit a Box-Jenkins time series model to the data (see Box et al. 2016). Based on the autocorrelation function (ACF) and the partial autocorrelation function (PACF), we identify a random walk as the most appropriate model for the error term (see, e.g., Shumway and Stoffer 2016 on ARMA processes, ACFs and PACFs). The forecast model for PC1 is therefore

$$p_1(y) = -4{,}551.43 + 731.96\ln(s_y) + 0.41l_y - 0.32u_y + 0.58f_y + 355.66\ln(b_y) + r_y + \varepsilon_y, \tag{1}$$

with $\varepsilon_y \sim \mathcal{NID}(0, 0.37^2)$, $\ln(s_y)$ being the natural logarithm of the mean standard legal retirement age, $l_y$ being the mean legal retirement age after being insured 45 years, $u_y$ being the mean legal retirement age for unemployed persons, $f_y$ being the mean legal female retirement age and $b_y$ being the mean legal retirement age of mineworkers in year $y$, as calculated in Appendix A. $r_y$ is the difference between the actual value of PC1 in period $y$ and its mean estimate according to Model 1.3. The forecast of PC1 with 75% PIs is illustrated in Figure 3.

**Figure 3. Forecast of Principal Component 1 with 90% PI**



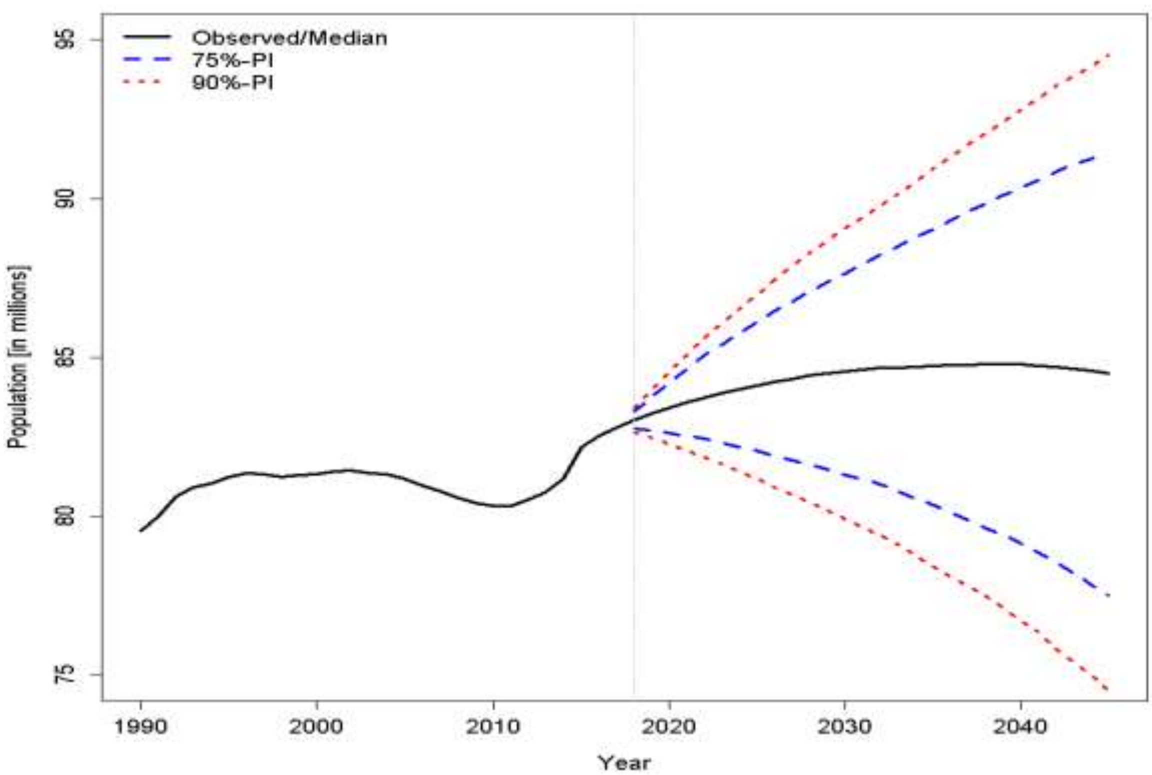

Source: Own calculation and design.

The remaining 15 PCs are assumed to be random walk processes, as they show no clear trending behavior and this allows for including the surplus risk generated by them reasonably well. The fitted PC models are used for future simulation of the 10,000 trajectories until 2040 via Wiener processes (see, e.g., Vanella 2018 on these). In this way, the stochasticity of all variables is considered in the forecast model (Vanella 2017). The trajectories of the PCs can easily be re-transformed into trajectories of the ASSPRs (Vanella 2018). These are multiplied by the trajectories resulting from the probabilistic population forecast for Germany conducted by Vanella and Deschermeier (2020). In this way, trajectories of the pensioner numbers are derived through 2040.

## 5 Results

We will now present selected results of the population forecast by Vanella and Deschermeier (2020), which constitutes the basis of the pension forecast conducted in the present contribution. Figure 4 shows the forecast of the future total population through 2045 with 75% and 90% PIs.

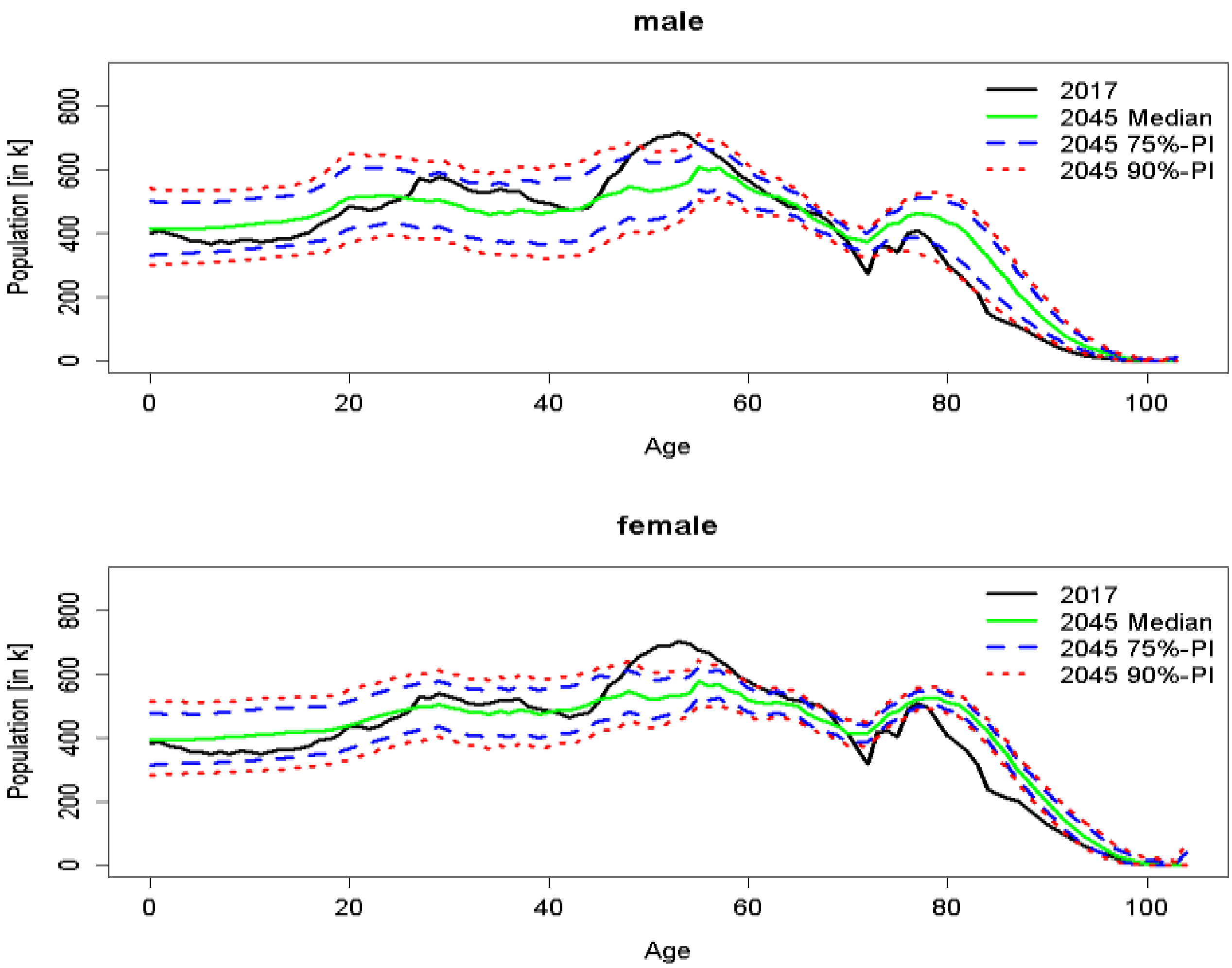
**Figure 4. Population until 2045 with 75% and 90% PIs**




Source: Vanella and Deschermeier (2020).

There is a high probability that the total population will increase over the forecast horizon. The population in the median forecast for December 31st, 2040 will be slightly below 85 million. In light of the pension fund, the population structure is of high relevance. Figure 5 compares the estimated age- and sex-specific population on December 31st, 2017 with the median forecast and the 75% and 90% PIs for each age and both sexes. The uncertainty in the future population is substantial, to a large extent due to the high stochasticity in international migration. This accentuates why it is advantageous for forecasting population and pensions on a probabilistic basis.

**Figure 5. Population by Sex and Age on Dec 31, 2017 and 2040**




**Source: Vanella and Deschermeier (2020).**

Obviously, the increase in the population results from clear growth in the population in the pension age group, whereas the population in the typical labor age group is expected to decrease by then. The old-age dependency ratio resulting from the population structure is often used as a representative statistic for future pressure on the DRV, as illustrated in Section 3. Table 2 gives a selection of the simulation results of Vanella and Deschermeier (2020) by three age groups.

**Table 2. Forecast Population (in millions) for Selected Years and Three Age Groups with 75% PIs**

| Year | Young Median | Young 75% PI Lower Bound | Young 75% PI Upper Bound | Working Age Median | Working Age 75% PI Lower Bound | Working Age 75% PI Upper Bound | Old Median | Old 75% PI Lower Bound | Old 75% PI Upper Bound |
|---|---|---|---|---|---|---|---|---|---|
| 2017 | 15.252 | | | 51.804 | | | 15.736 | | |
| 2021 | 15.573 | 15.354 | 15.799 | 51.588 | 51.178 | 52.012 | 16.428 | 16.357 | 16.500 |
| 2025 | 16.122 | 15.675 | 16.563 | 50.755 | 49.958 | 51.568 | 17.233 | 17.064 | 17.398 |
| 2029 | 16.642 | 15.948 | 17.337 | 49.396 | 48.224 | 50.559 | 18.446 | 18.181 | 18.715 |
| 2033 | 17.031 | 16.065 | 17.985 | 47.845 | 46.346 | 49.325 | 19.812 | 19.439 | 20.188 |
| 2037 | 17.086 | 15.853 | 18.311 | 47.065 | 45.233 | 48.871 | 20.630 | 20.127 | 21.118 |
| 2041 | 16.974 | 15.510 | 18.481 | 47.352 | 45.211 | 49.513 | 20.389 | 19.744 | 21.013 |
| 2045 | 16.835 | 15.124 | 18.609 | 47.829 | 45.340 | 50.382 | 19.844 | 19.020 | 20.603 |

Source: Vanella and Deschermeier (2020).

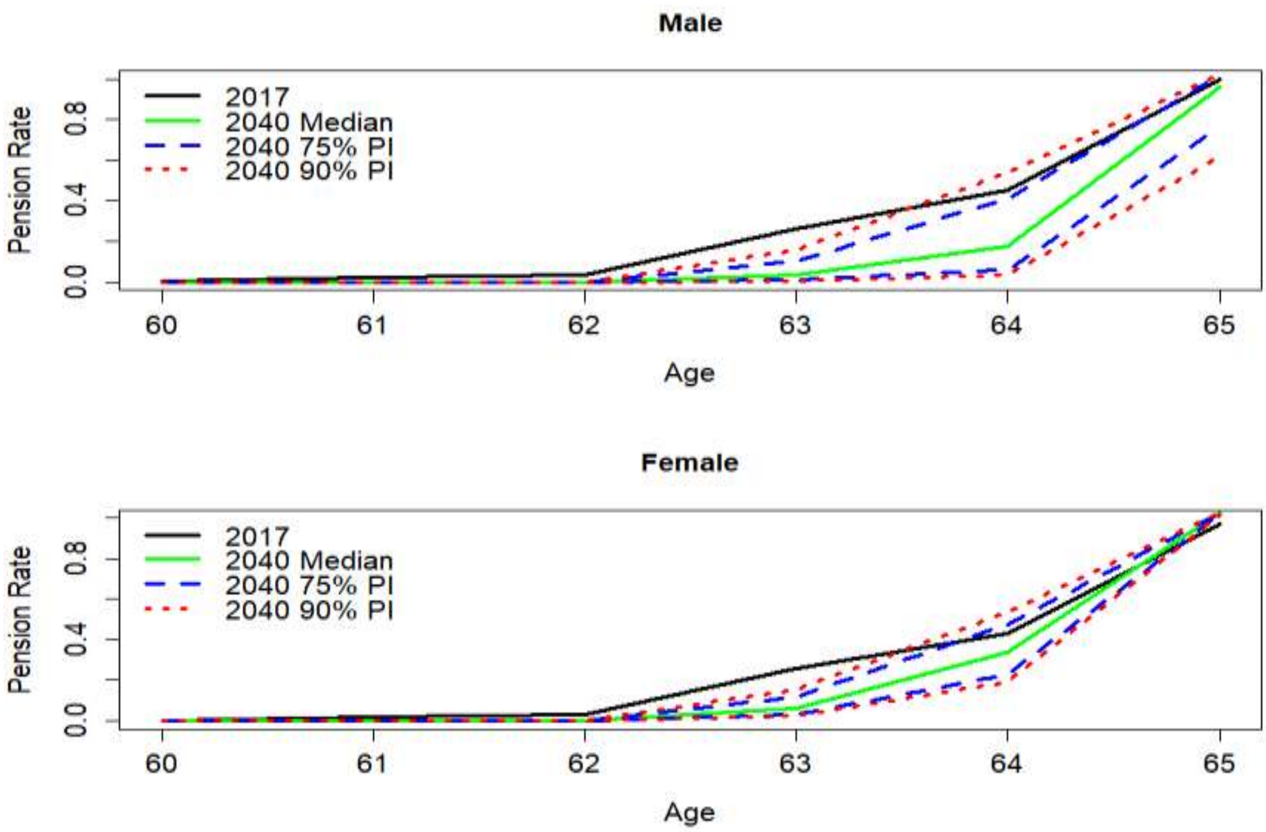
In our context, the forecasts of the working-age population[7]and the old-age population[8]. At high probability, we observe a strongly increasing old-age dependency ratio due to a decreasing

[7] Defined by Vanella and Deschermeier (2020) as persons aged 20-66 years of age.
[8] The old-age population here is defined as the population aged 67 and above.

working-age population[9] until the late 2030s alongside an increasing population in the pension age group.

Our modeling approach provides more insight into the actual pension numbers because the predicted population at this stage is multiplied by the age- and sex-specific risks of pension claim estimated by our PC time series method. The trajectories of the PCs are transformed back into trajectories of the ASSPRs, as mentioned above. The trajectories can be used to estimate quantiles of the forecast to construct PIs. Figure 6 illustrates the ASSPRs for old-age pensions at year-end 2017 in comparison to the predicted ASSPRs in the median trajectory with 75% and 90% PIs at the end of the forecast horizon.

**Figure 6. Age- and Sex-specific Pension Rates in 2017 and 2040**

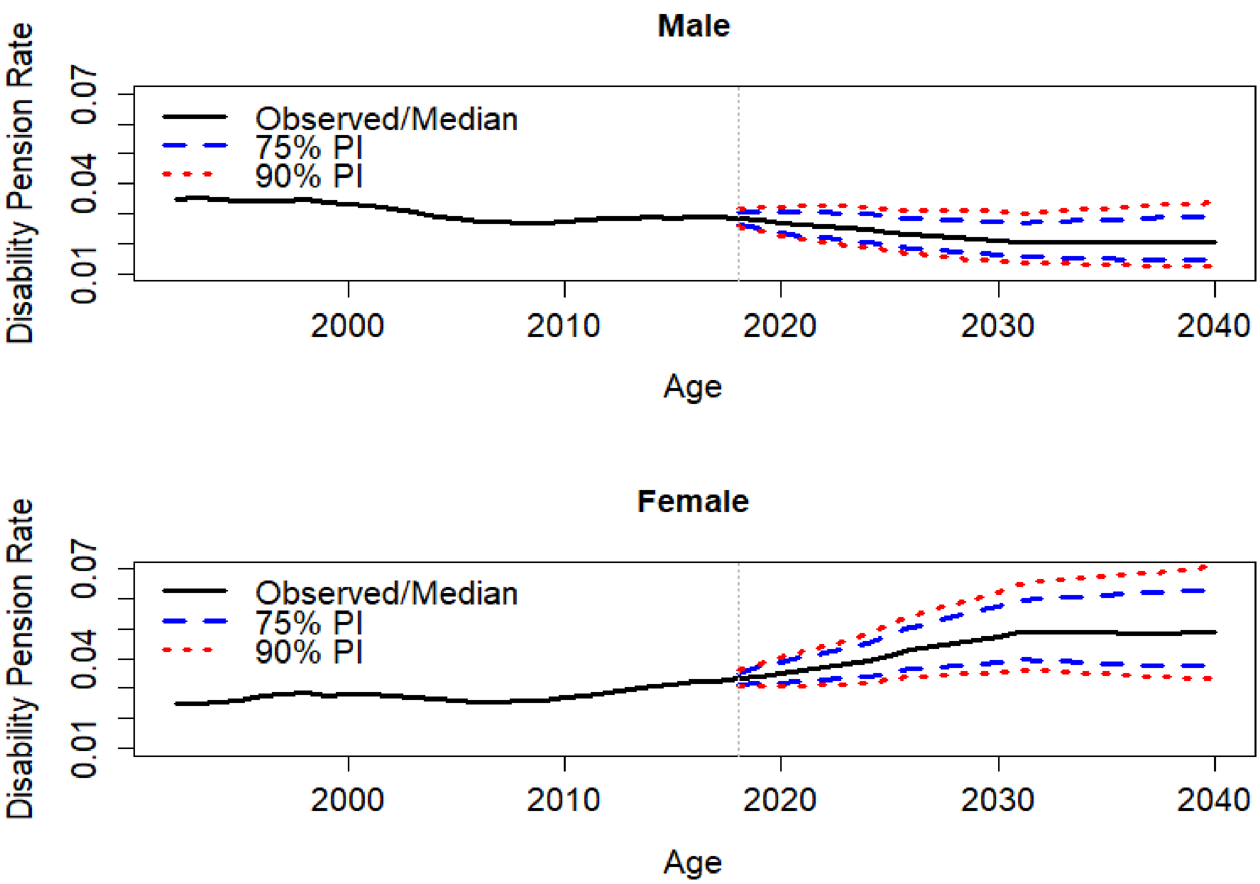



Source: Statistikportal der Rentenversicherung (2020a); Human Mortality Database (2019); Own calculation and design.

[9] Remember, that this does not even include labor force participation (see, for instance, Fuchs et al. 2018 on this), but simply refers to the age group.

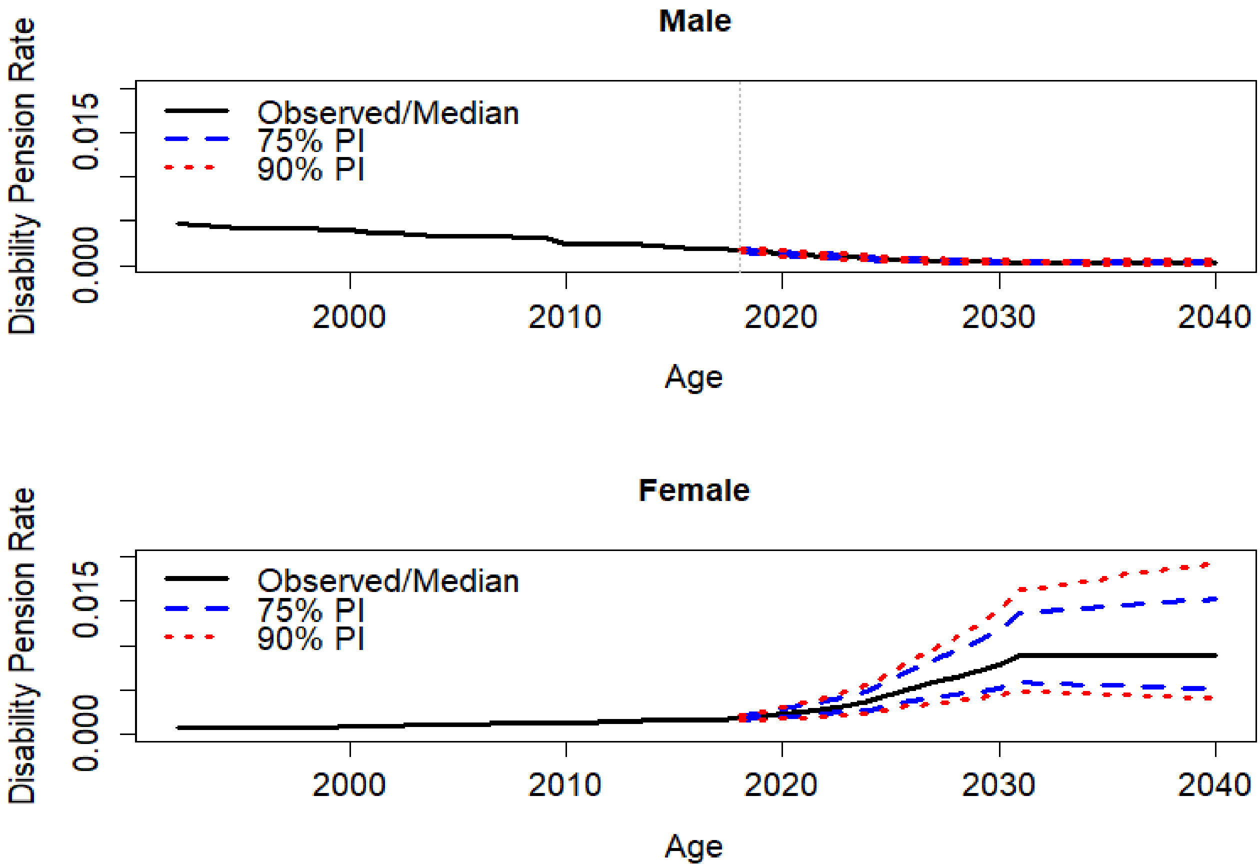
For the age group of people under 65 years, a decrease in the prevalence of old-age pension claims is probable as a result of the pension reform of 2007 described in Section 2, as these reforms push the standard legal retirement age to 67 years. For the age group older than that, the changes for the males will be subtle, whereas the ASSPRs of the females will almost certainly increase. This stems from the high labor force participation rates of the female population born since the baby-boom years (Fuchs et al. 2018). The preceding generations participated less in the labor market because their primary profession was mostly motherhood and housekeeping (Hertrampf 2008).

**Figure 7. Forecast of Full Disability Pension Rate by Sex**




Source: Deutsche Rentenversicherung Bund (2018); Forschungsportal der Deutschen Rentenversicherung (2018); Statistikportal der Rentenversicherung (2020b); Human Mortality Database (2019); Own calculation and design.

**Figure 8. Forecast of Partial Disability Pension Rate by Sex**

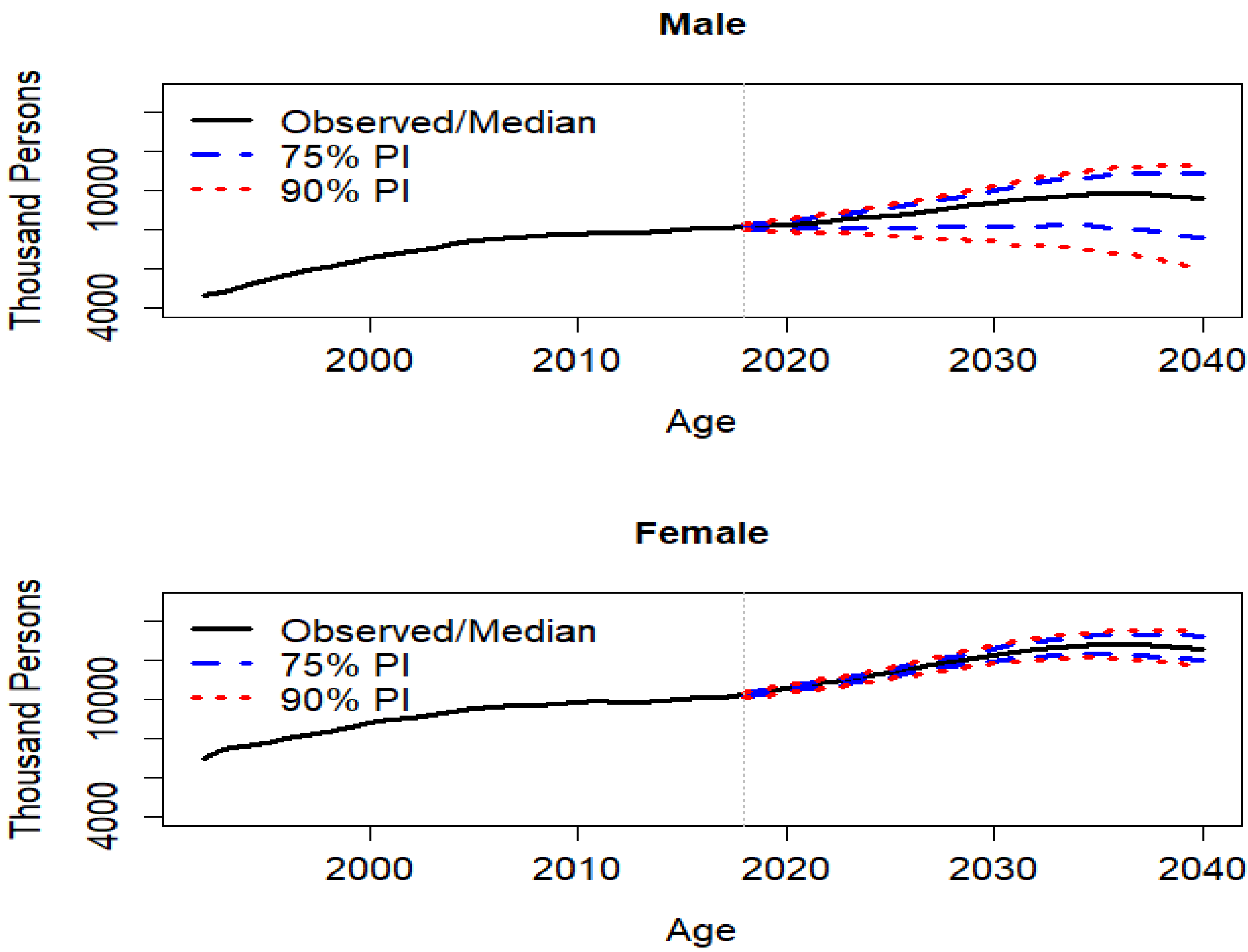



Source: Deutsche Rentenversicherung Bund (2018); Forschungsportal der Deutschen Rentenversicherung (2018); Statistikportal der Rentenversicherung (2020b); Human Mortality Database (2019); Own calculation and design.

Figures 7 and 8 give the past and future overall rates of disability pensions by sex and type of pension. For both types of disability pension, we will observe decreasing overall rates for the males and increases for the females. This can be explained by decreasing age-specific disability risks for the males due to healthier life circumstances and a decreasing share of persons working in the physically exhausting fields of work, which more often precede disabilities (Rodriguez Gonzalez et al. 2015). The trends for females are much different. First, the susceptibility to serious disabilities in the high age groups for the females excels the males' strongly (Vanella et al. 2020). Second, due to the increasing labor force participation rates of the females, they are to a higher degree eligible to disability pensions in comparison to the preceding generations.

Multiplication of the ASSPRs derived in this study with the age- and sex-specific population estimated by Vanella and Deschermeier (2020) results in forecasts of the future pension numbers. Figure 9 illustrates the resulting forecast of the total numbers of old-age retirees by sex.

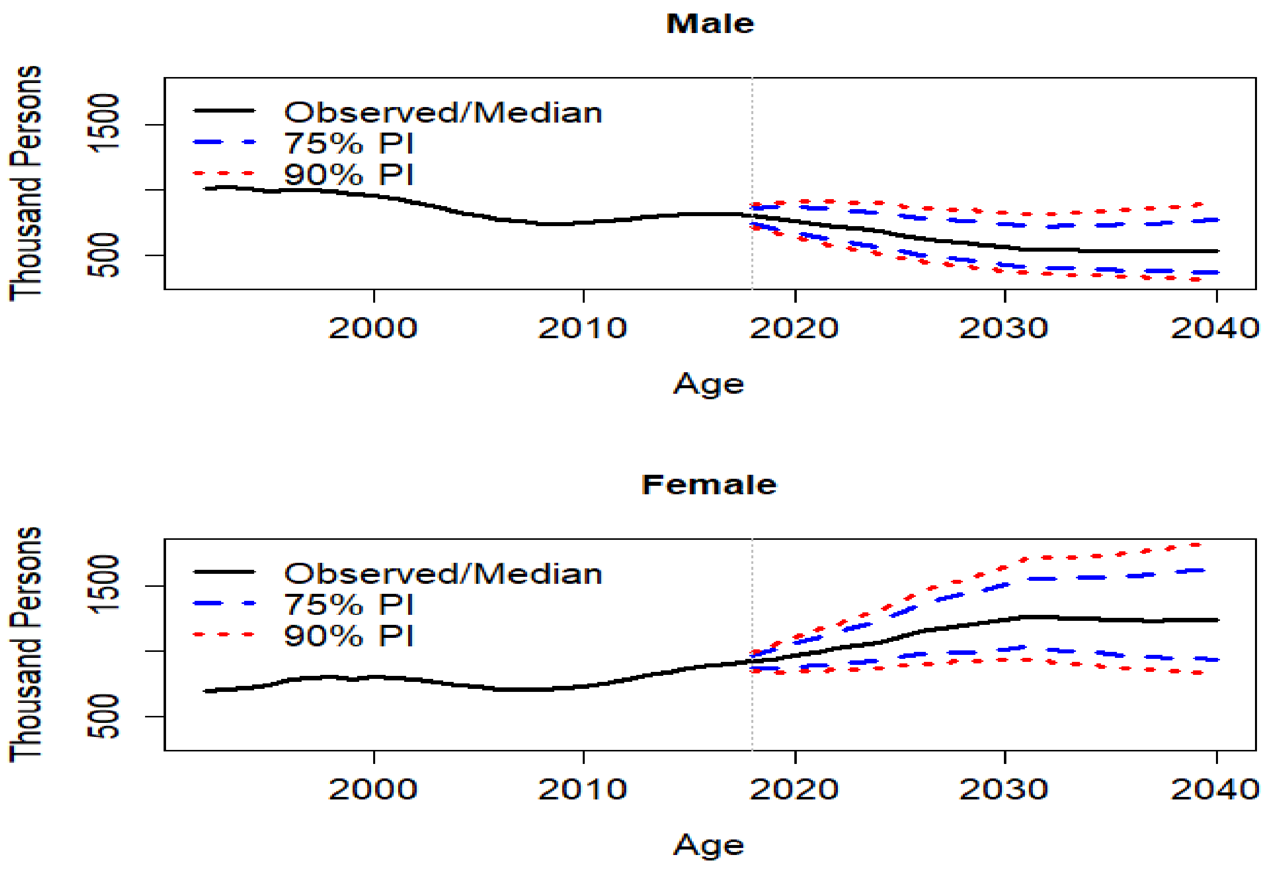
**Figure 9. Forecast of Old-Age Pensioners by Sex until 2040**




Source: Destatis (2018a); Statistikportal der Rentenversicherung (2020a); Vanella and Deschermeier (2020); Own calculation and design.

In the mean, we observe a monotonically increasing number of old-age pensioners for both sexes until the mid-2030s. The increase is especially large until the late 2020s, the period in which the strongest birth cohorts reach their respective retirement ages. After this point, there is a high probability that the total number of retirees will increase further, but at decreasing rates. This trend is caused by slightly decreasing birth cohorts entering their retirement ages, combined with the effects of the pension reforms since 1992, which imply lower age-specific old-age pension rates. The decreasing trend after the mid-2030s echoes the weaker birth cohorts since the 1970s, which can also be observed in Table 2. Overall, we see that the number of old-age pensioners will increase from 8.1 million to 9.9 million in the median for the males and from 10.1 to 12.8 million for the females between 2017 and 2036, the year with the predicted maximum for both genders. These results include demographic trends and the labor market participation effect. These results show the massive increase in retirees occurring over the forecast horizon. The increase in legal retirement ages by two years obviously does not suffice to address demographic development from the perspective of the DRV.

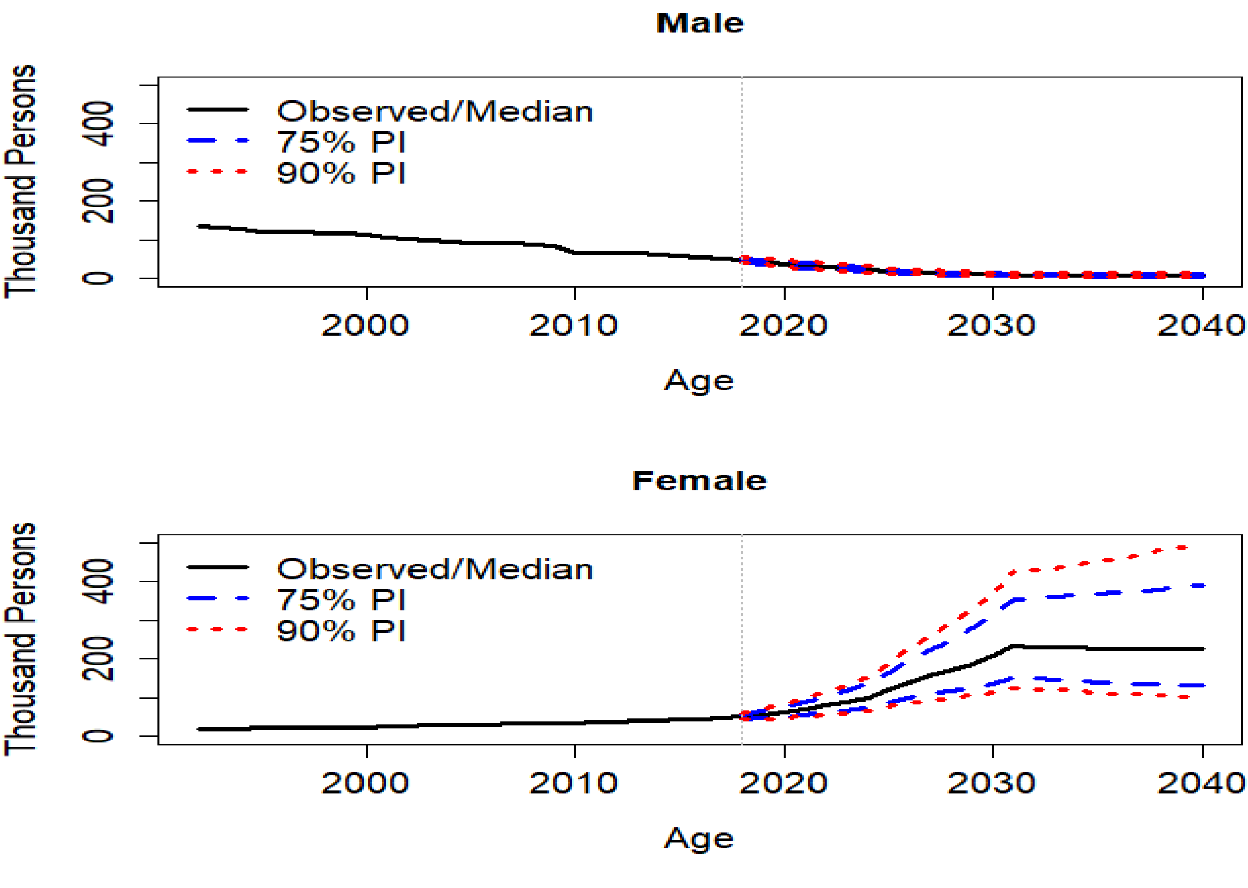
**Figure 10. Forecast of Full Disability Pensions**




Source: Deutsche Rentenversicherung Bund 2018; Forschungsportal der Deutschen Rentenversicherung 2018; Statistikportal der Rentenversicherung 2020b; Vanella and Deschermeier 2020; Own calculation and design.

Figures 10 and 11 show the cumulated forecasts of the numbers of disability pensions for fully and partially disabled persons by sex, respectively. The long-term trend for males is negative because the relative prevalence of disability decreases as illustrated in Figure 8. On the other hand, the increase in the legal retirement age means, c.p., an increase in the risk of disability pension claims. These trends are superimposed on the demographic trends for the females; therefore, increasing numbers of pensioners can be expected until the early 2030s. After that point, the strong birth cohorts enter the legal retirement age, so the disability pension numbers will probably decrease again slightly because of the decrease in the population numbers in the respective age group.

**Figure 11. Forecast of Partial Disability Pensions**




Source: Deutsche Rentenversicherung Bund 2018; Forschungsportal der Deutschen Rentenversicherung 2018; Statistikportal der Rentenversicherung 2020b; Vanella and Deschermeier 2020; Own calculation and design.

It can be concluded that the pension reforms that increase the legal retirement rates not only contain the increase in old-age retiree numbers but also increase the numbers of disability pensions until the early 2030s. Especially for the females, the increase in the legal retirement age might lead to a sharp increase in the number of cases in which a disability pension will be claimed. This is an effect of increasing female labor force participation rates in combination with the increasing legal retirement age, as there will be more women active in the labor market and therefore "eligible" for disability pensions; in the past, these women might have retired earlier.

To conclude, we see that a trivial analysis based on simple statistics such as the old-age dependency ratio does not suffice for a thorough forecast of the demand for statutory pension payments. An age-specific and joint forecast of old-age and disability pensions is needed for a full understanding of the real sensitivity of the pension system to reforms and demographic developments. Moreover, a stochastic approach includes the high uncertainty of the complex system of interacting population trends, labor market effects, and the regulations of the pension system.

## 6 Discussion

Even though our main result concerning an increasing number of future pensioners is very robust, our approach has some limitations. For example, the model does not consider age-specific pension rates of disability rates. This approach was tested as well, but did not give plausible loadings for all variables. Therefore, disability pensions are only discriminated by sex and type of pension. Moreover, the model does not include widow and orphan pensions. There are two reasons for this: First, regarding disabled persons under 60, inclusion in the model could give false indications of sensitivity to retirement ages.[10] Second, the data for this type of pension is not available in the form needed to fit our model. Third, we would need data or strong assumptions on nuptial behavior, eradicating the advantages of the chosen probabilistic approach to some degree.

Further studies might include these types of pensions in their analyses. To provide a full picture of not only the numbers of pensions but also their volumes, an enhanced pension model should include all kinds of pensions covered by the DRV as well as the development of the labor market. A joint model for the labor market and pensions would present a meaningful extension, as the labor market and the pension system are basically two sides of the same coin and influence one another heavily. Moreover, the present contribution was restricted to persons instead of economic entities such as monetary units. Such deeper analyses require forecasts of economic development as well. Because our pension model is fully probabilistic, the associated economic model should also be probabilistic. Drawing stochasticity from one source only, as done in previous studies, would create a biased picture of reality by creating some kind of pseudo-stochasticity. Further research might add forecasts addressing the financial effects using a probabilistic economic model and might elaborate on the approaches presented in Section 3 within a probabilistic framework.

## 7 Conclusions

The present study showed the effect of future demographic development in Germany on the numbers of old-age and disability pensioners of the public pension system. Due to the aging of the baby-boom generation, we expect the numbers of old-age pensioners to increase by almost

[10] Those effects of course do not exist because persons do not “decide to die” based on the pension policy regime.

5 million persons, from 18 million in 2017 to 23 million in 2036. An increase holds even under increasing legal retirement ages, as adopted in the 2007 pension reform. Stochastic modeling for trajectories with high mortality rates equally shows increasing pensioner numbers. The pension reforms targeting obvious demographic trends do help mitigate the effects of the aging process to some extent but are far from sufficient.

Further reforms concerning the three basic parameters of the DRV in Germany are thus inevitable: the pension contribution rate, the pension level, and the legal retirement age. Furthermore, proposals regarding the financing option, such as a shift to a tax-funded system or the implementation of state-owned funds, are likewise being discussed. Furthermore, demography and labor market policy could offer another option for the long-term stabilization of the pension system. A larger number of retirees means that there is a need for a proportional increase in the labor force, assuming that the labor market offers enough jobs to support this increase. Because fertility influences the labor market only after approximately 20 years, a short- or mid-term effect can only be achieved by either decreasing emigration of the labor force or increasing the immigration of qualified workers, who can be integrated into the labor market quickly.[11]

Improvements on our modeling approach, as indicated in Section 6, might be considered in further studies, enhancing the model by a more detailed economic approach, which takes the pension formula into account an predicts the future payments of the DRV.

[11] Guest 2008, for example, also discusses measures to stimulate labor force participation rates of the elderly population, the fertility rate and higher immigration, alongside other measures like superannuation or health and care policy.

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## List of Abbreviations

| | |
|---|---|
| ACF | autocorrelation function |
| AIC | Akaike's Information Criterion |
| ARIMA | autoregressive integrated moving average |
| ASFR | age-specific fertility rate |
| ASSPR | age- and sex-specific pension rate |
| BIC | Bayesian Information Criterion |
| DDR | German Democratic Republic |
| c.p. | ceteris paribus |
| DRV | Deutsche Rentenversicherung |
| MAC | mean age at childbearing |
| PACF | partial autocorrelation function |
| PCA | principal component analysis |
| PI | prediction interval |
| RKI | Robert Koch Institute |
| RRG | Rentenreformgesetz |
| RÜG | Renten-Überleitungsgesetz |
| RV-AltAnpG | Rentenversicherungs-Altersgrenzenanpassungsgesetz |
| TFR | total fertility rate |
| WFG | Wachstums- und Beschäftigungsförderungsgesetz |

## Appendix A: Mean Retirement Ages

### Table 3. Past, Current, and Future Mean Annual Legal Retirement Ages

| Year | Stand-ard | 35 Years Insured | 45 Years Insured | Severely Disabled | Unem-ployed | Women | Mine-workers |
|---|---|---|---|---|---|---|---|
| 1992 | 65.000 | 63.000 | 63.000 | 60.000 | 60.000 | 60.000 | 60.000 |
| 1993 | 65.000 | 63.000 | 63.000 | 60.000 | 60.000 | 60.000 | 60.000 |
| 1994 | 65.000 | 63.000 | 63.000 | 60.000 | 60.000 | 60.000 | 60.000 |
| 1995 | 65.000 | 63.000 | 63.000 | 60.000 | 60.000 | 60.000 | 60.000 |
| 1996 | 65.000 | 63.000 | 63.000 | 60.000 | 60.000 | 60.000 | 60.000 |
| 1997 | 65.000 | 63.000 | 63.000 | 60.000 | 60.292 | 60.000 | 60.000 |
| 1998 | 65.000 | 63.000 | 63.000 | 60.000 | 60.792 | 60.000 | 60.000 |
| 1999 | 65.000 | 63.000 | 63.000 | 60.000 | 61.292 | 60.000 | 60.000 |
| 2000 | 65.000 | 63.292 | 63.292 | 60.292 | 61.792 | 60.292 | 60.000 |
| 2001 | 65.000 | 63.792 | 63.792 | 60.792 | 62.292 | 60.792 | 60.000 |
| 2002 | 65.000 | 64.292 | 64.292 | 61.292 | 62.792 | 61.292 | 60.000 |
| 2003 | 65.000 | 64.792 | 64.792 | 61.792 | 63.292 | 61.792 | 60.000 |
| 2004 | 65.000 | 65.000 | 65.000 | 62.292 | 63.792 | 62.292 | 60.000 |
| 2005 | 65.000 | 65.000 | 65.000 | 62.792 | 64.292 | 62.792 | 60.000 |
| 2006 | 65.000 | 65.000 | 65.000 | 63.000 | 64.792 | 63.292 | 60.000 |
| 2007 | 65.000 | 65.000 | 65.000 | 63.000 | 65.000 | 63.792 | 60.000 |
| 2008 | 65.000 | 65.000 | 65.000 | 63.000 | 65.000 | 64.292 | 60.000 |
| 2009 | 65.000 | 65.000 | 65.000 | 63.000 | 65.000 | 64.792 | 60.000 |
| 2010 | 65.000 | 65.000 | 65.000 | 63.000 | 65.000 | 65.000 | 60.000 |
| 2011 | 65.000 | 65.000 | 65.000 | 63.000 | 65.000 | 65.000 | 60.000 |
| 2012 | 65.083 | 65.083 | 65.083 | 63.000 | 65.083 | 65.083 | 60.292 |
| 2013 | 65.159 | 65.159 | 65.159 | 63.000 | 65.159 | 65.159 | 60.538 |
| 2014 | 65.235 | 65.235 | 63.000 | 63.000 | 65.235 | 65.235 | 60.614 |
| 2015 | 65.311 | 65.311 | 63.000 | 63.292 | 65.311 | 65.311 | 60.689 |
| 2016 | 65.386 | 65.386 | 63.167 | 63.538 | 65.386 | 65.386 | 60.765 |
| 2017 | 65.462 | 65.462 | 63.333 | 63.614 | 65.462 | 65.462 | 60.841 |
| 2018 | 65.538 | 65.538 | 63.500 | 63.689 | 65.538 | 65.538 | 60.917 |
| 2019 | 65.614 | 65.614 | 63.667 | 63.765 | 65.614 | 65.614 | 61.000 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 2020 | 65.689 | 65.689 | 63.833 | 63.841 | 65.689 | 65.689 | 61.167 |
| 2021 | 65.765 | 65.765 | 64.000 | 63.917 | 65.765 | 65.765 | 61.300 |
| 2022 | 65.841 | 65.841 | 64.167 | 64.000 | 65.841 | 65.841 | 61.433 |
| 2023 | 65.917 | 65.917 | 64.333 | 64.167 | 65.917 | 65.917 | 61.567 |
| 2024 | 66.000 | 66.000 | 64.500 | 64.300 | 66.000 | 66.000 | 61.700 |
| 2025 | 66.167 | 66.167 | 64.667 | 64.433 | 66.167 | 66.167 | 61.833 |
| 2026 | 66.300 | 66.300 | 64.833 | 64.567 | 66.300 | 66.300 | 62.000 |
| 2027 | 66.433 | 66.433 | 65.000 | 64.700 | 66.433 | 66.433 | 62.000 |
| 2028 | 66.567 | 66.567 | 65.000 | 64.833 | 66.567 | 66.567 | 62.000 |
| 2029 | 66.700 | 66.700 | 65.000 | 65.000 | 66.700 | 66.700 | 62.000 |
| 2030 | 66.833 | 66.833 | 65.000 | 65.000 | 66.833 | 66.833 | 62.000 |
| 2031 | 67.000 | 67.000 | 65.000 | 65.000 | 67.000 | 67.000 | 62.000 |
| 2032 | 67.000 | 67.000 | 65.000 | 65.000 | 67.000 | 67.000 | 62.000 |
| 2033 | 67.000 | 67.000 | 65.000 | 65.000 | 67.000 | 67.000 | 62.000 |
| 2034 | 67.000 | 67.000 | 65.000 | 65.000 | 67.000 | 67.000 | 62.000 |
| 2035 | 67.000 | 67.000 | 65.000 | 65.000 | 67.000 | 67.000 | 62.000 |
| 2036 | 67.000 | 67.000 | 65.000 | 65.000 | 67.000 | 67.000 | 62.000 |
| 2037 | 67.000 | 67.000 | 65.000 | 65.000 | 67.000 | 67.000 | 62.000 |
| 2038 | 67.000 | 67.000 | 65.000 | 65.000 | 67.000 | 67.000 | 62.000 |
| 2039 | 67.000 | 67.000 | 65.000 | 65.000 | 67.000 | 67.000 | 62.000 |
| 2040 | 67.000 | 67.000 | 65.000 | 65.000 | 67.000 | 67.000 | 62.000 |

Source: RRG (1992); WFG (1996); RRG (1999); RV-AltAnpG (2007); Bundesregierung (2013:72); RVLeistVerbG (2014); Own calculation and design.

## Appendix B: Correlation Matrix of Mean Retirement Ages

**Table 4. Correlation Matrix of Different Legal Retirement Ages over the Period 1992 to 2016**

| | Standard | 35 Years Insured | 45 Years Insured | Severely Disabled | Unem-ployed | Women | Mine-workers |
|---|---|---|---|---|---|---|---|
| **Standard** | 1 | 0.87 | 0.52 | 0.82 | 0.75 | 0.78 | 0.96 |
| **35 Years Insured** | 0.87 | 1 | 0.72 | 0.99 | 0.97 | 0.96 | 0.89 |
| **45 Years Insured** | 0.52 | 0.72 | 1 | 0.7 | 0.72 | 0.64 | 0.47 |
| **Severely Disabled** | 0.82 | 0.99 | 0.7 | 1 | 0.99 | 0.98 | 0.86 |
| **Unem-ployed** | 0.75 | 0.97 | 0.72 | 0.99 | 1 | 0.98 | 0.79 |
| **Women** | 0.78 | 0.96 | 0.64 | 0.98 | 0.98 | 1 | 0.84 |
| **Mine-workers** | 0.96 | 0.89 | 0.47 | 0.86 | 0.79 | 0.84 | 1 |

Source: Own calculation and design.

## Appendix C: Comparison of Selected Pension Projections

### Table 5. Overview of Selected Studies on Pension Forecasting

| Study | Baseline Data | Methods and Assumptions | Results | Countries | Forecast Horizon |
|---|---|---|---|---|---|
| **Alho and Nikander (2004)** | Smoothed Age- and sex-specific mortality rates (ASSMRs) over preceding 30-year period<br><br>Smoothed/interpolated/extrapolated Age-specific fertility rates (ASFRs) for females aged 15-49 in 2002<br><br>Estimated overall net migration and age pattern for 1990-2000<br><br>Estimated Jump-off population on January 1, 2003 | Age-, sex-, and country-specific rates of decline in the ASSMRs assumed by linear extrapolation until 2030, after that constant ASSMRs in point forecast<br><br>Assumed future total fertility rates (TFRs) in 2050, linear interpolation for the intermediate years; Mean age at childbearing (MAC) assumed to increase to 31 years by 2017, constant age schedule thereafter<br><br>Net migration constant for ten years, then linear increase to presumed ultimate level<br><br>Simulate 3,000 trajectories for each demographic component by AR(1) models including auto- and cross-correlations | Stochastic forecast of age- and sex-specific population (ASSP)<br><br>Stochastic forecast of age-dependency ratio | 19 EU and Schengen countries | 2004-2050 |
| **Ahn, Alonso-Meseguer and García (2005)** | Smoothed ASSMRs 1998-2002<br><br>ASFRs in 2002<br><br>Estimated overall net migration and age pattern for 1990-2000 | Population forecast model similar to Alho and Nikander (2004) with 1,500 trajectories<br><br>LFPR assumed to increase exponentially until a stated maximum (average of EU countries), thereafter kept constant | Stochastic forecast of ASSP<br><br>Deterministic projection of | Spain | 2004-2050 |

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|---|---|---|---|---|---|
| | Estimated Jump-off population on January 1, 2003<br><br>Labor force participation rate (LFPR), employment and unemployment rate in 2000<br><br>Data on unemployment benefits in 2000 from Spanish Labor Force Survey | Derive working-age population from population and LFPR forecasts<br><br>Unemployment rate assumed to decrease linearly to 4.5% in 2015, constant thereafter<br><br>Derivation of employed and unemployed population<br><br>Labor productivity growth assumed to increase to 2 in 2019, constant thereafter<br><br>Projection of GDP as sum of growth rate in employed population number and labor productivity<br><br>Wage increases assumed equal at same rate as labor productivity<br><br>Forecast of persons entering old-age pension or other types of pensions annually<br><br>Calculation of pension contributions as share of labor income | macroeconomic development<br><br>Pseudo-probabilistic projection of financial balance of pension system | | |
| **Lipps and Betz (2005)** | Numbers of deaths by age and sex 1954-2000<br><br>Numbers of births by mother's age (15-49) 1973-2000<br><br>Population by sex and age 1954-2000 | Forecast of ASSMRs following Lee and Carter (1992)<br><br>TFR assumed random walk process; MAC forecast by logistic growth model; age schedule assumed Gaussian<br><br>Net migration number assumed AR(1) process<br><br>500 trajectories sampling from estimated demographic models | Stochastic forecast of ASSP<br><br>Stochastic forecast of old-age dependency ratio (OADR) | East and West Germany separately | 2001-2050 |

| | | | | | |
|---|---|---|---|---|---|
| **Giang and Pfau (2008)** | Mortality rates by five-year age groups 1999-2005<br><br>Fertility rates by five-year age groups (15-49) 1990-2005<br><br>Net migration structure of Japan 2005<br><br>Estimated Jump-off population 2005 by five-year age groups and sex<br><br>Population active contributor age structure from Vietnam Household Living Standards Survey 2004<br><br>LFPR from annual surveys 1996-2005<br><br>Quinquennial Urbanization projections by UN 2005-2050<br><br>Inflation rates 1994-2005<br><br>Real investment return for pension fund assets 1996-2005<br><br>Real wage growth 1992-2005 | Forecast of ASSMRs following Lee and Carter (1992)<br><br>Forecast of ASFRs following Lee (1993); sex ratio of births assumed 1.06:1 for boys to girls as derived from past data; long-term TFR assumed 1.85, following UN assumptions<br><br>Net migration constant<br><br>LFPRs from 2005 assumed constant over forecast horizon<br><br>Projected active labor force derived from population forecast and LFPRs<br><br>Statutory pension coverage rate assumed to increase to 66% by 2105<br><br>Mean retirement ages assumed 57 and 51 years for males and females, respectively<br><br>Average length of employment from past data<br><br>Active contributors at average retirement ages assumed to change status into pensioner<br><br>Economic variables stochastically simulated with log-normal distributions | Pseudo-stochastic pension fund forecast (1,000 iterations); migration and labor market variables deterministic | Vietnam | 2005–2105 |
| **Härdle and Myšičková (2009)** | ASSMRs 1956-2006<br><br>ASFRs (15-49) 1950-2006 | Forecast of ASSMRs following Lee and Carter (1992)<br><br>Forecast of ASFRs following Lee (1993) | Stochastic population forecast (5,000 iterations) | Germany | 2007-2058 |

| | | | | | |
|---|---|---|---|---|---|
| | Age- and sex-specific immigration and emigration<br><br>ASSP on January 1, 2007 | Total immigration and emigration by sex modeled by fit AR(1) models<br><br>Age structure of migrants estimated by estimated kernel density<br><br>Status Quo scenario for labor market participation, income pension system | Stochastic OADR, taking pension reform of 2007 into account<br><br>Projection of pension premium rate and average pension level deterministic by nature, taking stochastic population into account | | |
| **Wilke (2009)** | ASSMRs after World War II<br><br>ASSP on December 31, 2005<br><br>Population projections by Destatis<br><br>LFPRs from micro census<br><br>Danish LFPRs | Different scenarios regarding future development of mortality, fertility and migration<br><br>Different scenarios about labor force participation<br><br>Different scenarios about unemployment rates<br><br>Different assumptions about economic growth and development of wages<br><br>Different assumptions about retirement age<br><br>Different scenarios for disability risks | Wide range of possible deterministic scenarios for future development of pension fund, contribution rates and pension levels taking German pension system fully into account | Germany | 2006-2100 |

| | | | | | |
|---|---|---|---|---|---|
| **Werding (2013)** | ASSMRs 2000-2008<br><br>ASFRs (15-49) 2008<br><br>Age- and sex-specific immigration and emigration 2008<br><br>ASSP on December 31, 2008<br><br>LFPRs by sex and age (15-64) 1991-2010<br><br>LFPRs by sex and age (65-74) 2000, 2005 and 2010<br><br>Labor force, social insured employed and unemployed persons by qualification from micro census 1991-2010<br><br>Data on wages by sex and qualification<br><br>Estimated unemployment rates by qualification<br><br>Economic data | Forecast of ASSMRs following Lee and Carter (1992)<br><br>TFR assumed constant on 2008 level; fixed age schedule following Gaussian function for ASFRs<br><br>Total immigration and emigration assumed to increase to 800,000 and 650,000 to 2020 respectively, constant thereafter; age schedules similar to 2008<br><br>Female LFPRs converge to males'<br><br>Deterministic projection economic, labor market and educational variables | Deterministic projections of numbers of active insured and number and structure of pensioners, contribution rates and pension levels | Germany | 2009-2060 |
| **European Commission (2018)** | Demographic, labor market and economic data | Scenario analyses for the variety of variables | Deterministic pension expenditure projections | 28 EU member states | 2016-2070 |
| **OECD (2018)** | Demographic, labor market and economic data | Scenario analyses for the variety of variables | Deterministic pension expenditure projections | OECD-/G20-countries | 2017-2060 |

| | | | | | |
|---|---|---|---|---|---|
| **Our study** | Deaths by sex and age 1952-2016<br><br>Births by age of mother (13-52) 1968-2016<br><br>Age- and sex-specific immigration and emigration 1990-2016<br><br>Immigration and emigration by age group, sex and nationality<br><br>ASSP 2010-2016<br><br>Pension numbers by age, sex and type (old-age, disability) 1992-2016<br><br>Forecast of age- and sex-specific survival rates following Vanella (2017)<br><br>Forecast of ASFRs following Vanella and Deschermeier (2019)<br><br>Forecast of age- and sex-specific net migration following Vanella and Deschermeier (2018a)<br><br>Forecast of ASSP from Vanella and Deschermeier (2018b) | Forecast of pension rates by age, sex and pension type (old-age, full disability, partial disability) by time series forecast of principal component model<br><br>Derivation of pension numbers from forecast pension rates and population<br><br>Monte Carlo simulation (10,000 trajectories) | Stochastic forecast of pension rates by age and sex for old-age pensions, full disability pensions and partial disability pensions<br><br>Stochastic forecast of numbers of old-age pensioners, fully disabled and partially disabled aged 60 and older | Germany | 2017-2040 |

Source: Own representation.